The Theory of Invariance
A perspective of absolute space and time

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Abstract
In this article, by using fundamental concepts in classical mechanics, we derive equations describing gravitational red shift and Doppler effect for light as well as equations describing the relations among mass, momentum, and energy including mass-energy equivalence. Although our equations are different than those in Newtonian mechanics or special relativity, they yield results that are approximate results calculated with Newton mechanics or special relativity for values of velocity, which are much less than speed of light. Since the concepts in classical mechanics are not separated from the perspective of absolute space and time, this theory is named the theory of invariance.

Keywords: mass-energy equivalence, light Doppler effect, gravity, red shift, barycenter

1. Introduction

In 17th century, classical mechanics developed by Sir Isaac Newton and other natural philosophers became an accurate theory describing the motion of macroscopic objects under the action of forces. However, in 1905, the scientist Albert Einstein published a theory which later was called the theory of special relativity. A revolution in science started with the perspective of relative space and time. The equation E = mc2 derived from special relativity has become famous throughout the world [1, 2]. Is it important to investigate if the famous equation also holds in the perspective of absolute space and time? Can we derive the equation E = mc2 from concepts in classical mechanics? If yes, what does this equation tell us about the nature of absolute space and time?

2. Light under the effect of gravity

Let us consider the following conceptual experiment:

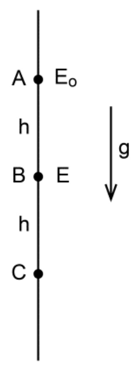

Figure 1: Light travel down from A to B and C in uniform gravitation *g*.

A photon with energy E_o is emitted at a point A in a uniform gravitational field *g*. Points B and C are positioned below the point A such that AB = BC = *h*. The photon energy measured at point B is *E*. The energy *E* and the height *h* are variable amounts. Hence, the ratio E/E_o can be written as a function of the height, $\varphi(h)$, as follows:

$$\frac{E}{E_o} = \varphi(h), \text{ and } \frac{E_o}{E} = \varphi(-h).$$

Hence,

$$\varphi(h).\varphi(-h) = 1.$$

We also have

$$\varphi(2h) = \varphi(h).\varphi(h).$$

Therefore, $\varphi(h)$ is an exponential function:

$$\varphi(h) = e^{kh}.$$

Thus,

$$\frac{E}{E_o} = e^{kh}, \tag{1}$$

and

$$\Delta E = E - E_o = E_o\left(e^{kh} - 1\right). \tag{2}$$

Because the energy of a light ray is proportional to its frequency, Equation (1) can also be interpreted as a ratio of the frequencies of the photon:

$$\frac{f}{f_o} = e^{kh}. \tag{3}$$

In real experiments, if $gh << c$, the ratio of the frequencies is measured as

$$\frac{f}{f_o} \approx 1 + \frac{gh}{c^2}. \tag{4}$$

Comparing Equations (3) and (4), we obtain:

$$k = \frac{g}{c^2}.$$

Substituting $k = g/c^2$ into Equations (1), (2), and (3), we obtain:

$$\frac{E}{E_o} = \exp\left(\frac{gh}{c^2}\right), \tag{5}$$

$$\Delta E = E_o\left[\exp\left(\frac{gh}{c^2}\right) - 1\right], \tag{6}$$

and

$$\frac{f}{f_o} = \exp\left(\frac{gh}{c^2}\right). \tag{7}$$

This equation describes the effect of gravity on light. However, in the Universe, there is no uniform gravitational field, only gravitational fields around astronomical bodies. For a spherical body of rest mass M_o, the frequency of a light ray f_o emitted at a distance R_o from the center of the body will be changed to f as the light ray comes to a distance R from the center of the body:

$$f = f_o \exp\left[\frac{GM_o(R_o - R)}{R_o R c^2}\right], \tag{8}$$

where G the gravitational constant.

Hence, if the light ray f_o approaches infinity, then its frequency will be reduced to f_∞:

$$f_\infty = f_o \exp\left(-\frac{GM_o}{R_o c^2}\right). \tag{9}$$

This equation implies that the frequency f_∞ approaches zero as R_o approaches zero. This consequence indicates that if a black hole exists, then it has no dimensions, and no event horizon around it.

3. Mass-energy equivalence

Let us consider an object m that is dropped freely from a point A in a uniform gravitational field g. Points B and C are positioned below point A such that AB = BC = h.

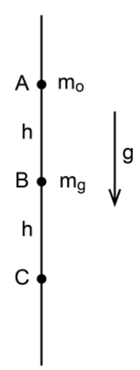

Figure 2: Object m fall freely from A to B and C in uniform gravitational field g.

Call m_o the rest mass and m_g the gravitational mass of the object m. The gravitational mass m_g and the height h are variable amounts. Hence, we can write the ratio of the masses as a function of the height, $\varphi(h)$, as follows:

$$\frac{m_g}{m_o} = \varphi(h), \text{ and } \frac{m_o}{m_g} = \varphi(-h).$$

Hence,

$$\varphi(h).\varphi(-h) = 1.$$

We also have

$$\varphi(2h) = \varphi(h).\varphi(h).$$

Therefore, $\varphi(h)$ is an exponential function:

$$\varphi(h) = e^{kh}.$$

Thus,

$$m_g = m_o\varphi(h) = m_o e^{kh}. \tag{10}$$

Call F the gravitational force that is exerting a pull on the object m in the gravitational field g:

$$F = m_g g.$$

From the definition of work, W, we obtain:

$$W = \int_0^h F dh = \int_0^h g m_g dh.$$

Substituting Equation (10) into the equation above, we obtain:

$$W = \int_0^h g m_o e^{kh} dh$$

$$W = g m_o \frac{1}{k}\left(e^{kh} - 1\right). \tag{11}$$

Now, let us consider the object m that "decays" into two "pieces" of light precisely when the object is dropped. One piece emits upwards, and the other piece emits downwards. The upwards-emitting piece immediately hits a mirror and reflects downward with the other piece.

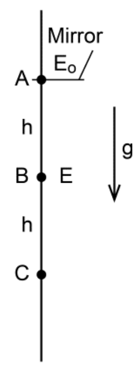

Figure 3: Object m "decay" into light precisely when dropped.

Comparing Equations (6) and (11), let us pay attention at the portions of [exp(gh/c^2)-1] and (e^{kh}-1) in the equations, respectively, we obtain:

$$k = \frac{g}{c^2}.$$

Comparing Equations (6) and (11), let us pay attention at the portions of (E_o) and (gm_o/k) in the equations, respectively, we obtain:

$$E_o = gm_o \frac{1}{k}.$$

Substituting $k = g/c^2$ into the equation above, we obtain:

$$E_o = m_o c^2. \tag{12}$$

This equation describes the mass-energy equivalence for an object at rest. Interestingly, this equivalence is exactly the same as that in special relativity.

Now, substituting $k = g/c^2$ into Equation (10), we obtain:

$$m_g = m_o \exp\left(\frac{gh}{c^2}\right). \tag{13}$$

Substituting Equation (12) into Equation (5), we obtain:

$$E = m_o c^2 \exp\left(\frac{gh}{c^2}\right). \tag{14}$$

Substituting Equation (13) into Equation (14), we now obtain:

$$E = m_g c^2. \tag{15}$$

Equations (13) and (14) describe the gravitational mass m_g and the mass-energy equivalence for the object m when it is falling through point B, respectively (See figure 2). Mass-energy equivalence for an object and its gravitational mass can also be expressed in its velocity. These relations will be derived in the following sections.

4. Doppler effect

Let us consider the following conceptual experiment:

Imagine a light source S and an observer O. The light source emits a flash of light towards the observer O. Call f_o the frequency of the flash received by the observer when he is at rest with respect to the light source. Call f the frequency of the flash received by the observer when he is moving at a velocity v towards the light source. The frequency f and the velocity v are variable amounts. Hence, we can write the ratio of the frequencies as a function of the velocity, $\varphi(v)$, as follows:

$$\frac{f}{f_o} = \varphi(v) \text{ , and } \frac{f_o}{f} = \varphi(-v).$$

Hence,

$$\varphi(v).\varphi(-v) = 1.$$

We also have

$$\varphi(2v) = \varphi(v).\varphi(v).$$

Therefore, $\varphi(v)$ is an exponential function:

$$\varphi(v) = e^{kv}.$$

Thus,

$$\frac{f}{f_o} = e^{kv}. \tag{16}$$

In real experiments, if $v << c$, then the ratio of the frequencies is measured as follows:

$$\frac{f}{f_o} \approx 1 \pm \frac{v}{c}. \tag{17}$$

Comparing Equations (16) and (17), we obtain:

$$k = \pm \frac{1}{c}.$$

Substituting $k = \pm 1/c$ into Equation (16), we obtain:

$$\frac{f}{f_o} = \exp\left(\pm \frac{v}{c} \right). \tag{18}$$

This equation describes the Doppler effect for light. And because the energy of a light ray is proportional to its frequency, Equation (18) can also be interpreted as

$$\frac{E}{E_o} = \exp\left(\pm \frac{v}{c} \right). \tag{19}$$

5. Total energy, gravitational mass, kinetic energy, potential energy, and linear momentum

5.1. Total energy

Let us consider the following experiment:

An object m is allowed to “decay” into two “pieces" of light when an observer is moving towards the object at a velocity v. Call m_o and E_o the rest mass and the rest energy of the object m, respectively. Applying Equation (19) to the experiment, the observer receives a total quantity of energy E,

$$E = \frac{1}{2} E_o \left[\exp\left(\frac{v}{c} \right) + \exp\left(-\frac{v}{c} \right) \right].$$

Using Equation (12), we then obtain:

$$E = \frac{1}{2} m_o c^2 \left[\exp\left(\frac{v}{c} \right) + \exp\left(-\frac{v}{c} \right) \right]$$

$$E = m_o c^2 \cosh \frac{v}{c}. \tag{20}$$

This equation describes the total energy of an object m moving at a velocity v.

5.2. Gravitational mass

Comparing Equations (15) and (20), we obtain:

$$m_g = m_o \cosh \frac{v}{c}. \tag{21}$$

In invariance, the rest mass is different from the gravitational mass. The rest mass of an object is unchanged. The gravitational mass is dependent on velocity and is therefore an indicator of kinetic energy as described in the following section.

5.3. Kinetic energy

Kinetic energy is the difference between total energy and rest energy:

$$KE = E - E_o.$$

Substituting Equations (15) and (12) into the equation above, we obtain:

$$KE = \left(m_g - m_o \right) c^2.$$

Using Equation (21), we then obtain:

$$KE = m_o c^2 \left(\cosh \frac{v}{c} - 1 \right). \tag{22}$$

This is the invariant kinetic energy equation. It can be expanded as

$$KE = m_o c^2\left[\frac{1}{2!}\left(\frac{v}{c}\right)^2 + \frac{1}{4!}\left(\frac{v}{c}\right)^4 + \frac{1}{6!}\left(\frac{v}{c}\right)^6 + \ldots\right].$$

Hence, for low values of velocity v, this equation approaches the Newtonian mechanics kinetic energy equation.

$$KE \approx \frac{1}{2} m_o v^2 \text{ for } v << c.$$

5.4. Potential energy

Let us return to section 3. Substituting $k = g/c^2$ into Equation (11), we obtain:

$$W = m_o c^2\left[\exp\left(\frac{gh}{c^2}\right) - 1\right]. \quad (23)$$

By the law of conservation of energy, Equation (23) also describes the potential energy of an object m at rest at height h in a gravitational field g,

$$PE = m_o c^2\left[\exp\left(\frac{gh}{c^2}\right) - 1\right]. \quad (24)$$

This is the invariant potential energy equation. It can be expanded as

$$PE = m_o c^2\left[\frac{gh}{c^2} + \frac{1}{2!}\left(\frac{gh}{c^2}\right)^2 + \frac{1}{3!}\left(\frac{gh}{c^2}\right)^3 + \ldots\right].$$

Hence, for low values of gh, this equation approaches the Newtonian mechanics potential energy equation.

$$PE \approx m_o gh \text{ for } gh << c^2.$$

In general, the potential energy of an object m at rest at a distance R_1 with respect to a distance R_2 from a spherical object M is

$$PE = m_o c^2\left\{\exp\left[\frac{U(R_1) - U(R_2)}{c^2}\right] - 1\right\},$$

where $U(R) = -\frac{GM_o}{R}$, where G the gravitational constant.

5.5. Linear momentum

From the experiment described in subsection 5.1, we can also determine the total quantity of momentum p of the two pieces of light with respect to the observer:

$$p = \frac{1}{2}\frac{E_o}{c}\left[\exp\left(\frac{v}{c}\right) - \exp\left(-\frac{v}{c}\right)\right]$$

$$p = \frac{E_o}{c}\sinh\frac{v}{c}.$$

Substituting Equation (12) into the equation above, we obtain:

$$p = m_o c \sinh\frac{v}{c}. \quad (25)$$

In vector denotation, this equation is written as

$$\mathbf{p} = m_o \mathbf{v}\frac{\sinh(v/c)}{(v/c)}. \quad (26)$$

This is the invariant linear momentum equation. It can be expanded as

$$\mathbf{p} = m_o\mathbf{v}\left[1+\frac{1}{3!}\left(\frac{v}{c}\right)^2+\frac{1}{5!}\left(\frac{v}{c}\right)^4+\ldots\right].$$

Hence, for low values of velocity v, this equation approaches the Newtonian mechanics momentum equation.

$$\mathbf{p} \approx \mathrm{m_o}\mathbf{v} \text{ for } v << c.$$

Now from Equations (20) and (25), we obtain:

$$E^2 - p^2c^2 = m_o{}^2c^4\left[\cosh^2\left(\frac{v}{c}\right)-\sinh^2\left(\frac{v}{c}\right)\right]$$

$$E^2 = p^2c^2 + m_o{}^2c^4. \tag{27}$$

Even though the total energy and linear momentum described by Equations (20) and (25) are different from those values in special relativity, it is interesting that the total energy of a particle of rest mass m_o described by Equation (27) is exactly the same as that in special relativity.

From Equations (20), (21), and (25), we also recognize that the relations among the energy, linear momentum scalar, and gravitational mass of a moving object m can be described as follows:

$$p = \frac{dE}{dv}\text{ , and } m_g = \frac{dp}{dv}.$$

6. Free falls

6.1. Velocity

Applying the law of conservation of energy to the equations of potential energy (24) and kinetic energy (22), we obtain:

$$m_oc^2\left[\exp\left(\frac{gh}{c^2}\right)-1\right] = m_oc^2\left(\cosh\frac{v}{c}-1\right)$$

$$\cosh\frac{v}{c} = \exp\left(\frac{gh}{c^2}\right). \tag{28}$$

This equation describes the velocity v of an object dropped falling freely from a height h in a gravitational field g. The equation can be expanded as

$$1+\frac{1}{2!}\left(\frac{v}{c}\right)^2+\frac{1}{4!}\left(\frac{v}{c}\right)^4+\ldots = 1+\frac{gh}{c^2}+\frac{1}{2!}\left(\frac{gh}{c^2}\right)^2+\ldots.$$

Hence, for low values of gh, this equation approaches the Newtonian mechanics equation:

$$v^2 \approx 2gh \text{ for } gh << c^2.$$

6.2. Acceleration

Differentiating both sides of Equation (28) with respect to time, we obtain:

$$\frac{1}{c}\sinh\frac{v}{c}\frac{dv}{dt} = \frac{g}{c^2}\exp\left(\frac{gh}{c^2}\right)\frac{dh}{dt}$$

$$a\sinh\frac{v}{c} = \frac{g}{c}\exp\left(\frac{gh}{c^2}\right)v.$$

Substituting Equation (28) into the equation above, we obtain:

$$a\frac{\sinh(v/c)}{(v/c)} = g\cosh\frac{v}{c} \tag{29}$$

$$a\tanh\frac{v}{c} = g\left(\frac{v}{c}\right). \tag{30}$$

This equation describes the acceleration of an object which is falling at a velocity v in a gravitation field g. The equation can be expanded as

$$a\left[1-\frac{1}{3}\left(\frac{v}{c}\right)^2+\frac{2}{15}\left(\frac{v}{c}\right)^4-\ldots\right]=g\ .$$

Hence, for low values of velocity v, this equation approaches the Newtonian mechanics viewpoint about the relation between acceleration and gravitation.

$$a \approx g \text{ for } v << c, \text{ and } a = g \text{ for } v = 0.$$

7. Inertial mass

Let us consider an object m falling freely downward in a gravitational field g. Call m_i and m_g the inertial mass and the gravitational mass of the object, respectively. From the definition of inertial mass, $m_i = F/a$ and from the definition of gravitational mass, $m_g = F/g$, we obtain:

$$\frac{m_i}{m_g} = \frac{g}{a}. \tag{31}$$

Substituting Equations (21) and (30) into Equation (31), we obtain:

$$m_i = m_o\frac{\sinh(v/c)}{(v/c)}. \tag{32}$$

Hence, the equation of linear momentum (26) can also be written as

$$\mathbf{p} = m_i\,\mathbf{v}. \tag{33}$$

In invariance, the inertial mass is a function of velocity. In addition, linear momentum can be defined as the product of inertial mass and velocity.

8. Barycenter and gravitational force

Let us imagine two astronomical objects M_1 and M_2 orbiting around each other. Call R_1 and R_2 the distances from M_1 and M_2 to their barycenter, respectively. In Newtonian mechanics, the position of the barycenter and the gravitational force between M_1 and M_2 are as follows:

$$M_1R_1 = M_2R_2\ , \text{ and } F=\frac{GM_1M_2}{R^2}, \text{ [3]}$$

where $R = R_1 + R_2$, and G the gravitational constant.

Because the theory of invariance is based on classical concepts and perspective, invariant expressions of barycenter and gravitational force are very similar with those in Newtonian mechanics. However, gravitational masses are used in the expressions above instead of masses, because, by the definition of gravitational mass, gravitational mass is corresponding to gravitational force. Hence, the position of the barycenter and the gravitational force are described by the following expressions:

$$M_{1g}R_1 = M_{2g}R_2, \tag{34}$$

and

$$F=\frac{GM_{1g}M_{2g}}{R^2}, \tag{35}$$

where

$$M_{1g} = M_{1o} \cosh\frac{v_1}{c}\text{, and } M_{2g} = M_{2o} \cosh\frac{v_2}{c},$$

where v_1 and v_2 the velocities of M_1 and M_2 on their orbit, respectively. (See section 5.2, Equation (21)).

9. Summary

- The different masses of an object *m* are its
  Rest mass m_o, which is unchanged,
  Inertial mass $m_i = m_o \dfrac{\sinh(v/c)}{(v/c)}$, and
  Gravitational mass, which can be expressed as
  $m_g = \dfrac{dp}{dv}$, and $m_g = m_o \cosh\dfrac{v}{c}$.

- The linear momentum of an object *m* can be expressed as
  $\mathbf{p} = m_i \mathbf{v}$, $p = \dfrac{dE}{dv}$, and $p = m_o v \dfrac{\sinh(v/c)}{(v/c)}$.

- The mass-energy equivalence is
  $E = m_g c^2$.

- The potential energy of an object *m* at rest at height *h* in a gravitational field *g* is
  $PE = m_o c^2 \left[\exp\left(\dfrac{gh}{c^2}\right) - 1\right]$.
  In general, the potential energy of an object *m* at rest at a distance R_1 with respect to a distance R_2 from a spherical object *M* is
  $PE = m_o c^2 \left\{\exp\left[\dfrac{U(R_1) - U(R_2)}{c^2}\right] - 1\right\}$, where $U(R) = -\dfrac{GM_o}{R}$.

- The kinetic energy of an object *m* moving at a velocity *v* is
  $KE = m_o c^2 \left(\cosh\dfrac{v}{c} - 1\right)$.

- The total energy of a particle *m* is
  $E^2 = p^2 c^2 + m_o{}^2 c^4$.

- The Doppler effect for light is
  $f = f_o \exp\left(\pm\dfrac{v}{c}\right)$.

- The gravitational red/blue shift effect for light is
  $f = f_o \exp\left(\pm\dfrac{gh}{c^2}\right)$.

- A black hole, if it exists, is a point with no volume and no event horizon.

- The relation between inertial mass and gravitational mass is
$$\frac{m_i}{m_g} = \frac{\tanh(v/c)}{(v/c)}.$$

- The relation between acceleration and gravitation is
$$\frac{g}{a} = \frac{\tanh(v/c)}{(v/c)}.$$

- The position of the barycenter between two objects M_1 and M_2 is described as
$$M_{1g} R_1 = M_{2g} R_2.$$

- The gravitational force is
$$F = \frac{G M_{1g} M_{2g}}{R^2}.$$

10. Conclusion

We studied the theory of invariance, which is based on the perspective of absolute space and time. Our analysis produces invariant equations which yield results that are approximate results calculated with either Newtonian mechanics or special relativity. In addition, we reproduce two of Einstein's equations of special relativity: $E_o = m_o c^2$ and $E^2 = p^2c^2 + m_o^2c^4$. These outcomes indicate that the theory of invariance can provide a distinctive view of the natural world, and the perspective of absolute space and time is an appropriate perspective in progressions of understanding reality.

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