The Solution of the Problem of Relation Between Geometry and Natural Sciences

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Abstract. The work is devoted to solution of an actual problem – the problem of relation between geometry and natural sciences. Methodological basis of the method of attack is the unity of formal logic and of rational dialectics. It is shown within the framework of this basis that geometry represents field of natural sciences. Definitions of the basic concepts "point", "line", "straight line", "surface", "plane surface", and "triangle" of the elementary (Euclidean) geometry are formulated. The natural-scientific proof of the parallel axiom (Euclid's fifth postulate), classification of triangles on the basis of a qualitative (essential) sign, and also material interpretation of Euclid's, Lobachevski's, and Riemann's geometries are proposed.

Keywords: physics, geometry, engineering, philosophy of science

Introduction

Sciences differ from each other in the point that they abstract the different aspects of reality. The general (common) theoretical basis of all sciences is the unity of formal logic and of rational dialectics (philosophical formalism), and criterion of truth is practice (i.e. material activity of people). The mathematics takes a particular place among other sciences: it abstracts the common aspect of reality – quantitative relations which arise in the process of measurements of properties of material objects. It is obvious that the properties, signs (for example, energy, weight, speed, extent, surface, plane, curvature, circle, straightness etc.) of objects do not exist irrespective of objects: there are not qualities and quantities but only the objects possessing qualities and quantities. Therefore, criterion of truth of pure (abstract) mathematics (i.e. mathematics abstracted, separated from properties of material objects) is not in itself. The criterion of truth is in natural sciences studying properties of material objects. Natural sciences use pure mathematics by the way of material interpretation (application). The criterion of truth of the interpreted (applied) mathematics is practice.

As is well known, the problem of relation between geometry and natural sciences – 21^{st} century's urgent problem of philosophy and of natural sciences – was not solved in 20^{th} century. The explanation is that researches on the foundations of geometry, carried out by N. Lobachevski, Janos Bolyai, B. Riemann, D. Hilbert, F. Klein, G. D. Birkhoff, A. Tarski, etc. in $19\text{-}20^{th}$ centuries, and creation of the theory of relativity by A. Einstein in 20^{th} century led to origination of two points of view on geometry. The traditional understanding of geometry in the sense of Euclid's "Elements" which existed more than two thousand years was forked on opposite aspects: mathematical (absolute) aspect and physical (relative) aspect. These aspects can be characterized as follows.

1. In 19^{th} century, the so-called absolute geometry arose, i.e. modern mathematical (absolute, logical, not philosophical, separated from practice) point of view from which geometrical concepts and geometrical axioms are considered now [1]. Unlike Euclid's text "Elements", there are no descriptions of geometrical objects (i.e. there are no definitions of concepts of geometrical objects) in modern lists of axioms of Euclidean geometry. It is supposed that there are three groups of the objects named "points", "straight lines", and "planes" concerning which some completely certain conditions are observed. The objects in

the axioms and relations between these objects can be chosen by any way but only with observance of requirements of axioms. In other words, the modern geometry breaks off relation between theory and practice: the modern geometry operates with concepts which express some of the properties separated (abstracted) from the objects. Therefore, the modern geometry cannot answer the following important question: Why does the axiom V (parallel axiom) is not a consequence of axioms I–IV in the list of Hilbert's axioms [2]? In order to answer this question, it is necessary to consider two essential circumstances. Firstly, from the rational-dialectic point of view, it is impossible to separate (to abstract) properties from objects – property carriers – because properties of objects do not exist separately from objects. Secondly, from the formal-logical point of view, the list of Hilbert's axioms is neither correct nor complete because it does not contain definitions of geometry basic concepts "point", "line", "straight line", "surface", "plane surface", and "triangle".

2. In 20th century, modern physical (relative, logical, philosophical, connected with practice) approach to understanding of essence of geometry has arisen. It has been proposed by Einstein in connection with creation of the theory of relativity. In spite of erroneousness of the theory of relativity [3], Einstein's approach does not contradict the sense of Euclid's text "Elements" and is as follows: "Among of all sciences, mathematics is held in special respect because its theorems are absolutely true and incontestable whereas other sciences' laws are fairly disputable and there is always danger of their refutation by new discoveries. However, mathematics propositions are based not upon real objects, but exceptionally on objects of our imagination. In this connection, there is a question which excited researchers of all times. Why is possible such excellent conformity of mathematics with real objects if mathematics is only product of the human thought which have been not connected with any experience? Can the human reason understand properties of real things by only reflection without any experience? In my opinion, the answer to this question is in brief as follows: if mathematics theorems are applied to reflection of the real world, they are not exact; they are exact if they do not refer to the reality. Mathematics itself can say nothing about real objects. However, on the other hand, it is also truth that mathematics in general and geometry in particular have its origin in the fact that there is necessity to learn something about behaviour of materially existent objects. It is clear that from system of concepts of axiomatic geometry it is impossible to obtain any judgements about such really existent objects which we call by practically solid bodies. In order to such judgements were possible, we should deprive geometry of its formal-logical character having compared the empty scheme of concepts of axiomatic geometry to real objects of our experience. For this purpose, it is enough to add only such statement: solid bodies behave in sense of various possibilities of a mutual position as bodies of Euclidean geometry of three measurements; thus, theorems of Euclidean geometry enclose the statements determining behaviour of practically solid bodies. The geometry added with such statement becomes, obviously, natural science; we can consider it actually as the most ancient branch of physics. Its statements are based essentially upon empirical conclusions and not just on the logical conclusions. We will call further the geometry added in such a way as "practical geometry" unlike "purely axiomatic geometry" [4]. However, Einstein's approach has not been correctly analyzed and grounded in works of contemporary scientists (for example, in Adolf Grünbaum's work [5]). Besides, this approach is not generally accepted because it does not contain a methodological key to solution of the problem of relation between geometry and natural sciences.

Therefore, the purpose of the present work is to propose analysis of the problem of relation between geometry and natural sciences within the framework of the correct methodological basis – unity of formal logic and of rational dialectics.

1. Methodological basis of the analysis

The principle of unity of formal logic (i.e. the science of laws of correct thinking) and of rational dialectics (i.e. the dialectics based on rational thinking) represents methodological basis of the correct analysis. The methodological basis contains the general arguments (premises) for the deductive proof of theoretical prepositions. The general arguments (premises) are as follows.

1. According to the principle of materiality of the Nature, the Nature is a system of material objects (particles, fields, bodies). Research of systems is carried out within the framework of the system approach. The system approach is a line of methodology of scientific cognition which is based on treatment of objects as systems. The system approach is a concrete definition of main principles of rational dialectics.

2. System (i.e. whole, made of parts; connection) is a set of the elements which are in relations and connections with each other forming certain integrity, unity. One marks out material and abstract systems. In modern sciences, a research of any systems is carried out within the frameworks of various special theories of systems.

3. According to rational dialectics, a material object has qualitative and quantitative determinacy (i.e. qualitative and quantitative aspects). The unity qualitative and quantitative determinacy (aspects) of object is called as property (measure) of object (for example, physical, chemical, and geometrical properties). Properties, signs (for example, energy, weight, speed, extent, surface, plane, curvature, circle, straight-linearity etc.) of objects do not exist independently of objects: there are not qualities and quantities, but only the objects having qualities and quantities. In other words, property (measure) is an inalienable characteristic of material object and belongs only to material object.

4. Pure (abstract) mathematics studies the quantitative determinacy separated (abstracted) from qualitative determinacy (i.e. from properties, signs) of material object. Therefore, pure (abstract) mathematics has no qualitative (for example, physical, chemical, geometrical) sense, meaning. The criterion of the validity of pure (abstract) mathematics is not in itself. It is in natural sciences studying material objects. These sciences use pure (abstract) mathematics by its material interpretation (application). The material interpretation is that the identity relation between concepts “material object” and “mathematical object” is established. The criterion of the validity of the interpreted (applied) mathematics is practice.

5. Natural sciences study properties (measures) of material objects (for example, physical, chemical, and geometrical properties). A mathematical formalism is an instrument of quantitative studying of material objects. The applied mathematics (i.e. the mathematics applied to studying of material objects) leads to mathematical relations of natural sciences. The mathematical relations of natural sciences describe properties of material objects, contain the reference to these objects and, consequently, have qualitative (natural-scientific) sense, meaning. From formal-logical point of view, mathematical (quantitative) operations over an equation do not lead to change of qualitative determinacy.

6. From formal-logical point of view, the natural-scientific concept and mathematical concept can be compared with each other only if there are logical relations (for example, the identity relation) between them. Therefore, the application of mathematics to the description (studying) of properties of material object is possible only in the case if used mathematical concepts are identical to used natural-scientific concepts, i.e. if the identity relation between natural-scientific concepts (characterizing material object) and mathematical concepts (characterizing mathematical object) is established.

7. The description of qualitative and quantitative determinacy (aspects) of material object is obeyed to formal logic laws. According to the identity law, the left and right sides of mathematical equation should belong to the same qualitative determinacy (aspect) of material object. And according to the contradiction law, the left and right sides of mathematical equation should not belong to different qualitative determinacy (aspects) of material object.

8. A material object has set of properties. If the form of macroscopical object (i.e. relation between sizes and mutual position of parts of macroscopical object) represents a unique essential sign of object, the macroscopical object is called as geometrical object (body). From this point of view, geometry (as the science of geometrical properties of system of material objects) is not a part (section) of mathematics. The essence of geometry is that geometry is a section of natural sciences studying geometrical properties of system of macroscopical material objects by means of the theory of systems and mathematical formalism. (A method of construction (designing, synthesis) used in geometry as the way of studying is not mathematics). For example, geometry is the component part of the science of strength and of deformability of elements of constructions and details of machines [6]. Bases of this engineering science have been formulated by Galileo Galilei in its book, «Discorsi e Dimostrazioni matematiche» (Leiden, 1638), L. Euler, and many other scientists.

9. Studying of geometrical properties is based on application of the theory of systems and mathematics (mathematical formalism). The basic proposition of the theory of systems is formulated as follows: properties of system are not a consequence of properties of its elements. From the formal-logical point of view, application of mathematics is permitted by the identity law only in the case if there is the identity relation between concepts "geometrical object (body)" and "mathematical object".

10. One of the first and important conditions of application of mathematics to description of geometrical body is that it is necessary to establish the identity relation between mathematical concept "zero" and geometrical concept "zero object (body)". In other words, it is necessary to define the concept "zero object (body)" and to consider the macroscopical "zero object (body)" as a zero form. Since a material object has three measurements, a geometrical object (body) also represents a relation of three sizes.

11. Movement is a change in general. Movement of material object is a transitions of object from one states in others. There are physical, chemical, geometrical and other states of a material object. In general case, environment influences (affects) on states of objects. The set of geometrical states of material object is called as geometrical space of object.

12. If environment influence (effect) can be neglected, then the form of material object is invariant relative to position of this object in the environment. In this case, geometrical states (i.e. geometrical space) of this object are simply positions of this object relative to other objects. This geometrical space is called as Euclidean space of the object.

13. If environment (physical fields) has force influence (effect) on the form of the material object, then the material object can take set of forms (geometrical states). In this case, the geometrical states of this object represent non-Euclidean space.

14. Axiomatic construction of science is the present stage of development of rational thinking. Axiom is an elementary proposition of the theory, expressing the empirical (experimental) fact.

The formulated arguments (premises) allow to define the basic concepts of geometry.

2. Definition of the basic concepts of elementary geometry

The basic concepts of elementary (Euclidean) geometry – "point", "line", and "surface" – are defined as follows.

1. The concept "geometrical object (body)" expresses an essential sign of material object. It is the general and concrete concept. The volume of the general concept is expressed in the form of a logical class. This class is the highest one (i.e., it is genus).

2. Point is "material point", "zero object (body)". "Material point", "zero object (body)" is the geometrical object (i.e. the macroscopic body or the macroscopic part of body) which has three sizes. These three sizes are always considered (assumed) equal to zero. "Zero object (body)" is not divided into parts (i.e. it has no parts) and represents zero form

(reference point, origin of form). The concept "point" is specific and concrete concept. "Nonzero geometrical object" (i.e. the macroscopic body or the macroscopic part of body) contains set of points and, consequently, is a geometrical place of points.

3. Line is the geometrical object (i.e. the macroscopic body or the macroscopic part of body) which has three sizes. Two of these three sizes are always considered (are assumed) equal to zero. The concept "line" is specific and concrete concept. This concept does not contain assertion of method of ordering of points of line.

4. Surface is the geometrical object (i.e. the macroscopic body or the macroscopic superficial part (layer) of body) which has three sizes. One of these sizes is always considered (is supposed) equal to zero. The concept "surface" is specific and concrete concept. This concept does not contain assertion of method of ordering of points of surface.

These basic concepts permit to define the derivative (subordinated) concepts "plane" ("plane surface"), "circle", "circular line", and "straight line". As is known, the general form of logical definition of concepts is definition through the nearest genus and specific difference. The general form of definition is complete form of definition if specific difference can be defined. The specific concepts "plane" ("plane surface"), "circle", "circular line", and "straight line" can be defined only by means of genetic definition. In accordance with formal logic, genetic definition is the special form of definition, which shows how given object arises. Therefore, genetic definitions of the concepts "plane" ("plane surface"), "circle", "circular line", and "straight line" must show how the ordering of points of plane, circle, and straight line arises. The genetic definitions of the basic concepts are as follows.

(a) Plane (plane surface) is the surface arising as the geometrical place of points at arrangement of these points on equal (identical) distance from two fixed pole points which do not belong to a surface. In other words, plane is the surface representing the geometrical place of points which are equidistant from two pole points which do not belong to a surface. These pole points are called as poles of plane.

(b) The part of the plane bounded (limited) by a circular line is called as circle. Circular line is the closed curve which arises as a geometrical place of points on a plane at arrangement of these points on equal (identical) distance from one fixed pole point belonging to a plane. In other words, circular line is the closed curve representing a geometrical place of points on a plane which are equidistant from one pole point belonging to a plane. This pole point is called as pole (or centre) of circle (circular line). There is the following relation between a circle (circular line) and its pole: the pole defines (generates) set of nonintersecting (concentric) circular lines; the generated circle (circular line) characterizes one and only one pole. Consequences are as follows: the existence of system "plane and one pole on it" represents a necessary condition of origin (existence) of a circular line and a circle; circular line is the carrier of curvature property; curvature property is one of manifestations of properties of the system "plane and one pole on it"; concept "curvature" is an abstract concept.

(c) Straight line is the line which arises as a geometrical place of points on a plane at arrangement of these points on equal (identical) distance from two fixed pole points belonging to a plane. In other words, straight line is the line representing a geometrical place of points of a plane, which are equidistant from two pole points belonging to a plane. These pole points are called as poles of straight line. There is the following relation between a straight line and its poles: given pair of poles defines (generates) one and only one straight line; the generated straight line characterizes a geometrical place of poles. Consequences are as follows: existence of the system "plane and two points-poles on it" represents a necessary condition of existence of a straight line; two arbitrary points of a plane define a straight line (in other words: the straight line passes through two arbitrary points; it is possible to pass one and only one straight line through two arbitrary points); a straight line is the carrier of straightness property; straightness property is one of manifestations of properties of the system “plane and two poles on it”; concept "straightness" is an abstract concept.

3. Properties of system of geometrical objects

Elementary geometrical objects – point, straight line, circular line, circle, and plane – are elements of any synthesized geometrical objects (systems): flat angles, triangles, etc. Properties of the synthesized system are not a logical consequence of properties of elements. Therefore, studying of the synthesized system is an experimental determination of properties of this system. From this point of view, properties of system are expressed in the form of axioms. Axioms are the elementary theoretical propositions expressing empirically (experimentally) studied properties of system.

3.1. Flat angle as system of two intersecting straight lines

If two straight lines (elements) on a plane is united (connected) by the common point, (i.e. by the point at which straight lines intersect), then the synthesized system (the constructed, designed geometrical figure) is called as a flat angle. The point of connection (intersection) of straight lines is called as a vertex of angle, and the straight lines bounded by vertex is called as sides of angle. A measure of angle is degrees or radians. The angle which has coincident sides is called as zero angle (it have measure 0° , or 0π). If sides of angle form straight line, then the angle is called as straight angle (it have measure 180°, or 1π). If value of angle equals to half of straight angle, then the angle is called as right angle (it have measure 90°, or $\pi/2$). The basic property of a angle as a system is as follows: value of angle does not depend on lengths of its sides (elements).

3.2. Parallel straight lines as subsystem

If: (a) the straight line passing through two points of a plane represents a geometrical place of poles of straight lines; (b) one of poles is fixed, and another is variable, – then there exists set (system) of the straight lines on a plane, defined (generated) by variable pole. All these straight lines are connected by the parallelism relation.

Two straight lines are connected by the parallelism relation and are called as parallel if the distance d_{ij} between them represents difference of the distances of poles of i th and j th straight lines: $d_{ij} = \left| d_i - d_j \right|$, $i, j = 0, 1, 2, \ldots$ where d_i and d_j are the distances of poles of i th and j th straight lines, respectively. As $d_{ij} = 0$ under $i = j$, parallel straight lines coincide under $i = j$

Consequences are as follows: (1) the parallelism relation between two given straight lines is established by means of third straight line passing through poles and connecting (intersecting) these two given straight lines; parallel lines form the right angle with the third line passing through poles; (2) parallelism relation between two straight lines is a property of the system of three straight lines; (3) parallel straight lines are not intersected; (4) if: the straight line passing through two points of a plane is unique and represents a geometrical place of poles of parallel straight lines; for any point which does not lie on the geometrical place of poles there are a fixed poles; the fixed poles determine a unique straight line, – then one and only one straight line which is parallel to given parallel straight lines passes through arbitrary point of the plane (this statement expresses invariance property of a right angle at movement).

Thus, these consequences express experimentally studied properties of system. The elementary theoretical propositions expressing experimentally studied properties of system are axioms. It means that Hilbert’s axiom V (parallelism axiom) [2] is not a logical consequence of axioms I–IV because property of system of geometrical elements is not a consequence of properties of its elements.

3.3. Triangle as system of three intersecting straight lines

If the sides of flat angle are connected (intersected) by straight line, the synthesized system (the constructed geometrical figure) is called as triangle. The points of intersection of straight lines (i.e. three points A, B, C) are called as vertexes of triangle. Straight lines (i.e. three straight lines – segments AB, BC, CD) bordered by vertexes are called as sides of triangle. Existence of interior angles (i.e. elements $\angle CAB \equiv \alpha$, $\angle ABC \equiv \beta$, $\angle BCA \equiv \gamma$) of triangle leads to arising of the essential sign (parameter) of system: the sums $S = \alpha + \beta + \gamma$. Value of S can be determined by only means of experimental studying of properties of triangle as system.

The experimental device represents the following material design: material triangle which has vertexes A, B, C as joints. The joints permit to change the parameters of elements: values of angles α, β, γ and lengths of sides of the triangle. In other words, joints permit structural ("inner") movement of triangle (i.e. transitions from one structural state into others). Structural movement of triangle is reduced to two elementary movements of its sides: to the "shift along a straight line" and to the "rotation round a point"). Statement of the problem is as follows: it is necessary to show experimentally that property S of triangle (as system) does not depend on properties (parameters) of elements of a triangle. (In other words, it is necessary to show that S is the invariant of the structural movement of a triangle).

The results of the experiment are as follows:

(a) if α is independent variable, then $S(\alpha) = \alpha + \beta(\alpha) + \gamma(\alpha)$ is linear function of α; $0 \le \alpha \le \pi$;

(b) if $\alpha \to 0$, then $\beta + \gamma \to \pi$;

(c) if $\alpha \to \pi$, then $\beta + \gamma \to 0$;

(d) $0 \le (\beta + \gamma) \le \pi$;

(f) area (as a variable) is not essential sign of a triangle;

(g) unlike reasoning of A.M. Legendre, it is not assumed in experiment that "sides of triangle increase infinitely" (N. Lobachevski) [6]. Therefore, it is possible "to conclude from this that approaching of opposite sides to the third side under decrease of two angles is necessarily finished with transmutation of other angle into two right angles" (N. Lobachevski) [6].

The results of the experiment signify that S represents the sum of adjacent angles α and $(\beta + \gamma)$. Hence, $S = \pi$.

Thus, property of a rectilinear triangle as system is that the sum S of interior angles of a rectilinear triangle is equal to π and it does not depend on properties of its elements, i.e. S is invariant of movement of rectilinear triangle. This experimental fact represents equivalent of Euclid's Vth postulate (or the axiom V in the list of Hilbert's axioms).

4. Natural-scientific classification of triangles and of geometries

As is known, the generally accepted mathematical classification of geometries is based on not qualitative determinacy of a triangle but quantitative determinacy – value of the sum S of the interior angles of the flat triangle: $S_E = \pi$ (Euclidean geometry), $S_L < \pi$ (Lobachevski geometry), $S_R > \pi$ (Riemann geometry). From this formal-logical point of view, such classification is incorrect, and natural-scientific classification of triangles on the basis of qualitative determinacy (essential sign) is correct. The correct classification of triangles represents a key to natural-scientific (material) interpretation of Euclidean geometry, Lobachevski geometry, and Riemann geometry.

Classification of triangles on the basis of the essential sign – qualitative determinacy of the sides of the triangle – is carried out as follows.

1. Concepts of line, of segment of line (element), and of triangle are defined.
2. The class of lines is divided into two nonintersecting classes: the class of straight lines and the class non-straight lines. (Division is carried out on the basis of curvature sign: if the line does not possess the curvature sign, then it is a straight line; if the line possesses the curvature sign, then it is a curve line).
3. The class of triangles is divided into two nonintersecting classes: the class of rectilinear triangles and the class of non-rectilinear (curvilinear) triangles (Fig. 1).
4. The triangle is a rectilinear triangle if all its sides are formed by segments of straight line (Fig. 1).
5. The triangle is non-rectilinear (curvilinear) even if one side is formed by segment of curve line.
6. The class of curvilinear triangles is divided into two nonintersecting subclasses: the subclass of flat curvilinear triangles and the subclass of non-flat curvilinear triangles (Fig. 1).
7. The subclass of flat curvilinear triangles is divided into three subclasses: two nonintersecting subclasses (i.e., subclass of convex curvilinear triangles and subclass of non-convex (concave) curvilinear triangles) and one mixed subclass (Fig. 1).

The above-stated classification is a basis for experimental studying of quantitative determinacy of triangles. According to experimental data, the true statements are as follows (Fig. 1):

(a) the sum of the interior angles of any triangle is either $S = \pi$, or $S \neq \pi$;
(b) the sum of the interior angles of a rectilinear triangle satisfies to the relation $S_E = \pi$;
(c) the sum of the interior angles of a flat concave curvilinear triangle satisfies to the relation $S_L < \pi$;
(d) the sum of the interior angles of a flat convex curvilinear triangle satisfies to the relation $S_R > \pi$;
(e) the sum of the interior angles of the flat convexo-concave curvilinear triangle concerning the mixed subclass can satisfy to any relations: $S < \pi$, $S = \pi$, $S > \pi$.

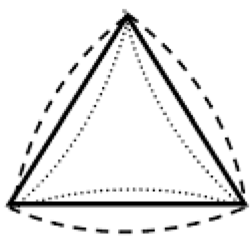

Figure 1. The basic types of flat triangles:
rectilinear triangle (it is represented by continuous line), $S_E = \pi$;
concave triangle (it is represented by dotted line), $S_L < \pi$;
convex triangle (it is represented by dashed line), $S_R > \pi$.

These statements can be is briefly formulated in the form of the following theorem of sum of interior angles of flat triangle.

Theorem. *If flat triangle is rectilinear one, then* $S = \pi$; *if flat triangle is convex one, then* $S_L < \pi$; *if flat triangle is convex one, then* $S_R > \pi$.

The converse theorem is not true.

Thus, concepts "geometry of rectilinear triangle", "geometry of flat concave triangle", and "geometry of flat convex triangle" are identical to concepts "Euclidean geometry", "Lobachevski geometry", and "Riemann geometry", respectively.

5. Natural-scientific (engineering) meaning of Lobachevski function

As it is known, Lobachevski function [1] is of fundamental importance in non-Euclidean geometry. One should consider deformation of infinite rectilinear rod to elucidate natural-scientific (material, engineering) meaning of Lobachevski function (Fig. 2).

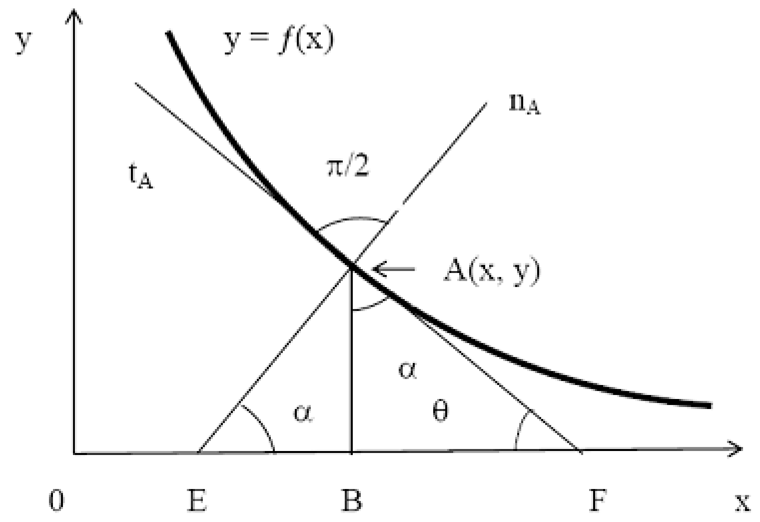

Figure 2. Rod deformation and Lobachevski function:
$y = f(x)$ is the axis of the bent rod;
t_A, n_A are the tangent and the normal to the curve at the point $A(x, y)$, respectively;
AB is the perpendicular to the axis x ;
θ is the slope angle of the tangent at the point $A(x, y)$; ;
$\alpha = \Pi(y)$ is the parallelism angle (Lobachevski function).

Let, according to the course "Resistance of materials" [7], following conditions are satisfied:

1) the flat bend of the rod takes place under the influence of external forces;
2) the direct problem is to find the equation of the bent axis of the rod;
3) deformation is completely determined by position of the cross-section of the rod at any point $A(x, y)$ of line $y = f(x)$;

4) position of cross-section section of the rod at the point $A(x, y)$ is completely determined by the following quantities: (a) rod deflection – the perpendicular AB to the axis x, i.e. the coordinate y of the point $A(x, y)$; (b) the rotation angle θ of cross-section at the point $A(x, y)$, i.e. the angle θ formed by the axis x and the tangent t_A at the point $A(x, y)$;
5) the angle θ is a function of AB in the rectangular triangle AFB, i.e. there is the certain functional dependence between $\theta(x)$ and y at each point $A(x, y)$;
6) the function $\theta(x)$ is monotonous and continuous one;
7) the relation $\alpha + \theta = \pi/2$ represents the connection of the angle θ with the angle α formed by the axis x and the normal n_A to the axis of the rod at the point $A(x, y)$ (i.e., the angle α is a function of AB in the rectangular triangle ABE);
8) conditions of end fixity of the rod are as follows: $\theta \to 0$ under $x \to \infty$, and
$\theta \to \pi/2$ under $y \to \infty$.
Then the following existence theorem of Lobachevski function is true.
Theorem. *If function $\alpha = \Pi(y)$ is determined for every positive y, decreases monotonously and is continuous, $\Pi(y) \to \pi/2$ under $y \to 0$, and $\Pi(y) \to 0$ under $y \to \infty$, then the angle α at the point $A(x, y)$ is a parallelism angle with respect to the axis x, and the function $\alpha = \Pi(y)$ represents Lobachevski function* [3, 4].

From this it follows that natural-scientific (material, engineering) meaning of Lobachevski function is that it characterizes position of cross-section of the deformed rod.

6. Discussion

The concepts “Euclidean geometry”, “Lobachevski geometry”, and “Riemann geometry” have natural-scientific meaning only in the case of material interpretation of triangles. If to take into consideration existence of the environment which can have a force effect (influence) on a material rectilinear triangle and to consider the complete system “material rectilinear triangle + environment”, then it becomes clear that effect (influence) of external forces leads to deformation (stretching, contraction, bending, and torsion) of the sides of the rectilinear triangle. The rectilinear triangle is plain-deformed (plane-strain) under the influence of the external forces if force vectors are on the plane of the rectilinear triangle. In this case, either Lobachevski geometry or Riemann geometry or geometry of the mixed type is realized (Fig. 1). The rectilinear triangle is non-plain-deformed under the influence of the external forces if force vectors are not on the plane of the rectilinear triangle. Then the rectilinear triangle becomes non-plain one and, consequently, “geometry of non-plain triangle” is realized. The rectilinear triangle is not deformed if external forces have no influence on it. In this case, Euclidean geometry is realized (Fig. 1).

From the point of view of an experimental research of the complete system “material triangle + environment”, the “question of true geometry of the Universe” [8] is incorrect. Really, as rectilinear triangles can be formed, for example, with light beams, rigid rods etc. (which are subjects to influence of the environment to a variable degree), the results of an experimental research will represent set of various true geometries: “geometry of the triangle formed by light beams”, “geometry of the triangle formed by rigid rods” etc.

From the natural-scientific point of view, the solution of the problem of logical consistency Euclidean geometry, Lobachevski geometry, and Riemann geometry is trivial one. Really, Euclidean geometry, Lobachevski geometry, and Riemann geometry are mutually consistent, because, in accordance with practice, existence of Lobachevski curvilinear triangle and of Riemann curvilinear triangle does not contradict existence of Euclidean rectilinear triangle.

And, at last, about so-called absolute geometry. As is known, the absolute geometry is Euclidean geometry without the fifth postulate [1]. Therefore, the material essence of absolute geometry is “geometry of elements” of material system: geometry of elements “point”, “line”, “straight line”, and “surface”.

Conclusion

Thus, the solution of the problem of relation between geometry and natural sciences within the framework of correct methodological basis – the unity of formal logic and rational dialectics – shows that geometry represents section (part) of natural sciences. Geometry as section (part) of natural sciences uses mathematical and philosophical formalisms and also leans on experiment. Criterion of validity of geometrical statements is practice. From this point of view, practice represents not only a basis of correct classification of triangles and of geometries, the material interpretation of Euclidean geometry, of Lobachevski geometry, and of Riemann geometry, but also a starting point of axiomatic (system) construction of sciences. The axiomatic construction of sciences is the present stage of development of rational thinking.

The rational thinking based on the unity of formal logic and of rational dialectics leads to the new results received in the present work. These results are as follows.

1) Genetic definitions of the basic concepts “point”, “line”, “straight line”, and “surface” of elementary (Euclidean) geometry are formulated.

2) It is shown that the list of Hilbert’s axioms is incomplete because it does not contain definition of the concept “triangle”.

3) The natural-scientific proof of the parallelism axiom (Euclid’s fifth postulate), classification of triangles on the basis of the qualitative (essential) sign, and also material interpretations of Euclidean geometry, of Lobachevski geometry, and of Riemann geometry are proposed.

These results are of fundamental importance for elucidation of essence of multidimensional geometry, and also for progress in science and engineering.

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