How an effective "cosmological constant" may affect a minimum scale factor, to avoid a cosmological singularity.(Breakdown of the First Singularity Theorem)

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We once again reference Theorem 6.1.2 of the book by Ellis, Maartens, and MacCallum in order to argue that if there is a non zero initial scale factor, that there is a partial breakdown of the Fundamental Singularity theorem which is due to the Raychaudhuri equation. Afterwards, we review a construction of what could happen if we put in what Ellis, Maartens, and MacCallum call the measured effective cosmological constant and substitute $\Lambda \to \Lambda_{Effective}$ **in the Friedman equation.** I.e. there are two ways to look at the problem, i.e. after $\Lambda \to \Lambda_{Effective}$**,** set Λ_{Vac} as equal to zero, and have the left over Λ as scaled to background cosmological temperature, as was postulated by Park (2002) or else have Λ_{Vac} as proportional to $\Lambda_{Vac} \sim 10^{38} GeV^2$ which then would imply using what we call a 5 dimensional contribution to Λ as proportional to $\Lambda \approx \Lambda_{5D} \sim -\mathrm{const}/\mathrm{T}^{\beta}$. We find that both these models do not work for generating an initial singularity. Λ removal as a non zero cosmological constant is most easily dealt with by a Bianchi I universe version of the generalized Friedman equation. The Bianchi I universe case almost allows for use of Theorem 6.1.2. But this Bianchi 1 Universe model **almost** in fidelity with Theorem 6.1.2. requires a constant non zero shear for initial fluid flow at the start of inflation which we think is highly unlikely.

Key words: Raychaudhuri equation, Fundamental Singularity theorem, Bianchi I universe, effective cosmological parameter

1. Introduction

The present document is to determine what may contribute to a nonzero initial radius, i.e. not just an initial nonzero energy value, as Kauffman's paper would imply, and how different models of contributing vacuum energy, initially may affect divergence from the first singularity theorem. The choices of what can be used for an effective cosmological constant will affect if we have a four dimensional universe in terms of effective contributions to vacuum energy, or if we have a five dimensional universe. The second choice will probably necessitate a tie in with Kaluza Klein geometries, leaving open possible string theory cosmology. In order to be self contained, this paper will give partial re productions of Beckwith's (2013) earlier paper, but the 2nd half of this document will be completely different, ie. When considering an effective cosmological constant. With four different cases. The last case is unphysical, even if it has, via rescaling zero effective cosmological constant, due to an effective 'fluid mass' M_{eff}

2. Looking at the First Singularity theorem and how it could fail

Again, we restate at what is given by Ellis, Maartens, and MacCallum. (2012) as to how to state the fundamental singularity theorem

Theorem 6.1.2 (Irrotational Geodestic singularities) *If* $\Lambda \le 0$, $\rho + 3p \ge 0$, *and* $\rho + p > 0$ *in a fluid flow for which* $\dot{u} = 0$, $\omega = 0$ *and* $H_0 > 0$ *at some time* s_0*, then a spacetime singularity , where either* $\ell(\tau) \to 0$ *or* $\sigma \to \infty$*, occurs at a finite proper time* $\tau_0 \le H_0$ *before* s_0.

As was brought up by Beckwith, (2013), if there is a non zero initial energy for the universe, a supposition which is counter to ADM theory as seen in Kolb and Turner (1991), then the supposition by Kauffman (2012) is supportable with evidence. I.e. then if there is a non zero initial energy, is this in any way counter to Theorem 6.1 above? We will review this question, keeping in mind that. $\ell(\tau)\to 0$ is in reference to a scale factor, as written by Ellis, Maartens, and MacCallum. (2012), vanishing.

3. Looking at how to form $\ell(\tau)\neq 0$ for all scale factors.

What was done by Beckwith (2013) involved locking in the value of Planck's constant initially. Doing that locking in of an initial Planck's constant would be commensurate with some power of the mass within the Hubble parameter, namely M_0,

$$\hbar \propto \sqrt{M_0} \tag{1}$$

We would argue that a given amount of mass, M_0 would be fixed in by initial conditions, at the start of the universe and that if energy, is equal to mass (E = M) that in fact locking in a value of initial energy, according to the dimensional argument of $E \sim \hbar\cdot\omega$ that having a fixed initial energy of $E \sim \hbar\cdot\omega$, with Planck's constant fixed would be commensurate with, for very high frequencies, ω of having a non zero initial energy, thereby confirming in part Kauffmann(2012) , as discussed in Appendix A, for conditions for a non zero lower bound to the cosmological initial radius. If so then we always have $\ell(\tau)\neq 0$. We will then next examine the consequences of $\ell(\tau)\neq 0$. I.e. what if $\ell(\tau)=a(\tau)$ for a FLRW cosmology?

4. $\ell(\tau)\neq 0$ and what to look for in terms of the Raychaudhuri-Elders equation for $\ell(\tau)=a(\tau)$ at the start of cosmological expansion in FLRW cosmology

We will start off with $\ell(\tau)=a(\tau)=a_{initial}e^{\tilde{H}\tau}$ with $\tilde{H}$ an initial huge Hubble parameter

$$3\ddot{a}/a=-4\pi G\cdot(\rho+3p)+\Lambda \Rightarrow \dot{a}^2-8\pi G\rho a^2-\Lambda a^2-const=0 \tag{2}$$

Equation (2) above becomes, with $\ell(\tau)=a(\tau)=a_{initial}e^{\tilde{H}\tau}$ introduced will lead to

$$a_{initial}^2=const/\left[\tilde{H}^2-\left[8\pi G\rho+\Lambda\right]\right]\Rightarrow a_{initial}=\sqrt{const/\left[\tilde{H}^2-\left[8\pi G\rho+\Lambda\right]\right]} \tag{3}$$

5. Analyzing Eq. (3) for different candidate values of Λ, with $\Lambda\to\Lambda_{Effective}$ for three cases.

The equation to look at if we have $\Lambda\to\Lambda_{Effective}$ put into Eq. (3) is to go to, instead to looking at

$$\Lambda\to\Lambda_{Effective}=\Lambda_{Vac}+\Lambda \tag{4}$$

Case 1, set $\Lambda_{Vac}=0$, and $\Lambda=\Lambda_{start-value}T^{\beta}$ such that in the present era with T about 2.7 today

$$\Lambda_{Effective}=\Lambda_{Vac}+\Lambda\sim 10^{-83}GeV^2\,\text{(today)} \tag{5}$$

This would change to , if the temperature T were about $10^{32}Kelvin\sim 10^{19}GeV$

$$\Lambda_{Effective}=\Lambda_{Vac}+\Lambda\sim 10^{38}GeV^2\,\text{(Plank era)} \tag{6}$$

The upshot, is that if we have Case 1, we will not have a singularity if we use Theorem 6.1

Case 2, set $\Lambda_{Vac}=0$, and such that $\Lambda=\Lambda_{start-value}$ in the present era with T about 2.7 today
The upshot, is that if we have Case 2, we will not have a singularity if we use Theorem 6.1
Unless $\Lambda=\Lambda_{start-value}$ is less than or equal to zero. In reality this does not happen, and we have

$$\Lambda_{Effective}=\Lambda_{Vac}+\Lambda\sim 10^{-83}GeV^2\,\text{(always)} \tag{7}$$

Case 3, set $\Lambda_{Vac}\sim 10^{38}GeV^2$, and set $\Lambda\approx\Lambda_{5D}\sim -\text{const}/\text{T}^{|\beta|}$ for all eras. Such that

$$\Lambda_{Effective}=\left[\Lambda_{Vac}\sim 10^{38}GeV^2\right]+\left[\Lambda_{5D}\sim -\text{const}/\text{T}^{|\beta|}\right]\sim 10^{-83}GeV^2(today) \tag{8}$$

$$\Lambda_{Effective} = \left[\Lambda_{Vac} \sim 10^{38} GeV^2\right] + \left[\Lambda_{5D} \sim -\mathrm{const}/\mathrm{T}^{|\beta|}\right] \sim 10^{38} GeV^2 (\mathrm{Planck-era}) \quad (9)$$

The only way to have any fidelity as to this theorem 6.1 would be to eliminate the cosmological constant entirely. There is, one model where we can, in a sense "remove" a cosmological constant, as given by Ellis, Maartens, and MacCallum. (2012), and that is the Bianchi I universe model, as given on page 459.

6. Bianchi I universe in the case of $\omega = p/\rho = -1 \Leftrightarrow \rho = const$

In this case, we have pressure as the negative quantity of density, and this will be enough to justify writing

$$3\frac{\dot{\ell}^2}{\ell^2} = \frac{\Sigma^2}{\ell^6} + \frac{M}{\ell^{3\omega+3}} + \Lambda\Big|_{\omega=-1} = \frac{\Sigma^2}{\ell^6} + \left(M + \Lambda\right) \quad (10)$$

If $\ell(\tau) = \ell_{initial} e^{\tilde{H}\tau}$, we can re write Eq.(10) as, if the sheer term in fluid flow, namely Σ is a non zero constant term.(I.e. at the onset of inflation, this is dubious)

$$\ell^6 = \Sigma^2 \Big/ \left[3\tilde{H}^2 - \left[M + \Lambda\right]\right] \quad (11)$$

In this situation, we are speaking of a cosmological constant and we will collect $\left[M + \Lambda\right] \equiv M_{eff}$ such that

$$\ell^6 = \Sigma^2 \Big/ \left[3\tilde{H}^2 - M_{eff}\right] \quad (12)$$

If we speak of a fluid approximation, this will lead to for Planck times looking at $\ell \sim \ell_{initial}$ so we solve

$$\ell = \Sigma^{1/3} \Big/ \left[3\tilde{H}^2 - M_{eff}\right]^{1/6} \quad (13)$$

The above equation no longer has an effective cosmological constant, i.e. if matter is the same as energy, in early inflation, Eq. (13) is a requirement that we have, effectively, for a finite but very large $\tilde{H}^2$

$$\tilde{H}^2 >> M_{eff}/3 \quad (14)$$

7. Use of Thermal history of Hubble parameter equation represented by Eq.(14)

Ellis, Maartens, and MacCallum. (2012) treatment of the thermal history will then be, if $100 < g_*(T) < 1000$

$$\tilde{H}^2 \approx (1.7)^2 g_*(T) \cdot \frac{T^4}{M_p^2} \quad (15)$$

Then we have for Eq. (14),if the value of Eq.(15) is very large due to Plank temperature values initially

$$(1.7)^2 g_*(T) \cdot \frac{T^4}{M_p^2} >> M_{eff}/3 \quad (16)$$

This assumes that there is an effective mass which is equal to adding both the Mass and a cosmological constant together. In a fluid model of the early universe. This is of course highly unphysical. But it would lead to Eq. (13) having a non zero but almost infinitesimally small Eq. (13) value. The vanishing of a cosmological constant inside an effective (fluid) mass, as given above by $\left[M + \Lambda\right] \equiv M_{eff}$ means that if we treat Eq. I5 above as ALMOST infinite in value, that we ALMOST can satisfy Theorem 6.1 as written above. The fact that $100 < g_*(T) < 1000$, i.e. we do not have infinite degrees of freedom, means that we get out of having Eq. (15) become infinite, but it comes very close.

8. Use of Thermal history of Hubble parameter equation represented by Eq.(3) and an effective cosmological parameter.

Case 1, if $\Lambda_{Vac} = 0$. But the cosmological parameter has a temperature dependence. Is the following true when the temperatures get enormous?

$$(1.7)^2 g_*(T)\cdot\frac{T^4}{M_p^2} >> 8\pi G\rho_{initial} + \Lambda_{start-Value}T^{\beta} \quad (17)$$

Not necessarily,. It could break down.

Case 2, set $\Lambda_{Vac}=0$, and such that $\Lambda=\Lambda_{start-value}$ (cosmological constant). Then we have

$$(1.7)^2 g_*(T)\cdot\frac{T^4}{M_p^2} >> 8\pi G\rho_{initial} + \Lambda_{start-Value} \quad (18)$$

Yes, but we have problems because the cosmological parameter, while still very small is not zero or negative. So theorem 6.1.2 above will not hold. But it can come close if the initial value of the cosmological constant is almost zero.

Case 3, when we can no longer use $\Lambda_{Vac}=0$. Is the following true ? When the Temperature is Planck temp?

$$(1.7)^2 g_*(T)\cdot\frac{T^4}{M_p^2} >> \Lambda_{Effective} = \left[\Lambda_{Vac}\sim 10^{38}GeV^2\right]+\left[\Lambda_{5D}\sim -\text{const}/\,\text{T}^{|\beta|}\right]\sim 10^{38}GeV^2(\text{Planck}-\text{era}) \quad (19)$$

Almost certainly not true. Our section eight is far from optimal in terms of fidelity to Theorem 6.1.

We are close to Theorem 6.1.2 on our section seven. But this requires a demonstration of the constant value of the following term, in section 7 , namely in the Bianchi universe model, that the sheer term in fluid flow, namely Σ is a non zero constant term.(I.e. at the onset of inflation, this is dubious). If it, Σ, is not zero, then even close to Planck time, it is not likely we can make the assertion mentioned above. In Section 7.

9. Conclusion: Non singular solutions to cosmological evolution require new thinking.

For section 7 above we have almost an initial singularity, if we replace a cosmological constant with $[M+\Lambda]\equiv M_{eff}$, And we also are assuming then, a thermal expression for the Hubble parameter given by Ellis, Maartens and Mac Callum as a $(1.7)^2 g_*(T)\cdot\frac{T^4}{M_p^2}$ term which is almost infinite in initial value. Our conclusion is that we almost satisfy Theorem 6.1 if we assume an initially almost perfect fluid model to get results near fidelity with the initial singularity theorem (Theorem 6.1). This is dubious in that it is unlikely that Σ, as a shear term is not zero, but constant over time, even initially.

The situation when we look at effective cosmological “constants” is even worse. I.e. Case 1 to Case 3 in section eight no where come even close to what we would want for satisfying the initial singularity theorem (theorem 6.1)

We as a result of these results will in future work examine applying Penrose’s CCC cosmology to get about problems we run into due to the singularity theorem cosmology as represented by Theorem 6.1 above.

Appendix A: Indirect support for a massive graviton

We follow the recent work of Steven Kenneth Kauffmann, which sets an upper bound to concentrations of energy, in terms of how he formulated the following equation put in below as Eq. (A1). Equation (A1) specifies an inter-relationship between an initial radius R for an expanding universe, and a “gravitationally based energy” expression we will call $T_G(r)$ which lead to a lower bound to the radius of the universe at the start of the Universe’s initial expansion, with manipulations. The term $T_G(r)$ is defined via Eq.(A2) afterwards. We start off with Kauffmann’s

$$R\cdot\left(\frac{c^4}{G}\right)\geq\int_{|r''|\leq R} T_G(r+r'')\,d^3r'' \quad (A1)$$

Kauffmann calls $\left(\frac{c^4}{G}\right)$ a "Planck force" which is relevant due to the fact we will employ Eq. (A1) at the initial instant of the universe, in the Planckian regime of space-time. Also, we make full use of setting for small r, the following:

$$T_G\left(r+r''\right) \approx T_{G=0}\left(r\right) \cdot const \sim V(r) \sim m_{Graviton} \cdot n_{Initial-entropy} \cdot c^2 \qquad \text{(A2)}$$

I.e. what we are doing is to make the expression in the integrand proportional to information leaked by a past universe into our present universe, with Ng style quantum infinite statistics use of

$$n_{Initial-entropy} \sim S_{Graviton-count-entropy} \qquad \text{(A3)}$$

Then Eq. (A1) will lead to

$$\begin{aligned} & R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G\left(r+r''\right) d^3 r'' \approx const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \\ & \Rightarrow R \cdot \left(\frac{c^4}{G}\right) \geq const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \\ & \Rightarrow R \geq \left(\frac{c^4}{G}\right)^{-1} \cdot \left[const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right]\right] \end{aligned} \qquad \text{(A4)}$$

Here, $\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \sim 10^5$, $m_{Graviton} \sim 10^{-62}\, grams$, and

1 Planck length = l_{Planck} **= 1.616199 × 10^{-35} meters**

where we set $l_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$ with $R \sim l_{Planck} \cdot 10^{\alpha}$, and $\alpha > 0$. Typically $R \sim l_{Planck} \cdot 10^{\alpha}$ is about $10^3 \cdot l_{Planck}$ at the outset, when the universe is the most compact. The value of *const* is chosen based on common assumptions about contributions from all sources of early universe entropy, and will be more rigorously defined in a later paper.

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