Some Identities Involving four Squares II

Edigles Guedes

April 15, 2013

(306 years of the birth of Leonhard Euler)

ABSTRACT. We continue to develop some algebraic identities related to the power three as: $(a^2c^2+a^2d^2+b^2c^2+b^2d^2)^3=[ac(a^2-3b^2)(c^2-3d^2)]^2+[ad(a^2-3b^2)(3c^2-d^2)]^2+[bc(3a^2-b^2)(c^2-3d^2)]^2+[bd(3a^2-b^2)(3c^2-d^2)]^2$.

I. Identities

Lemma 1. *For $a,b\in\mathbb{R}$, then*

$$(a^2+b^2)^3=[a(a^2-3b^2)]^2+[b(3a^2-b^2)]^2.$$

Proof. In previous paper [1, p. 3], we proof that

$$(1)\qquad [2(x^2+y^2)]^3=[2(x+y)(x^2-4xy+y^2)]^2+[2(x-y)(x^2+4xy+y^2)]^2.$$

Expanding the left-hand side of (1), we have

$$(2)\ [(x+y)^2+(x-y)^2]^3=[2(x+y)(x^2-4xy+y^2)]^2+[2(x-y)(x^2+4xy+y^2)]^2.$$

If we set $x+y=a$ and $x-y=b$, then

$$(3)\qquad x=\frac{a+b}{2},\quad y=\frac{a-b}{2}.$$

Substituting (3) in (2), we obtain

$$(a^2+b^2)^3=[a(3b^2-a^2)]^2+[b(3a^2-b^2)]^2$$
$$=[a(a^2-3b^2)]^2+[b(3a^2-b^2)]^2.\ \square$$

Theorem 1. *For $a,b,c,d\in\mathbb{R}$, then*

$$(a^2c^2+a^2d^2+b^2c^2+b^2d^2)^3$$
$$=[ac(a^2-3b^2)(c^2-3d^2)]^2+[ad(a^2-3b^2)(3c^2-d^2)]^2$$
$$+[bc(3a^2-b^2)(c^2-3d^2)]^2+[bd(3a^2-b^2)(3c^2-d^2)]^2.$$

Proof. Using the Lemma 1, we put

$$(4)\qquad (a^2+b^2)^3=[a(a^2-3b^2)]^2+[b(3a^2-b^2)]^2,$$

$$(5)\qquad (c^2+d^2)^3=[c(c^2-3d^2)]^2+[d(3c^2-d^2)]^2.$$

Multiplying (4) by (5), we complete the proof. $\square$

REFERENCES

[1] Guedes, Edigles, *Some Algebraic Identities Involving four Square*, available at http://viXra.org/abs/1304.0083.