The theory of electrodynamic space-time relativity (Revision 2)

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Abstract: The theory of electrodynamic space-time relativity (TESTR) is the study of the transformation of time and space between two electrodynamic inertial frames of reference, which have both inertial velocity difference and electric potential difference. It is a fundamental space-time theory of theoretic physics based on the Einstein's special theory of relativity, the electric potential limit postulate and the high-precision experimental facts of the inversion proportional square law of Coulomb's force. It founded new physical space-time concepts, for example, the electrodynamic space-time is five-dimensional which is composed of quaternion space, and time. It also proposed some new basic concepts of physics, such as electric potential limit, quaternion electric potential and etc., revealing the inherent relationships between electric potential-velocity and time-space. This paper discusses in detail the process of establishing the theory of complex electrodynamic space-time relativity and theory of quaternion electrodynamic space-time relativity as well as their various conversions and transformations. At the same time, the basic effects of TESTR were discussed, obtaining the equations for superposition of quaternion velocity and potential of TESTR. It predicts some important new space-time change effects, which provides theoretical basis for the experimental validation of TESTR. This is the fundamental theory of the author's series.

Keyword: theory of electrodynamic space-time relativity, theory of complex electrodynamic space-time relativity, theory of quaternion electrodynamic space-time relativity, theory of electric potential relativity, special theory of relativity, electric potential limit, complex speed, complex electric potential, quaternion velocity, quaternion electric potential, quaternion, electrodynamic space-time, five-dimensional space-time, the time expansion effect of electric potential, electrodynamic space-time effects

The two pillars of modern physics are quantum mechanics and the theory of relativity which were both proven through numerous experiments. However, on the fundamental understandings, such as the "actuality" of physics, there are profound contradictions between quantum mechanics and general relativity. For the past century, many attempts to unite quantum mechanics and general relativity have been unsuccessful. At the same time, problems such as dark matters and dark energy also cannot be explained by current theories, revealing flaws in the current fundamental theory of physics.

It is well-known that the Dirac equation of quantum mechanics was built upon the relationship between energy and momentum of the special theory of relativity (STR). The general theory of relativity was also developed through advancement of the basic postulates of the STR. If there are unsolvable conflicts between quantum mechanics and general relativity, the root cause may be related to special theory of relativity. However, STR has been well proven through abundant experiments, and its correctness is sufficiently confirmed. Hence one cannot help but to doubt the completeness of special theory of relativity. In another word, our current understanding of time, space, and momentum may not be complete. To discover what the "incompleteness" of STR is, and to establish more complete space-time relativity, are of great significance for deepening our understanding on the basic physical concepts such as space-time, matter and motion. This will solve a series of basic problems in current physics and accelerate developments of physics.

Part 1. The theory of complex electrodynamic space-time relativity (TCESTR)

Einstein's special theory of relativity is based on two basic postulates [1] about inertial motion, that is:

1. The principle of relativity: physical laws should be the same in every inertial frame of reference
2. The principle of invariant speed of light: in any inertial system, the speed of light in vacuum is a constant.

According to these two postulates, the famous Lorentz transform equations can be derived.

$$X' = \gamma(X - V_X t) \tag{1}$$

$$Y' = Y \tag{2}$$

$$Z' = Z \tag{3}$$

$$t' = \gamma\left(t - \frac{V_X}{C_0^2}X\right) \tag{4}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{V_X^2}{C_0^2}}} \tag{5}$$

Here, the Lorentz transformation equations are scalar equations, where $V_X$is the speed along x-axis of inertial frame $\Sigma_R'(X', Y', Z', t')$ relative to the inertial frame $\Sigma_R(X, Y, Z, t)$. C_0 is the speed of light. X, Y, Z, t and X', Y', Z', t' are the length and time of the observing system and the observed system, respectively.

Upon analyzing the basic postulates of the special theory of relativity, a question can be asked: is our understanding of motion complete? Is there another physical reference frame besides the inertial reference frame, where the same the physical laws still hold in each frame of reference when it is under different states? In this physical reference frame, the state of such reference frame cannot be determined through any experiment. Does the related physical parameters have limit? And would such limit lead to the relativistic effect of time and space? After further study, it was found that equipotential bodies are of such physical reference systems. Based on Coulomb's inverse proportional square law and its high-precision experiments [2], we know that within a closed conductor of any shape, the electric potential at any point is the same regardless of how much electric charge its surface carries. In addition, the interior electric field strength E is zero. This is a corollary of Gauss' law of electric field.

Thus, a thought experiment can be carried out: there are two identical closed metal carriages A and B, and they are motionless relative to each other and are insulated from each other with only carriage B being grounded. Suppose there is an electrode of an ultra-high voltage static electric generator connected to B, and the other electrode is connected to the carriage A. Let the electric potential of the ground be zero. Once the generator starts running and continues to charge the carriage A with electrical charges (either positive or negative), the electric potential on the surface and in the interior of the carriage increases with it. When the surface electric charges reach Q, the interior and surface electric potential ϕ is the same everywhere. At the same time, the electric field intensity is zero. Therefore, when an observer is isolated inside carriage A, it is impossible for the observer to know the values of electric potential or whether it is positive or negative relative to carriage B through any experiment. Same can be said for the observer in the carriage B where the electric potential is zero, even when there is a very high electric potential difference between them. Please note that same experimental results can be obtained in the inertial frames of reference. Therefore, we can put forth two postulates of equipotential reference system:

3. Relative principle of electric potential: physical law has the same form in any electric equipotential frame of reference;

4. Postulate of electric potential limit: in any equipotential frame of reference, electric potential limit is a constant Φ_0 at any point in vacuum.

The value for such electric potential limit Φ_0 only can be determined by experiment. Its possible value could be the Planck voltage which is 1.04295×10^{27} volts, or of similar magnitude.

There seems to have conflicts between the postulate of electric potential limit and the current electromagnetism. However, there is not any experiment that proves whether or not electric potential has limit. In addition, electromagnetic theory is not perfect, as there are difficulties that cannot be overcome by the current theory itself, such as that the energy of a point charge is infinite. Hence, it is logical to propose such postulate that electric potential limit.

If the postulate is true, the current Maxwell's equations must be modified. When electric potential is far lower than the electric potential limit, the modified Maxwell's equations will revert back to the current Maxwell's equations (this will be discussed in another paper).

Special theory of relativity and the current Maxwell's equations are covariant. Hence the applicable conditions of the STR are the same, such that electric potential is much lower than the electric potential limit. Therefore, the space-time theory of physics must be further developed.

By comparing the relative principle of electric potential and the postulate of electric potential limit with the two postulates of the special theory of relativity, it can be seen that their forms are very similar. When the two postulates are established, it leads to the possibility of a new theory of relativity such is the electric potential relativity. It has symmetric relationship with the STR. Further reasoning will reveal that both STR and the theory of electric potential relativity are special cases of a higher level theory of relativity. This higher level relativity theory would be the study of space-time relationship between two frames of reference with both velocity difference and electric potential difference. In order to establish such "higher level theory of relativity", both the electric charge state (electric potential) and the motion state (speed) of the same frame of reference must be unified. In modern physics, however, the concepts of electric potential and speed have no direct correlation. Through in-depth study, it was discovered that in order to solve this contradiction, new physical concepts such as imaginary motion, complex motion and etc., must be introduced.

To investigate this problem, suppose at random point P in the equipotential space:

$\beta = \frac{\phi}{\Phi_0}$

Where Φ_0 is the electric potential limit, ϕ is electric potential of point P, and β is the ratio between the electric potential and the electric potential limit. Even though equipotential frame of reference is very similar with inertial frame of reference, but the equipotential frame of reference is stationary in the three-dimensional space. It cannot be defined as a regular inertial reference frame. Equipotential reference frame will be defined as the inertial frame of reference of the imaginary speed. Because equipotential space-time and inertial space-time are homogeneous and isotropic, therefore corresponding relationship exists between electric potential and imaginary speed. The electric potential limit Φ_0 is equivalent to the imaginary speed of light $C_0 i$. The potential ϕ is equivalent to the imaginary speed $V_\phi i$. From the perspective of complex number, both C_0 and V_ϕ are scalar. That is:

$\beta = \frac{V_\phi i}{C_0 i}$

Where, imaginary number $i = \sqrt{-1}$,

Hence the following formula is established:

$$\frac{\phi}{\Phi_0} = \frac{V_\phi i}{C_0 i} \qquad (6)$$

Let, $V_\phi i = K\phi$ (7)

Here, $K = \frac{C_0}{\Phi_0} i$, is an imaginary constant, and can be named as the electrodynamic conversion factor.

To further investigate more general space-time relationship, we must extend our concept of motion and space-time from the real number domain to the complex number domain. The more general motion can be abstractly understood as the motion state in the complex plane. If the motion in the coordinate reference system possesses both real and imaginary motion, then we call its motion state the complex motion state. The frame of reference situated in the complex motion state is called complex inertial electrodynamic frame of reference; it has both equipotential and inertial motion. The theory to describe this space-time relation of such motion is called the theory of complex electrodynamic space-time relativity.

As shown in Figure 1, let there be two complex coordinate systems of reference $\Sigma_C(X, F, t)$and $\Sigma_C'(X', F', t')$in the same complex plane. Their imaginary axis F and F′, real axis X and X′ are all parallel to each other. Since in complex number there is no vector, therefore all the relevant physical parameters of motion must be scalars quantities such as speed and distance. Let $\Sigma_C(X, F, t)$ be the stationary observing reference frame that is, the imaginary speed is zero (the electric potential is zero) and real speed is also zero. While $\Sigma_C'(X', F', t')$ is the observed reference frame and in complex motion state relative to $\Sigma_C(X, F, t)$, its complex speed is V_w, its imaginary speed is $V_\phi i$, and real speed is V_X, and θ is the complex angle.

$$V_w = V_X + V_\phi i = |V_w|\, e^{\theta i} \qquad (8)$$

The modulus of complex speed is: $|V_w| = \sqrt{{V_X}^2 + {V_\phi}^2}$ (9)

Where, $V_\phi = \frac{\phi C_0}{\Phi_0}$ (10)

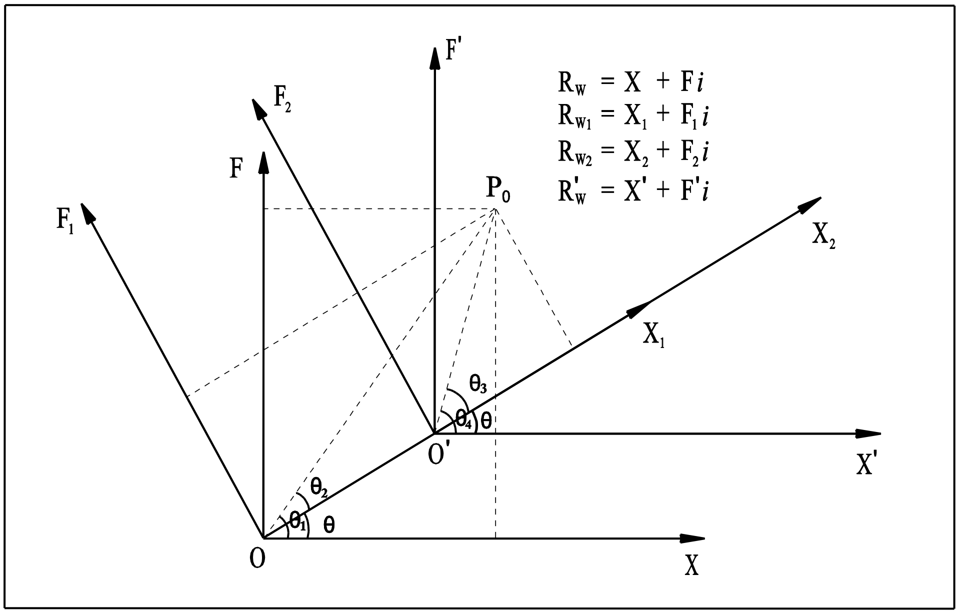

Figure 1

Since real electric potential can be converted into imaginary speed, then by symmetry principle, real speed can be converted into imaginary electric potential $\phi_X i$. Therefore, $\phi_X i = V_X \frac{1}{K} = -\frac{V_X \Phi_0}{C_0} i$, where real electric potential ϕ and imaginary electric potential $\phi_X i$ together forms complex electric potential ϕ_w:

$$\phi_w = \phi + \phi_X i \tag{11}$$

The modulus of complex electric potential is:

$$|\phi_w| = \sqrt{\phi^2 + {\phi_X}^2} \tag{12}$$

Where, $\phi_X = -\frac{V_X \Phi_0}{C_0}$ (13)

Multiplying both sides of the equation (11) with K, and comparing with equation (8), we get:

$$V_w = K\phi_w \tag{14}$$

This shows that complex speed and the complex electric potential are inter-convertible. Therefore the complex reference frame of electric potential is a type of complex electrodynamic inertial reference frame. To take one step further, the above four basic postulates will be combined into two basic postulates of TCESTR. Since complex numbers cannot be compared in magnitude, but their modulus can, hence we have:

5. The principle of the complex electrodynamic space-time relativity: physical law has the same form in any complex electrodynamic inertial frame of reference;

6. The postulate of complex electrodynamic time-space limit: in any complex electrodynamic inertial reference system, the limit of complex speed's modulus of any point in vacuum is a constant, C_0; or the limit of complex electric potential's modulus of any point in vacuum is a constant , Φ_0.

Where, C_0 is the speed of light in the vacuum with zero electric potential, and is equal to 299,792,458 m/sec. The limit of complex electric potential's modulus Φ_0 can only be determined by experiment.

One of the inference can be derived from postulate 6 is that in the frame of reference where electric potential is not zero, the speed of light $C_0{}'$ in the real three-dimensional space is less than C_0. However, the complex speed's modulus of the light in the complex space is C_0. (This is proven in subsequent paper regarding expansion of the Maxwell's equations).

When $\mathrm{V_X} = 0,\ \mathrm{V_w} = \mathrm{V_\phi} i$, the two above postulates become the postulates of the theory of electric potential relativity. When $\mathrm{V_\phi} = 0,\ \mathrm{V_w} = \mathrm{V_X}$, the two above postulates become the fundamental postulates of the special theory of relativity. Hence, special theory of relativity and the theory of electric potential relativity are two special cases of TCESTR.

The special theory of relativity is commonly referred to the motion of the observed system relative to the observing system along an axis and is called one-dimensional special theory of relativity. In fact, such motion can have two-dimensional or three-dimensional forms and the corresponding the special theory of relativity become more intricate but also more universal. There are already detailed discussions in literatures on this matter, showing that in the real space two-dimensional [3] and three-dimensional special theory of relativity [4] in arbitrary direction can be derived through rotation and translation of the coordinate system of one-dimensional special theory of relativity; or three-dimensional special theory of relativity can be derived through vector transformation of one-dimensional special theory of relativity[5], [6]. Although they are difference in their derivation methods, there is a common theme, that is, through the use of one-dimension special theory of relativity and appropriate mathematical approach, the higher real form of special theory of relativity can be derived. Therefore, based on the above mentioned postulates and one-dimension special theory of relativity, TCESTR may be derived by a way of transforming the complex coordinate system.

As shown in Figure 1, let there be a random point $\mathrm{P_0}$ be in the complex plane. In different complex coordinate systems, it can be represented by different coordinate parameters. In the complex coordinate system $\Sigma_\mathrm{C}(\mathrm{X,F,t})$, the coordinate of $\mathrm{P_0}$ is represented by complex distance $\mathrm{R_w} = \mathrm{X} + \mathrm{F}i$ with complex angle is θ_1. In the coordinate system $\Sigma_\mathrm{C}{}'(\mathrm{X',F',t'})$, the coordinate of point $\mathrm{P_0}$ is represented as complex distance $\mathrm{R_w}{}' = \mathrm{X}' + \mathrm{F}'i$ with complex angle θ_4. Refer to method of two dimensional coordinate transformation of real numbers [3], and expanding it into the complex planar space, reference system $\Sigma_\mathrm{C}{}'(\mathrm{X',F',t'})$ can be obtained through three times coordinate transformations from reference system $\Sigma_\mathrm{C}(\mathrm{X,F,t})$.

(1) In the complex coordinate system $\Sigma_\mathrm{C}(\mathrm{X,F,t})$, its time is t, rotating the coordinate system by θ degree counterclockwise, we get complex coordinate system $\Sigma_\mathrm{C1}(\mathrm{X_1,F_1,t_1})$, , whose time is $\mathrm{t_1}$;

(2) Complex coordinate system $\Sigma_\mathrm{C1}(\mathrm{X_1,F_1,t_1})$ is translated along axis $\mathrm{X_1}$ of real number in the quantity equal to the modulus of the complex speed $|\mathrm{V_w}|$, we get complex coordinate system $\Sigma_\mathrm{C2}(\mathrm{X_2,F_2,t_2})$., whose time is $\mathrm{t_2}$;

(3) Complex coordinate system $\Sigma_\mathrm{C2}(\mathrm{X_2,F_2,t_2})$. is rotated by θ degree clockwise, we get complex coordinate system $\Sigma_\mathrm{C}{}'(\mathrm{X',F',t'})$, whose time is $\mathrm{t'}$.

The detailed derivation steps, numbered corresponding to the above, are as follow:

(1) Rotate the coordinate system $\Sigma_C(X, F, t)$ by θ degree counter clockwise, making the real axis X_1 of the coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$ pass through the origin O' of the reference system $\Sigma_C{}'(X', F', t')$. Point P_0 in $\Sigma_C(X, F, t)$ complex coordinate system has complex distance R_w, whose complex angle is θ_1. Point P_0 in the coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$ has complex distance R_{w_1}, whose complex angle is θ_2. That is,

$$R_w = X + Fi = |R_w|e^{\theta_1 i} \tag{15}$$

$$R_{w_1} = X_1 + F_1 i = |R_{w_1}|e^{\theta_2 i} \tag{16}$$

Also, $|R_w| = |R_{w_1}|$, therefore,

$$R_{w_1} = R_w\, e^{(\theta_2-\theta_1)i}$$

$$R_{w_1} = R_w\, e^{-\theta i} \tag{17}$$

$$R_{w_1} = (X + Fi)(\cos\theta - i\sin\theta) \tag{18}$$

From equations (18) and (16),

$$X_1 = X\cos\theta + F\sin\theta \tag{19}$$

$$F_1 = F\cos\theta - X\sin\theta \tag{20}$$

(2) Let $\Sigma_{C1}(X_1, F_1, t_1)$ be the stationary frame of reference whose time is t_1. $\Sigma_{C2}(X_2, F_2, t_2)$ is the moving reference system whose time is t_2, and real axes X_1 and X_2 are overlapping. The reference system $\Sigma_{C2}(X_2, F_2, t_2)$ moves along the positive X_1 direction relative to $\Sigma_{C1}(X_1, F_1, t_1)$ with the magnitude of $|V_w|$, hence $F_2 = F_1$. Since X_1 is the real number axis of $\Sigma_{C1}(X_1, F_1, t_1)$, it can be thought as the real motion. This is the same state that the special theory of relativity studies. Hence the Lorentz transform equations (1), (2), (4) and (5) can be directly used, but the parameters in the equations need to be replaced by the parameters of $\Sigma_{C1}(X_1, F_1, t_1)$ and $\Sigma_{C2}(X_2, F_2, t_2)$.

Because the time is scalar quantity, and isotropic in the complex space, that is time is not related to the rotary mathematic transformation of the coordinates, therefore $t = t_1$, $t_2 = t'$. We get,

$$X_2 = \gamma(X_1 - |V_w|t) \tag{21}$$

$$F_2 = F_1 \tag{22}$$

$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2}X_1\right) \tag{23}$$

Where, $$\gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}} \tag{24}$$

(3) In complex coordinate reference system $\Sigma_{C2}(X_2, F_2, t_2)$., P_0 is represented by complex distance R_{w_2} with complex angle θ_3, rotating clockwise θ degree, we get coordinate system $\Sigma_C{}'(X', F', t')$. In $\Sigma_C{}'(X', F', t')$, P_0 is represented by complex distance $R_w{}'$ with complex angle θ_4:

$$R_{w_2} = X_2 + F_2 i = |R_{w_2}|e^{\theta_3 i}$$

$$R_w{}' = X' + F' i = |R_w{}'|e^{\theta_4 i}$$

And also, $|R_w{}'| = |R_{w_2}|$, hence,

$$\mathrm{R_w}' = \mathrm{R_{w_2}}\, e^{(\theta_4-\theta_3)i} = \mathrm{R_{w_2}}\, e^{\theta i} \tag{25}$$

Because $\mathrm{R_{w_2}} - \mathrm{R_{w_1}} = \mathrm{X_2} - \mathrm{X_1}$, therefore,

$$\mathrm{R_w}' = (\mathrm{R_{w_1}} - (\mathrm{X_1} - \mathrm{X_2}))\ e^{\theta i} \tag{26}$$

Substituting equation (17) into equation(26),

$$\mathrm{R_w}' = \mathrm{R_w} - (\mathrm{X_1} - \mathrm{X_2})\, e^{\theta i}$$

Because $\mathrm{V_w} = |\mathrm{V_w}| e^{\theta i}$, therefore,

$$\mathrm{R_w}' = \mathrm{R_w} - \mathrm{V_w}\frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|} \tag{27}$$

Let $\mathrm{t_w} = \frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|}$ (28)

$\mathrm{t_w}$ is a real number, and can be called the system time of complex electrodynamic inertial reference frames. It is not a special time of any particular reference frame, but related to the time of both the observing system and the observed system. It is the "time" needed to solve some fundamental physic problems.

To sum up the above discussion, we obtain the basic equations of TCESTR as the following:

$$\mathrm{R_w}' = \mathrm{R_w} - \mathrm{V_w}\mathrm{t_w} \tag{29}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{|\mathrm{V_w}|}{C_0^{\,2}}\mathrm{X_1}\right) \tag{30}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\mathrm{V_w}|^2}{C_0^{\,2}}}}$ (31)

$$\mathrm{t_w} = \frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|} \tag{32}$$

$$\mathrm{X_2} = \gamma(\mathrm{X_1} - |\mathrm{V_w}|\mathrm{t}) \tag{33}$$

$$\mathrm{X_1} = \mathrm{X}\cos\theta + \mathrm{F}\sin\theta \tag{34}$$

Depending on different applications, the above relationship can be further expanded into different forms.

According to equation (8), $\cos\theta = \frac{\mathrm{V_X}}{|\mathrm{V_w}|}$, $\sin\theta = \frac{\mathrm{V_\phi}}{|\mathrm{V_w}|}$. Substituting them into equation (34), we get:

$$\mathrm{X_1} = \frac{1}{|\mathrm{V_w}|}(\mathrm{X}\mathrm{V_X} + \mathrm{F}\mathrm{V_\phi}) \tag{35}$$

By substituting equation (32), (33) and (35) into equation (29), we can obtain expanded equations describing TCESTR with complex speed,

$$\mathrm{R_w}' = \mathrm{R_w} - \frac{\mathrm{V_w}}{|\mathrm{V_w}|}\left((1-\gamma)\mathrm{X_1} + |\mathrm{V_w}|\gamma\mathrm{t}\right) \tag{36}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{|\mathrm{V_w}|}{C_0^{\,2}}\mathrm{X_1}\right) \tag{37}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\mathrm{V_w}|^2}{C_0^{\,2}}}}$

Speed and electric potential has a corresponding relationship. Therefore, we can also obtain another extended equations describing TCESTR with complex electric potential. This will be discussed later in the quaternion electrodynamic space-time relativity.

By decomposing (36) into equations of real number and imaginary number, we obtain:

$$X' = X - \frac{V_X}{|V_w|}\left((1-\gamma)X_1 + |V_w|\gamma t\right) \quad (38)$$

$$F'i = Fi - i\frac{V_\phi}{|V_w|}\left((1-\gamma)X_1 + |V_w|\gamma t\right) \quad (39)$$

Therefore when $V_\phi = 0$, then $|V_w| = V_X$, according to (35) can get $X_1 = X$, Substituting the equations into (38)(39)(30) and (31), the special theory of relativity can be obtained:

$$X' = \gamma(X - V_X t)$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right)$$

$$F' = F$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}}$$

let $V_X = 0$, then $|V_w| = V_\phi$, according to (35) can get $X_1 = F$, Substituting the equations into (38)(39)(30) and (31), a new special theory of relativity can be obtained:

$$F'i = \gamma\left(Fi - \frac{C_0}{\Phi_0}\phi i t\right) \quad (40)$$

$$X' = X \quad (41)$$

$$t' = \gamma\left(t - \frac{\phi}{\Phi_0 C_0}F\right) \quad (42)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{\phi^2}{{\Phi_0}^2}}} \quad (43)$$

This is the aforementioned the theory of electric potential relativity (TEPR), whose form can be seen here as symmetric to that of the special theory of relativity, but they have different physical meaning (will be discussed in another paper).

Part 2. The theory of quaternion electrodynamic space-time relativity (TQESTR)

TCESTR describes relationship between the space-time and the state of complex motion of the reference frames in the complex plane, but the real axis in the complex plane is one-dimensional. In reality, our real number space is three-dimensional. Therefore two-dimensional space of TCESTR must be expanded into four dimensions. In mathematics, the higher form of complex number is quaternion. Thus TCESTR can be developed into TQESTR. Although the elements of quaternion can have many variations, they can all be converted uniformly into quaternion velocity or quaternion electric potential. Their corresponding frame of reference can be called equi-quaternion velocity or equi-quaternion electric potential frames of reference, or in general be called the quaternion electric inertial frame of reference.

Hence, the two basic postulates 5 and 6 of TCESTR can be further expanded into the basic postulates of TQESTR:

7. Relative principle of quaternion electrodynamic space-time: in any quaternion electrodynamic inertial reference system, physical laws have the same form;

8. Postulate of quaternion electrodynamic space-time limit: in any quaternion electrodynamic inertial reference system, the limit of quaternion velocity's modulus of any point in vacuum is a constant, C_0; or the limit of quaternion electric potential's modulus of any point is a constant , Φ_0.

Where C_0 is the speed of light in the vacuum with zero electric potential, and is equal to $299{,}792{,}458\ \mathrm{m/ses}$. The limit of quaternion electric potential's modulus Φ_0 only can be determined by experiment.

According to the basic equation (29) of TCESTR, it can be expanded and separated into real number equation and imaginary number equation:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_\phi i \mathrm{t_w} \tag{44}$$

$$\mathrm{X}' = \mathrm{X} - \mathrm{V_X t_w} \tag{45}$$

Notice that equation (45) is a real scalar expression. However, the fact is that the observed reference frame $\Sigma_\mathrm{C}'(\mathrm{X}', \mathrm{F}', \mathrm{t}')$ moves along the real axis X of the observing reference frame $\Sigma_\mathrm{C}(\mathrm{X}, \mathrm{F}, \mathrm{t})$ with vector velocity $\mathbf{V_x}$. Let its unit vector be **i** , because axes X' and X are the same direction as **i**. Multiply both sides of equation (45) by **i**, equation (45) becomes a vector equation:

$$\mathbf{X}' = \mathbf{X} - \mathbf{V_x}\mathrm{t_w} \tag{46}$$

Equations (44) and (46) describe the physical nature more objectively than the complex equation (29). So the physical quantities about motion will also use vector, such as velocity, displacement and etc. Equation (46) can be expanded into three-dimensional. If the vector $\mathbf{X}'$ and $\mathbf{X}$ in equation (46) are defined as vector $\mathbf{r}'$ and $\mathbf{r}$ in the three-dimensional space $\mathrm{X}', \mathrm{Y}', \mathrm{Z}'$ and $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, corresponding velocity $\mathbf{V_x}$ is defined as $\mathbf{V_r}$**,** and have the same direction as $\mathbf{r}'$**,** $\mathbf{r}$ and $\mathbf{V_r}$. Let $\mathbf{i}, \mathbf{j}, \mathbf{k}$ be the unit vectors in the coordinate of three-dimensional space along the axis $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, respectively, we have:

$$\mathbf{r} = \mathrm{X}\mathbf{i} + \mathrm{Y}\mathbf{j} + \mathrm{Z}\mathbf{k} \tag{47}$$

$$\mathbf{r}' = \mathrm{X}'\mathbf{i} + \mathrm{Y}'\mathbf{j} + \mathrm{Z}'\mathbf{k} \tag{48}$$

$$\mathbf{V_r} = \mathrm{V_X}\mathbf{i} + \mathrm{V_Y}\mathbf{j} + \mathrm{V_Z}\mathbf{k} \tag{49}$$

$\mathbf{X}', \mathbf{X}, \mathbf{V_x}$ becomes $\mathbf{r}', \mathbf{r}, \mathbf{V_r}$**.** At the same time, let $\mathrm{t_w}$ become $\mathrm{t_q}$ accordingly. Therefore, equation (44) and (46) can be transformed into:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_\phi i \mathrm{t_q} \tag{50}$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{V_r}\mathrm{t_q} \tag{51}$$

Equation (50) and (51) form a quaternion space system composed of three real number vectors and one imaginary number, and is called type Y_i quaternion space. This expands the state of motion of the reference system in the complex plane into motion of the type Y_i quaternion space, that is, the reference system $\Sigma_\mathrm{Q}'(\mathrm{X}', \mathrm{Y}', \mathrm{Z}', \mathrm{F}', \mathrm{t}')$ moves with quaternion velocity $\mathrm{V_q}$ relative to the reference frame $\Sigma_\mathrm{Q}(\mathrm{F}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{t})$. Suppose R'_q and $\mathrm{R_q}$ are quaternion displacement coordinates at any point $\mathrm{P_0}$ in the inertial reference system $\Sigma_\mathrm{Q}'(\mathrm{X}', \mathrm{Y}', \mathrm{Z}', \mathrm{F}', \mathrm{t}')$ and $\Sigma_\mathrm{Q}(\mathrm{F}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{t})$ in the Y_i type quaternion space, respectively, then:

$R_q = \mathbf{r} + Fi = X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k} + Fi$ (52)

The modulus of R_q is $|R_q|$, that is $|R_q| = \sqrt{X^2 + Y^2 + Z^2 + F^2}$ (53)

$R'_q = \mathbf{r}' + F'i = X'\mathbf{i} + Y'\mathbf{j} + Z'\mathbf{k} + F'i$ (54)

The modulus of R'_q is $|R'_q|$, i.e., $|R'_q| = \sqrt{X'^2 + Y'^2 + Z'^2 + F'^2}$ (55)

Type Y_i quaternion velocity V_q and its modulus $|V_q|$ are:

$V_q = \mathbf{V_r} + V_\phi i = V_X\mathbf{i} + V_Y\mathbf{j} + V_Z\mathbf{k} + V_\phi i$ (56)

$|V_q| = \sqrt{{V_r}^2 + {V_\phi}^2} = \sqrt{{V_X}^2 + {V_Y}^2 + {V_Z}^2 + {V_\phi}^2}$ (57)

Where $i = \sqrt{-1}$

Adding equation (50) with (51), and substituting (52), (54) and (56) into it to obtain equation (58). Also, replacing X', X_1, X_2, X and $|V_w|$ of equations (30), (31), (32), (33) and (34)with r', r_1, r_2, r and $|V_q|$, the quaternion basic equations can be obtained for TQESTR：

$R_q{}' = R_q - V_q t_q$ (58)

$t' = \gamma\left(t - \frac{|V_q|}{{C_0}^2} r_1\right)$ (59)

where， $\gamma = \frac{1}{\sqrt{1 - \frac{|V_q|^2}{{C_0}^2}}}$ (60)

$r_1 = r\cos\theta + F\sin\theta$ (61)

$r_2 = \gamma\left(r_1 - |V_q|t\right)$ (62)

$t_q = \frac{(r_1 - r_2)}{|V_q|}$ (63)

Depending on the need, the above basic equation can be transformed into different variations.

Because， $\cos\theta = \frac{V_r}{|V_q|}$， $\sin\theta = \frac{V_\phi}{|V_q|}$ ， $r = X\frac{V_X}{V_r} + Y\frac{V_Y}{V_r} + Z\frac{V_Z}{V_r}$， then，

$r_1 = \frac{1}{|V_q|}(XV_X + YV_Y + ZV_Z + FV_\phi)$ (64)

The r_1 can also be expressed with quaternions or four-dimensional vector’s dot product.

Hence the quaternion velocity equations can be obtained for TQESTR,

$R_q{}' = R_q - \frac{V_q}{|V_q|}\left((1-\gamma)r_1 + |V_q|\gamma t\right)$ (65)

$t' = \gamma\left(t - \frac{|V_q|}{{C_0}^2} r_1\right)$ (66)

Where, $\gamma = \frac{1}{\sqrt{1 - \frac{|V_q|^2}{{C_0}^2}}}$ (67)

When $V_\phi = 0$, $R_q = \mathbf{r}$， $R_q{}' = \mathbf{r}'$, $V_q = \mathbf{V_r}$ can be obtained based on equations (52), (54) and (56), according to equation (64), then ,

$r_1 = \frac{\mathbf{r}\cdot\mathbf{V_r}}{|\mathbf{V_r}|}$ (68)

Therefor the special theory of relativity in any direction $\mathbf{r}(\mathbf{x},\mathbf{y},\mathbf{z})$ in three-dimensional space can be obtained [5], [6]:

$$\mathbf{r}' = \mathbf{r} - \frac{\mathbf{V_r}}{|\mathbf{V_r}|}\left((1-\gamma)\frac{\mathbf{r}\cdot\mathbf{V_r}}{|\mathbf{V_r}|} + |\mathbf{V_r}|\gamma t\right)$$

$$t' = \gamma\left(t - \frac{\mathbf{r}\cdot\mathbf{V_r}}{C_0{}^2}\right)$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\mathbf{V_r}|^2}{C_0{}^2}}}$

Because imaginary speed V_ϕ and real electric potential ϕ are inter-convertible, and real velocity can be converted into imaginary electric potential vector, hence the quaternion velocity equations can be converted into another type of quaternion electric potential equations. The quaternion is composed of one real and three imaginary vectors. It is called H-type quaternion. Through further analysis, both Y_i type quaternion and H-type quaternion are a type of dual quaternion. Rules of operation between the two dual quaternions are identical to ordinary quaternion [7].

Let H-type quaternion electric potential be ϕ_h, then:

$$\phi_h = \frac{V_q}{K} = (V_X\mathbf{i} + V_Y\mathbf{j} + V_Z\mathbf{k} + V_\phi i)\frac{\Phi_0}{C_0}(-i) \tag{69}$$

Let $\phi = \frac{V_\phi\Phi_0}{C_0}$, $\phi_X = \frac{V_X\Phi_0}{C_0}$, $\phi_Y = \frac{V_Y\Phi_0}{C_0}$, $\phi_Z = \frac{V_Z\Phi_0}{C_0}$, then:

$$\phi_h = \phi + (\phi_X\mathbf{i} + \phi_Y\mathbf{j} + \phi_Z\mathbf{k})(-i) \tag{70}$$

This shows that H-type quaternion potential ϕ_h is composed of one scalar electric potential and three imaginary components of the vector electric potential. If the H-type quaternion potential ϕ_h is divided by speed of light C_0, a new kind of electromagnetic four-dimensional potential $\mathbf{A}_\beta$. Its physical significance will be further explored. It is expressed as:

$$\mathbf{A}_\beta = \frac{\phi_h}{C_0} = \left(\frac{\phi}{C_0}, \; -i\mathbf{A}\right) \tag{71}$$

Where, ϕ is the electric potential, $\mathbf{A}$ is the magnetic vector potential.

According to equation (69) from the definition of ϕ_h, we get:

$$\frac{V_q}{|V_q|}(-i) = \frac{\phi_h}{|\phi_h|} \tag{72}$$

Because $|V_q| = \frac{C_0}{\Phi_0}|\phi_h|$ (73)

Substituting equations (72)(73) into (65)(66)(67), the quaternion electric potential equations can be obtained for TQESTR:

$$R_q{}' = R_q - i\frac{\phi_h}{|\phi_h|}\left((1-\gamma)r_1 + \frac{C_0}{\Phi_0}|\phi_h|\gamma t\right) \tag{74}$$

$$t' = \gamma\left(t - \frac{|\phi_h|}{\Phi_0 C_0}r_1\right) \tag{75}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\phi_h|^2}{\Phi_0{}^2}}}$, (76)

Part 3. Discussion on the basic effects of the theory of electrodynamic space-time relativity

Although the theory of electrodynamic space-time relativity (TESTR) have various expression forms, but the forms can all be obtained through conversion and simplification of basic the equations (58), (59), (60), (61), (62) and (63). Therefore, they will be the main focus for the discussions of the basic effects of TESTR. The effects are called electrodynamic space-time effect.

1. Superposition principle of TESTR

By analyzing the above equations, one can see that the equations (62) and (59) have the same form as that of Lorentz transform equations (1) and (4). Therefore, the equations(62)(59) and (60) can also be written in the form of hyperbolic functions [5],

$$r_2 = r_1\cosh(\varphi) - C_0 t\sinh(\varphi) \quad (77)$$

$$C_0 t' = -r_1\sinh(\varphi) + C_0 t\cosh(\varphi) \quad (78)$$

Where, $\cosh(\varphi) = \frac{1}{\sqrt{1-(\frac{|V_q|}{C_0})^2}}$ (79)

Because, $\cosh^2(\varphi) - \sinh^2(\varphi) = 1$ (80)

From(79) and(80) it can be obtained: $\tanh(\varphi) = \frac{|V_q|}{C_0}$ (81)

Assume: $\varphi = \varphi_1 + \varphi_2$, $\frac{|V_{q1}|}{C_0} = \tanh(\varphi_1)$ and $\frac{|V_{q2}|}{C_0} = \tanh(\varphi_2)$

$$\frac{|V_q|}{C_0} = \tanh(\varphi_1 + \varphi_2) \quad (82)$$

According to equation(82), the formula for superposition of velocity of TESTR can be obtained:

$$|V_q| = \frac{|V_{q2}|+|V_{q1}|}{1+\frac{|V_{q1}||V_{q2}|}{{C_0}^2}} \quad (83)$$

Where, $|V_{q1}| = \sqrt{{V_{X1}}^2 + {V_{Y1}}^2 + {V_{Z1}}^2 + {V_{\phi 1}}^2}$ (84)

$$|V_{q2}| = \sqrt{{V_{X2}}^2 + {V_{Y2}}^2 + {V_{Z2}}^2 + {V_{\phi 2}}^2} \quad (85)$$

When the direction of motion of the reference frame is along the X axis, that is $|V_q| = V_X$, $|V_{q1}| = V_{X1}$, $|V_{q2}| = V_{X2}$, all other terms are zero. By substituting the above-mentioned formula of superposition, the formula for velocity superposition of special theory of relativity can be obtained:

$$V_X = \frac{V_{X2}+V_{X1}}{1+\frac{V_{X1}V_{X2}}{{C_0}^2}} \quad (86)$$

According to the equation (73), the quaternion velocity equations can be converted into quaternion electric potential equations. Substituting equation (73) into (83), The formula for superposition of quaternion electric potential of TESTR can be obtained:

$$|\phi_h| = \frac{|\phi_{h2}|+|\phi_{h1}|}{1+\frac{|\phi_{h1}||\phi_{h2}|}{{\Phi_0}^2}} \quad (87)$$

When the only the scalar electric potential is considered, $|\phi_h| = \phi$, $|\phi_{h1}| = \phi_1$, $|\phi_{h2}| = \phi_2$, other terms are all zero. By substituting the above-mentioned formula, the formula for superposition of scalar electric potential can be obtained：

$$\phi = \frac{\phi_2+\phi_1}{1+\frac{\phi_1\phi_2}{\Phi_0{}^2}} \tag{88}$$

Thus a conclusion that differs from the modern physics is deduced: the superposition of electric potential is nonlinear. However, if the electric potential is far lower than the electric potential limit， the equation (88) will change back to linear equation of the current electromagnetics, that is $\phi = \phi_2 + \phi_1$.

2. The time relationship of TESTR

(1) Time effects

In any electrodynamic inertia frame of reference, time is isotropic. The time measured by the clock put in the real space of the inertial frame is the time of the electrodynamic inertial frame. If two events happen at two different moments t_1 and t_2 at the same location r_1 of the electrodynamic stationary inertial reference frame, their time difference is $\Delta t = t_2 - t_1$; for the moments $t_1{}'$and $t_2{}'$corresponding to the electrodynamic relative inertial reference frame, the time difference is $\Delta t' = t_2{}' - t_1{}'$. From equations(59), (60) and (57), the equation of quaternion velocity-time change effect of TESTR is:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{V_X{}^2+V_Y{}^2+V_Z{}^2+V_\phi{}^2}{C_0{}^2}}}\Delta t \tag{89}$$

It shows that the expansion effect of the time is not just about speed, but with potential.

The expression for the quaternion electric potential-time change effect of TESTR is:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{\phi_X{}^2+\phi_Y{}^2+\phi_Z{}^2+\phi^2}{\Phi_0{}^2}}}\Delta t \tag{90}$$

In the reference frame of complex electrodynamic inertia, i.e., there are only electric potential ϕ and one dimensional speed V_X, from $V_\phi = \frac{C_0}{\Phi_0}\phi$, it can be obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{V_X{}^2}{C_0{}^2}-\frac{\phi^2}{\Phi_0{}^2}}}\Delta t \tag{91}$$

In(91), when $\phi = 0$, the formula for time expansion of special relativity is obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{V_X{}^2}{C_0{}^2}}}\Delta t \tag{92}$$

In(91)， $V_X = 0$, the formula for the time expansion effect of electric potential relativity theory is obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{\phi^2}{\Phi_0{}^2}}}\Delta t \tag{93}$$

This is a new important physical effect, called it the potential time expansion effect.

(2) The system time

The system time t_q is a new concept in TESTR. It unified the observe reference time t and the observed reference time t' together. And let basic equations of TESTR become very concise form.

From equations (63), (62) and(59), it can be obtained:

$$t_q = \left((1-\gamma)\frac{C_0{}^2}{|V_q|^2} + \gamma\right)t - \left(\frac{1}{\gamma} - 1\right)\frac{C_0{}^2}{|V_q|^2}t' \tag{94}$$

3. Space change effect of TESTR

In the stationary electrodynamic reference frame Σ_Q, event 1 (R_{q1}, t_1) and event 2 (R_{q2}, t_2) occur, the corresponding events in the moving electrodynamic frames of reference $\Sigma_Q{}'$are $(R_{q1}{}', t_1{}')$ and $(R_{q2}{}', t_2{}')$. V_q is the relative quaternion velocity of two reference frames. From equation (94), $t_{q1} = f(t_1, t_1{}')$ and $t_{q1} = f(t_2, t_2{}')$ can be obtained, therefore from equation (58) it can be known that:

$R_{q1}{}' = R_{q1} - V_q t_{q1}$

$R_{q2}{}' = R_{q2} - V_q t_{q2}$

So there is, $\Delta R_q{}' = \Delta R_q - V_q \Delta t_q$ (95)

$$\Delta t_q = \left((1-\gamma)\frac{C_0{}^2}{|V_q|^2} + \gamma\right)\Delta t - \left(\frac{1}{\gamma} - 1\right)\frac{C_0{}^2}{|V_q|^2}\Delta t' \tag{96}$$

From equation (59), $\Delta t' = \gamma\left(\Delta t - \frac{|V_q|}{C_0{}^2}\Delta r_1\right)$ (97)

Let there be an abstract "ruler" in the quaternion space, whose length is obtained through the measurement of its two end points. In the stationary electrodynamic frame of reference, the ruler is measured to be ΔR_q, and this measuring process is completed in time Δt. However, in the moving electrodynamic frame of reference, where the quaternion velocity is V_q, the measuring of its two end points must simultaneous. Therefore, $\Delta t' = 0$, and its length is measured to be $\Delta R_q{}'$. Substituting $\Delta t' = 0$ into equations (96) (97), obtains:

$$\Delta t_q = \frac{\left(1-\frac{1}{\gamma}\right)\Delta r_1}{|V_q|} \tag{98}$$

By substituting(98) into(95), the equation of space change effect of TESTR can be obtained:

$$\Delta R_q{}' = \Delta R_q - \frac{V_q}{|V_q|}\left(1 - \frac{1}{\gamma}\right)\Delta r_1 \tag{99}$$

Where $\Delta r_1 = \Delta r\cos\theta + \Delta F\sin\theta$ (100)

Equation (99) is expanded into the equations of imaginary and vector equations of real number:

$$\Delta F' i = \Delta F i - i\frac{V_\phi}{|V_q|}\left(1 - \frac{1}{\gamma}\right)\Delta r_1 \tag{101}$$

$$\Delta \mathbf{r}' = \Delta \mathbf{r} - \frac{\mathbf{V_r}}{|V_q|}\left(1 - \frac{1}{\gamma}\right)\Delta r_1 \tag{102}$$

When $V_\phi \neq 0$, $V_r = 0$, $\theta = \frac{\pi}{2}$, $\Delta r_1 = \Delta F$, there is:

$$\Delta \mathbf{r}' = \Delta \mathbf{r} \tag{103}$$

That is, in any space of relative stationary static electric potential, the real space length is not related to the scalar electric potential. It can be called the real space conservation effect of electric potential.

When $V_\phi = 0$, $\mathbf{V_r} \neq 0$, $|V_q| = V_r$, $\theta = 0$, $\Delta r_1 = \Delta r$

$$\Delta r' = \frac{1}{\gamma}\Delta r \tag{104}$$

This is the formula for length contraction of the special theory of relativity.

4. Predictions and verifications of the theory

The above derivations indicate that in the theory of electrodynamic space-time relativity, time and space are not only related to velocity but also related to electric potential. Electric potential and velocity may be interconverted from each other. In principle, different experiments can be designed to verify the theory. Amongst them, the effect of electric potential time expansion can be more easily tested.

Equation (93) shows that under completely stationary conditions, when there is sufficiently high electric potential difference between two reference frames, their time difference will also be found. If the potential is higher, this effect will be more obvious. Therefore, an experiment may be designed: after calibration, two clocks of high precision T_c and T_d are put separately in two identical metallic closed rooms C and D, which are isolated and motionless relative to each other. Room C is grounded and let its electric potential be zero. An electric potential generator of super high voltage is used to charge room D and maintain the super high electric potential ϕ comparing to room C. Equation (93) indicates that given long enough time, after electricity of room D is totally discharged, the electric potential in room D becomes zero again. Then two clocks T_c and T_d are put together to compare their respective reading of the elapsed time, Δt and $\Delta t'$. It can be found that there is a time difference between time Δt and $\Delta t'$ recorded by T_c and T_d that is induced by electric potential ϕ, and $\Delta t < \Delta t'$. Comparing the measured values and the values calculated by equation (93) will verify the theory. If the experiment proves that the theoretic calculation is correct, then from the experiment data, the magnitude of the electric potential limit Φ_0 can be obtained by the equation:

$$\Phi_0 = \frac{\phi}{\sqrt{1 - \frac{\Delta t^2}{\Delta t'^2}}} \tag{105}$$

It would be extremely difficult to achieve very high electric potential ϕ in laboratory. However, the advantage of this experimental setup is that through adding the experimental time and improving the accuracy of the time measurement, it lowers the required the electric potential and increases the possibility of success in the experiment. Base on the theory of electrodyanmic space-time relativity, many new physical effects can be derived, which in turn, can prove the validity of the theory through experiments based on the said effects.

This shows the theory of electrodynamic space-time relativity (TESTR) is the generalized name for the family of relativity theories. Special theory of relativity is a special case of this family. The family of relativity theories includes the theory of electric potential relativity, the theory of complex relativity, the theory of quaternion relativity and other combinations and transformations of relativity theories. These theories of relativity have close relationships, each describing a corresponding space-time.

Space-time is the foundation and core of physics. The theory establishes a new concept, that our space-time is five dimensional which is composed of quaternion space and time or electrodynamic space-time. TESTR has a variety of forms of expression. The theory of electricdynamic space-time relativity unites inertial motion, electromagnetic motion, time and space. Because TESTR predicted some new electrodynamic space-time effects, some basic theories of physics (such as the Maxwell's equations and etc.) will be modified accordingly. They will derive number of new physical effects. These effects will be rigorously tested through scientific experiments. If future experiments show the theory is correct, its impact on the physics will be comprehensive and profound. Subsequent papers will further explore applications and development of the theory.

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