

Flux Divergence Method of solving Einstein equations

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Trying to solve Einstein Equation with new integral of motion (conserved quantity) for system. The integrals of motion are important. Let us say, that discovery of new one means the Nobel Prize. Using them the "unsolvable" problems solve. Several dust collapse solutions satisfy the new formulas. I solved also collapse of perfect liquid ball with unexpected result. After the perfect liquid I have got result(s) for the real liquid. Critically discussed known conserved quantities. Revealed the nature of Dark Energy. Cancelled the cyclic Universe hypothesis (World will never be shrinking).

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I. INTRODUCTION

There is Newman-Janis complex transformation method [1], which not allways working. And explained is only for Schwarzschild starting spacetime [2] (and not quite clearly, for my opinion). From Schwarzschild metric the method gives Kerr metric [3], from Reissner-Nordström metric gives the Kerr-Newman metric [4]. The development of method: [5].

So the precedent is there: the theory, what not allways working, not quite explained, but still is accepted in a top journal. Therefore I propose to publish following new idea. I use spacetimes, where the norm of a timelike vector is negative. I use comma as index to show partial derivative. I use semicolon as index to show covariant derivative (the one with Christoffel symbols).

II. THE IDEA

By coordinate transformation one can allways turn the spacetime so, that g_{00} is independent from x^0 . Secondly, let the spacetime satisfies the energy conditions (i.e. be physically reasonable) throughout the space and time, hereby the Einstein equations let include the Dark Energy as part of Geometry law (i.e. sit on left hand side); namely $G^\mu_\nu + \kappa\Lambda\delta^\mu_\nu = \kappa T^\mu_\nu$, the $\kappa = 8\pi G/c^4$. Is sufficiently, but not necessary, that $T_0^1 = T_0^2 = T_0^3 \equiv 0$. Then certainly the following quantity

$$NP = - \int T_0^0 \sqrt{-g} dV, \quad (1)$$

where $dV = dx^1 dx^2 dx^3$ (in spherical coordinates $dV = \frac{1}{c} dr d\theta d\phi$), is independent from x^0 . The integral can have finite or infinite ranges. It is the System Integral of Motion. The *integrals of motion* are important. Let us say, that discovery of new one means the Nobel Prize (*NP*). Using them the "unsolvable" problems solve.

Even if several above conditions are violated, nevertheless the *NP* may occur as constant. You shall assume, that it leads to the natural solution, till the opposite is proved.

III. EXAMPLES

Marshall's corrected dust ball collapse [6] spherical-symmetric metric

$$ds^2 = -d\tau^2 + \left(\frac{\partial W(\tau, R)}{\partial R} \right)^2 dR^2 + W^2(\tau, R)(d\theta^2 + \sin^2 \theta d\phi^2), \quad (2)$$

where

$$W(\tau, R) := R \left(1 - \frac{3}{2} \tau \exp\left(\frac{3}{2}(1-R)\right) \right)^{2/3}, \quad (3)$$

the centre has $R = 0$, the surface has $R = 1$, the gravitational mass is $M = 1/2$. After transformation $\tau = f(u)$ the spacetime violates $NP = const$, if is not $f(u) = C_1 u + C_2$. In last case the $NP = const$ holds. Moreover, if $C_1 = 1$ the $NP = M = 1/2$.

I propose to study plane-symmetric dust collapse

$$ds^2 = -dt^2 + A^2(t, z) dz^2 + W^2(t, z)(dx^2 + dy^2) \quad (4)$$

and the axis-symmetric dust collapse

$$ds^2 = -dt^2 + \left(\frac{\partial W(t, r)}{\partial r} \right)^2 dr^2 + W^2(t, r) d\phi^2 + B^2(t, r) dh^2. \quad (5)$$

One can try to find exact solutions, but I left the problem. All these two forms have matter tensor $T^\mu_\nu = \text{diag}(-\rho, 0, 0, 0)$, where the ρ is function of two coordinates. From matter equation

$$T^\mu_{\nu;\mu} = 0 \quad (6)$$

follows, that

$$\rho = \rho_0 \sqrt{\frac{g_0}{g}}, \quad (7)$$

where index 0 means values at initial moment $t = t_0$. Thus, the $\rho \sqrt{g}$ is truly time-independent: $NP = const$.

In case of flat metric of Universe

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \quad (8)$$

the choices $R(t) = \sqrt{t}$ and $R(t) = t$ satisfies the weak and null energy conditions (if Dark Energy is matter).

From Synge argument (reading the T_ν^μ from $g_{\mu\nu}$) holds $T_\nu^\mu = \text{diag}(-F(t), A(t), A(t), A(t))$. The Einstein theory only allows knowing the natural $F(t)$ get $A(t)$ and $R(t)$. I believe the $NP = \text{const}$ selects from all possible functions $F(t)$ the historical, the natural one.

Looking at left hand of Einstein equations, the energy conditions are more hard to satisfy, if the Dark Energy is Geometry. For any causal (i.e. timelike or null) vector must be

$$T_{\mu\nu}U^\mu U^\nu = G_{\mu\nu}U^\mu U^\nu + \kappa\Lambda U_\nu U^\nu \geq 0. \quad (9)$$

Indeed, occurs negative addition with Λ . Shall we write minus sign before the Λ ? No, putting the minus is against the observations. But choices $R(t) = \sqrt{t}$ and $R(t) = t$ violate energy conditions after certain point in time. So no wonder, that $NP = \text{const}$ is violated: these are not the true choices and truly, the Dark Energy is Geometry.

Consider Friedmann Universe with $NP = \text{const}$, Dark Energy is Geometry and free choice of closed $k = +1$, open $k = -1$ and flat $k = 0$ Geometry. Then the $(R_{,t})^2 \geq 0$ never turns to zero. In any variation of measured parameters. So the Universe is not cyclic: it expands and never will be shrinking.

I took Marshall's dust ball collapse and made coordinate transformation $\tau = f(t, R)$. The $NP = \text{const}$ holds, if $f(t, R) = t f_1(R) + f_2(R)$. This means, that g_{tt} is time-independent and the energy conditions are not violated in all space and time. Note, that making the $g_{tt} = -1$ in all spacetime doesn't give gravitational mass of stable liquid ball $NP \neq M$, however in case of Marshall's dust-ball collapse the $NP = M$.

I have derived (compare with [7]) the solution of stable liquid ball of constant density ρ is (the Dark Energy is ignored)

$$ds^2 = -A(r) dt^2 + \frac{dr^2}{B(r)} + r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (10)$$

joint on surface to vacuum

$$B(r) = 1 - \frac{8}{3} \pi \rho r^2,$$

$$A(r) = \frac{5}{2} - 6 R^2 \pi \rho -$$

$$\frac{1}{2} \sqrt{(3 - 8 R^2 \pi \rho)(3 - 8 \pi \rho r^2)} - \frac{2}{3} \pi \rho r^2,$$

$$p(r) = \rho \left(-1 + \sqrt{\frac{B(R)}{A(r)}} \right),$$

with R as star radius and $p(r)$ is pressure, the perfect liquid has $T_\nu^\mu = \text{diag}(-\rho, p, p, p)$. The *natural units* set gravitational constant and light-speed to 1. Turns out, that down after $R = 2.25 M$ there are singularities of $p(r)$. Thus, solution holds, if $R > 2.25 M$.

IV. FIRST DERIVATION

I was thought, that divergence is the intensity of springs (sources). So I define the flux of a tensor as integral over its divergents. Demanding, that flux shall be invariant, I get covariant divergence, where invariant 4-volume element is $d\Omega := \sqrt{-g} dx^0 dV$. Suppose the time axis changes $\Delta t = C_1 \Delta u$, where units are $[t] = [\text{sec}]$, $[u] = [\text{betasec}]$, $[C_1] = [\text{sec/betasec}]$. The time-etalon is redefined and renamed. So the flux Φ as leaked mass per time changes:

$$\Phi_u := \frac{\Delta m}{\Delta u} = C_1 \frac{\Delta m}{\Delta t} = C_1 \Phi_t. \quad (11)$$

Moreover, calculations show, that

$$NP_u = C_1 NP_t. \quad (12)$$

So, demand of invariance is demand of unchanging time-axis, in particular $C_1 = 1$. Thus transformation formula has

$$\frac{\partial x^\nu}{\partial u} = (1, 0, 0, 0). \quad (13)$$

Therefore, following flux is invariant:

$$\Phi = \int F_{\nu;\mu}^\mu \sqrt{-g} dx^0 dV, \quad (14)$$

where $\nu = 0$.

The invariant flux can be defined alternatively. It's the projections of the tensor on surface vectors. The invariant scalar products are

$$F_0^\nu d\sigma_\nu^{(1)} = \sqrt{-g} F_0^0 dV, \quad (15)$$

$$F_0^\nu d\sigma_\nu^{(2)} = \sqrt{-g} dx^0 F_0^1 dx^2 dx^3. \quad (16)$$

It is because the contravariant component $i = 0, 1$ in F_0^i transform like dx^i inside the formula of invariant [7] 4-volume element $d\Omega = \sqrt{-g} dx^0 dx^1 dx^2 dx^3 = inv$.

Thus, we demand the rightness of both flux definitions, so the Flux Divergence Theorem:

$$\oint F_0^\nu d\sigma_\nu = \int F_{\nu;\mu}^\mu \Big|_{\nu=0} d\Omega. \quad (17)$$

The insertion of very first metric and F has satisfied the theorem. Indeed, take $F_\nu^\mu = A^\mu A_\nu$, where $A^\mu = (t, r, 0, 0)/(1+r)^4$, $ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$, where $g_{tt} = -1 - 1/(1+r)$, $g_{rr} = (2+r)/(3+r)$. A container with volume Ω : upper plane has $t = t_2$, lower plane has $t = t_1$, sides have $r = R$. The spacetime violates the weak energy condition, but g_{tt} is time-independent.

Then Eq.(17) is satisfied only if on upper plane $d\sigma_\mu = (d\sigma, 0, 0, 0)$, where $d\sigma := \sqrt{-g} dr d\theta d\phi$, on lower plane $d\sigma_\mu = (-d\sigma, 0, 0, 0)$, on sides $d\sigma_\mu = (0, d\psi, 0, 0)$, where $d\psi := \sqrt{-g} dt d\theta d\phi$.

If it is $F_\nu^\mu := \delta_\nu^\mu$, then there is counterexample: dust ball collapse.

V. SECOND DERIVATION

What is physical meaning of this NP ? If metric is diagonal, the

$$dM = -T_0^0 \sqrt{-g_{11}g_{22}g_{33}} dV \quad (18)$$

is energy (not density) measured by local observer. The

$$M = \sum \sqrt{-g_{00}} dM = \text{const}. \quad (19)$$

is quantity, what conserves during evolution; like the integral of motion of test particle in Schwarzschild spacetime [8]

$$E \sqrt{-g_{00}} = \text{const}, \quad (20)$$

where E is total energy of test particle, measured locally by stationary observer. As you see, the dM must be valid energy of system. Thus, at least all energy conditions must be satisfied.

VI. WHO WANTS BUCKET OF LIQUID?

A liquid-ball. Distant observer slowly takes the water amount dm using a bucket on very long cord and distributes it throughout the Universe. He loses on his efforts dE_0 energy to pull the bucket. The local stationary observers at the surface measure the energy of the thin layer dR (outside the ball is Schwarzschild metric)

$$dm = T_0^0 \sqrt{\frac{g}{g_{tt}}} dV = 4\pi \rho \frac{R^2 dR}{\sqrt{1-2M/R}} = \frac{dM}{\sqrt{1-2M/R}}. \quad (21)$$

There is energy conservation

$$dm - dE_0 = dm \sqrt{1 - \frac{2M}{R}} = dM. \quad (22)$$

Thus, integral gives

$$\Delta m - \Delta E_0 = M, \quad (23)$$

where amount of gotten liquid is

$$\Delta m = \int \frac{dM}{\sqrt{1-2M/R}}. \quad (24)$$

Liquid ball solution Eq.(10) of constant density provides

$$M = \frac{4}{3} \pi \rho R^3. \quad (25)$$

I calculated the NP , if on ball solution is made $g_{uu} = -1$ by coordinate transformation $t = u f(r)$. Then holds exactly:

$$\Delta m = NP, \quad (26)$$

where Δm and NP are calculated in different coordinates.

Conclusion: in stationary case the $\Delta E_0 = NP - M$ is work needed to be done to get needed substance of amount NP . The ΔE_0 is height of potential barrier needed to overcome.

But also in unstationary, Marshall's dust collapse spacetime (2). In distant past $\tau \rightarrow -\infty$ the dust would fill all spacetime $r := W(\tau, 1) \rightarrow \infty$. Thus, the dust is distributed around the world even without use of bucket. Therefore, the obstacle to distribute the dust is zero $NP - M = 0$.

VII. MORE ON APPLICATION TO UNIVERSE

In General Relativity from geometrical reasons (s.c. Synge argument) the "metric" of isotropic, homogeneous Universe is

$$ds^2 = -c^2 dt^2 + R^2(t) \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right). \quad (27)$$

Inserting it into Einstein Equations, I get true form of matter tensor:

$$T_\nu^\mu = \text{diag}(-F(t), A(t), A(t), A(t)). \quad (28)$$

The physical meaning of $F(t)$ is energy density, because stationary observer $u^\mu = (1, 0, 0, 0)$ measures it $\delta = F(t)$, see Eq.(50). The physical meaning of $A(t)$ is not fixed: we don't know the matter tensors of dark matter and perhaps electromagnetic fields, but sum of all factors produces the Eq.(28). Applying $NP = \text{const}$ we get

$$F(t) = F_0 \left(\frac{R_0}{R(t)} \right)^3. \quad (29)$$

From Eqs.(29),(6) and established $R_{,t} \neq 0$ the $A(t) \equiv 0$. You try to reject my paper, on the believe, that $A(t)$ is pressure and, thus, can't remain zero? Then the zero is less strange, than the negative pressure of the believed Universe dynamics. Hereby in fixation of $A(t) \equiv 0$ I see the *fine tuning*, we talk about.

I use modified Einstein Equations, where "dark energy" is expressed through geometric term on the left, see Wikipedia:

$$G_\nu^\mu + \kappa \Lambda \delta_\nu^\mu = \kappa T_\nu^\mu, \quad (30)$$

where G_ν^μ is s.c. Einstein's tensor. The Geometry Lagrangian is then $L = \hat{R} + \Lambda$, where \hat{R} is s.c. scalar curvature. Thus now the dark energy is part of Geometry law. It's not materia, which on the right sides of Eqs. (30) and (33). Thus,

$$2R R_{,tt} + (R_{,t})^2 + kc^2 = R^2 c^2 \kappa \Lambda, \quad (31)$$

$$-3R (R_{,t})^2 / c^2 - 3kR + R^3 \kappa \Lambda = -\kappa F_0 (R_0)^3. \quad (32)$$

Taking derivative from second equation (32) gives first one (31), so if second is satisfied also holds first one.

Let us assume, that dark energy is matter, i.e. holds

$$G_{\nu}^{\mu} = \kappa T_{\nu}^{\mu}, \quad (33)$$

where $F_0 = \rho_0 c^2 + \Lambda$, here ρ_0 includes *dark matter* and ordinary, baryonic matter. Then

$$2R R_{,tt} + (R_{,t})^2 + kc^2 = 0, \quad (34)$$

$$-3R (R_{,t})^2/c^2 - 3kR = -\kappa F_0 (R_0)^3. \quad (35)$$

Taking derivative I get first equation (34). Structure of first equation shows, that must be $k = -1$ (because is established $R_{,tt} > 0$). This is not case of the popular flat Universe model, also it is not the Biblical model.

A. Comparison with experiments

From Wikipedia (also Russian one) the set of parameters, I use, is:

$$\Lambda \approx 5.98 \times 10^{-10} \text{ J/m}^3, \quad F_0 \approx \frac{27.2}{72.8} \Lambda, \quad F_0 \approx \frac{27.2}{72.8} \Lambda + \Lambda, \quad (36)$$

$$H_0 \approx 69.32 \frac{\text{km/s}}{\text{Mpc}}, \quad (37)$$

where the higher F_0 is for "dark energy is matter" variant. Then this variant, that dark energy is materia, is excluded: any possible curves are close to line (1) dis-matching the experimental points (see Fig.1). So, the dark energy is Geometry.

In such case can we distinguish between different k ? Possibly. The line (2) is not perfect: entire 4 points are out of "standart" range. But curve (3) with $R_0 = 27.1$ Gpc leaves only one point out. Note, that already 1.5×27.1 Gpc is "bad" match: 3 points out. This line is closer to line (2). Taking higher the R_0 we get even more closer to line (2). If you take $k = 0$ then solutions are at line (2), hereby my eye can not distinguish lines of different R_0 . If $k = -1$ solution is higher then line (2), so the higher mismatch than the line (2).

But the set Eq.(37) with $R_0 = 27.1$ Gpc, $k = 1$ violates Einstein's equation (32) at present time $t = t_0 = 0$. But if to replace Λ with 1.3 percent higher value, then Eq.(32) is satisfied and the solution is between (2) and (3) lines, so still having better match, than line (2). Hereby case 5×27.1 Gpc closely resembles line (2), being partly higher than it.

Conclusion: the Biblical model more beautifully matches the data.

B. Biblical model

Let us draw a circle in the closed Friedmann universe [10]. The idea is to assume squared interval

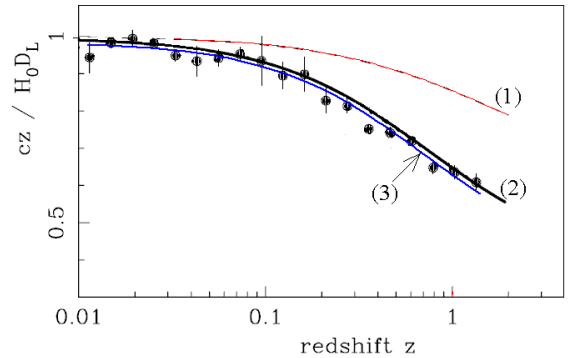


FIG. 1: Picture from internet. My numerical blue line (3) more perfectly matches the experimental points. Analytic solution (1) is from assumptions, that dark energy is matter and $k = 0$. It matches the $\Omega_m = 1$ line of Ref.[9]. Line (2) is the thick line $\Omega_m = 0.3$, $k = 0$ from Ref.[9].

$ds^2 = -c^2 dt^2 + R^2(d\xi^2 + \sin^2\xi(d\theta^2 + \sin^2\theta d\phi^2))$. Scale factor R expands with time t . We get the perimeter of the circle $L = 2\pi R_0 \sin\xi = 30$ cubit and diameter $2a = 2R_0\xi = 10$ cubit, where metric spatial coordinate $\xi = \pi/6$ and scale factor $R_0 = 30/\pi$ cubit. The 30 and 10 are in 1 Kings 7:23.

The molten sea was observed in Solomon's temple. The diameter of the observable universe is $2a = 28.3$ Gpc [11]. Thus, using the equation for the diameter $2a = 2R_0\xi$ again, at the present time, the $R = R_0 \approx 27.1$ Gpc.

The topology scale d of hypothetical non-trivial topology in [11, 12] is not the geometrical R . These specialists distinguish geometry and topology. Note, that we have also "topological censorship theorem".

Because the most severe constraint cannot reach 28 Gpc, (read [12]), the line of termination will be always on the very edge of the Biblical Model. In case 27.1 Gpc is wrong, I will take the next possible number between 27.1 Gpc and 28 Gpc (but the current Biblical model would be disproved). Why? Take into account that God is not forcing us into a Biblical World. So, as I can accept, He left the Biblical Model in balancing between proof and disproof.

C. Singularity

Let's find the moment of virtual inflation. In Biblical model the edge of observable Universe has $\xi = \pi/6$. This corresponds to parameter $t \approx -14.0995$ billion years, scale factor $R(t) \approx 53$ Mpc and maximum cosmological redshift $z_m \approx 512$. The singularity has $t \approx -14.101$ (it is almost the $1/H_0$).

Taking not the Biblical model: $R_0 = 20 \times 27.1$ Gpc we get $\xi \approx 0.02611$, singularity has $t = -13.8992$, the relict "light" has $t = -13.8982$, the $R(t) \approx 799$ Mpc and $z_m \approx 677$.

Indeed, the Wikipedia tells, that the redshift of relict

photons is in the order of thousand, however found Ia supernovas have $z < 2$ some quasars have $z \approx 6$. Of course, not all relict light is hitting telescope right now, so the sphere of relict light's ξ origin is really the "observable Universe".

In these two models ($R_0 = 27.1$ Gpc and $R_0 = 20 \times 27.1$ Gpc) the relict light is at million years after singularity, however in Wikipedia it is allmost half million years. So there is new result in present paper: the scientific world does not know our integral of motion $NP = const$.

VIII. OTHER INTEGRALS OF MOTION

Let's start with F^{ikl} , which

$$F^{ikl} = -F^{ilk}. \quad (38)$$

Then

$$\frac{\partial^2 F^{ikl}}{\partial x^k \partial x^l} \equiv F^{i,kl} = 0. \quad (39)$$

Integrating

$$\int F^{i,kl} dV \equiv \int \left(\frac{\partial F_{,l}^{i0l}}{\partial x^0} + \frac{\partial F_{,l}^{i1l}}{\partial x^1} + \frac{\partial F_{,l}^{i2l}}{\partial x^2} + \frac{\partial F_{,l}^{i3l}}{\partial x^3} \right) dV, \quad (40)$$

where $dV := dx^1 dx^2 dx^3$. Such kind of calculations is well shown in [13], page 876. If holds

$$\int \frac{\partial F_{,l}^{i1l}}{\partial x^1} dV + \int \frac{\partial F_{,l}^{i2l}}{\partial x^2} dV + \int \frac{\partial F_{,l}^{i3l}}{\partial x^3} dV = 0, \quad (41)$$

then

$$\int \frac{\partial F_{,l}^{i0l}}{\partial x^0} dV = 0. \quad (42)$$

If the ranges of integration are time-independent (or do not influence), then

$$\frac{\partial}{\partial x^0} \int F_{,l}^{i0l} dV = 0. \quad (43)$$

Only then I have "System Integral of Motion"

$$P^i = \int F_{,l}^{i0l} dV = const. \quad (44)$$

It can be, but not necessary, the energy-momentum vector.

From Landau's book [8] (see the *energy-momentum pseudotensor* paragraph)

$$F^{ikl} := \frac{c^4}{16\pi G} [(-g)(g^{ik}g^{lm} - g^{il}g^{km})]_{,m}. \quad (45)$$

Landau formula (applied to my liquid ball) gave negative $P^0 \approx -0.002897$, which absolute value is not gravitational mass $M \approx 0.015451$. Negative energy can not

be, such energy is against the energy conditions. But in Landau's book the P^0 equals the energy (in $c = 1$ units system). But it turned out negative and with wrong absolute value (further read Sections X and IX).

Eq.(5) in Ref.[14], the "Einstein complex" gives

$$F_i^{kl} := \frac{c^4}{16\pi G} \frac{g_{is}}{\sqrt{-g}} [(-g)(g^{ks}g^{lm} - g^{ls}g^{km})]_{,m}. \quad (46)$$

Using it in above formulas simply picture index i being written below (i.e. as covariant one).

Eq.(11) in Ref.[14], the Møller formula gives

$$F_i^{kl} := \frac{c^4}{8\pi G} \sqrt{-g} g^{km} g^{lj} (g_{ij,m} - g_{im,j}). \quad (47)$$

Generally, variants to think out is infinitely many. More examples. Because Ricci tensor is symmetric like g^{ik}

$$F^{ikl} := \psi(x^0, x^1, x^2, x^3) [R^{ik}R^{lm} - R^{il}R^{km}]_{,m} \quad (48)$$

with arbitrary, tunable function ψ of spacetime coordinates (or simply $\psi \equiv 1$ if you choose). Important could become things for matter tensor \mathbf{T} , for example

$$F^{ikl} := [T^{ik}T^{lm} - T^{il}T^{km}]_{,m} \quad (49)$$

Landau's integral of motion (from Eq.(45)), if instead of g^{ik} there is combination of tensors $aG^{ik} + bR^{ik} + dg^{ik} + const^{ik}$ and instead of $(-g)$ a tunable function $w(t, x) := W(t)f(x)$. Also Eq.(48), if $\psi = w(t, x)$ and instead of R^{ik} is above combination. Tuning the W , I still have not found restriction for Universe scale factor $R(t)$ (considered simplest, $-dt^2 + R^2(t)(dx^2 + dy^2 + dz^2)$ metric). The wanted equation would with combination of $NP = const$ theoretically determine the experimental constants $k, H_0, R_0, \rho_0, \Lambda$. At least, one could prove, that $NP = const$ is law in nature of Universe. The curvature tensor product $F^{iskl} = W(t)f(x)R^{iskl}$ with antisymmetric pair kl was also checked with no result. Conclusion: the initial point is any antisymmetric thing F^{ik} . I found in components, that this theory says, what $f(t) := F_{,1}^{10} + F_{,2}^{20} + F_{,3}^{30}$ is time independent, if $df(t)/dt = 0$. Thus, it is identity expressions. This method does not constrain the metric functions. As also Landau case showed. The known conservation laws [8, 14], "system integrals of motion" (Einstein, Møller and Landau) are just identity forms, may be usefull in theoretical considerations. Like the very useful identity is $g_{,\mu}^{\nu\mu} = 0$, which however does not constrain the metric functions. Indeed, we started with identity $F_{,i,k}^{ik} = 0$, which holds for any metric. Thus, it and its consequences does not constrain the metric functions. But, my Gauss flux theorem with arbitrary \mathbf{F} works not for any metric in Sygne argument. Thus, it is not an identity and may constrain the metric functions.

IX. ON ENERGY IN GENERAL RELATIVITY

The isolated system has gravitational mass as integral of motion. Like in the Birkhoff's theorem. The total energy also conserves. I recognize the energy as gravitational mass (times c^2) of an asymptotically flat system. In General Relativity the gravitational mass equals the inertial mass (theoretical proof is in [8]) and inertial mass times c^2 is energy [8], according to Special Relativity. Looking at electron we recognizing it: the space integral of energy density T_0^0 does not provide the experimental value of energy even in "Minkowski spacetime" approximation (integrating outside sphere of s.c. classical radius). The Tolman's formula uses other diagonal elements of matter tensor and provides correct energy of electron. Indeed, because electron is splitted [15], it is not a point; and observed perfect sphere [16] of, indeed, classical radius (as reported in media) has sharp boundaries, which "fur coat" of virtual particles around pointlike electron can not have. Here I can not see the place for Higgs boson.

The local energy density δ observer measures the known way: $\delta = T_{\mu\nu} u^\mu u^\nu$, for observer at rest

$$\delta = -T_0^0. \quad (50)$$

X. AGAIN ON GAUSS THEOREM

In Landau's book [8]

$$\oint X^{ik} dS_k = \int X_{,k}^{ik} dx^0 dx^1 dx^2 dx^3. \quad (51)$$

Here on left is box

$$A_0 < x^0 < B_0, \quad A_1 < x^1 < B_1, \quad A_2 < x^2 < B_2,$$

$$A_3 < x^3 < B_3. \quad (52)$$

With accompanying number complects $dS_k = (dx^1 dx^2 dx^3, 0, 0, 0)$ at B_0 , $dS_k = (-dx^1 dx^2 dx^3, 0, 0, 0)$ at A_0 , $dS_k = (0, dx^0 dx^2 dx^3, 0, 0)$ at B_1 , $dS_k = (0, -dx^0 dx^2 dx^3, 0, 0)$ at A_1 , $dS_k = (0, 0, dx^0 dx^1 dx^3, 0)$ at B_2 , $dS_k = (0, 0, -dx^0 dx^1 dx^3, 0)$ at A_2 , $dS_k = (0, 0, 0, dx^0 dx^1 dx^2)$ at B_3 , $dS_k = (0, 0, 0, -dx^0 dx^1 dx^2)$ at A_3 .

It is in Cartesian coordinates t, x, y, z the closed box. But looking from our room on box of spherical coordinates $x^\mu = (t, r, \theta, \phi)$ in formula (52) it is not closed surface. The Gauss theorem, which likes to use the Landau, uses just the closed surface. So the Landau's result holds only for Cartesian coordinates (in which the metric is asymptotically Minkowskian).

Indeed, example of liquid ball, shows, that in spherical coordinates Landau formula for P^0 does not give the gravitational mass. The last we get after coordinate transformation $x = r \cos\phi \sin\theta$, $y = r \sin\phi \sin\theta$,

$z = r \cos\theta$ and integrating also outside the ball. In Landau formula the curved vacuum gives nonzero, negative addition to gravitational mass. But there are "positive mass theorem" [17], which says, that mass can not be negative; I believe, even that of curved vacuum. The Møller formula [14] seems to give gravitational mass in every coordinate system, but in case of dust collapse it does not working.

XI. COLLAPSE OF BALL OF PERFECT LIQUID

Took Schwarzschild-kind metric with two unknown functions $g_{tt}(t, r)$ and $g_{rr}(t, r)$. Then by coordinate transformations $r = W(\tau, R)$, $t = t(\tau, R)$ was put to zero velocity component $u^R \equiv 0$ and the 4-velocity of liquid elements has only one nonzero component u^τ . Also eliminated the metric non-diagonal component $g_{\tau R} \equiv 0$.

$$ds^2 = -A^2(\tau, R) d\tau^2 + B^2(\tau, R) dR^2 + W^2(\tau, R)(d\theta^2 + \sin^2\theta d\phi^2). \quad (53)$$

$$T_\nu^\mu = \text{diag}(-\rho(\tau, R), p(\tau, R), p(\tau, R), p(\tau, R)). \quad (54)$$

Then the $X_1 := -T_\tau^\tau \sqrt{-g} = W^2 \rho A B$. After it is made coordinate transformation to make g_{00} independent from x^0 , namely $\tau = \tau(u, R)$, so that $\tau_{,u} = \sqrt{f(R)/A}$, where $f(R)$ is arbitrary positive function or nonzero constant. Then $X_2 := T_u^u \sqrt{-g} = |\tau_{,u}| X_1$. In this simple transformation $T_u^R = T_u^\theta = T_u^\phi \equiv 0$. The $NP = \text{const}$ results in independence from the $x^0 = u$, i.e. $X_2 = \text{const}$ (because the $f(R)$ is arbitrary). Thus,

$$T_u^u = \frac{S(R)}{W^2 B},$$

where $S(R)$ we can find from values at initial moment $u = u_0$. And the back transformation gives

$$T_\tau^\tau = T_u^u = -\rho(\tau, R) = -\rho_0(R) \frac{W_0^2 B_0}{W^2 B}.$$

From matter equations (6) if $p(\tau, R) \neq 0$ the $\rho(\tau, R) = \rho_0(R)$. Thus, there is no singularity or contraction. But it must be. Therefore while collapse of non-equilibrium the $p(\tau, R) \equiv 0$. It is like World Trade Center has being collapsed 11 September with free fall acceleration (like a dust cloud).

If you take a stable perfect liquid ball and by addition of small amounts of liquid make it to collapse, then the gravitational mass (and, thus, the total energy) does not change. Indeed, in Schwarzschild coordinates the stable ball solution and the dust collapse (at initial moment, when all velocities $u^r \equiv 0$) has the same $g_{rr} = 1/(1 - 8\pi\rho r^2/3)$.

Vasili K. showed definition-description of perfect liquid: it is incompressible one <http://u.to/GetSAw>. How perfectly my formula has shown it and predicted the pressure drop in collapse of real liquid!

XII. VISCOUS LIQUID [7] BALL COLLAPSE

For simplicity I consider viscousness as constants in space and time. The matter tensor T_{ν}^{μ} is diagonal. If "first viscousness" η is zero, then the pressure can not be zero. Hereby if also "second viscousness" ζ is zero, then the pressure must be zero. That show the produced differential equations if viscousness is incorporated. But if you from the beggining consider the perfect liquid $\zeta = \eta = 0$, then you get nonzero pressure for stable ball. Thus, the perfect liquid approach can be very wrong.

I considered $\eta = 0$ case. There is exact solution (Marshall's metric (2) above), the matter tensor is dust-like $T_{\nu}^{\mu} = \text{diag}(-\rho(\tau, R), 0, 0, 0)$, but the pressure is nonzero. However is negative (if $\zeta > 0$) and becoming singular. Thus in my approach, if first viscousness is zero, then

the collapse is inevitable: there is no stationary solution (because $T_1^1 = T_2^2 = T_3^3 \equiv 0$).

The $\zeta \neq 0, \eta \neq 0$ case was considered. Result: if first viscousness $\eta \neq 0$ then the matter tensor certainly has nonzero components besides the T_0^0 , i.e. does not hold $T_1^1 = T_2^2 = T_3^3 \equiv 0$. Thus, there can exist the stable solution. It was done by inserting Eqs.(2),(3) into matter equations (6). Indeed, if would $T_1^1 = T_2^2 = T_3^3 \equiv 0$ the formulas for metric reduce to dust collapse case. The last does not happens. The pressure and the elements T_1^1, T_2^2, T_3^3 have functional additions with ζ or η before them (however after finding the metric functions some constant may be eliminated).

My thesis: the viscousness causes the pressure and stabilizes the ball, at least perfect liquid does not provide correctly the solution.

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