Electron's Mass and Electric Charge, which one changes with velocity ?

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Abstract
This article presents two contrary opinions on the variability of two fundamental properties of the electron: its mass and its electric charge .
Mainstream physics considers the mass of the electron as varying with its velocity while its electric charge as a fundamental constant of physics (Section 1) .
On the other hand , other physicists maintain the opposite opinion : the mass of the electron is constant in all physical conditions (Section 2).
In the latter case , a plausible expression which describes the variability of the electric charge of the electron in external fields can be searched (Section 3) .
A thought experiment is proposed to show the variability of the electric charge of the electron by a variable magnetic field (Section 4) .
Keywords : relativistic increase in mass , longitudinal & transversal masses , extended model of the electron , screened electron , fine-structure constant , Lamb shift .

1. Mainstream physics : the mass of an elementary particle (e.g. an electron) varies with its velocity .

This idea appeared to come from the theory of the electron of Lorentz in which he proposed (1904) his model of the electron as a uniform spherical surface charge . When this electron moved through the "ether " , its transverse dimension remained unchanged , but its length in the motion direction was contracted and the variation of mass with velocity was derived as

$$m = m_0 (1 - v^2/c^2)^{-1/2} \qquad (1)$$

Later, the same expression (1) was derived by Einstein in his Special Theory of Relativity (1905) .

We can also find various expressions which describe the variation of the mass with velocity in contemporary textbooks . For example , in the topic " ***Relativistic Force*** " in " *Introduction to the Theory of Relativity* " by **Bergmann** [1] (1976)   we can read :

" *In general , the force thus defined is not parallel to the acceleration . It is parallel only when the acceleration is either parallel or perpendicular to the velocity .*
When it is parallel , it takes the form

$$f = (1 – u^2 / c^2)^{-3/2} \ m \ a \qquad (2)$$

When the force and velocity are orthogonal , it becomes
$$f = (1 - u^2 / c^2)^{-1/2} \, m \, a \qquad\qquad\qquad (3)$$

The coefficients of the acceleration on the right-hand sides of these two equations are occasionally referred to as " longitudinal mass" and " transversal mass ", respectively".

So , the longitudinal mass $m_1 = (1 - u^2 / c^2)^{-3/2} m = \gamma^3 m$ $\qquad\qquad (4)$
and the transversal mass $m_t = (1 - u^2 / c^2)^{-1/2} m = \gamma \, m$ $\qquad\qquad (5)$

where $\gamma = (1 - u^2 / c^2)^{-1/2}$ depends on the velocity of the particle in the external field .

In experimental physics , the concept of velocity-dependent mass is firmly expressed by the term **"relativistic increase in mass "** as we can see in the following two topics in the "*Encyclopedia of Physics*" by **Lerner and Trigg** [2] (2005) .

In the topic " *Mass* " (page 1448) the author **Van den Akker** wrote :
" If the reader is concerned about the <u>relativistic increase in mass</u> of a high-speed spacecraft , it is of interest to calculate that , for each metric ton , this is , at escape velocity 11180 m/s , 7×10^{-7} kg , or 0.7 mg " .

In the topic " *Cyclotron* " (page 492) two authors **Judd** and **Baron** wrote :
" The small decrease of B and (particularly for light ions) the <u>relativistic increase of mass</u> ($m = m_0 + T / c^2$, with m_0 the rest mass and c the speed of light) both act to decrease an ion's frequency $\omega = QeB / m$ as its energy and radius increase" .

For the electron $Q = 1$, e is the electric charge of the electron , the frequency ω is

$$\omega = eB / m \qquad\qquad\qquad (6)$$

The operation of the cyclotron requires ω to be kept constant to maintain the cyclotron frequency ; this is done in the azimuthally-varying-field (AVF) cyclotrons by introducing the azimuthal variations in the magnetic field (page 493) .

" In these machines the magnetic field (average along a full turn of an ion's orbit) increases with increasing radius just enough <u>to offset the relativistic mass increase</u> , matching the cyclotron frequency to that of the oscillator at all ion energies ".

This explanation means that the magnetic field **B** is designed to increase with the velocity of the electrons to offset the relativistic increase of the mass m , while the electric charge e is considered constant . We call this opinion " *mainstream* " because it appeared early in the history of physics and lasts until to-day .

2. The opposite concept : the mass of an elementary particle is always constant .

 Like in politics , physicists almost always confront with the opposition !
Here , the opposite concept is that *the mass of an elementary particle is always constant*
in all physical conditions.
Let us see how physicists confirmed the constancy of the mass of a particle .

i) **Okun**[3] , ' The concept of mass' , Physics Today ,1989
*" In the modern language of relativity theory there is one mass , the Newton mass m ,
which does not vary with velocity ".*

 ii) **Sternheim & Kane**[4] , 'General Physics ' , 1991
*" The correct definition of the relativistic momentum of an object of mass m and velocity
v is $p = mv (1 - v^2 / c^2)^{-1/2}$. In this equation , m is the ordinary mass of the object
as measured by an observer in its rest frame . (Some books refer to this quantity as the
rest mass and also define a velocity-dependent mass . We do not do this)" .*

iii) **Marion & Thornton**[5] , 'Classical Dynamics of Particles and Systems',1995 , p.555
*" Scientists spoke of the mass increasing at high speeds . We prefer to keep the concept of
mass as an invariant , intrinsic property of an object . The use of two terms relativistic
and rest mass is now considered old-fashioned . We therefore always refer to the mass m ,
which is the same as the rest mass ".*

iv) **Kacser**[6]**,** ' Encyclopedia of Physics ' , by Lerner & Trigg , 2005, topic : "Relativity,
   Special Theory"
 *" Mass – a notational issue - yet profoundly important . In many relativity presentations
(but generally not in Einstein's own works) , a misleading set of mass definitions was
created – rest mass , relativistic mass (an abomination), transverse mass , etc . It has
been strongly and correctly argued by Okun   that these confusions should not be
propagated . So here I will use m as the one-and-only mass of a particle being what is
often called the rest mass and written m_0 . This mass m (by others often called m_0 or
the rest mass) is the same as the Newtonian mass at low velocities . Most important , m
is a scalar or invariant , it has the same value for all observers of the particle, and is a
constant parameter for the particle . It is to be determined by experiment, and by use of
relativistic dynamics " .*

v) **Adler**[7] , Am. J. Phys. 55, (1987) ; " Does mass really depend on velocity , dad ? "
In the letter from Albert Einstein to Lincoln Barnett , 19 June 1948 , Einstein wrote in
German : *" It is not good to introduce the concept of mass $M = m / (1 - v^2/c^2)^{1/2}$ of a
moving body for which no clear definition can be given . It is better to introduce no other
mass concept than ' the rest mass ' m . Instead of introducing M , it is better to mention
the expression for the momentum and energy of a body in motion ."*

We notice that among the quotations from **the Encyclopedia of Physics** by Lerner &
Trigg , the mass was considered differently by different physicists : **Van der Akker, Judd**

and Baron believed in the relativistic mass increase , while **Kacser** rejected all concepts such as relativistic , longitudinal or transverse masses .
We therefore come to the conclusion that in the contemporary physics there exists an obvious contradiction in the concept of mass as to whether it is constant or varying with velocity .

Now , if the mass m is considered constant, then Eq.(6) tells us that the electric charge e must decrease to keep ω constant while the magnetic field **B** increases.

3. Search for a mathematical expression that describes the variation of the electric charge of the electron in external field .

In AVF cyclotrons , the magnetic field **B** is designed to increase in order to keep the angular frequency   ω of the electron constant

$$\omega = eB / m = \text{constant} \tag{6}$$

To keep ω   constant while **B** increasing , either m has to increase or e has to decrease by appropriate factors . One way or the other is mathematically equivalent as we can see in the following argument .

If it is the increase of the mass that keeps ω constant (while **B** increasing) , we can use either the longitudinal mass m_l [Eq.(4)] or the transversal masse m_t [Eq.(5)] to express the increase of mass with velocity :

Using $m_l = \gamma^3 m$, Eq, (6) becomes $\omega = eB / m_l = eB / (\gamma^3 m) = \text{constant}$ $\qquad$ (7)

Using $m_t = \gamma m$, Eq, (6) becomes $\omega = eB / m_t = eB / (\gamma m) = \text{constant}$ $\qquad$ (8)

where $\gamma = (1 - u^2 / c^2)^{-1/2}$ which increases with the velocity u .

Eqs. (7) and (8) mean that the mass m   is increased by the factor γ^3 or γ respectively , such that ω can remain constant (while **B** increasing) .

Eq.(7) can be rewritten as $\omega = eB / (\gamma^3 m) \equiv (\gamma^{-3} e)\, B / m = \text{constant}$ $\qquad$ (9)

Eq.(8) can be rewritten as $\omega = eB / (\gamma m) \equiv (\gamma^{-1} e)\, B / m = \text{constant}$ $\qquad$ (10)

The right hand sides of Eqs.(9) and (10) (after the sign ≡) show that the electric charge is decreased by the factor γ^{-3} or γ^{-1} respectively , such that ω can remain constant (while **B** increasing) . So , Eq.(6) helped us move from the increase of mass to the decrease of charge .

From Eqs.(9) and (10) we can deduce two expressions for the *effective charge* q of the electron :

From (9) : $\omega = q\,B\,/\,m\ =\ $ constant , where $q = \gamma^{-3}\,e$ (11)
From (10) : $\omega = q\,B\,/\,m\ =\ $ constant , where $q = \gamma^{-1}\,e$ (12)

We notice that in Eq.(11) the factor γ^{-3} is derived from the longitudinal mass m_l , meanwhile in Eq. (12) the factor γ^{-1} is derived from the transversal mass m_t . Because of different applying fields (electric or magnetic) and direction of motion of the electron in these fields , the factor γ in two Eqs.(11) and (12) have *different exponent*s : -3 and -1 .
This remark suggests that the exponent of the factor γ depends on the applying field and the direction of motion of the electron in the field .

And hence , to generalize the expression of the effective electric charge we ascribe to the factor γ **the general exponent –N** to replace two exponents -3 and –1 ; where N $\geq$ 0 is a positive real number representing the applying field . This *heuristic reasoning* unites two particular equations (11) and (12) into the general equation (13)

$$q =\ \ \gamma^{-N}\,e\ \ =\ (\,1 - u^2/c^2\,)^{N/2}\,e$$ (13)

In Eq.(13) , let's set $e \equiv q_0$ and $u \equiv v$ to get the familiar form of Eq.(14) , the shape of which is illustrated in the following figure (Fig.1)
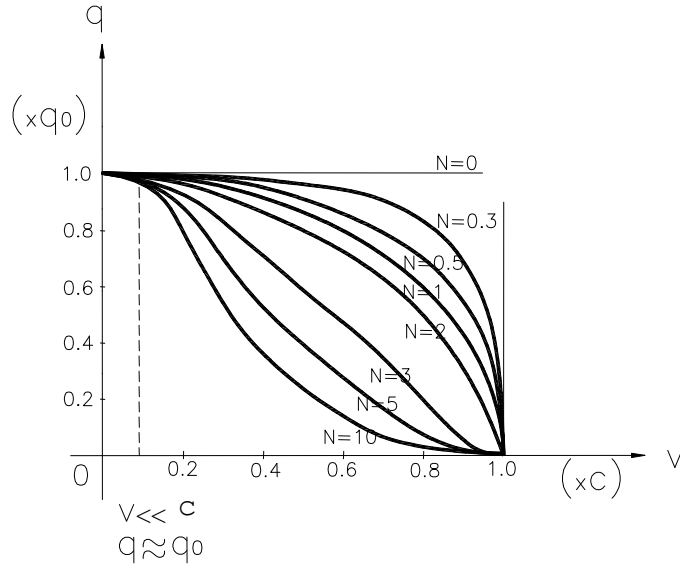
$$q =\ \ \gamma^{-N}\,q_0\ \ =\ (\,1 - v^2/c^2\,)^{N/2}\,q_0$$ (14)

Since $0 < v < c$ and $N \geq 0$, the factor $(1 - v^2 / c^2)^{N/2}$ varies in the interval $(0 , 1)$; and hence , q varies in the interval $0 < q < q_0$ where $q_0 \equiv e$. This means that when the electron is subject to an external electric or magnetic field , its effective charge q drops below q_0 . From Fig.1 we notice the following remarkable points :

(i) when $N = 0$, **q = q$_0$** for all velocities : this is the case when the electron moves in free space , ($N = 0$ means there is no applying field) .

(ii) at low velocities ($v << c$) , **q $\approx$ q$_0$** for all values of N (i.e., for all applying fields).

Let's note that this is the case of the oil-droplet experiment of **Millikan** [8] in which electrons (on oil droplets) fell down at velocity of **a fraction of a millimeter per second in the electric field of 6000 volts per cm** . And hence , Millikan's experiments could only give the unique value $q \approx q_0$ ($\equiv e$) for the electric charge . Mainstream physicists accepted this value e as the constant of the electric charge in all physical conditions because after Millikan , they could not perform any other experiments at higher velocities or in stronger fields which might have given the electric charge of the electron other effective values different from e . *

(iii) at high velocities near c : $v \rightarrow c$, **q $\rightarrow$ 0** for all values of N .

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*   It is interesting to read the following comment that **Millikan** made on his experiment of oil-droplets :
In order to be able to measure very accurately the force acting upon the charged oil-droplet it was necessary to give it about a centimeter of path in which the speed could be measured . This is one of the most important elements in the design , the overlooking of which has caused some subsequent observers to fall into error ... The field strength too , about 6,000 volts per cm , was vital , and new in work of anything like this kind . It was the element which turned possible failure into success . Nature here was very kind . She left only a narrow range of field strengths within which such experiments as these are all possible ". (Millikan's Nobel lecture , 1924)

So , the numerical value of e ($= 1.602 \times 10^{-19}$ C) was measured by the Millikan 's experiment under specific conditions of speed and field strength . If the experiment had been performed at different conditions , (at higher speeds , near c , and in stronger electric fields) , would it have given other values for e ?
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 Note on the consequence of the Eq.(14) on the electric force **Fe** and the magnetic force **Fm** produced on the electron by constant fields **E** and **B** :

If the electric charge q varies with v according to Eq.(14) then :
  ▪ $Fe = (1 - v^2 / c^2)^{N/2} q_0 E$: Fe decreases with v and tends to zero as $v \rightarrow c$.

  ▪ $Fm = (1 - v^2 / c^2)^{N/2} q_0 v B$ (**v** $\perp$ **B**) : Fm increases with v , reaches its maximum at $v = c (N + 1)^{-1/2}$ then decreases and tends to zero as $v \rightarrow c$.

For $v << c$, $Fe \approx q_0 E$ and $Fm \approx q_0 v B$: these are familiar non-relativistic expressions of Fe and Fm .

4 . A thought experiment to show the variability of the electric charge of the electron in a variable magnetic field .

If the electric charge of the electron is an effective one which varies with the applying field , we can figure out an experiment to demonstrate this variability (Fig.2) .
In this thought experiment we keep the velocity of the electrons unchanged while we change the intensity of the magnetic field **B** in the solenoid by changing the intensity of the current I .
- An electron gun produces electrons with various velocities at the point A .
- A velocity selector allows only electrons with velocity **v** to travel to the point B .
- A solenoid produces a uniform magnetic field **B** along its axis which coincides with the trajectory of the electron beam . The intensity **B** of the magnetic field can be regulated by the current I . Since **v** ∥ **B** , there is no net (magnetic) force produced on the electron , so electrons travel with constant velocity **v**   through the solenoid to the point C . And hence , there is no change in the mass and the kinetic energy of the electron with velocity .
- A detector , which can be a thick block of silver bromide (photographic emulsion), is installed at the exit C of the solenoid to detect the changing of the electric charge q of the electron when **B** changes its intensity .

At the point of entrance C on the detector, the velocity of the electron is v , and its effective charge is q , which is expected to decrease when the intensity of the magnetic field **B** increases .
Since the energy loss per unit distance[4] in the medium of the detector is proportional to q^2 / v^2 , that is $\Delta K \propto q^2 / v^2$. Now if we increase the intensity of **B** (N increases) , according to the curves in Fig. 1 , the effective electric charge q   (at the velocity v) will drop ; and hence ΔK decreases , resulting in a deeper penetration of electrons into the block of photographic emulsion .
In short , when we change the intensity of **B**, if the depth of penetration changes in response to the change of **B** , this proves that q varies with the applying magnetic field.
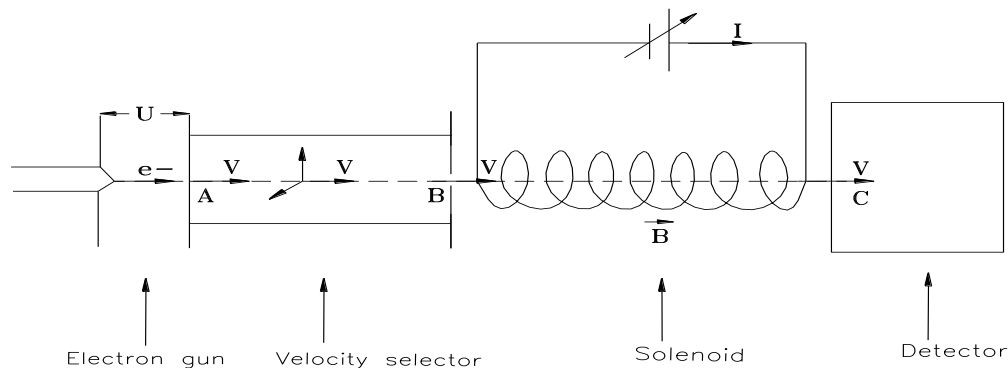

Fig 4

(Correction: this is Fig.2 , not Fig.4)

5 . Summary & Conclusion

Sections 1 and 2 recalled two opposite opinions on the mass of the electron from the current literature : whether or not it depends on the velocity .
Section 3 presented the standpoint of the author of this article : the mass of the electron is invariable , its electric charge is variable . This opinion leads to the general equation (13) or (14) which illustrates the variation of the effective electric charge with velocity and external field . This is an attempt to show that the concept of varying electric charge can replace the concept of varying mass in interpreting physical phenomena which involve mass and charge .
The thought experiment described in section 4 would be helpful for the justification of the concept of effective electric charge .

Although physicists have speculated on the variability of the electric charge in certain physical phenomena (presented in the following appendices) , they are not ready to confirm this variability as a fact . Therefore , the variability or the constancy of the mass and the electric charge of the electron still remains as a *foundational problem in the modern physics* for decades . This situation needs to be changed and justified .

Appendices

In the modern physics there are experimental phenomena which are related to the variability of the electric charge of the electron :
   A) the fine-structure constant α ,
   B) the Rutherford's nuclear experiment ,
   C) the Lamb's shift

A . The fine-structure constant α

Nowadays , physicists consider the dimensionless fine-structure constant $\alpha = e^2 / 4\pi\varepsilon_0 \hbar c$ as a "running" coupling constant which varies with the energy at which it is measured .
Bekenstein   and **Uzan**   have investigated the variability of α and its consequences on the variation of the electric charge .
 Bekenstein [9]   wrote : " *Since $\alpha = e^2 / \hbar c$, where e is the electron charge , α variability means that e depends on the spacetime point . ... Thus every particle charge can be expressed in the form $e = e_0 \in(x^\mu)$, where e_0 is a constant characteristic of the particles and $\in$ a dimensionless universal field .*" (p. 1529)

Uzan [10] came to an affirmative conclusion :
 " *For instance, theory of varying fine-structure constants can be implemented either as a theory with varying electric charge or varying speed of light .*" (p.409)
 " *Based on recent astrophysical observations of quasars , it has been claimed that the fine- structure constant has evolved .*"(p.448)

B . The Rutherford's nuclear experiment

" Rutherford' s experiment, in which he scattered alpha particles by atomic nuclei, showed that the equation $F = qq'/(4\pi\varepsilon r^2)$ is valid for charged particles of nulear dimensions down to separations of about 10^{-12} cm . Nuclear experiments have shown that the forces between charged particles do not obey the equation for separations smaller than this ." [11]

The Coulomb's force equation becomes invalid at distance smaller than 10^{-12} cm because at these short distances the mutual electric field created by two charges q and q' becomes strong enough to change their initial magnitude q and q' to different values (other than q and q') which are unknown to the experimenters . And consequently , the actual force which gives the experimental results is no longer equal to $F = qq'/(4\pi\varepsilon r^2)$ given by Coulomb' s law . Therefore , the invalidity of the Coulomb's force equation at very short distances could find the explanation in the variation of the effective electric charge under the action of the mutual field created by the very charges q and q' .

C . The Lamb shift

The Lamb shift is a manifestation of the invalidity of Coulomb's law at short distances . In 1947 Lamb succeeded in measuring the small energy difference between two energy levels $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ of hydrogen atom . In his Nobel lecture (1955) Lamb [12] pointed out the reason for the splitting of these two energy levels as follows :

" The exact coincidence in energy of the $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ states is a consequence of the assumed Coulomb law of attraction between electron and proton. Any departure from this law would cause a separation of these levels".

French and **Taylor** wrote in their textbook[13]   :

" The existence of this splitting (now known as the Lamb shift) means , in effect , that the electrostatic interaction between proton and electron is not described with complete accuracy by Coulomb' s law" – although there is no implication that Coulomb' s law fails as a statement of the force between ideal point charges . The modified result can be understood in terms of the theory called quantum electrodynamics ".

Therefore , it is the variability of the electric charge that makes the Coulomb's law inaccurate at very short distances and hence the Lamb shift .

D . Why can the electric charge of the electron change ?

From the phenomena cited above we wonder why the electron can change its electric charge in external field . The plausible answer is because **it is not a rigid point charge** , **but an extended and structured particle** ; and hence its electric charge is affected by the action of the external field . **Lamb** said in his Nobel lecture that *" the electron does not behave like a point charge as implied in Dirac' s equation ... According to relativity theory , energy and mass are proportional to one another . In order to obtain a finite*

mass of the electron on a purely electromagnetic basis , it was necessary to assign an extended structure to the electron”.

An extended model for the electron based on the image of the *screened electron* was proposed and discussed in the article " **A new extended model for the electron** "[14] , trying to explain the variability of the electric charge of the electron in external field .

In this article , we came to the equation (13) : $q = \gamma^{-N} e = (1 - u^2/c^2)^{N/2} e$ while

other physicists expressed their speculation on the variability of the electric charge by different expressions :

Bekenstein wrote [9]:

" Thus every particle charge can be expressed in the form $e = e_0 \in (x^\mu)$, where e_0 is a constant characteristic of the particles and $\in$ a dimensionless universal field ."

Rohrlich wrote in the topic of renormalization [15] :

“The effective charge e , which is the physical (renormalized) charge , is defined to be

$$ e = Z_1 Z_2^{-1} Z_3^{-1/2} e_o $$

where Z_i are renormalization constants .”

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