On the Invalidity of the Hawking-Penrose Singularity 'Theorems' and Acceleration of the Universe from Negative Cosmological Constant

Stephen J. Crothers
(Queensland, Australia)
thenarmis@gmail.com

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ABSTRACT

Hawking and Penrose proposed "A new theorem on spacetime singularities … which largely incorporates and generalizes the previously known results" which they claimed "implies that space-time singularities are to be expected if *either* the universe is spatially closed *or* there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface)" and that their 'Theorem' applies if four certain physical conditions are satisfied**.** Hartle, Hawking and Hertog have proposed a quantum state with wave function for the Universe which they assert "raises the possibility that even fundamental theories with a negative cosmological constant can be consistent with our low-energy observations of a classical, accelerating universe." They also relate this concept to string cosmology. It is however proven in this paper that the Hawking-Penrose Singularity 'Theorem' and accelerated expansion of the Universe with negative Λ are invalid because they are based upon demonstrably false foundations relating to Einstein's field equations, trapped surfaces, and the cosmological constant.

I. INTRODUCTION

Hawking and Penrose (1970) proposed in their paper '*The singularities of gravitational collapse and cosmology*' what they claim is a theorem which,

> "*implies that space-time singularities are to be expected if either the universe is spatially closed or there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface)*".

They also assert that their Theorem applies if the following conditions are also satisfied,

"(i) Einstein's equations hold (with zero or negative cosmological constant), (ii) the energy density is nowhere less than minus each principal pressure nor less than minus the sum of the three principal pressures (the 'energy condition'), (iii) there are no closed timelike curves, (iv) every timelike or null geodesic enters a region where the curvature is not specifically aligned with the geodesic."

To disprove the Hawking-Penrose Singularity Theorem requires only disproof of one of the conditions the Theorem must satisfy. Nonetheless, all of the required conditions are proven invalid herein.

In their paper '*Accelerated Expansion from Negative Λ*', Hartle, Hawking and Hertog (2012) have proposed a quantum state of the Universe with a wave function in a theory of gravitation which they claim,

> "**…** *raises the possibility that even fundamental theories with a negative cosmological constant can be consistent with our low-energy observations of a classical, accelerating universe.*"

This they maintain has implications for cosmological models which rely on String theory:

> "*Treating the classical behavior of our universe as an emergent property of a fully quantum mechanical treatment of cosmology opens up new possibilities for building models of inflation in string theory.*"

Hertog (Grossman 2012) states further,

> "*We have a new route towards constructing string theory models of our world.*"

In addition they imbue Big Bang creation with quantum cosmology:

> "*… quantum cosmology enables us not just to calculate quantum processes near the big bang but also gives us a deeper understanding of our universe at the classical level today.*"

However, the arguments adduced by all these authors have no valid basis in science bearing in mind that their concepts and arguments are based on the General Theory of Relativity which is easily proven to violate the usual conservation of energy and momentum and to not predict the black hole (Crothers 2005, 2008, 2010, 2012, 2012b), with or without cosmological constant, as proven herein. It therefore matters not whether the cosmological constant is positive, zero or negative since the General Theory of Relativity is in conflict with experiment on a deep level, rendering it invalid and hence the Big Bang creation cosmology with its alleged expansion of the Universe fallacious, along with the conditions stipulated by Hawking and Penrose (1970) for their Singularity Theorem.

II. INVALIDITY OF SINGULARITY THEOREM

By way of preamble it is curious that Hawking and Penrose (1970) and Penrose (2002) repeatedly refer to the "*pull of gravity*" and "*gravitational collapse*". This implies the Newtonian concept of a force of gravitational attraction. However, in General Relativity gravity is *not* a force that produces a pull or attraction on anything because Einstein's gravitation is due to a curvature of spacetime induced by the presence of matter.

> "*Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move.*" (Carroll and Ostlie 1996)

Matter according to Einstein (1916) consists of mass and electromagnetic radiation, and by implication from the latter, also charge. Furthermore, 'gravitational collapse' is not explained by Hawking and Penrose in terms of either Newtonian theory or General Relativity. They merely alleged the existence, without qualification, of the unobserved process of irresistible "*gravitational collapse*", which is the case in the literature generally to the present day. For example, in the 'Introduction' of their paper, Hawking and Penrose (1970) remark,

> "*An important feature of gravitation, for very large concentrations of mass, is that it is essentially unstable. This is due, in the first instance, to its r^{-2} attractive character. … The pull of gravity is readily counteracted by other forces. … a star of mass greater than 1.3 times that of the Sun, … cannot sustain itself against its own gravitational pull, so a gravitational collapse ensures. … some form of gravitational collapse may be taking place in quasars, or perhaps in the centres of (some?) galaxies. Finally, on the scale of the universe as a whole, this instability shows up again in those models for which the expansion eventually reverses, and the entire universe becomes involved in a gravitational collapse.*"

Hawking and Pernose (1970) also remark,

> "*In the reverse direction in time there is also the 'big bang' initial phase which is common to most relativistic expanding models.*"

In Section '3. THE THEOREM', of their paper, Hawking and Penrose (1970) say,

> "*We expect trapped surfaces to arise when a gravitational collapse of a localized body (e.g. a star) to within its Schwarzschild radius takes place, which does not deviate too much from spherical symmetry.*"

In the same section of their paper they write Einstein's field equations as,

$$R_{ab} - \frac{1}{2} R g_{ab} = -K T_{ab} \qquad (1)$$

which they number as equation (3.5) therein and in which the constant $K > 0$. It is this form of Einstein's field equations upon which Hawking and Penrose (1970) rely for their Singularity Theorem, although they also remark in relation to eq. (1) above,

"*To incorporate a cosmological constant λ, we would have to replace T_{ab} with $T_{ab} + \lambda K^{-1} g_{ab}$.*"

In this case eq. (1) becomes,

$$R_{ab} - \frac{1}{2} R g_{ab} = -K T_{ab} - \lambda g_{ab} \qquad (2)$$

Consider the notion of a trapped surface. The reference to "*gravitational collapse of a localized body (e.g. a star) to within its Schwarzschild radius*" necessarily refers to the formation of a black hole related to Einstein's so-called *static* field equations for *empty space*, i.e. when $T_{\mu\nu} = 0$ (Dirac 1996, Eddington 1960, Foster and Nightingale 1995, Tolman 1997), given by,

$$R_{\mu\nu} = 0 \qquad (3)$$

Now the black hole allegedly associated with eq. (3) is alone in its universe. Indeed,

"*Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time.*" (Matzner 2001)

In fact, all alleged black hole solutions to Einstein's field equations pertain to a spatially infinite Universe which is eternal, contains only one mass, is not expanding, and is asymptotically flat (Crothers 2008, 2010). Hilbert's solution, Schwarzschild's (1916) actual solution, and the Reissner-Nordström (charged) black hole, are both very easily seen to be asymptotically flat. The Kerr solution (rotating black hole), and the Kerr-Newman (charged and rotating) black hole are also asymptotically flat (Penrose 2002) (note that Penrose's metric (1) is metric (4) given below):

"*The Kerr-Newman solutions … are explicit asymptotically flat stationary solutions of the Einstein-Maxwell equation (λ = 0) involving just three free parameters m, a and e. As with the metric (1), the mass, as measured asymptotically, is the parameter m (in gravitational units). The solution also possesses angular momentum, of magnitude am. Finally, the total charge is given by e. When a = e = 0 we get the Schwarzschild solution.*" (Penrose 2002)

However, a localised body such as a star is *not* the only mass present in the actual Universe and since it is present amidst very many stars and galaxies its associated spacetime is not asymptotically flat. So the notion that a localised body such as a star can form a 'Schwarzschild' black hole, and hence a trapped surface at its 'Schwarzschild radius', or indeed any type of black hole, by means of irresistible gravitational collapse, is in conflict with the defining characteristics of a black hole because the said star and its associated black hole with trapped surface by means of gravitational collapse is not alone in a Universe that is spatially infinite, eternal, asymptotically flat, and not expanding. Furthermore, the notion of a black hole with its trapped surface is also in conflict with Big Bang cosmology because the Hot Big Bang pertains to a Universe that is said to be finite both spatially and temporally, is expanding, is not asymptotically flat, and contains radiation and many masses, including many black holes some of which are claimed to be primordial.

Since a star is not the only mass in the Universe and since there are no known solutions to Einstein's field equations for two or more masses and no existence theorem by which it can even be asserted that the field equations contain latent solutions for two or more masses (McVittie 1978), on what theoretical basis are the many stars and radiation in the Relativistic universe postulated to begin with? It clearly cannot be from General Relativity. It is in fact achieved by means of invalidly applying Newton's theory and its associated Principle of Superposition to General Relativity. But then "*gravitational collapse*" cannot be due to Newtonian gravitation, given the resulting black hole, which does not occur in Newton's theory of gravitation. And a Newtonian universe cannot "collapse" into a non-Newtonian universe such as that of the lonesome black hole or a cosmological singularity. Oppenheimer and Snyder (1939) began with a Newtonian universe owing to the *a priori* presence of many stars, and allowed a star therein to collapse into a black hole to form a non-Newtonian universe, which is impossible, and also maintained that the black hole so formed is present in a universe that contains other masses in contradiction to the defining characteristics of the black hole itself. Hawking and Penrose (1970), Hawking (2002) Penrose (2002), Misner, Thorne and Wheeler (1970), Chandrasekhar (1972), do the very same. According to Oppenheimer and Snyder (1939),

> *"When all thermonuclear sources of energy are exhausted, a sufficiently heavy star will collapse. … this contraction will continue indefinitely. … the radius of the star approaches asymptotically its gravitational radius."*

It is also routinely claimed by the proponents of the black hole that not only does it exist in multitudes, they can collide and merge, be components of binary systems, and consume other non black hole matter from the Universe, and that primordial black holes existed in the early Universe, shortly after the Big Bang; but all this is in conflict with the fact that all black hole models pertain to a spatially infinite, eternal, asymptotically flat, non-expanding universe that contains only one mass. In addition, it has been shown elsewhere (Crothers 2008, 2010, 2012) that Einstein's field equations (3) for static empty spacetime actually contain *no matter* by mathematical construction and that Einstein's field equations (1) cannot in fact reduce to equations (3).

Owing to the foregoing, the notion of a trapped surface enunciated by Hawking and Penrose (1970) and Penrose (2002) has no valid basis in General Relativity whatsoever and so their Singularity Theorem is invalid.

Consider next the so-called "*Schwarzschild radius*", also called the "*gravitational radius*". Hilbert's metric (Abrams 1989, Antoci 2001) is usually given as,

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$d\Omega^2 = \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$0 \le r \qquad (4)$$

where m is claimed to represent the mass of a body that can undergo irresistible gravitational collapse to form a black hole and trapped surface. Schwarzschild's (1916) actual metric is different to Hilbert's metric and contains no black hole. In expressions (4) according to Penrose (2002),

> *"The quantity m is the mass of the body …"*

In expression (4) the speed of light c and Newton's gravitational constant G are both set equal to unity. This practice is rather misleading and so with c and G written explicitly so that nothing is hidden, expression (4) becomes,

$$ds^2 = c^2\left(1 - \frac{2Gm}{c^2 r}\right)dt^2 - \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$0 \le r \qquad (5)$$

where $d\Omega^2$ is as that given in expressions (4). Consider the two terms g_{00} and g_{11} extracted from expressions (5), thus,

$$g_{00} = \left(1 - \frac{2Gm}{c^2 r}\right) \qquad g_{11} = -\left(1 - \frac{2Gm}{c^2 r}\right)^{-1} \qquad (6)$$

It is asserted routinely that when $r = 2Gm/c^2$ there is a 'coordinate singularity' and when $r = 0$ there is 'physical singularity'. In the case of $r = 2Gm/c^2$ it is also routinely asserted that (Dirac 1996),

$$g_{00} = (1-1) = 0 \qquad g_{11} = \frac{-1}{(1-1)} = \frac{-1}{0} = -\infty \qquad (7)$$

and that a trapped surface is produced in the course of gravitational collapse (Hawking and Penrose 1970, Penrose 2002), so that the quantity $r = 2Gm/c^2$ is the 'Schwarzschild radius' of the black hole, that is, the 'radius' of the event horizon of the black hole. Penrose (2002) says in relation to expression (4),

> *"The radius r = 2m is referred to as the Schwarzschild radius of the body."*

However, we note that in expressions (7) there is division by zero in the case of g_{11}; but division by zero is *undefined* in mathematics. It is also noted that not only is division by zero permitted to generate the trapped surface and the 'Schwarzschild radius' for the event horizon of the black hole, division by zero is also alleged to produce -∞. This too is false. Since g_{11} is undefined at $r = 2Gm/c^2$ expression (5) is also undefined at this value, and so no physical entity can be assigned to this value of r.

In the case of r = 0 we obtain from expressions (6),

$$g_{00} = \left(1 - \frac{2Gm}{0}\right) \quad g_{11} = \frac{-1}{\left(1 - \frac{2Gm}{0}\right)} \tag{8}$$

We note once again that division by zero results; and not once but twice! In this case both g_{00} and g_{11} are undefined and so expression (5) is undefined. Nonetheless, the proponents of the black hole again permit division by zero and assign to this value of r an infinitely dense point-mass singularity for the 'Schwarzschild' black hole. Indeed, according to Hawking (2002),

> "*The work that Roger Penrose and I did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole.*"

Furthermore, by virtue of expressions (5), (7) and (8) it is also incorrectly claimed that 1/∞ = 0 (Misner, Thorne and Wheeler 1970), not by any limiting process but by actual division by ∞. Owing to expressions (7) and (8), the trapped surface is again invalidated and hence also the Hawking-Penrose Singularity Theorem.

Consider further the expression for the 'Schwarzschild radius' of the trapped surface (i.e. event horizon),

$$r = \frac{2Gm}{c^2} \tag{9}$$

Solving equation (9) for c we obtain,

$$c = \sqrt{\frac{2Gm}{r}} \tag{10}$$

We immediately recognise that this is Newton's expression for escape velocity. It is from this expression that it is so often claimed that the escape velocity of a black hole is the speed of light c in vacuum or even greater than c. Chandrasekhar (1972) says,

> "*Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it. ... A trapped surface is one from which light cannot escape to infinity.*"

In the Dictionary of Geophysics, Astrophysics and Astronomy (Matzner 2001), one finds the following assertions:

> "***black hole*** *A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*
>
> $$v_{esc} = \sqrt{\frac{2GM}{R}},$$
>
> *where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole. The required radius R_{BH} follows from setting v_{esc} equal to c*:
>
> $$R_{BH} = \frac{2GM}{c^2}$$
>
> *"In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression R_{BH} holds for the surface of a black hole. The surface of a black hole at R_{BH} is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon."*

According to Hawking (2002),

> *"Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. Its boundary is called the event horizon. It coincides with the paths of the light*

rays that just fail to escape from the black hole."

In the Collins Encyclopaedia of the Universe (2001) there occurs the following assertion,

"***black hole*** *A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light*."

However, it is also claimed on the other hand, that nothing at all, including light, can even leave the black hole. Chandrasekhar (1972) states,

"*The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge*."

Hawking (2002) says,

"I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole."

Taylor and Wheeler (2000) assert,

"*... Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon ... and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter*."

Equations (9) and (10) actually have nothing to do with the black hole at all; they are related only to Newton's theory of gravitation and equation (9) is the critical radius for the formation of the theoretical Michell-Laplace dark body, which is *not* a black hole since it does not possess the signatures of the black hole and is a Newtonian theoretical object, not a Relativistic theoretical object (Crothers 2006, 2008, 2010, McVittie 1978). The theoretical Michell-Laplace dark body forms when,

$$r < \frac{2Gm}{c^2}$$

Nonetheless it is routinely but incorrectly claimed that the Michell-Laplace dark body is a black hole (Misner, Thorne and Wheeler 1970). According to Hawking and Ellis (1973),

"Laplace essentially predicted the black hole…"

In The Cambridge Illustrated History of Astronomy (Hoskin 1997) it is asserted that,

"*Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work*."

Chandrasekhar (1972) reiterates,

"That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that $\frac{1}{2}v^2 > GM/R$; *and it cannot escape if* $v^2 < 2GM/R$. *On the basis of this last inequality, Laplace concluded that if* $R < 2GM/c^2 = R_s$ *(say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*

"*By a curious coincidence, the limit* R_s *discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass."*

But it is not surprising that General Relativity gives the same R_s "*discovered by Laplace*" because the Newtonian expression for escape velocity is deliberately inserted *post hoc* into Hilbert's solution by the astrophysical scientists.

Now if the black hole has an escape velocity *c*, then, by definition, light can escape, contrary to the frequent claim that it cannot. Moreover, material bodies can leave the black hole but not escape since in Relativity theory no material body can acquire the speed *c* and because ‘escape velocity’ does not mean bodies cannot leave, only that if they leave they cannot escape. If the escape velocity of the black hole is greater than *c* then light cannot escape and no material body can escape, on the basis that no material body can acquire the speed of light in vacuum, but once again that does not mean that material bodies and light cannot *leave*, only that they cannot escape. Yet black hole theory also maintains that neither light nor material bodies can even leave the event horizon of the black hole. The notion of black hole escape velocity is just a play on the words ‘escape velocity’ (McVittie 1978). The black hole has no escape velocity. This fact also invalidates the Hawking-Penrose concept of ‘trapped surface’ and hence the Hawking-Penrose Singularity Theorem.

That the Michell-Laplace dark body is not a black hole is easy to prove. The Michell-Laplace dark body possesses an escape velocity, whereas the black hole has no escape velocity; masses and light can leave the Michell-Laplace dark body, but nothing can leave the black hole; it does not require irresistible gravitational collapse, whereas the black hole does; it has no infinitely dense singularity, whereas the black hole does; it has no event horizon, whereas the black hole does; there is always a class of observers that can see the Michell-Laplace dark body, but there is no class of observers that can see the black hole; the Michell-Laplace dark body persists in a space which by consistent theory contains other Michell-Laplace dark bodies and other matter and they interact with themselves and other matter, but the spacetime of all types of black hole pertain to a universe that contains only one mass (but actually contains no mass by mathematical construction) and so cannot interact with any other masses; the space of the Michell-Laplace dark body is 3-dimensional and Euclidean, but the black hole is in a 4-dimensional non-Euclidean spacetime; the space of the Michell-Laplace dark body is not asymptotically flat whereas the spacetime of the black hole is asymptotically flat. Thus the Michell-Laplace dark body does not possess the characteristics of the black hole and so it is not a black hole.

Furthermore, although expression (10) contains only one mass term, it is implicitly a *two-body* relation: one body escapes from another body. But all alleged black hole solutions to Einstein’s field equations pertain to an infinite universe that contains only one mass, and so Newton’s expression for escape velocity *cannot* rightly appear in any mathematical expression alleging a black hole. Indeed, Newton’s expression for escape velocity is inserted *post hoc* into Hilbert’s solution (5) in order to satisfy the misleading words “*gravitational field outside a body*” or “*point-mass*” when in fact there is no mass present since expression (3) contains no matter by mathematical construction because the energy-momentum tensor is set to zero for no material sources present (Crothers 2008, 2010, 2012, 2012b) bearing in mind that Einstein’s field equations,

> “*... couple the gravitational field (contained in the curvature of spacetime) with its sources*.” (Foster and Nightingale 1995)

> “*Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field.*” (Landau and Lifshitz 1951)

It is clear that indeed Newton’s expression for escape velocity is included in Hilbert’s solution (5) by an invalid application of Newton’s theory of gravitation, and from it the fallacious notions of black hole escape velocity and black hole trapped surface at the ‘Schwarzschild radius’ are obtained, along with multiple black holes and black holes with other forms of matter by application of the Principle of Superposition, which is valid in Newton’s theory but which *does not* hold in General Relativity:

> *“The Einstein equations are nonlinear. Therefore for gravitational fields the principle of superposition is not valid.”* (Landau and Lifshitz 1951)

In direct violation of this fact, it is also routinely claimed that the black hole exists in multitudes, that black holes can collide and merge and interact with other matter, be components of binary systems, and can form by gravitational collapse in a universe that contains many masses and radiation. In other words, Newtonian theory is inadmissibly employed to produce many masses, including multiple black holes, in a non-

Newtonian General Relativistic universe. According to Chandrasekhar (1972),

"*From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.*

"*In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease; it can at best remain unchanged (if the conditions are stationary*).

"*Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass ½M, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass M. Then Hawking's theorem requires that*

$$16\pi\overline{M}^2 \geq 16\pi\left[2\left(\frac{1}{2}M\right)^2\right] = 8\pi M^2$$

or

$$\overline{M} \geq \frac{M}{\sqrt{2}}.$$

Hence the maximum amount of energy that can be released in such a coalescence is $M\left(1-1/\sqrt{2}\right)c^2 = 0.293M^2$".

Hawking (2002) says,

"*Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes*."

According to Schutz (1990),

"*... Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can never bifurcate spontaneously into two smaller ones*.

"*Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved*."

Carroll and Ostlie (1996) remark,

"*The best hope of astronomers has been to find a black hole in a close binary system. … If a black hole coalesces with any other object, the result is an even larger black hole. … If one of the stars in a close binary system explodes as a supernova, the result may be either a neutron star or a black hole orbiting the companion star. … the procedure for detecting a black hole in a binary x-ray system is similar to that used to measure the masses of neutron stars in these systems. … What is the fate of a binary x-ray system? As it reaches the endpoint of its evolution, the secondary star will end up as a white dwarf, neutron star, or black hole*."

In the case of Hilbert's solution (4) and (5) the quantity *r* has never been correctly identified by astrophysics. It has been variously and vaguely called a *"distance"*, "*the radius*", the "*radius of a 2-sphere*", the "*coordinate radius*", the "*radial coordinate*", the "*Schwarzschild r-coordinate*", the "*radial space coordinate*", the "*areal radius*", the "*reduced circumference*", "*the shortest distance a ray of light must travel to get to the center*", and even "*a gauge choice: it defines the coordinate r*". In the particular case of $r = 2Gm/c^2$ it is invariably called the "*Schwarzschild radius*" or the "*gravitational radius*". Dirac (1996) says in relation to Hilbert's solution in the form of expression (4),

"It would seem that r = 2m gives a minimum radius for a body of mass m."

According to Penrose (2002), in relation to expression (4),

"The radius r = 2m is referred to as the Schwarzschild radius of the body."

However, none of the foregoing various and vague conceptions of r are correct because the *irrefutable* geometrical fact is that r, in the spatial section of Hilbert's solution (4) and (5), is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, and as such it is not a distance at all, let alone the 'radius', in Hilbert's solution. In Newton's theory the quantity r in expressions (9) and (10) *is* the radius and since it is incorporated, invalidly, into Hilbert's solution, it is misinterpreted as the radius in Hilbert's solution as well. It is easy to prove that in Hilbert's solution r is not even a distance. Consider the surface in the spatial section of expressions (4) and (5). It is given by the First Fundamental Quadratic Form,

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{11}$$

The Gaussian curvature K of a surface can be calculated by,

$$K = \frac{R_{1212}}{g} \tag{12}$$

where R_{1212} is a component of the Riemann tensor of the first kind and g is the determinant of the metric tensor. The calculation can be made with the assistance of the following relations,

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma}R^{\lambda}_{.\,\nu\rho\sigma}$$

$$R^{1}_{.212} = \frac{\partial\Gamma^1_{22}}{\partial x^1} - \frac{\partial\Gamma^1_{21}}{\partial x^2} + \Gamma^k_{22}\Gamma^1_{k1} - \Gamma^k_{21}\Gamma^1_{k2}$$

$$\Gamma^i_{ij} = \Gamma^i_{ji} = \frac{\partial\left(\frac{1}{2}\ln\left|g_{ii}\right|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i} \quad (i \neq j)$$

and all other Γ^i_{jk} vanish. Making the calculation gives for expression (11),

$$K = \frac{1}{r^2}$$

and so

$$r = \frac{1}{\sqrt{K}}$$

which proves that r is neither a radius nor a distance in Hilbert's solution (4) and (5). Hence the 'Schwarzschild radius' is the radius of *nothing* in Hilbert's metric because it is not even a distance therein. This once again invalidates the concept of a trapped surface as proposed by Hawking and Penrose and hence also invalidates their Singularity Theorem.

Remarkably, in relation to Hilbert's metric, Celotti, Miller and Sciama (1999) make the following incorrect assertion:

"The 'mean density' $\bar{\rho}$ of a black hole (its mass M divided by $\frac{4}{3}\pi r_s^3$) is proportional to $1/M^2$"

where r_s is the so-called 'Schwarzschild radius'. The volume they adduce for a black hole cannot be obtained from Hilbert's solution: it is a volume associated with the Euclidean 3-space pertaining to Newton's theory: and the 'Schwarzschild radius' is not even a distance in Hilbert's solution, as proven above. Furthermore, at the 'Schwarzschild radius' Hilbert's metric is *undefined*. Misner, Thorne and Wheeler (1970) also incorrectly implicitly assert that the volume of a star according to Hilbert's solution is given, by $4\pi r^3/3$ via the expression,

"total mass-energy inside radius r ≡ m(r)
$$= \int_0^r 4\pi r^2 \rho\, dr\text{"}$$

where $\rho = \rho(r)$ is the "*density of total mass-energy*".

It has been shown (Crothers 2005, 2010) that the actual radius R_p for Hilbert's metric is given by,

$$R_p = \sqrt{r(r-\alpha)} + \alpha\ln\left(\frac{r + \sqrt{r-\alpha}}{\sqrt{\alpha}}\right)$$

where α is a constant given by $\alpha = 2Gm/c^2$. However, as explained above, the equality $\alpha = 2Gm/c^2$ is produced by an invalid application of Newton's theory by which Newton's expression for escape velocity is inserted into Hilbert's metric to obtain a source mass, escape velocity and 'Schwarzschild radius'. Hilbert's metric actually contains no matter in any case and is therefore physically meaningless (Crothers 2008, 2010, 2012, 2012b).

It is common practice to write Einstein's field equations compactly by means of Einstein's tensor $G_{\mu\nu}$ defined by,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

so that equations (1) become,

$$G_{\mu\nu} = -K T_{\mu\nu} \tag{13}$$

However, this form of Einstein's field equations is incorrect and so equations (1) and (13) *cannot* reduce to $R_{\mu\nu} = 0$ when $T_{\mu\nu} = 0$. When the conservation of energy and momentum is accounted for correctly, and bearing in mind that $R_{\mu\nu} = 0$ has no physical meaning because it contains no matter by mathematical construction, Einstein's field equations must take the following form (Crothers 2008, 2010, 2012, Lorentz 1916, Levi-Civita 1917),

$$\frac{G_{\mu\nu}}{K} + T_{\mu\nu} = 0 \tag{14}$$

Inclusion of the cosmological constant λ gives,

$$\frac{\left(G_{\mu\nu} + \lambda g_{\mu\nu}\right)}{K} + T_{\mu\nu} = 0 \tag{15}$$

This expression can be written in terms of mixed tensors as follows,

$$\frac{\left(G_{\nu}^{\mu} + \lambda g_{\nu}^{\mu}\right)}{K} + T_{\nu}^{\mu} = 0 \tag{16}$$

Compare this now to Einstein's (1916) expression for the total energy and momentum of his gravitational field given by,

$$\left(t_{\sigma}^{\mu} + T_{\sigma}^{\mu}\right) = E \tag{17}$$

The term t_{σ}^{μ} is Einstein's pseudo-tensor, which he calls *the energy components of the gravitational field* (Einstein 1916, Pauli 1981), and it is defined as follows (Levi-Civita 1917),

$$\sqrt{-g}\, t_{\nu}^{\mu} = \frac{1}{2}\left[\delta_{\nu}^{\mu} L - \left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\nu}^{\sigma\beta}\right] \tag{18}$$

where L is given by,

$$L = -g^{\alpha\beta}\left(\Gamma_{\alpha\kappa}^{\gamma}\Gamma_{\beta\gamma}^{\kappa} - \Gamma_{\alpha\beta}^{\gamma}\Gamma_{\gamma\kappa}^{\kappa}\right)$$

Since the pseudo-tensor is not a tensor, Einstein could not take a tensor divergence of equation (17) and so he took an ordinary divergence (Einstein 1916), thus,

$$\frac{\left(t_{\sigma}^{\mu} + T_{\sigma}^{\mu}\right)}{\partial x_{\sigma}} = 0 \tag{19}$$

and since this is zero Einstein (1916) asserts,

> *"Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied."*

However, it is noted that not only is expression (16) the correct form of Einstein's field equations with cosmological constant, it is also a total energy and momentum expression where the energy components of the gravitational field are in place of Einstein's pseudo-tensor actually given by the term,

$$\frac{\left(G_{\nu}^{\mu} + \lambda g_{\nu}^{\mu}\right)}{K} \tag{20}$$

Einstein's pseudo-tensor does not describe *the energy components of the gravitational field* for the following two reasons:

(1) t_{ν}^{μ} is not a tensor and so it does not meet Einstein's requirement that all the equations of physics be tensorial in General Relativity;

(2) Setting $\nu = \mu$ in expression (18) produces an invariant by the contraction of the pseudo-tensor, thus,

$$t_{\mu}^{\mu} = t = \frac{L}{\sqrt{-g}}$$

which is easily shown to be a first-order intrinsic differential invariant, i.e. an invariant that depends *solely* upon the components of the metric tensor and their first derivatives (Crothers 2008, 2010, 2012, Levi-Civita 1917). However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita (1900), inventors of the tensor calculus, proved that such invariants *do not exist*! Hence Einstein's pseudo-tensor is a meaningless collection of mathematical symbols and cannot be used to make any calculations or to represent any physical entity.

Taking the tensor divergence of the left side of expression (16) produces zero and there is conservation of energy and momentum, but since expression (16) is also an energy equation the total energy is always zero and so the usual conservation of energy and momentum is violated (Crothers 2008, 2010, 2012, Pauli 1981) and gravitational energy cannot be localised i.e. there are no Einstein gravitational waves (Crothers 2008, 2010, 2012, Levi-Civita 1917). Thus, Einstein's field equations are in conflict with experiment on a deep level, rendering them invalid. Hence *all* the conditions specified for the Hawking-Penrose Singularity Theorem are invalidated. Note that by expressions (16) and (20) this result holds whether λ is negative, zero, or positive.

III. Invalidity of Acceleration with Negative Λ

In the abstract of their paper, Hartle, Hawking and Hertog (2012) say that,

> *"Wave functions specifying a quantum state of the universe must satisfy the constraints of general relativity, in particular the Wheeler-DeWitt equation (WDWE)."*

However, it follows immediately from Section II above that the foundations of the quantum state with wave function containing negative λ proposed by Hartle, Hawking and Hertog (2012) are also invalidated, since the "*constraints of general relativity*" are meaningless. Hartle, Hawking and Hertog (2012) also say that,

> *"… even theories with a negative cosmological constant can predict accelerating classical histories.*
>
> *"At the level of the wave function there is a close connection between asymptotic Lorentzian de Sitter (dS) spaces and Euclidean anti-deSitter (AdS) spaces."*

But since the presence of λ in expressions (16) and (20) makes no difference whatsoever to the violation of the usual conservation of energy and momentum by General Relativity, the cosmological constant has no physical meaning and its inclusion in the quantum state as a negative constant proposed by the three aforementioned authors is also meaningless. Similarly de Sitter spaces and anti-de Sitter spaces have no physical meaning.

The claim (Hartle, Hawking and Hertog 2012) that,

> *"The observed classical expansion of our universe is accelerating at a rate consistent with a positive cosmological constant of order $\Lambda \sim 10^{-123}$ in Planck units"*

finds no justification in either General Relativity or Newtonian gravitation and so the *"observed classical expansion"* cannot be interpreted in terms of General Relativity as due to Big Bang cosmology.

The Cosmic Microwave Background (CMB) is ubiquitously employed to justify Big Bang cosmology on empirical grounds (Hawking and Penrose 1970). However, it is now known that the CMB is not cosmic. Its true source is the oceans of the Earth (Robitaille 2007, 2009) owing to the hydrogen bonds in water (Robitaille 2009b) which cause the oceans to emit radiation in the microwave and far-infrared bands, which are scattered by the atmosphere to produce an isotropic signal from an isotropic source. It is now also known that COBE and WMAP spectra and images are not scientifically useful (Robitaille 2007, 2009).

IV. Conclusion

The black hole is not predicted by General Relativity or by Newton's theory of gravitation, and it has no theoretical basis. The Hawking-Penrose 'trapped surface' and Singularity Theorem are invalid. All singularity theorems for General Relativity are invalid. General Relativity violates the usual conservation of energy and momentum and is therefore in conflict with experiment on a deep level, making it invalid. Consequently the black hole, Einstein gravitational waves, and the Big Bang Cosmology have no theoretical basis. The Cosmic Microwave Background is not the afterglow of the birth of the Universe from a Big Bang *creatio ex nihilo* or otherwise. The Hartle, Hawking, Hertog quantum wave function with negative cosmological constant is invalid.

DEDICATION

I dedicate this paper to my late beloved brother:

Paul Raymond Crothers
12th May 1968 – 25th December 2008

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