Special Relativity With Apparent Simultaneity

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Abstract

We consider a model of special relativity in which standard simultaneity is replaced by an alternative defined per observer by the direct appearance of simultaneity. The postulates of special relativity are interpreted to permit it, using a corresponding measure of distance chosen so that the measurement of light's speed remains invariant with a value of c. The relativistic Doppler effect and Lorentz transformation of time are derived from direct observations without consideration of a delay of light. Correspondence of the model with SR is further shown by finding a displaced observer whose measure of apparent simultaneity is identical to a given observer's measure of standard simultaneity. The advantages of apparent simultaneity include unifying apparent delay of light with relative simultaneity, and unifying changes to relative simultaneity with change in observer position. With speculative interpretation the model implies an equivalence of time and distance.

Introduction

In the definitive work [1] on special relativity (SR), Albert Einstein acknowledges that an assumption is required in order to define a common time between two separated clocks. The assumption used—which defines *standard simultaneity*—is that the time required for a light signal to travel from an observer O to an observer P is the same as the time required for a signal to travel from P to O. While it is generally acknowledged that this remains an assumption, it is consistent with observation and is used in the physical representation of time throughout modern physics. Einstein justifies the assumption by its practical benefit of independence of observer standpoint with the clock, but it is tempting to presume that the assumption has a fundamental physical basis, since it is consistent with observations of an invariant speed of light and with a classical interpretation thereof.

This paper hypothesizes that the delay of observed light inherent in standard simultaneity is an unnecessary classical assumption, and it is possible to avoid it while maintaining an assumption of an invariant speed of light. We define *apparent simultaneity* such that events are simultaneous according to a particular observer if they appear simultaneous in direct observation. Apparent simultaneity is defined in [2] equivalently as "backward light cone simultaneity".

Two models of SR will be considered: The *classical model* of SR as described by Einstein, including the assumption of standard simultaneity, and the *apparent model* with standard simultaneity replaced by an assumption of apparent simultaneity. *SR* alone will then refer to aspects that do not assume a particular simultaneity.

Assuming equivalence of all inertial frames and constant speed of light, it is classically absurd that a fixed transmission of light could take a different time depending on how it is observed. However, it is by the very nature of relativity that mutually consistent observers may disagree on the timing of events, and this will allow us to circumvent classical reasoning. It may also be argued that since the transmission and reception of a direct signal appear simultaneous to the receiver, the duration of the signal appears instantaneous, thus constituting "action at a distance." There is however only relative instantaneity—no single observer sees signals between O and P appearing instantaneous in both directions—without any two-way instantaneous interaction, and without implied violation of causality.

In the context of an argument in favour of conventionality of simultaneity, the apparent model corresponds with a Reichenbach ε-value of 1. Rather than arguing in favour of a choice of valid simultaneities for a given observer, let us assume a fixed simultaneity per observer in each model considered, but allow that different observers can measure simultaneity differently. Conventionality becomes superfluous as it can be expressed entirely in terms of relativity of simultaneity and a choice of observer.

Our development of apparent simultaneity differs from that of [2], in which a variant speed of light is assumed.

Assumptions and Conventions Used

Assume that SR and the classical model are precisely consistent with reality, without assuming that standard simultaneity is the only consistent definition of simultaneity.

The first postulate of relativity—that all physical laws are the same regardless of inertial reference frame—is assumed without interpretation. The second postulate—that the speed of light is c as measured in all inertial frames of reference regardless of the motion of the light's source—is assumed to be true on condition that any measurement of velocity which assumes a specific simultaneity is only considered valid for models that employ that simultaneity. Without yet defining a measure of velocity for apparent simultaneity, let us assume that any such measure is valid if it is functionally equivalent to classical model velocity for all physically possible velocities.

Proper times are assumed to be valid regardless of choice of simultaneity. Velocity of an observer moving relative to some reference, as measured by the observer using its ruler and local proper time, is assumed to not depend on a choice of simultaneity.

It is not assumed that different observers in an inertial frame must agree on the simultaneity of events. This paper considers only observations of direct straight-line light signals through a vacuum in flat Minkowski spacetime. Velocity is restricted to the x-axis.

The prototypical physical system used throughout this paper involves two observers named O and P. Observer P's coordinate system is analogous to the typical primed frame as per [3], except that the coordinate systems are configured symmetrically, with each observer's positive x-axis pointing toward the other's location. The x coordinate then corresponds to a measure of separation distance; positive velocity corresponds with increasing separation. Unless otherwise specified, all measurements are assumed to be according to O.

Two proper times measured by different clocks are simultaneous according to a given observer and definition of simultaneity if they start simultaneously and end simultaneously.

The variable t refers to a local proper time at O, and τ refers to a proper time at P, defined such that the two times are classically simultaneous.

1 Definition of Apparent Time

Definition 1.1. The *apparent time* $\tau_{\mathrm{A}}(t)$ at P according to observer O, is the proper time at P that is apparently simultaneous with local duration t.

This is the time that can be seen to elapse on P's remote clock while directly observed for a local duration of t. If P's clock appears to be zero when O's clock is zero, then the apparent time at P coincides with the time that appears on a directly observed image of its clock.

1.1 Apparent Distance

Because distance in SR is measured at a given instant that is defined by standard simultaneity, the apparent model requires an alternative measure of distance that uses apparent simultaneity. To correspond with measurements of apparent time, this metric is given the following corresponding features: *a*) it is a measurement made at P using P's measuring devices, and *b*) the measurement event and its observation at O appear simultaneous to O. The following definition achieves these features:

Definition 1.2. The *apparent distance* $x_{\mathrm{A}}(t)$ between O and P, according to O at time t, is the location on O's x-axis at which P appears at time t, measured using a ruler in P's inertial frame.

Both apparent time and distance can be thought of as observations of a remote measurement. Note that apparent distance does not refer to appearance using O's measuring devices alone, independent of any motion of P, as might be classically intuited. Instead, it makes use of a moving object's moving ruler, which will appear distorted by relativistic effects.

Consider a location on O's ruler at a distance of x. If P coincides with this location while moving, with its own measure of distance length-contracted relative to O's by a factor of ϵ, then

$$x_{\mathrm{A}} = \epsilon x. \tag{1}$$

According to SR, ϵ equals the reciprocal of the Lorentz factor. Extending P's ruler along its x-axis through O, the ruler will locally appear length-contracted by ϵ.

Apparent distance is completely determined in the single instant of observation. Due to the invariance of c and the uniformity of flat spacetime at the moment of observation, the classical distance of an event is also completely determined in the single instant of observation, in agreement with SR. Classically, according to O, the entire distance to P is length-contracted by the same factor that P's standard ruler is length-contracted, at every moment, including the instant of observation. Even as P's ruler may change with relative acceleration during the flight time of incoming photons, the change in length contraction is accompanied by a change in relative classical simultaneity that ensures the consistency of P's ruler along the entire length of separation at any given instant. Regardless of simultaneity, both apparent and classical distance traveled by an incoming photon depends on only the instant of observation rather than on what happens over any extended duration of inbound travel.

1.2 Conversion Between Classical and Apparent Time

For a given time t, we can convert between τ and τ_A simply by compensating for the classical travel time of incoming light.

Proposition 1.3. *If P is moving at a fixed velocity relative to O, and is observed at time t_0, then the time that appears on P's clock is*

$$\tau_{A0} = \tau_0 - \frac{x_A(t_0)}{c}, \tag{2}$$

where an event at P at time τ_0 is classically simultaneous with an event at O at time t_0.

Proof. Suppose that P appeared to be at a distance of x on O's ruler, in observations arriving at time t_0. Then according to the classical model, light has traveled a lightlike interval with spatial distance x according to O. This interval is has a spatial distance of x_A when measured by P, by definition of apparent time. According to O, while P measures light traveling a distance of x_A, a time of x_A/c will pass according to P's clocks. Therefore the classical time τ_0 must be that much later than the time that appears in direct observations. □

Proposition 1.4. *If P is moving at a fixed velocity relative to O, the apparent time τ_A at P corresponding to a local time t is given by:*

$$\tau_A = \tau - \frac{d_A}{c}, \tag{3}$$

where d_A is the change in apparent distance to P that occurs during time t.

Proof. Let τ_{A0} be the time that appears on P's clock in observations made by O at time 0, and let τ_{A1} be the time that appears at time t. Then the time τ_A that appears to elapse during time t is equal to $\tau_{A1} - \tau_{A0}$. Using Eqn (2), with additional variables set accordingly,

$$\tau_A(t) = \left(\tau_1 - \frac{x_{A1}}{c}\right) - \left(\tau_0 - \frac{x_{A0}}{c}\right), \tag{4}$$

and Eqn (3) follows, where d_A equals $x_{A1} - x_{A0}$ and τ equals $\tau_1 - \tau_0$. □

It might be possible to relax the condition of fixed velocity, but the restriction is fine for this paper.

2 Relativity

The first measurement of the finite speed of light, by Ole Rømer in 1676, was not a direct measurement of the timing of light signals, but rather an interpretation of observations of a clock appearing to tick at different rates [4]. The clock in this case is the orbit of Io, whose period appears from Earth to respectively decrease or increase depending on whether we are moving toward or away from it. As universal time was assumed in Rømer's era, the appearance of a modified rate of time would naturally be considered illusory, and attributing it to a changing delay of light is reasonable.

Since SR established the relativity of time, the appearance of modified rates of time is partially attributed to delay of light, and partially to relativistic effects, depending on velocity. We will calculate the appearance of relativistic time dilation while not making any assumptions regarding delay of light.

Io demonstrates the principle of relativity in that there is no physical reason to prefer either Earth or Jupiter's exclusive motion relative to the other. However, it makes a poor prototype for understanding relativistic mechanics because there is no easily inspected mechanical connection between the two moving bodies. We can consider an ideal clock which appears to behave exactly as Io does at a given velocity, and construct a mechanical coupling between time and distance at that velocity. The validity of the following thought experiments can be verified in accordance with SR.

Consider a clock at remote observer P, which includes a hand that rotates once per unit of time. Let P and its clock move at a proper velocity of w along O's ruler, which spans O's x-axis. The clock and ruler can be physically connected using a rack and pinion, fixing the rack to the ruler and the pinion to the clock hand. With cogs spaced one unit apart, and a pinion chosen with $|w|$ cogs on it, then the clock can move freely at proper velocity w with the gears meshing properly (assuming appropriate clock orientation and choice of units). Assume that this measuring apparatus is available in all descriptions of P in this section.

Lemma 2.1. *Neglecting the passage of proper time, a change in apparent distance of d_{A} will correspond to a change in apparent time of $-d_{\text{A}}/c$.*

Proof. This follows from Eqn (3) with a neglected duration τ of zero, and is consistent with any directly observed difference in time between a remote and local clock, such as between Io and a clock on Earth. □

The Lemma holds for both models, regardless of whether the effect is attributed to a changing delay of light or left unexplained. While it is verified by assuming that the classical model is valid, the apparent model requires acceptance that the Lemma could be based on experimental observation alone.

Lemma 2.2. *Neglecting classical relativistic effects, an apparent time τ_{A} of a clock that is moving at a fixed velocity of v relative to the observer, will correspond to a change in apparent distance of $v\tau_{\text{A}}$.*

Proof. By neglecting relativistic effects, we have that O's ruler and P's ruler share a unit length, and the proper velocity of P is equal to its velocity. If the gear attached to P's clock hand appears to turn τ_{A} times, then the gear must also appear to traverse $v\tau_{\text{A}}$ cogs along O's ruler, which is the same if measured using P's ruler. □

The second postulate is implicit in the lemmas, because they apply to any change of apparent time or distance, and do not depend on whether any other effect of relative velocity has already been considered.

In isolation, the lemmas describe the appearance of a clock at non-relativistic speeds. Relativistic effects emerge when the lemmas are recursively applied to each other, which we now consider. Given some finite time t which elapses at O, let $\tau_{\text{A}n}$ equal the apparent time associated with the nth recursive application of Lemma 2.1, and similarly let $d_{\text{A}n}$ represent the apparent distance associated with the nth application of Lemma 2.2. Letting $\tau_{\text{A}0}$ equal the apparent time that elapses at P excluding any application of Lemma 2.1, define

$$\tau_{\text{A}0} = \epsilon t, \tag{5}$$

such that the unknown factor ϵ includes any other possible relativistic effect that is not accounted for by the lemmas applied to P's clock.

Applying Lemma 2.2, we have

$$d_{\text{A}0} = v\tau_{\text{A}0} = v\epsilon t. \tag{6}$$

Applying Lemma 2.1, there is a corresponding change in apparent time which has not been accounted for in $\tau_{\text{A}0}$:

$$\tau_{\text{A}1} = -\frac{d_{\text{A}0}}{c} = -\frac{v}{c}\epsilon t, \tag{7}$$

to which we apply Lemma 2.2, and so on. By induction,

$$\tau_{\text{A}n} = \left(\frac{-v}{c}\right)^n \epsilon t, \tag{8}$$

Taking τ_{A} to be the sum of the sequence $\{\tau_{\text{A}n}\}$, we have

$$\tau_{\text{A}} = \sum_{n=0}^{\infty} \left(\frac{-v}{c}\right)^n \epsilon t, \tag{9}$$

$$\tau_{\text{A}} = \frac{1}{1+\frac{v}{c}}\epsilon t. \tag{10}$$

Similarly the sum of the sequence $\{d_{\text{A}n}\}$ is

$$d_{\text{A}} = \sum_{n=0}^{\infty} v \left(\frac{-v}{c}\right)^n \epsilon t \tag{11}$$

$$= v\tau_{\text{A}}. \tag{12}$$

Since $d_{\text{A}}/\tau_{\text{A}}$ equals v, it is a valid measure of velocity. Consistent with SR and the preceding analysis, $d_{\text{A}}/\tau_{\text{A}}$ is not measurable at a velocity of c, but in limit form it approaches c as v approaches c.

We proceed despite the unknown relativistic factor, by similarly analyzing a geared clock at O, configured symmetrically to the one at P. In accordance with SR and the principle of relativity, movement of P relative to O is equivalent to movement of O relative to P, with differences in appearance or measurements depending on choice of observer. Converse to the preceding analysis, we now consider a fixed apparent time τ_A that is observed to pass, and construct a sequence of partial times t_n that add up to t. Again let us use an unknown factor ϵ' to take care of any appearance of relativistic effects that are not accounted by applying the lemmas to O's clock, defining

$$t_0 = \epsilon' \tau_A \tag{13}$$

such that we may use the lemmas while assuming an absence of length contraction and time dilation, setting ϵ' to account for all neglected effects. With the entirety of τ_A already associated with t_0, but the apparent times associated with the lemmas not yet counted, we can construct $\{t_n\}$ as corrections to t_0 that ensure that the total apparent time observed remains τ_A.

Considering only the geared clock at O, while a time of t_0 elapses, O moves a distance of vt_0 relative to P. Equivalently, P moves the same distance relative to O, which by Lemma 2.1 corresponds to an apparent time of $-vt_0/c$. Including this, the total apparent time considered becomes $\tau_A - vt_0/c$. We can correct this by letting both clocks run for a duration of vt_0/c (which may be negative; letting the clocks run to the correct time must be considered only an abstract calculation, as the clock doesn't actually oscillate to settle on a proper time). Assuming no apparent time dilation, we have a correction of

$$t_1 = \frac{v}{c} \epsilon' \tau_A. \tag{14}$$

Repeating this reasoning for each of t_n, the sum of $\{t_n\}$ is

$$t = \sum_{n=0}^{\infty} \left(\frac{v}{c}\right)^n \epsilon' \tau_A \tag{15}$$

$$= \frac{1}{1 - \frac{v}{c}} \epsilon' \tau_A. \tag{16}$$

Combining Eqns (10) and (16), we have

$$(\frac{\tau_A}{t})^2 = \frac{\epsilon}{1 + \frac{v}{c}} \; \frac{1 - \frac{v}{c}}{\epsilon'}. \tag{17}$$

Here we have two unknown relativistic factors, one representing the contribution of relativistic effects not measured by P's clock in accordance with the lemmas, and the other representing the contribution of effects similarly not measured by O's clock. Proposing that there are no additional or hidden relativistic effects not measured by the combination of both observer's clocks, and that the difference in appearance of relativistic effects is completely described by the lemmas and their recursive application, let us hypothesize that the yet unknown effects of each observer relative to the other are the same and that ϵ and ϵ' are equal. Defining ϵ_A as the rate at which an observed clock appears to tick relative to a local clock, we get

$$\epsilon_A = \frac{\tau_A}{t} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}. \tag{18}$$

This equals the expected reciprocal relativistic Doppler factor. Thus the relativistic Doppler effect is deduced from direct observations and recursive application of Lemmas 2.1 and 2.2.

2.1 Correspondence With the Lorentz Transformation

Solving Eqn (9) for ϵ, using Eqn (18), produces the reciprocal Lorentz factor:

$$\epsilon = \sqrt{1 - \frac{v^2}{c^2}}. \tag{19}$$

Using the variables of the Lorentz transformation given in [3], the time t' corresponds to the proper time τ at x when

$$x = tv. \tag{20}$$

Limiting ourselves to this case, with $\tau_{\mathrm{A}} = \epsilon_{\mathrm{A}} t$ and $d_{\mathrm{A}} = \tau_{\mathrm{A}} v = \epsilon_{\mathrm{A}} x$, Eqn (3) gives

$$\tau = \epsilon_{\mathrm{A}} \left(t + \frac{x}{c} \right). \tag{21}$$

Incorporating Eqns (9), (18), and (20), we get

$$\tau = \frac{1}{\epsilon} \left(t - \frac{v}{c} \frac{x}{c} \right), \tag{22}$$

which is the Lorentz transformation of time.

It is enough to show correspondence with the Lorentz transformation in the case of proper time, as we can define the time of a clock at any other location using any means desired, since we have not assumed any specific coordination among clocks. Thus we may use Einstein synchronization to define a clock at any location, and use its proper time, to establish correspondence with the Lorentz transformation at that location. We need not admit that the coordination of the clocks according to standard simultaneity has any important physical meaning, as the apparent time of uncoordinated or alternatively coordinated clocks can be modelled equally well. Therefore the physical meaningfulness of the Lorentz transformation, for time at locations remote from its reference clock, depends entirely on the physical meaningfulness of standard simultaneity.

Using this derivation, the "mixing of time and space" inherent in the Lorentz transformation only emerges in the conversion from apparent time to classical time, indicating that it is a strictly interpretation-dependent aspect of SR.

2.2 Composition of Velocities

Consider an intermediary observer P' moving along its x-axis, which is also aligned with those of O and P. It has a velocity $v_{P'}$ relative to O, while P has a velocity of v_P relative to P' and v relative to O. While a time of t passes at O, let it observe $\tau_{\mathrm{A}}'(t)$ appear to pass at P', while $\tau_{\mathrm{A}}(\tau_{\mathrm{A}}'(t))$ appears to P' to pass at P. Both O and P' see the same proper time appear to pass at P relative to proper time $\tau_{\mathrm{A}}'(t)$ at P'. This can be confirmed by considering that a signal from an event at P sent to O will arrive at the same time as a similar signal that is relayed through P' without additional delay, regardless of choice of simultaneity. Applying Eqn (18) to the new variables, let

$$\epsilon_{\mathrm{A}}(v_{P'}) = \frac{\tau_{\mathrm{A}}'(t)}{t} \tag{23}$$

be the ratio of apparent time to local time at velocity $v_{P'}$ (as is observed by O), and let

$$\epsilon'_{\mathrm{A}}(v_P) = \frac{\tau_{\mathrm{A}}(\tau'_{\mathrm{A}}(t))}{\tau'_{\mathrm{A}}(t)} \tag{24}$$

be the ratio at velocity v_P (as is observed by P'). Then the ratio at the composite velocity v of P relative to O is

$$\epsilon_{\mathrm{A}} = \frac{\tau'_{\mathrm{A}}(\tau_{\mathrm{A}}(t))}{t} = \epsilon_{\mathrm{A}}(v_{P'})\epsilon'_{\mathrm{A}}(v_P). \tag{25}$$

Solving Eqn (18) for v gives

$$v = c\frac{1-\epsilon_{\mathrm{A}}^2}{1+\epsilon_{\mathrm{A}}^2}. \tag{26}$$

Substituting Eqn (25), then using Eqn (18), results in

$$v = \frac{v_{P'} + v_P}{1 + v_{P'}v_P/c^2}, \tag{27}$$

which is the correct composition of velocities, according to [1].

2.3 Interpretation

All of the preceding calculations of relativistic apparent time are consistent with SR, and with both models. Whether they are meaningful, and directly represents relativistic effects, or simply calculate their appearance, depends on the interpretation of the effects described in Lemma 2.1. If we assume standard simultaneity and that a delay of incoming light causes the effect, then the preceding analysis describes the appearance of clocks in SR, while the delay of light is hidden in the lemmas. However, there is no need to assume this interpretation, or any interpretational explanation of Lemma 2.1 for that matter, for we still properly derive SR's relativistic effects and transformations.

Considering Eqn (10) with respect to the classical model, the effects measured by the lemmas can be entirely attributed to delay of light, while relativistic effects can be entirely separated and contained in ϵ. The disadvantage of this is that the relativistic effect is measured only in conjunction with the delay of light, and there is no visible distinction between the two. The separation comes only from the assumption of standard simultaneity. The fact that the correct relativistic effects can be derived by treating them the same might need to be considered coincidental.

Conversely, by not separately considering delay of light and time dilation, all appearance of relativistic effects of time can consistently be treated as real relativistic effects, with any directly apparent delay between events handled by relativity of simultaneity (while a receiver sees transmission and reception events appearing simultaneous, generally other observers see a delay).

The convoluted nature of apparent distance is demonstrated by two observers P and P', that pass each other at different velocities, and have different apparent distances according to O, yet appear to be at the same location and at the same rest distance. On the other hand, apparent distance conveys the understanding that the two observers' rulers appear to have different unit lengths. It copes with strictly relativistic distances while abandoning the notion of rest distances to moving objects. Informally, in this sense the classical model emerges from measuring the movement of an object relative to a stationary observer, while the apparent model emerges from considering the movement of both object and observer relative to each other, inseparably. Considering Eqns (10) and (16) separately, in the apparent model each effectively expresses relativity incompletely.

Considering that P's clock gear turns ϵ_A times for every turn of O's identical gear, P's position on O's ruler must be a factor of ϵ_A times O's position on P's ruler. This ratio could define an alternative measure of distance, that relates the two moving lengths, instead of defining distance relative to O's rest ruler. In this case the ϵ factor disappears. This alternative of distance would require an alternative measure of speed as well. While other measures of distance may be more naturally suitable for the apparent model, the current definition—chosen for correspondence with the classical model—suffices for this paper. The physical significance of the different measures, and of the factors ϵ and ϵ_A, is seemingly somewhat interpretational anyway.

3 Reduction to the Classical Model of SR

The previous section showed that the apparent model follows from an assumption of the classical model's validity. This section shows that the classical model also follows from the apparent model.

Suppose that the adjustment made to the immediate observations by observer O to correct for a classical delay of light is equivalent to the adjustment needed to translate what O sees into what some other observer S sees. Then what is apparent to S is classically measured by O, and S could be used as a substitute measurement reference for O. Then it need not be a law of nature that multiple different definitions of simultaneity are consistent, nor a mere coincidence, but rather a consequence of the relativity of simultaneity measured by different observers, and the mutual consistency of multiple observers.

We will show that the above hypothesis holds, by finding for different cases a specially located observer S which directly observes the appearance of simultaneity of events as classically modelled for O. The apparent model reduces to the classical model, if for any observation by O, there is an observer S whose immediate observations can be substituted for O's classically delayed observations.

Lemma 3.1 (Local equivalence of classical and apparent models)**.** *In the case that O and P remain at a common location, τ is classically simultaneous with t if τ is apparently simultaneous with t.*

Proof. Let P be collocated with O. Then the apparent distance x_A between O and P is zero. By Proposition 1.3, τ_A and τ are equal. □

Lemma 3.2 (Rest frame correspondence of classical and apparent models)**.** *In case O and P are relatively at rest, and τ and t are classically simultaneous according to O, then τ and t are apparently simultaneous according to S, where S is located exactly midway between O and P.*

Proof. Let S be at rest relative to O and P and midway between them, equidistant to each. According to classical SR, a pair of signals that leave P and O simultaneously will each take the same amount of time to reach S, thus appearing to occur simultaneously at S. □

For the following Lemma, velocities are expressed as ratios of c.

Lemma 3.3 (Inertial motion correspondence of classical and apparent models). *Given P moving at a constant finite velocity β relative to O, where τ and t are classically simultaneous according to O, there is a reference point S such that τ and t are apparently simultaneous from S.*

Proof. Let P be moving inertially at a velocity β relative to observer O. We will construct S in such a way that the Lemma is satisfied.

First, we may assume that O and P are coincident at some instant, since P is moving collinear with O at a fixed non-zero velocity. Let S also coincide with O at the same instant and set its clock to zero then. By Lemma 3.1, S would observe events at P and O as simultaneous at that instant.

Let ϵ be the reciprocal of the Lorentz factor, i.e.

$$\epsilon = \sqrt{1-\beta^2} = \frac{\tau}{t}. \tag{28}$$

A satisfactory S must observe that the apparent rate of time seen passing at P relative to that seen passing at O equals the classical rate of time, according to O, passing at P relative to that at O. Letting τ_{A}' be the apparent time at P observed by S, and letting t_{A}' be the apparent time at O also observed by S, we must have that

$$\frac{\tau_{\text{A}}'}{t_{\text{A}}'} = \epsilon. \tag{29}$$

From S's perspective, S remains stationary while O and P move at fixed velocities. Let β_O and β_P be the respective velocities of O and P relative to S. Since β is the velocity of P relative to O, it must equal the composition of velocities of β_O and β_P.

Since we're using a symmetrical configuration, the velocity of O relative to S is the same as the velocity of S relative to O, which is given by the equation for composition of velocities. From Eqn (27) modified for velocities as ratios of c, this is

$$\beta = \frac{\beta_O + \beta_P}{1 + \beta_O \beta_P}. \tag{30}$$

Let ϵ_P equal $\sqrt{1-\beta_P^2}$ and ϵ_O equal $\sqrt{1-\beta_O^2}$. Expanding Eqn (29) using Eqn (10), we have

$$\epsilon = \frac{\epsilon_P \left(\frac{1}{1+\beta_P}\right) t_S}{\epsilon_O \left(\frac{1}{1+\beta_O}\right) t_S}. \tag{31}$$

Combining Eqns (30) and (31) gives a solution for β_O as a function of β. Discarding the solution where $|\beta_O| > 1$, we have

$$\beta_O = \frac{\beta}{2+\beta}. \tag{32}$$

Substituting Eqn (32) into Eqn (30), we also have

$$\beta_P = \frac{\beta}{2-\beta}. \tag{33}$$

We now know the physically possible respective velocities with which S must travel relative to O and P in order to keep P's clocks appearing to tick at a rate equal to a factor of ϵ times O's clock rate. Given that we have a location for S when its clock is zero, and a constant velocity at which S moves relative to O, we now have S's location relative to O at any time. Observer S will see that τ and t appear simultaneous. $\square$

For any valid $|\beta| < 1$, $|\beta_O|$ will be less than $|\beta|$. Thus S will always be located between O and P. Specifically, S will be closer to O when β is positive, and closer to P when β is negative.

The means of setting the clocks described in the proof does not apply if O and P are separated and at rest. However, as β approaches zero, β_O and β_P approach equality (at zero), and S approaches equidistance to each of O and P. Thus Lemma 3.2 and Lemma 3.3 together provide a formula for the location of S that is a continuous function of β.

Lemma 3.4 (A change in relative simultaneity corresponds to a change in observer coordinates)**.** *Let events E_{O1} and E_{O2} be events at O, occurring respectively before and after an instantaneous change in relative simultaneity with respect to P, and with a negligible proper time between the two events. Assume that O and P are inertial before the change in simultaneity, and are likewise after. Let E_{P1} and E_{P2} be events at P that are classically simultaneous according to O with E_{O1} and E_{O2} respectively. Let τ be the proper time from E_{P1} to E_{P2} (allowing that τ may be negative). Let event E_S at some observer S appear to S to be simultaneous with both E_{O1} and E_{P1}, and event $E_{S'}$ at some observer S' appear to S' to be simultaneous with both E_{O2} and E_{P2}.*

The change in classical simultaneity measured by O as a change in coordinate time of τ at P is equivalent to the change in apparent time at P measured by an abstract observer undergoing a change of coordinates from E_S to E'_S.

Proof. Suppose that the premise's variables and conditions hold. The preceding lemmas ensure that there are valid values for S and S'.

Let E_{P1} occur at time τ_1, and E_{P2} at time τ_2. In accordance with SR, any observers coincident with E_S will observe P appearing as it does to S, i.e. that the time that appears on P's clock is τ_1. Similarly any observer at E'_S will observe that the time that appears on P's clock is τ_2. Then any observer that passes through E_{P1} and E_{P2} agrees that P's clock appears to change by a total of $\tau_2 - \tau_1$ between the two respective events. $\square$

Note that the clocks of S and S' are irrelevant, and there is no requirement that a real observer can physically move between E_S to E'_S. If a real observer is able to, it will observe an apparent time of τ appear to elapse at P. A change in relative classical simultaneity is equivalent to switching from observation at S to that at S'. Since a change in classical simultaneity causes no real physical effect independent of O, there is no expectation that another real observer is physically able to measure its equivalent.

Theorem 3.5 (The apparent model reduces to the classical)**.** *Given a set of pairs of events, where each pair consists of one event at P and another at O which are classically simultaneous according to O, each pair will be apparently simultaneous according to some observer S, where S an abstract observer that is not limited to physically possible motion.*

Proof. This follows for any case of discrete velocities and instantaneous changes between them, from repeated application of Lemmas 3.1 through 3.4. The continuous case can be made by integration of the discrete case. □

Each of the observers labelled S in this section act as a substituted observer for O, directly observing what O is said to measure after compensating for a classical delay of light. For each event or object that O observes, a separate substitute observer can be defined.

Theorem 3.6. *A measurement of the classical speed of any light signal measured from any inertial frame, is equivalent to an apparent measurement of the signal from some location in the inertial frame, at which it appears to propagate at a rate of c.*

Proof. Consider the inertial frame of an arbitrary observer Q, and a light signal from P to O, as indirectly measured by Q.

Let points P' and O' share the inertial frame of Q, and let P' be coincident with P at the time of transmission, and O' be coincident with O at the time of reception. According to SR, the signal from P to O is equivalent to an identically measured signal from P' to O'.

Considering this equivalent signal, there is by Lemma 3.2 a point S from which the signal appears to have propagated at a rate of c, and which is also at rest relative to Q. Thus the classical speed of the signal, measured by Q to be c, is the same as the apparent rate of the signal measured by S. □

In any given inertial frame of reference, for any set of m observations of photons, there is a set $\{S_1, ..., S_m\}$, where each S_n observes the corresponding nth signal appear to propagate at c. This set of observers can be used by all observers in the inertial frame, as substitute measurement references, since all observers' measurements are mutually consistent. The adjustment required to modify O's direct measurements to match those of an appropriate S_n are equivalent to O compensating for a delay of incoming light equal to the distance to S divided by c. Finally, because the set $\{S_1, ..., S_m\}$ is specifically chosen to agree with the classical speed of light, any observer which uses a measurement reference equivalent to one from that set will measure the corresponding signal as having a speed of c.

Thus standard simultaneity and a constant observed speed of light remain valid in the apparent model, both mathematically and intuitively, because there is always a location from which they can be directly measured. The predictions of the Lorentz transformation in the case of inertial motion are real in the apparent model to the extent that a suitable inertial S is always physically possible. Conversely, changes to relative simultaneity that correspond to physically impossible changes to the location of S must be merely abstract. The property of Einstein synchronization of two clocks is apparent according to an appropriate observer S, but not to each of two observers located at the respective clocks, which have symmetrical viewpoints yet observe an apparent error in synchronization.

4 Interpretation

With relative apparent delay of light considered a relativistic effect, apparent relativity is location dependent. Using natural units, the conversion between the classical model and the apparent model is a conversion between time and distance in equal measure, and is equivalent to translating between measurements at different locations. Any change in relative distance corresponds with a change in apparent time. Including an axiom that any elapse of time can be measured by light traveling a set distance, the result is that time and distance can be measured as equivalent.

If we consider time and distance as little more than relations between events, and consider the paths of information traveling at the speed of light between events, then lengths can be understood as straight-line displacements between events, while times can be understood as cumulative distances traveled by some information. This interpretation correlates effectively with humanity's experience of time and distance. Velocity can then be treated as a measure of directness of information, with light traveling a direct route. If mass can be broken down into components that travel at the speed of light, those components travel an indirect route, meandering or curved according to an observer.

Equivalence of time and distance is easier to conceive if the sign of standard time is reversed. An observer can be considered to be at its origin both spatially and temporally, with the time of past events at an ever-increasing positive relative temporal distance. Then the time of a point event is meaningless except relative to another event, and a clock simply measures the elapsed time since some particular reference event. This concept is compatible with accepted interpretation of SR. However, it also implies that local time, and functioning clocks, require some external location in order to be defined.

The classical model implies that a photon travels progressively through all intervening space between its source and destination, which is no longer required or reasonable in the apparent model. Considering that the effects of a photon are only apparent at its source and destination, it consistently behaves as though leaping across the intervening space.

Conclusions

The apparent model and apparent simultaneity are consistent with the postulates and predictions of SR. The assumption of standard simultaneity defined per inertial frame with a delay of light defined per observer is equivalent to an assumption of apparent simultaneity defined per observer. Neither of the two models disprove the other. The differences between the models might represent little more than differences in respective observers' experience of reality.

The classical model's practical benefit of independence of observer location remains available in the apparent model, because a substituted observer S can represent the measurements of multiple observers in an inertial reference frame. The convenience of standard simultaneity in observer-independent systems also remains, because the predictions of such an observer S can be made abstractly, employing the Lorentz transformation without needing to specify the location or existence of S. Where standard simultaneity is inconvenient, such as in describing astronomical observations from Earth's particular vantage point, or in interpreting observer effects in quantum mechanics, the apparent model is beneficial.

Several practical benefits and simplifications of SR are provided by the apparent model, including *a*) use of direct measurements without modification for unobserved delays of light, *b*) unification of measurable delays of light and relative simultaneity, and *c*) lack of modifiable relative simultaneity of events. With this model, there is no need to distinguish between what is real and what is seen. There is no need for the notion of events which have happened but are yet to have any possibly measurable effect. There is no notion of slow inbound photons, or the unintuitive and unmeasurable action of reverting those photons via a change in relative simultaneity.

Ultimately, both the classical model and the apparent model emerge from observations as they appear. The apparent model requires less interpretation of those observations.

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