Lie Algebrized Gaussians for Image Representation

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Abstract

*We present an image representation method which is derived from analyzing Gaussian probability density function (*pdf*) space using Lie group theory. In our proposed method, images are modeled by Gaussian mixture models (GMMs) which are adapted from a globally trained GMM called universal background model (UBM). Then we vectorize the GMMs based on two facts: (1) components of image-specific GMMs are closely grouped together around their corresponding component of the UBM due to the characteristic of the UBM adaption procedure; (2) Gaussian* pdf*s form a Lie group, which is a differentiable manifold rather than a vector space. We map each Gaussian component to the tangent vector space (named Lie algebra) of Lie group at the manifold position of UBM. The final feature vector, named Lie algebrized Gaussians (LAG) is then constructed by combining the Lie algebrized Gaussian components with mixture weights. We apply LAG features to scene category recognition problem and observe state-of-the-art performance on 15Scenes benchmark.*

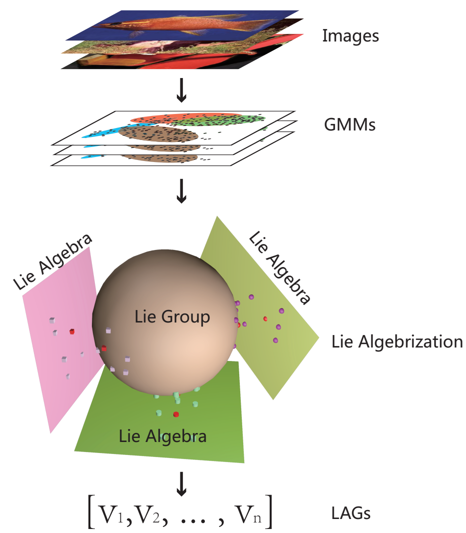


Figure 1. Illustration of LAG feature extraction procedure. Firstly, images are modeled by GMMs over local patch-level features. Each component of GMMs is represented as a point in the Lie group manifold formed by Gaussian *pdf*s. Since components from different GMMs are closely grouped together, we vectorize them by mapping them to tangent space of Lie group. Finally, we combine vectors of each component into our final LAG feature.

1. Introduction

Image representation (feature) is one of the most important tasks in computer vision. Recently Gaussian mixture models (GMMs), which have been widely used for audio representation [13] in speech recognition community, have been adopted to describe images [19][20]. Compared with the popular histogram image representation, GMMs have some attractive advantages (*e.g.* soft assignment, flexible to capture spatial information) and show better performance in many visual recognition applications [12][19][20][18]. One of the major problems of GMMs is that they do not form a vector space and can not convert to vectors trivially. Various vectorization methods for GMM representation have been developed in speech recognition community [13][2][11] and adopted to image classification applications [20]. The problem is clear: mapping elements in a space formed by Gaussian probability density functions (*pdf*s) to a vector space. However, none of the existing solutions take the properties of Gaussian function space into consideration. To do so, a fundamental question should be answered: what kind of space do Gaussian *pdf*s form? Recently, Gong *et al*. [6] theoretically point out that Gaussian *pdf*s are isomorphic to a special kind of affine matrices which form a Lie group. A Lie group is a differentiable manifold which is different from ordinary vector spaces. The structure of the manifold can be analyzed using Lie group theory. Therefore, we can vectorize GMMs to more effective image descriptors by taking the Lie group properties of Gaussian *pdf* space into consideration.

In this paper, we propose a novel image representation by investigating the problem of vectorization GMMs via analyzing Gaussian *pdf* space. Figure 1 gives an overview of our proposed method. The procedure of feature extraction is summarized as the following four major steps.

- *First*, images are modeled as GMMs over dense sampled patches. We employ a maximum a posteriori (MAP) which is used in [13][19][20] to estimate the GMMs: We train global GMM called universal background model (UBM) on the whole image corpus then adapt it to each image. Such a UBM adaptation GMM training approach is much more efficient and effective than the ordinary expectation maximization (EM) algorithm.

- *Then*, we parameterize each component of the GMMs to a upper triangular definite affine transformation (UTDAT) matrix. UTDAT matrices are isomorphic to Gaussian *pdf*s. Since UTDAT matrices form a Lie group (which means Gaussian *pdf*s form a Lie group too), the Gaussian components are points in the Lie group manifold. In figure 1, the Lie group manifold is represented by a sphere surface and Gaussian components are represented by points on the surface. Because GMMs trained by UBM-MAP have the same number of components as the UBM and each component is just a little shift from its corresponding component of the UBM, components are closely grouped together around the corresponding component of the UBM (represented by red points) in the manifold.

- *Next*, we utilize characteristic of UBM adapted GMMs to vectorize their components (*i.e.* UTDAT matrices) by local mapping. To be precise, we map Gaussian components to the tangent space of the Lie group manifold at the point of corresponding component of the UBM. Since the Gaussian components are locally grouped together around the UBM, the mapping preserves the local structure of the manifold. The tangent spaces, which are termed as *Lie algebra*s, are ordinary vector spaces.

- *Finally*, we derive our combined vector formula of GMM by approximating the inner product of GMMs using a sum of product kernel of mixture weights and Lie algebrized components. The final vectorized GMM, which is termed as Lie algebrized Gaussians (LAGs), is an effective vector representation of the original image and is suitable for well known machine learning algorithms.

We apply our proposed LAG feature to scene category recognition. Discriminant nuisance attribute projection (NAP) [17] is employed to reduce the intra-class variabilities. Then a simple nearest centroid (NC) classifier is adopted to perform the classification task. Our method show better performance than state-of-the-art methods on 15Scenes benchmark dataset. To be precise, we get 88.4% average accuracy. Furthermore, experimental results show that our Lie algebrization approach is superior to the widely used KullbackLeibler (KL) divergence based vectorization method.

The remaining of this paper is arranged as follows. In section 2, we present a short review of the related work. Section 3 describe the technical detail of LAG. In section 4, the method for image classification using LAG feature is given. Experimental results on 15Scenes dataset are reported in section 5. Conclusions are made and some future research issues are given.

2. Related Work

In recent years bag-of-features (BoF) image representation has been widely investigated in visual recognition systems. Inspired by the bag-of-word (BoW) idea in text information retrieval, BoF treats an image as an collection of local feature descriptors extracted at densely sampled patches or sparse interest points, encodes them into discrete "visual words" using K-means vector quantization (VQ), then builds a histogram representation of these visual words [14]. One major problem of BoF approach is that spatial order of the local descriptors is discarded. Spatial information is important for many visual recognition applications (*e.g.* scene categorization and object recognition). To overcome this problem, Lazebnik *et al.* propose a BoF extension called spatial pyramid matching (SPM) [10]. In the SPM approach, an image is partitioned into $2^l \times 2^l$ sub-images at different scale level $l = 0, 1, 2 \ldots$. Then BoF histogram is computed for each sub-image. Finally, a vector representation is formed by concatenating all the BoF histograms. Because of its remarkable improvements on several image classification benchmarks like Caltech-101 [4] and Caltech-256 [9], SPM has become a standard component in most image classification systems.

On the other hand, GMMs are widely used for speech signal representation and have become a standard component in most speaker recognition systems [11][2][13]. In GMM based speech signal representation, low-level features are extracted at local audio segments, then a GMM is estimated on these features for each speech clip. Reynolds *et al.* [13] propose a novel GMM training method called universal background model (UBM) adaptation. UBM adaptation employ a maximum a posterior (MAP) approach instead of normally used maximum likelihood (ML) approach, *e.g.* expectation maximization (EM). UBM adaption produces more discriminative GMM representations and is more efficient than ML estimation. Compared with BoF histogram representation, GMMs encode the local features in a continuous probability distribution using soft assignment instead of hard vector quantization. Zhou *et al.* [20] adopt GMM to image representation and report superior performance than SPM in several image classification applications. The problem of GMM representation is that GMMs do not form a vector space and can not be converted to vectors trivially.

To get effective vector representation, one should vectorize GMMs according to the structure properties of the space they formed. However, none of the existing approaches (*e.g*. [11][2]) take the structure properties of GMM space into consideration.

Feature space structure analysis is a new computer vision topic investigated in recent years. Tuzel *et al*. [15] analyze the space structure formed by covariance matrices in a cascade based object detection scenario. A boosting algorithm is used to train the node classifiers on covariance features for the detection cascade. Since covariance matrices form a Riemannian manifold, they are mapped to tangent space at their mean point before feeding to the weak learner of each boosting iteration. Compared with treating covariance matrices as vectors trivially, significant improvement is gained by taking the Riemannian manifold property of covariance feature space into consideration during machine learning. Gong *et al*. [6] derive a Lie group distance measure for Gaussian *pdf*s by analyzing the structure of a special kind of affine transformation matrix which is isomorphic to Gaussian *pdf*. It has been found empirically that Lie group based Gaussian *pdf* distance is superior to the widely used Kullback-Leibler (KL) divergence [8][7].

In this paper, we derive a feature descriptor by analyzing UBM adapted GMMs using Lie group theory. Compared with covariance [15] and Gaussian descriptor [6], our proposed LAG descriptor is a kind of holistic descriptors rather than local descriptors. Compared with previous GMM based audio and image representation method, our proposed method takes the structural properties of UBM adapted GMMs into consideration. Experiment results on scene recognition prove the effectiveness of our method.

3. Lie Algebrized Gaussians for Image Representation

3.1. Image Modeling Using UBM adapted GMM

We extract local features within densely sampled patches and represent an image using the probability distribution of its local features. Specifically, kernel descriptors [1] are computed for each patch. The distribution of local features within an image are modeled by a GMM. Let s denote patch-level feature vector. The *pdf* of s is modeled as

$$p(s|\Theta) = \sum_{k=1}^{K} \omega_k \boldsymbol{\mathcal{N}}(s; \mu_k, \Sigma_k) \tag{1}$$

where K denotes number of Gaussian components. $\mathcal{N}$ is multivariate normal *pdf*. ω_k, μ_k and Σ_k are the weight, mean vector and covariance matrix of the k_{th} component. For efficiency consideration, we restrict Σ_k to be a diagonal matrix. $\Theta \equiv \{\omega_k, \mu_k, \Sigma_k\}_{k=1,2,\ldots,K}$ denotes the whole parameter set of GMM.

The descriptive capability of GMM increases with the number of Gaussian components K. Normally, hundreds of Gaussians are required to build an effective representation. Compared with the number of parameters, however, the number of patches is small and insufficient to train a GMM using a conventional EM approach. Moreover, EM is time-consuming for GMMs with hundreds of Gaussians. To overcome these problems, we employ a UBM adaptation approach [13] to estimate the parameters. The adaptation contains two steps: Firstly, a global GMM (*i.e*. UBM) is trained using patches from the training set. Then, the parameters of each image-specific GMM are adapted from the UBM using a one iteration maximum a posterior approach as follows

$$\omega_k = [\alpha_k n_k / T + (1 - \alpha_k)\bar{\boldsymbol{\omega}}_k]\gamma \tag{2}$$

$$\mu_k = \alpha_k \mathbf{E}_k(s) + (1 - \alpha_k)\bar{\boldsymbol{\mu}}_k \tag{3}$$

$$\sigma_k^2 = \alpha_k \mathbf{E}_k(s^2) + (1 - \alpha_k)(\bar{\boldsymbol{\sigma}}_k^2 + \bar{\boldsymbol{\mu}}_k^2) - \mu_k^2 \tag{4}$$

where T is the number of patches in a specified image. $\bar{\boldsymbol{\omega}}_k$, $\bar{\boldsymbol{\mu}}_k$ and $\bar{\boldsymbol{\sigma}}_k$ are the weight, mean and standard deviation of the k_{th} mixture of UBM. ω_k, μ_k and σ_k are the weight, mean and standard deviation of the k_{th} mixture of image-specific GMM. The scale factor, γ, is computed over all adapted mixture weights to ensure they sum to unity. α_k is the adaptation coefficient used to control the balance between UBM and image-specific GMM. n_k, $\mathbf{E}_k(s)$ and $\mathbf{E}_k(s^2)$ are the sufficient statistics of s used to compute mixture weights, mean and covariance.

$$n_k = \sum_{t=1}^{T} \mathbf{Pr}(k|s_t) \tag{5}$$

$$\mathbf{E}_k(s) = \frac{1}{n_k} \sum_{t=1}^{T} \mathbf{Pr}(k|s_t) s_t \tag{6}$$

$$\mathbf{E}_k(s^2) = \frac{1}{n_k} \sum_{t=1}^{T} \mathbf{Pr}(k|s_t) s_t^2 \tag{7}$$

where $\mathbf{Pr}(k|s_t)$ is the posterior probability that the t_{th} patch belongs to the k_{th} Gaussian components.

$$\mathbf{Pr}(k|s_t) = \frac{\bar{\boldsymbol{\omega}} \boldsymbol{\mathcal{N}}(s_t; \bar{\boldsymbol{\mu}}_k, \bar{\boldsymbol{\sigma}}_k)}{\sum_{m=1}^{K} \bar{\boldsymbol{\omega}}_m \boldsymbol{\mathcal{N}}(s_t; \bar{\boldsymbol{\mu}}_m, \bar{\boldsymbol{\sigma}}_m)} \tag{8}$$

For each mixture, a data-dependent adaptation coefficient α_k is used, which is defined as

$$\alpha_k = \frac{n_k}{n_k + r} \tag{9}$$

where r is a fixed control value to give penalty to mixtures with lower posterior probability.

The parameters of image-specific GMM encode the distributions of local patch-level features from a specified image, thus can be used as an effective visual representation

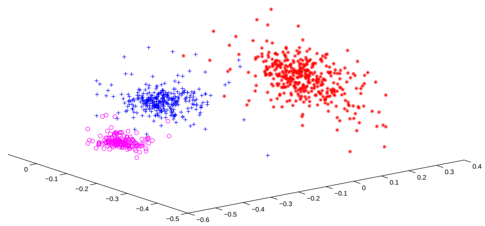


Figure 2. Components of UBM adapted GMMs are closely grouped together. We choose 3 dimension of 3 components from different GMMs and plot them in 3-d Euclidean space. Each point represent a Gaussian component. Different color and marker indicate different component index (*i.e.* first, second or third component of a GMM).

of that image. On one hand GMMs are continuous *pdf*s which can avoid vector quantization problems in discrete distribution estimation approach such as histograms. But on the other hand, GMMs are not vectors essentially thus are not suited to most well-known classifiers, especially linear classifiers. Of course we may simply concatenate the parameters as vector. But the structural information of the original GMM space are also regrettably discarded. The most straightforward way to use the structure information of GMM feature space is to identify what kind of space it is and then analyze it using existing theory. Although general GMMs are complex distributions whose space structure are difficult to be analyzed, we observe that UBM adapted GMMs have some special characteristics which can help us to analyze its structure.

In figure 2, we choose three dimensions of three components from UBM adapted GMMs trained on 15Scenes dataset and plot them as points in euclidean space. It can be observed that the components of these GMMs are closely grouped together around the components of UBM. Such a characteristic can be explained by the behavior of UBM adaptation procedure. In UBM adaptation, the MAP estimation contains one EM-like iteration only. Moreover, a adaptation coefficient prevents the resultant GMM shift too far from the prior distribution (*i.e.* UBM) in order to avoid under-fitting. Since components of image-specific GMMs are closely grouped together, they have correspondence across images. Therefore, we can analyze Gaussian components separately then fuse the results together. In the rest of this section, we show that Gaussian *pdf*s form a Lie group, then derive a vectorization method for GMM from analyzing Gaussian *pdf* space using Lie group theory.

3.2. Gaussian *pdf*s and Lie group

Let $\mathbf{x}_0$ denote a random vector which is standard multivariate Gaussian distributed (*i.e.* the mean and covariance are zero vector and identity matrix respectively). Let $\mathbf{x}_1 = A\mathbf{x}_0 + \mu$ be a resultant vector of an invertible affine transformation from $\mathbf{x}_0$. From the properties of multivariate Gaussian distribution, we can know that $\mathbf{x}_1$ is also multivariate Gaussian distributed. Furthermore, the mean vector and covariance matrix of $\mathbf{x}_1$ are μ and AA^T respectively. More generally speaking, any invertible affine transformation can produce a multivariate Gaussian distribution. Furthermore, if we restrict A to be upper triangular and definite, we can get an unique A given a arbitrary multivariate distribution by Cholesky decomposition, which means there is a bijection between Gaussian *pdf*s and upper triangular definite affine transformation (UTDAT). Therefore, Gaussian *pdf*s are isomorphic to UTDATs. Let M denote the matrix form of UTDAT which is defined as follow.

$$M = \begin{bmatrix} A & \mu \\ 0 & 1 \end{bmatrix} \tag{10}$$

We can analyze M instead of Gaussian *pdf*s.

Invertible affine transformations form a Lie group and matrix multiplication is its group operator. UTDAT which is a special case of invertible affine transformation is closed under matrix multiplication operation. Therefore, UTDAT is a subgroup of Invertible affine transformation. Since any subgroup of a Lie group is still a Lie group, UTDAT is a Lie group. In conclusion, Gaussian *pdf*s form a Lie group.

In mathematics, a Lie group is a group which is also a differentiable manifold, with the property that the group operations are compatible with the smooth structure. An abstract Lie group could have many isomorphic instances. Each of them is an representation of the abstract Lie group. In Lie group theory, matrix representation [16] is a useful tool for structure analysis. In our case, UTDAT is the matrix representation of the abstract Lie group formed by Gaussian *pdf*s. Specially, covariance matrices of our GMMs are diagonal thus A is diagonal too. Precisely, UTDAT of the k_{th} component is defined as follow.

$$M_k = \begin{bmatrix} \sigma_{k1} & & & & \mu_{k1} \\ & \sigma_{k2} & & & \mu_{k2} \\ & & \sigma_{k3} & & \mu_{k3} \\ & & & \ddots & \vdots \\ 0 & 0 & 0 & & 1 \end{bmatrix} \tag{11}$$

where $\mu_k(d)$ and $\sigma_k(d)$ are the mean and standard deviation of the d_{th} dimension of the k_{th} Gaussian component respectively.

3.3. Lie Algebrization of Gaussian components

As discussed before, components of UBM adapted GMM are closely grouped together around its corresponding component of UBM thus we can preserve most of their structure information by projecting them to the tangent space of Lie group at the point of the corresponding component. In mathematics, the tangent space of a manifold facilitates the generalization of vectors from affine spaces to general manifolds, since in the latter case one cannot simply subtract two points to obtain a vector pointing from one to the other. Analogous to a tangent plane of a sphere, a tangent space of a Lie group is a vector space. To best preserve the structure information of a collection of points in a manifold, the target vector space should be the tangent space at the mean points of the point set [15]. In our case, UBM is an approximation of all the image-specific GMMs thus we use components of UBM as mean Gaussian *pdf*s.

Let M_k and $\bar{\boldsymbol{M}}_k$ denote the k_{th} component (UTDAT matrix form) of an image-specific GMM and UBM. Let m_k denote the corresponding point in the tangent space projecting from M_k. The projection is accomplished via matrix logarithm.

$$\mathrm{m}_k = \log(\bar{\boldsymbol{M}}_k^{-1} M_k) \tag{12}$$

Note that here $\log$ is matrix logarithm rather than element-wise logarithm of a matrix. Since tangent space of an Lie group is a vector space, m_k is a vector thus we can unfold elements of m_k to a vector form.

Although we can project Gaussian components using equation (12), it is not efficient. The $\log$ operation in (12) requires Schur decomposition of $\bar{\boldsymbol{M}}_k^{-1} M_k$ [3], which is time-consuming. Fortunately, covariance matrices of Gaussian components are diagonal in our case thus we can develop a efficient scalar form of $\log$.

Here we derive our scalar form of UTDAT matrix logarithm. For diagonal Gaussian components, each dimension of transformation is independent. So we analyze the 1-d case of UTDAT logarithm first. Here we let M be a 1-d UTDAT matrix with the form

$$M = \begin{bmatrix} \sigma & \mu \\ 0 & 1 \end{bmatrix} \tag{13}$$

and let $K = M - I$ where I is a 2-d identity matrix. Using the series form of matrix logarithm, we have

$$\begin{aligned} m &= \log(M) && (14)\\ &= \log(I + K) && (15)\\ &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{K^n}{n} && (16)\\ &= \begin{bmatrix} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\sigma-1)^n}{n} & \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\mu(\sigma-1)^{n-1}}{n} \\ 0 & 0 \end{bmatrix} && (17)\\ &= \begin{bmatrix} \log(\sigma) & \mu\log(\sigma) \\ 0 & 0 \end{bmatrix} && (18) \end{aligned}$$

Note that we can always scale the matrix using the following equations in order to make sure the series convergent

$$\begin{aligned} \log A &= \log(\lambda(I + B)) && (19)\\ &= \log(\lambda I) + \log(I + B) && (20)\\ &= (\log\lambda) I + \log(B) && (21) \end{aligned}$$

Using the above equations, we have

$$\begin{aligned} \log(M_1^{-1} M_2) &= \log(\begin{bmatrix} \sigma_1^{-1} & -\mu_1\sigma_1^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_2 & \mu_2 \\ 0 & 1 \end{bmatrix}) && (22)\\ &= \log(\begin{bmatrix} \sigma_1^{-1}\sigma_2 & (\mu_2 - \mu_1)\sigma_1^{-1} \\ 0 & 1 \end{bmatrix}) && (23)\\ &= \begin{bmatrix} \log(\frac{\sigma_2}{\sigma_1}) & (\mu_2 - \mu_1)\frac{\log(\sigma_2) - \log(\sigma_1)}{\sigma_2 - \sigma_1} \\ 0 & 0 \end{bmatrix} && (24) \end{aligned}$$

Note that we use (18) to derive (24) from (23). Finally, we can get our projected Gaussian component m_k using the above equations.

$$\mathrm{m}_k = \begin{bmatrix} \log\frac{\sigma_{k1}}{\bar{\boldsymbol{\sigma}}_{k1}} & & & \frac{(\mu_{k1} - \bar{\boldsymbol{\mu}}_{k1})\log\frac{\sigma_{k1}}{\bar{\boldsymbol{\sigma}}_{k1}}}{\sigma_{k1} - \bar{\boldsymbol{\sigma}}_{k1}} \\ & \log\frac{\sigma_{k2}}{\bar{\boldsymbol{\sigma}}_{k2}} & & \frac{(\mu_{k2} - \bar{\boldsymbol{\mu}}_{k2})\log\frac{\sigma_{k2}}{\bar{\boldsymbol{\sigma}}_{k2}}}{\sigma_{k2} - \bar{\boldsymbol{\sigma}}_{k2}} \\ & & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

If we assume that σ always equals $\bar{\boldsymbol{\sigma}}$ (*i.e.* adapt mean only and keep covariance unchanged during UBM adaptation), m_k is reduced to $\hat{\mathrm{m}}$

$$\hat{\mathrm{m}}_k = \begin{bmatrix} \frac{(\mu_{k1} - \bar{\boldsymbol{\mu}}_{k1})}{\bar{\boldsymbol{\sigma}}_{k1}}, & \frac{(\mu_{k2} - \bar{\boldsymbol{\mu}}_{k2})}{\bar{\boldsymbol{\sigma}}_{k2}}, & \ldots \end{bmatrix} \tag{26}$$

using the fact

$$\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1 \tag{27}$$

Compared with m_k, $\hat{\mathrm{m}}_k$ represents each dimension using a scalar (while m_k uses a 2-d vector) and discards the covariance information of GMM.

3.4. Lie Algebrized Gaussians

After vectorization of Gaussian components, we fuse them together to get a vectorized GMM. We derive our vectorized GMM from a product kernel. Let a and b denote two GMMs, we use kernel function $f(a, b)$ defined as follow

$$f(a, b) = \sum_{k=1}^{K} f_\omega(\omega_k^a, \omega_k^b) f_{\mathrm{m}}(\mathrm{m}_k^a, \mathrm{m}_k^b) \tag{28}$$

where f_a and f_b are kernel functions for mixture weights and vectorized Gaussian *pdf*s. Using linear inner product $f_{\mathrm{m}}(x, y) = x^T y$ for vectorized Gaussian *pdf* and

$f_w(x,y) = \sqrt{xy}$ for mixture weights, we get

$$f(a,b) = \sum_{k=1}^{K} \sqrt{\omega_k^a}\sqrt{\omega_k^b}(\mathrm{m}_k^a)^T \mathrm{m}_k^b \tag{29}$$

$$= \sum_{k=1}^{K} (\sqrt{\omega_k^a}\mathrm{m}_k^a)^T (\sqrt{\omega_k^b}\mathrm{m}_k^b) \tag{30}$$

Using equation (30), we designed our final vector V_{lag} as

$$V_{lag} = [\sqrt{\omega_1}\mathrm{m}_1, \sqrt{\omega_2}\mathrm{m}_2, \ldots, \sqrt{\omega_K}\mathrm{m}_K] \tag{31}$$

The final vector V_{lag}, which is named Lie algebrized Gaussians (LAG), is an effective representation of the original image and is suitable for most known machine learning techniques. If we replace m_k with $\hat{\mathrm{m}}_k$ in (31), we can get a reduced LAG (rLAG) vector which has lower dimensionality but less discriminative.

4. Scene Category Recognition Using LAG Feature

We apply our LAG feature to scene category recognition. Scene recognition is a typical and important visual recognition problem in computer vision. Some digital cameras (*e.g.* Sony W170 and Nikon D3/300) are also starting to include "Intelligent Scene Recognition" modules to help selecting appropriate aperture, shutter speed, and white balance.

To address the scene recognition problem, we represent each image using our proposed LAG vector. Since SPM [10] have been proved empirically to be a useful component for various visual recognition system, we adopt it to our LAG based representation. Specifically, we divide image into sub-images in the same manner as SPM and extract LAG features for each sub-image, then combine these LAG vectors for image representation. Then we reduce within-class variability of LAGs using discriminant nuisance attribute projection [17]. For efficiency reasons, we employ a simple nearest centroid (NC) classifier to classify NAP projected LAG features into different scene categories.

5. Experimental Results

We test our method on the 15Scenes dataset [10]. The scene dataset contains fifteen scene categories, thirteen of them is provided by Fei-Fei *et al.* in [5]. Each scene category contains about 400 images. The size of each image is about 300×250 pixels. This dataset is the most comprehensive one for scene category recognition.

We extract kernel descriptors [1] on densely sampled patches for each image. Specifically, three types of kernel descriptors are used: color, gradient and LBP kernel descriptors. Large images are resized to be no larger than 300×300. 16×16 and 24×24 patches with 4 pixel step are used. The resultant kernel descriptors are reduced to 50-d using principal component analysis (PCA). Each 50-d vector is then combined with the normalized x-y spatial coordinates of the center of its patch window. Therefore, the final patch descriptors are 52-d which contains both appearance and spatial information of the patches. We model each image using GMMs with 512 components. Specifically, we divide each image into 1×1 and 2×2 pyramid-like sub-images and estimate a GMM for each sub-image (5 GMMs for an image in total). The corresponding 5 LAG vectors are concatenated to a single vector. To test scene recognition performance, we randomly select 100 images from each category for training and the rest for testing. The experiments are repeated 10 times and 88.4% average recognition accuracy is obtained. We assemble the performance of our algorithm and various state-of-the-art algorithms in table 1 for comparison. The results show that our method

Algorithm	Average Accuracy(%)
Histogram [5]	65.2
SPM [10]	81.4
HG [20]	85.3
KDES+LinSVM [1]	81.9
KDES+LapKSVM [1]	86.7
LAG+NC (this paper)	**88.4**

Table 1. Performance of different algorithms on 15Scenes dataset. Our LAG+NC method achieves the state-of-the-art performance.

outperform all the other algorithms. Kernel descriptor with Laplacian kernel SVM, which is the second best, obtains 86.7% average recognition accuracy. Note that our method uses a simple NC classifier rather than kernel machines. It is also observed that Laplacian kernel SVM boost the performance of kernel descriptors a lot from linear SVM (81.9%). So it is interesting to see what kind of kernel SVM can boost the performance of our LAG representation. However, our method is more practical because NC classifier is suitable for large scale dataset.

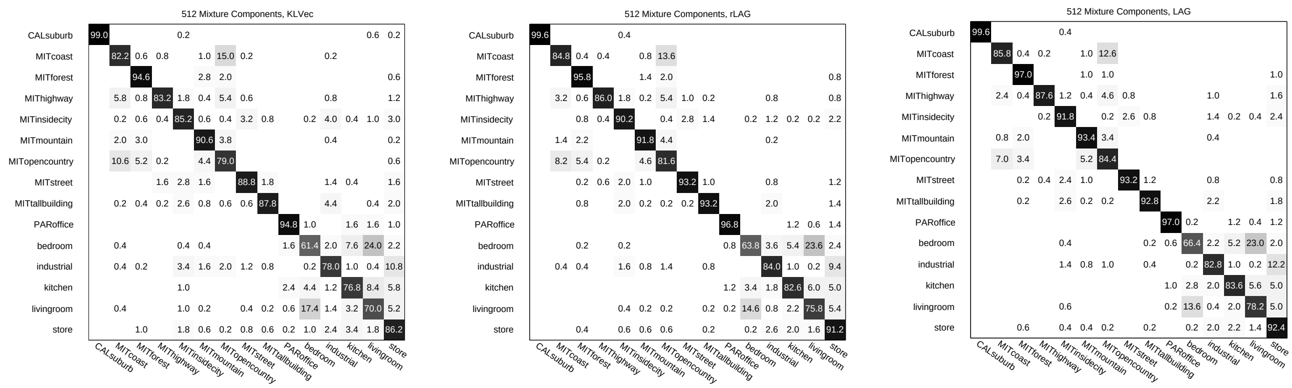
The third best algorithm [20] in table 1 using a KL divergence based vectorization together with a spatial information scheme called Gaussian map as image representation. KL divergence based vectorization (KLVec) is a most widely used GMM vectorization approach and has been empirically proved to be effective in many applications [2][18][19]. KLVec has the following form

$$V_{kl} = [\sqrt{\omega_1}\mu_1\bar{\boldsymbol{\sigma}}_1^{-1}, \sqrt{\omega_2}\mu_2\bar{\boldsymbol{\sigma}}_2^{-1}, \ldots, \sqrt{\omega_K}\mu_K\bar{\boldsymbol{\sigma}}_K^{-1}] \tag{32}$$

For a specified dimension (*e.g.* d_{th}) of a specified component (*e.g.* k_{th}), KLVec encodes the distribution as

$$\frac{\mu_{kd}}{\bar{\boldsymbol{\sigma}}_{kd}} \tag{33}$$

From (33), we can clearly see that our LAG feature is different from KLVec in two aspect: Firstly, KLVec discard



(a) (b) (c)

Figure 3. Confusion matrices of the three vectorization approaches: (a) KLVec, (b) reduced LAG and (c) LAG. The entry in the i_{th} row and j_{th} column is the percentage of images from the i_{th} class and classified as the j_{th} class.

covariance information of GMM. Secondly, mean vector in LAG is centralized by subtracting the corresponding mean of UBM. Furthermore, the difference between RLAG and KLVec is mean centralization only.

To compare KLVec with LAG and RLAG empirically, we implement both KLVec and test it in the same scenario with same parameter settings as LAG and rLAG. The average accuracies of the three vectors are presented in table 2.

We observe that LAG is significantly superior to KLVec (88.4% vs 83.8%). The reduced LAG achives 87.3% accuracy, which indicates that the centralization operation of LAG feature is important. The detailed confusion matrices of the three vectorization approaches are present in figure 3. Note that our system with KLVec obtain lower accuracy than the system in [20], the reason might be that some components (*e.g.* spatial Gaussian maps) is not included in our system.

Algorithm	Average Accuracy(%)
KLVec	83.84 ± 1.23
rLAG (this paper)	87.36 ± 0.95
LAG (this paper)	88.40 ± 0.96

Table 2. Comparison of different vectorization approach. The centralization operation of reduced LAG considerably improves the performance compared with KLVec. The covariance information in LAG vector is also useful for recognition, which improves another 1% from reduced LAG feature.

We test the three vectorization approaches (LAG, rLAG, KLVec) with different number of Gaussian mixture components and keep the same setting for the other parameters as described above. The results are shown in table 3. According to the table, LAG is always superior to rLAG and KLVec. Moreover, LAG gains fair performance when the number of Gaussian mixture components is just set to 32. This phenomenon demonstrates that the covariance matrix information which is represented by LAG is very useful.

6. Conclusion and Future Work

We analyze the structure of UBM adapted GMMs and derive a Lie group based GMM vectorization approach for image representation. Since Gaussian *pdf*s form a Lie group and components of UBM adapted GMMs are closely grouped together around UBM, we map each component of a GMM to tangent space (Lie algebra) of Lie group at the position of corresponding component of UBM. Such a kind of vectorization approach (named Lie algebrization) preserves the structure of Gaussian components in the original Lie group manifold. The final Lie algebrized Gaussians (LAG) features are constructed by combining Lie algebrized Gaussian components with mixture weights. We apply LAG to scene category recognition and achieve state-of-the-art performance on 15Scenes benchmark with a simple nearest centroid classifier. Experimental results also show that our vectorization approach is considerably superior to the widely used KL divergence based vectorization method.

There are several interesting issues about LAG based image representation we shall investigate in the future. Firstly, we shall apply LAG to other visual recognition problems, such as object recognition, action recognition. Secondly, it is interesting to develop a kernel classifier for GMM using its Lie group structure. Finally, applying LAG feature to audio representation and comparing it with KL divergence based vectorization is another interesting topic.

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	average accuracy(%)		
Mixture Number	LAG	rLAG	KLVec
32	86.41±0.87%	82.59±1.43%	82.59±1.15%
64	86.88±1.25%	84.76±1.19%	84.27±1.28%
128	87.55±1.37%	86.11±1.09%	84.57±1.64%
256	88.17±1.69%	87.03±1.16%	84.24±1.18%
512	88.40±0.96%	87.36±0.95%	83.84±1.23%
1024	87.73±1.38%	87.39±0.86%	83.39±1.30%

Table 3. Results for different Gaussian mixture number.

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