Primality Criteria for Specific Classes of Proth Numbers

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Abstract : Polynomial time prime testing algorithms for specific classes of Proth numbers are introduced .
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1 Introduction

In 1960 Kusta Inkeri provided uncoditional , deterministic , lucasian type primality test for Fermat numbers [1] . In this note we present lucasian type primality tests for specific classes of Proth numbers .

2 Main result

Conjecture 1.
Let $N = k \cdot 2^n + 1$, such that $n > 2$, k odd , $k < 2^n$ and
$k \equiv 5, 19 \pmod{42}, with\ n \equiv 0 \pmod{3}$, or
$k \equiv 13, 41 \pmod{42}, with\ n \equiv 1 \pmod{3}$, or
$k \equiv 17, 31 \pmod{42}, with\ n \equiv 2 \pmod{3}$, or
$k \equiv 23, 37 \pmod{42}, with\ n \equiv 0, 1 \pmod{3}$, or
$k \equiv 11, 25 \pmod{42}, with\ n \equiv 0, 2 \pmod{3}$, or
$k \equiv 1, 29 \pmod{42}, with\ n \equiv 1, 2 \pmod{3}$
Next , define sequence S_i :
$S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(5)$

where $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4} \right)^m + \left(x + \sqrt{x^2-4} \right)^m \right)$
, then
N is a prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.
Let $N = k \cdot 2^n + 1$, such that $n > 2$, k odd , $k < 2^n$ and
$k \equiv 1 \pmod{6}$ *and* $k \equiv 1,7 \pmod{10}$, *with* $n \equiv 0 \pmod{4}$, or
$k \equiv 5 \pmod{6}$ *and* $k \equiv 1,3 \pmod{10}$, *with* $n \equiv 1 \pmod{4}$, or
$k \equiv 1 \pmod{6}$ *and* $k \equiv 3,9 \pmod{10}$, *with* $n \equiv 2 \pmod{4}$, or
$k \equiv 5 \pmod{6}$ *and* $k \equiv 7,9 \pmod{10}$, *with* $n \equiv 3 \pmod{4}$
Next , define sequence S_i :
$S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(8)$
where $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4} \right)^m + \left(x + \sqrt{x^2-4} \right)^m \right)$
, then
N is a prime iff $S_{n-2} \equiv 0 \pmod{N}$

References

[1] Inkeri, K., Tests for primality, Ann. Acad. Sci. Fenn. A I 279, 119 (1960).