Generalization of Lucas-Lehmer-Riesel Test

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Abstract : Generalization of Lucas-Lehmer-Riesel primality test is introduced .
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1 Introduction

In number theory the Lucas-Lehmer-Riesel test [2] , is the fastest deterministic primality test for numbers of the form $k \cdot 2^n - 1$ with k odd and $k < 2^n$. The test was developed by Hans Riesel and it is based on Lucas-Lehmer test [1] .

2 Main result

Conjecture. *Let* $N = k \cdot b^n - 1$ *with* $b \equiv 0 \pmod 2$, $b \not\equiv 0 \pmod 3$,
b $\not\equiv 0 \pmod 5$, $k \equiv 1, 5 \pmod 6$, $k < b^n$, $n > 2$, *and*
$S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(4))$, *where*
$P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *then :*
N *is prime iff* : $S_{n-2} \equiv 0 \pmod N$

Maxima implementation

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b;k;n;
(s:2*chebyshev_t(b*k/2,chebyshev_t(b/2,2)),
for i from 1 thru n-2 do(s:mod(2*chebyshev_t(b,s/2),k*b^n-1)))$
(if(s=0) then print("prime") else print("composite"));
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References

[1] Crandall, Richard; Pomerance, Carl (2001), "Section 4.2.1: The Lucas-Lehmer test", Prime Numbers: A Computational Perspective (1st ed.), Berlin: Springer, p. 167-170
[2] Riesel, Hans (1969). "Lucasian Criteria for the Primality of $N = h \cdot 2^n - 1$". Mathematics of Computation (American Mathematical Society) 23 (108): 869-875