Special Relativity Predictions Using The Derivative

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Abstract

Assume a light pulse is emitted from the origins of two frames, F and F' that are in relative motion, when the origins are common. Based on the relativity and light postulates of special relativity (SR), all light beams travel at c. Therefore, the light pulse expands spherically from the origin of F if and only if it expands spherically from the origin of F'. However, by using the derivative, this paper will report a mathematical violation of this logic. Additionally, it will be proven SR predicts one spherical light wave (SLW) moves two different directions along a single line.

Keywords – Special Relativity, Light Sphere, Light Cone, Spherical light wave

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Introduction

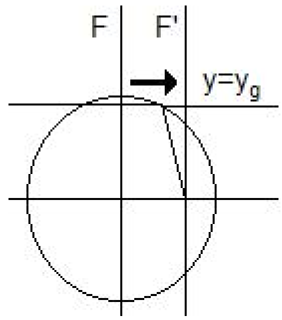
Assume two coordinate systems F and F' are in relative motion in the standard configuration and a light pulse is emitted from the origins when they are co-located. Under these conditions of SR the SLW expands spherically from the origin of F if and only if it expands spherically from the origin of F'. So, for example, if there is an observer at the origin of each frame communicating with each other, each would continuously report that the SLW is expanding spherically from the origin of their respective frame.

Thus, all possible calculations of SR must come to this conclusion. Now, if we assume the SLW expands spherically from the origin of F, the SLW spherical calculations for F’ do not have to agree on the expansion because the frames are in no way required to agree on the time at any SLW event. But, any calculation under SR that is consistent with the SLW expansion in F must imply the SLW expansion in F’ since that is physically what is going on in F’. Otherwise, if this rule is broken by SR, then SR would predict a falsehood in F’ and would therefore be inconsistent.

However, assume some fixed $y_g > 0$. Further assume the SLW in the view of the F frame is located in between the two origins and intersecting the line $y = y_g$. Under these conditions, this article will prove the expansion of the SLW in F causes the SLW to move closer to the F’ origin when the SLW is measured from its location along the line $y = y_g$ to the origin of F’. Therefore, there is a calculation under SR such that the expansion of the SLW in the context of F is inconsistent with the expansion of the SLW in the F’ frame. Further, it will be proven SR predicts that the one SLW, while it is in between the two origins, moves two different directions along the line $y = y_g$.

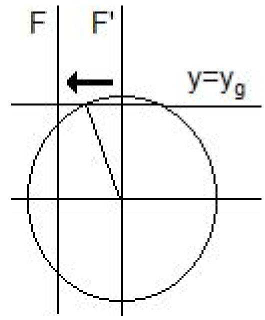
Method

Assume two coordinate systems F and F’ are in relative motion in the standard configuration and a light pulse is emitted from the origins when they are co-located. The following two figures represent the predictions of SR for the SLW given the unprimed frame values of $x > 0$, $y_g > 0$ with $y = y_g$, $z = 0$ and $y_g/c < t < y_g\gamma/c$.



In the view of the unprimed frame, the SLW must expand spherically from the origin of the frame by the light postulate. When the SLW is in between the two origins and intersecting the line $y=y_g$, the expanding SLW moves closer to the primed origin when measured from the line $y=y_g$. The arrow represents the direction of intersection of the SLW with the line $y=y_g$.

Figure 1



In the view of the primed frame, the SLW must expand spherically from the origin of the frame by the light postulate. When the SLW is in between the two origins and intersecting the line $y=y_g$, the expanding SLW moves further from the primed origin when measured from the line $y=y_g$. The arrow represents the direction of intersection of the SLW with the line $y=y_g$.

Figure 2

It will be proven that SR predicts the results in both figure 1 and figure 2. Therefore, SR predicts one SLW moves two different directions along the line $y = y_g$. To prove the results of figure 1 in the unprimed system, the equation for the SLW is $c^2t^2 = x^2 + y^2 + z^2$. Next, assume $x > 0$ and some fixed $y_g > 0$ with $y = y_g$ and $z = 0$ then we have, $c^2t^2 = x^2 + {y_g}^2$. So, $x = \sqrt{c^2t^2 - {y_g}^2}$. Also, the primed origin is located at vt for any time t in the unprimed frame. So, in the measurements of the unprimed frame, the distance to the primed origin from the location of the SLW intersecting the line $y = y_g$ given $y_g/c < t < y_g\gamma/c$ is $d' = \sqrt{(x - vt)^2 + {y_g}^2}$ with $x = \sqrt{c^2t^2 - {y_g}^2}$ or

$$d' = \sqrt{\left(\sqrt{c^2t^2 - {y_g}^2} - vt\right)^2 + {y_g}^2}\,.$$

To show the distance d' decreases as time increases between $y_g/c < t < y_g\gamma/c$, the partial derivative of d' with respect to time must be calculated and shown to be negative on that interval of time. So, calculate the partial derivative below.

$$\frac{\partial d'}{\partial t}=\frac{\left(\frac{c^2 t}{\sqrt{c^2t^2-y_g{}^2}}-v\right)\left(\sqrt{c^2t^2-y_g{}^2}-vt\right)}{\sqrt{\left(\sqrt{c^2t^2-y_g{}^2}-vt\right)^2+y_g{}^2}}.$$

Based on $\sqrt{c^2t^2-y_g{}^2}$ above, $y_g/c<t$ is immediate. Also since $c>v$, then $\left(\frac{c^2 t}{\sqrt{c^2t^2-y_g{}^2}}-v\right)>0$. Finally, if $t<y_g\gamma/c$ then $\left(\sqrt{c^2t^2-y_g{}^2}-vt\right)<0$. So, if $y_g/c<t<y_g\gamma/c$ then $\frac{\partial d'}{\partial t}<0$. Therefore, as the SLW expands away from the unprimed origin, time proceeds forward. As time proceeds forward on the interval $y_g/c<t<y_g\gamma/c$ with $x>0$ in the measurements of the unprimed frame, the distance of the SLW to the primed origin as measured from the line $y=y_g$ decreases since $\frac{\partial d'}{\partial t}<0$. Hence, the SLW moves in a direction closer to the primed origin when measured from the line $y=y_g$. So, the above SR calculations predict the results of figure 1. Additionally, this proves the expansion of the SLW in the context of the F frame is inconsistent with the expansion of the SLW in the F' frame. SR cannot come to any conclusion or calculation in which the expansion of the SLW in the F frame is inconsistent with the expansion of the SLW in the F' frame. Hence, the SR mandate that the SLW expands spherically from the origin of F if and only if the SLW expands spherically from the origin of F' is violated by SR with the above conditions.

Finally, it is shown that SR also predicts the conditions of figure 2, which represent the view of the primed frame. In that figure, as the SLW expands away from the origin of the primed frame and is located in between the two origins while intersecting the line

$y = y_g$, the SLW only moves further from the primed origin when measured from the line $y = y_g$.

The equation for the SLW in the primed frame is $c^2t'^2 = x'^2 + y'^2 + z'^2$. In the standard configuration, $y' = y$ and $z' = z$. Next, assume the same conditions $y' = y_g$ and $z = 0$. Then, below is the equation for x' given $x' < 0$.

$$x' = -\sqrt{c^2t'^2 - y_g^{\ 2}}\ .$$

Now, calculate the partial derivative below,

$$\frac{\partial x'}{\partial t'} = \frac{-c^2t'}{\sqrt{c^2t'^2 - y_g^{\ 2}}}\ .$$

Since $t' > 0$ then $\frac{\partial x'}{\partial t'} < 0$. Hence, given time only proceeds forward, as t' increases, x' decreases since $\frac{\partial x'}{\partial t'} < 0$. But with $x' < 0$, then a decreasing x' becomes more negative, so x'^2 increases and thus $d' = \sqrt{x'^2 + y_g^{\ 2}}$ increases. Therefore, in the view of the primed frame, the expanding SLW only moves further from the primed origin as it intersects the line $y' = y_g$ and is measured from the primed origin to the line $y' = y_g$. Thus, calculations of SR also predict the conditions of figure 2.

Therefore, there are conditions in which SR predicts the one SLW moves two different directions along the same line $y = y' = y_g$. So, with the example of the communicating observers at the origins of F and F’, F would send the message to the observer in F’ indicating that the SLW moves closer to the F’ origin when the SLW was in between the origins and intersecting the line $y = y' = y_g$. F’ on the other hand, would send the message to F indicating that F was absolutely wrong. F’ claims while the SLW was in between the origins and intersecting the line $y = y' = y_g$, the SLW only moves progressively further from the F’ origin when measured from its intersection with the line $y = y' = y_g$ to the primed origin. Therefore, an experiment such as this must produce results that contradict each other in order for SR to be true.

Conclusion

It was proven there are conditions in which SR violates the requirement that the SLW expands from the origin of F if and only if the SLW expands from the origin of F’. Also, it was shown for any line $y = y' = y_g > 0$, there are conditions in which one SLW moves two different directions along that line. Therefore, the truth of the light postulate in each frame produces an inconsistent physical outcome.

References

[1] Einstein A., in *The Principle of Relativity* (Dover, New York) 1952, p. 37.