A conjecture about a large subset of Carmichael numbers related to concatenation

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Abstract. Though the method of concatenation has it’s recognised place in number theory, is rarely leading to the determination of characteristics of an entire class of numbers, which is not defined only through concatenation. We present here a property related to concatenation that appears to be shared by a large subset of Carmichael numbers

Introduction: I was studying the primes of the form 12*k + 5 (*i.e.* the primes 5, 17, 29, 41, 53, 89, 101, 113, 137, 149, 173 and so on) when I noticed that the primes obtained through the concatenation of two of them are easily to find, especially the ones that end in the digits 29: 4129, 6529, 8929, 11329, 13729, 14929 and so on. When I looked on a certain subset of Carmichael numbers I observed an interesting property that appear to be common to the numbers from this subset (Observation) then I saw that the property is in fact common to a much larger subset of Carmichael numbers (Conjecture).

Observation: The numbers obtained through *deconcatenation* (I understand through this word the operation which is the reverse of concatenation) of the digits of the Carmichael numbers that have 29 as the last two digits and the respective two digits appear to be congruent to 5(mod 6) or to 2(mod 6).

I checked this property to the first 21 Carmichael numbers of the form 100*k + 29:

: for 1729 we have (17 – 5)/6 = 2;
: for 23382529 we have (233825 – 5)/6 = 38970;
: for 146843929 we have (1468439 – 5)/6 = 244739;
: for 172947529 we have (1729475 – 5)/6 = 288245;
: for 188516329 we have (1885163 – 5)/6 = 314193;
: for 246446929 we have (2464469 – 5)/6 = 410744;
: for 271481329 we have (2714813 – 5)/6 = 452468;
: for 484662529 we have (4846625 – 5)/6 = 807770;
: for 593234929 we have (5932349 – 5)/6 = 988724;
: for 934784929 we have (9347849 – 5)/6 = 1557974;
: for 958762729 we have (9587627 – 5)/6 = 1597937;

: for 1055384929 we have (10553849 – 5)/6 = 1758974;
: for 1688214529 we have (16882145 – 5)/6 = 2813690;
: for 1858395529 we have (18583955 – 5)/6 = 3097325;
: for 1942608529 we have (19426085 – 5)/6 = 3237680;
: for 6218177329 we have (62181773 – 5)/6 = 10363628;
: for 7044493729 we have (70444937 – 5)/6 = 11740822;
: for 10128932929 we have (101289329 – 5)/6 = 101289329;
: for 10387489729 we have (103874897 – 5)/6 = 17312482;
: for 11477658529 we have (114776585 – 5)/6 = 19129430.
: for 12299638429 we have (122996384 – 2)/6 = 20499397.

Note: I expressed this property in the way above so we can see yet another interesting pattern: many of the integers obtained through this operation have the sum of the digits equal to 29: 244739, 288245, 452468, 807770, 2813690, 3097325, 3237680, 10363628, 19129430.

Note: It would be interesting to see what kind of numbers we obtain if we reverse the operations above: let be x a number with the sum of the digits equal to 29, x*6 + 5 = y and z the number obtained through concatenation of y and 29:
: for x = 2999, y = 17999 and z = 1799929 prime;
: for x = 9299, y = 55799 and z = 1553*3593 semiprime;
: for x = 9929, y = 59579 and z = 373*15973 semiprime;
: for x = 9992, y = 59957 and z = 5995729 prime;
: for x = 3899, y = 23399 and z = 2339929 prime;
: for x = 3989, y = 23939 and z = 2393929 prime.
If we take x a number with the sum of the digits equal to another prime of the form 6*k – 1 instead 29, *i.e.* 41, and repeat the same operations from above, we obtain:
: for x = 59999, y = 359999 and z = 35999941 prime;
: for x = 99599, y = 597599 and z = 59759941 semiprime;
: for x = 99959, y = 599759 and z = 59975941 prime;
: for x = 99995, y = 599975 and z = 59997541 prime.
Even more than that, if we take x a number with the sum of the digits equal to 41, but we calculate z as the concatenation of y not cu 41 but with 29, we obtain:
: for x = 59999, y = 359999 and z = 35999929 semiprime;
: for x = 95999, y = 575999 and z = 57599929 prime;
: for x = 99599, y = 597599 and z = 59759929 prime;
: for x = 99959, y = 599759 and z = 59975929 semiprime;
: for x = 99995, y = 599975 and z = 59997599 semiprime.

We saw that, taking randomly 15 numbers with the property that sum of their digits is equal to a prime of the form 6*k – 1 (in fact not entirely random, because 2999 and 59999 are the smaller primes for which the sum of the digits is equal to 29, respectively 41), we obtained 9 primes and 6 semiprimes, so this direction of study seems to be prolific.

It is also interesting to see which are the smaller numbers with the property that the sum of their digits equals a prime p of the form 6*k - 1: these numbers are: 29 (for p = 11), 89 (for p = 17), 599 (for p = 23), 2999 (for p = 29), 59999 (for p = 41), 299999 (for p = 47), 899999 (for p = 53), 5999999 (for p = 59), 89999999 (for p = 71), 2999999999 (for p = 83), 8999999999 (for p = 89), 29999999999 (for p = 101) and so on. If we concatenate, for instance, the number 6*29999999999 + 5 with these numbers we obtain 17999999999929, 17999999999989, 179999999999599 (which are all semiprimes) and so on.

Conjecture: The numbers formed through deconcatanation of Carmichael numbers not divisible by 5 that ends in the digits that form a number of the form 6*k - 1 and the respective number are congruent to 2(mod 6) or to 5(mod 6).

I checked this property to the first few Carmichael numbers that ends in digits of this form (beside the cases that I already considered above):

: for 2821, where 821 ≡ 5(mod 6), we have 2 ≡ 2(mod 6);
: for 8911, where 11 = 5(mod 6), we have 89 ≡ 5(mod 6); but also 911 ≡ 5(mod 6), and we have 8 ≡ 2(mod 6);
: for 15841, where 41 ≡ 5(mod 6), we have 158 ≡ 2(mod 6);
: for 29341, where 41 ≡ 5(mod 6), we have 293 ≡ 5(mod 6); but also 341 ≡ 5(mod 6), and we have 29 ≡ 5(mod 6) and also 9341 ≡ 5(mod 6), and we have 2 ≡ 2(mod 6);
: for 41041, where 41 ≡ 5(mod 6), we have 410 ≡ 2(mod 6);
: for 52633, where 2633 ≡ 5(mod 6), we have 5 ≡ 5(mod 6);
: for 101101, where 101 ≡ 5(mod 6), we have 101 ≡ 5(mod 6);
: for 115921, where 5921 ≡ 5(mod 6), we have 11 ≡ 5(mod 6);
: for 126217, where 17 ≡ 5(mod 6), we have 1262 ≡ 2(mod 6);
: for 172081, where 2081 ≡ 5(mod 6), we have 17 ≡ 2(mod 6);
: for 188461, where 461 ≡ 5(mod 6), we have 188 ≡ 2(mod 6);
: for 252601, where 52601 ≡ 5(mod 6), we have 2 ≡ 2(mod 6);
: for 294409, where 4409 ≡ 5(mod 6), we have 29 ≡ 5(mod 6); but also 94409 ≡ 5(mod 6), and we have 2 ≡ 2(mod 6);
: for 314821, where 821 ≡ 5(mod 6), we have 314 ≡ 2(mod 6);
: for 334153, where 53 ≡ 5(mod 6), we have 3341 ≡ 5(mod 6);

: for 410041, where 41 ≡ 5(mod 6), we have 4100 ≡ 2(mod 6);
: for 488881, where 881 ≡ 5(mod 6), we have 488 ≡ 2(mod 6);
: for 512461, where 461 ≡ 5(mod 6), we have 512 ≡ 2(mod 6);
: for 530881, where 881 ≡ 5(mod 6), we have 530 ≡ 2(mod 6); but also 30881 ≡ 5(mod 6), and we have 5 ≡ 5(mod 6);
: for 658801, where 8801 ≡ 5(mod 6), we have 65 ≡ 2(mod 6);
: for 748657, where 8657 ≡ 5(mod 6), we have 74 ≡ 2(mod 6);
: for 838201, where 8201 ≡ 5(mod 6), we have 83 ≡ 2(mod 6);
: for 852841, where 41 ≡ 5(mod 6), we have 8528 ≡ 2(mod 6);
: for 1082809, where 809 ≡ 5(mod 6), we have 1082 ≡ 2(mod 6);
: for 1152271, where 71 ≡ 5(mod 6), we have 11522 ≡ 2(mod 6);
: for 1193221, where 221 ≡ 5(mod 6), we have 1193 ≡ 5(mod 6); but also 93221 ≡ 5(mod 6), and we have 11 ≡ 5(mod 6);
: for 1461241, where 41 ≡ 5(mod 6), we have 14612 ≡ 2(mod 6); but also 1241 ≡ 5(mod 6), and we have 146 ≡ 2(mod 6) and 61241 ≡ 5(mod 6), and we have 14 ≡ 2(mod 6);
: for 1615681, where 5681 ≡ 5(mod 6), we have 161 ≡ 5(mod 6);
: for 1773289, where 89 ≡ 5(mod 6), we have 17732 ≡ 2(mod 6); but also 73289 ≡ 5(mod 6), and we have 17 ≡ 2(mod 6).

We take now few bigger Carmichael numbers:

: for 998324255809, where 809 ≡ 5(mod 6), we have 998324255 ≡ 5(mod 6); but also 255809 ≡ 5(mod 6), and we have 998324 ≡ 2(mod 6) and 24255809 ≡ 5(mod 6), and we have 9983 ≡ 5(mod 6) and 324255809 ≡ 5(mod 6), and we have 998 ≡ 2(mod 6);
: for 998667686017, where 17 ≡ 5(mod 6), we have 9986676860 ≡ 2(mod 6); but also 6017 ≡ 5(mod 6), and we have 99866768 ≡ 2(mod 6) and 7686017 ≡ 5(mod 6), and we have 99866 ≡ 2(mod 6) and 67686017 ≡ 5(mod 6), and we have 9986 ≡ 2(mod 6) and 667686017 ≡ 5(mod 6), and we have 998 ≡ 2(mod 6);
: for 999607982113, where 113 ≡ 5(mod 6), we have 999607982 ≡ 2(mod 6);
: for 999629786233, where 233 ≡ 5(mod 6), we have 999629786 ≡ 2(mod 6); but also 6233 ≡ 5(mod 6), and we have 99962978 ≡ 2(mod 6) and 786233 ≡ 5(mod 6), and we have 999629 ≡ 5(mod 6) and 9786233 ≡ 5(mod 6), and we have 99962 ≡ 2(mod 6).

Note: From all the cases which appear until the Carmichael number 1773289 (we saw that for a single Carmichael number we can meet the conditions from hypothesis more than once), I only met one exception: for 162401, where 401 ≡ 5(mod 6), we have 162 ≡ 0(mod 6); I didn't change yet the statement from conjecture, waiting for at least one more counterexemple to set a pattern.

Conclusion: The results obtained for Carmichael numbers may have theoretical value, but for a more practical value, for instance to be helpful in a PRP test, let's see if these results can be extended for the class of Fermat pseudoprimes to base 2:
: for 341, where 41 ≡ 5(mod 6), we have 3 ≡ 3(mod 6);
: for 2047, where 47 = 5(mod 6), we have 20 ≡ 2(mod 6);
: for 2701, where 701 = 5(mod 6), we have 2 ≡ 2(mod 6);
: for 3277, where 77 = 5(mod 6), we have 32 ≡ 2(mod 6);
: for 4371, where 71 = 5(mod 6), we have 43 ≡ 1(mod 6).
Unfortunatelly, from the first 5 cases that we considered it becomes clear that the conjecture can't be extended on Poulet numbers. A resembling pattern seems not to exist in the case of prime numbers also, so this is a feature strictly of absolute Fermat pseudoprimes.