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The Radii of Baryons

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Abstract Considering the model in which the effective interaction between any two quarks of a baryon can be approximately described by a simple harmonic potential, and making use of the expression of the energy obtained in Cartesian coordinates for the above mentioned model, we find a general expression for the radii of baryons. We then apply the expression to some baryons and find very consistent values for the radii of baryons and an experimental confirmation for the ground state of Ξ^-.

Keywords Radii of Baryons, Baryon Spectroscoy

1. Introduction

There are various models that deal with the radii of baryons such as the Skyrme model[1], the $1/N_c$ expansion[2,3], chiral perturbation theory[4-7], lattice QCD [8,9], meson-baryon dynamics with chiral symmetry[10], chiral constituent quark model (χ CQM)[11] etc. However, none of these models has derived a formula for the radii of baryons. On the experimental side there are a couple of important works that have reported the radii of the ground states of some baryons[12,13,14].

The present work is an updated version of the pre-print[15]. It is based on reference[16] which calculated most energy levels of baryons by means of two simple formulas (one in Cartesian coordinates and another one in polar cylindrical coordinates) with which we can predict levels yet to be found. The formulas do not apply to levels resulting from hadronic molecules. One of the energy levels predicted was the 1st excited state of Λ_b (energy of 5.93 GeV) which has recently been found by CDF[17] with energy equal to (5919.5 ± 2.07) MeV.

2. Derivation of an Equation for the Radius of a Baryon

We make use of Eq. (1) below for the energies of baryons which was deduced in reference[16]

$$E_{nml} = h\upsilon_1(n+1) + h\upsilon_2(m+1) + h\upsilon_3(l+1) \tag{1}$$

where $n,m,k = 0,1,2,3,4,...$ and $h\upsilon_1, h\upsilon_2, h\upsilon_3$ are the masses of quarks. On the other hand it is well known that the average potential energy of a harmonic oscillator is half of the total energy, that is,

$$\left\langle \frac{1}{2}k\xi^2 \right\rangle = \frac{h\nu}{2}\left(n+\frac{1}{2}\right) \tag{2}$$

where $n = 0,1,2,3,...$ but since the 3 quarks are in a plane and as there are two spatial degrees of freedom in the plane for each quark, we have

$$\begin{aligned} E_{n,m,k} &= h\upsilon_1(n+1) + h\upsilon_2(m+1) + h\upsilon_3(k+1) = \\ &2\times\left[\left\langle \frac{1}{2}k_1\eta_1^2\right\rangle + \left\langle \frac{1}{2}k_2\eta_2^2\right\rangle + \left\langle \frac{1}{2}k_3\eta_3^2\right\rangle\right] = \\ &k_1\left\langle \eta_1^2\right\rangle + k_2\left\langle \eta_2^2\right\rangle + k_3\left\langle \eta_3^2\right\rangle \end{aligned} \tag{3}$$

where 1,2,3 refer to the 3 quarks of the baryons and $\eta_i^2 = \xi_{ij}^2 + \xi_{iq}^2$ in which j and q are the two orthogonal directions. Eq. (1) above was obtained considering three independent oscillators. Thus we can make the association

$$h\upsilon_i(n+1) = k_i\left\langle \eta_i^2\right\rangle \tag{4}$$

And defining the radius of a baryon as

$$r_{\eta_1\eta_2\eta_3} = \frac{1}{3}\left(\sqrt{\left\langle \eta_1^2\right\rangle} + \sqrt{\left\langle \eta_2^2\right\rangle} + \sqrt{\left\langle \eta_3^2\right\rangle}\right) \tag{5}$$

we obtain

$$r_{nml} = \frac{1}{3}\left(\sqrt{\frac{h\upsilon_1(n+1)}{k_1}} + \sqrt{\frac{h\upsilon_2(m+1)}{k_2}} + \sqrt{\frac{h\upsilon_3(l+1)}{k_3}}\right) \tag{6}$$

The consistency of Eq. (6) is proven with the calculation of r_{nml} for the ground state of Ξ^- and its agreement with the experimental value.

3. Calculation of the Radii of Baryons

The masses of quarks $h\upsilon_1, h\upsilon_2, h\upsilon_3$ were taken from

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Particle Data Group[18] as $m_u = m_d = 0.31$ GeV, $m_s = 0.5$ GeV, $m_c = 1.7$ GeV, $m_b = 5$ GeV, and $m_t = 174$ GeV. In the calculations below the average value $< r_{nml} >$ was calculated by performing the average of the different r_{nml} taking into account the several possible values for n, m, l.

Table 1. Radii for baryons N and Δ according to Eq. (8). $< r_{nml} >$ and r_{nml} have the same error bars

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)	$< r_{nml} >$
$0,0,0$	$0.93(N)$	$r_{000} = 0.83 \pm 0.02$	0.83
$n+m+l=1$	$1.24(\Delta)$	$r_{001} = 0.95 \pm 0.02$	0.95
$n+m+l=2$	1.55	$r_{002} = 1.04 \pm 0.03$	1.05
		$r_{011} = 1.06 \pm 0.03$	
$n+m+l=3$	1.86	$r_{003} = 1.10 \pm 0.03$	1.13
		$r_{012} = 1.14 \pm 0.03$	
		$r_{111} = 1.17 \pm 0.03$	
$n+m+l=4$	2.17	$r_{004} = 1.17 \pm 0.03$	1.22
		$r_{013} = 1.22 \pm 0.03$	
		$r_{022} = 1.23 \pm 0.03$	
		$r_{112} = 1.26 \pm 0.03$	
$n+m+l=5$	2.48	$r_{005} = 1.23 \pm 0.03$	1.30
		$r_{014} = 1.29 \pm 0.03$	
		$r_{023} = 1.31 \pm 0.03$	
		$r_{113} = 1.34 \pm 0.03$	
		$r_{122} = 1.35 \pm 0.03$	
$n+m+l=6$	2.79	$r_{006} = 1.29 \pm 0.03$	1.38
		$r_{015} = 1.35 \pm 0.03$	
		$r_{024} = 1.38 \pm 0.03$	
		$r_{033} = 1.39 \pm 0.03$	
		$r_{114} = 1.41 \pm 0.03$	
		$r_{123} = 1.43 \pm 0.03$	
$n+m+l=7$	3.10	$r_{007} = 1.34 \pm 0.03$	1.45
		$r_{016} = 1.40 \pm 0.03$	
		$r_{025} = 1.44 \pm 0.04$	
		$r_{034} = 1.45 \pm 0.04$	
		$r_{115} = 1.47 \pm 0.04$	
		$r_{124} = 1.48 \pm 0.04$	
		$r_{133} = 1.49 \pm 0.04$	
		$r_{223} = 1.51 \pm 0.04$	

Table 2. Radii for baryons Σ and Λ according to Eq. (10). $< r_{nml} >$ and r_{nml} have the same error bars

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)	$< r_{nml} >$
$0,0,0$	1.12	$r_{000} = 0.79 \pm 0.02$	0.79
$n+m=1, l=0$	1.43	$r_{010} = 0.90 \pm 0.02$	0.90
$0,0,1$	1.62	$r_{001} = 0.87 \pm 0.02$	0.87
$n+m=2, l=0$	1.74	$r_{020} = 0.98 \pm 0.02$ $r_{110} = 1.01 \pm 0.02$	0.99
$n+m=1, l=1$	1.93	$r_{011} = 0.99 \pm 0.02$	0.99
$n+m=3, l=0$	2.05	$r_{030} = 1.06 \pm 0.03$ $r_{120} = 1.10 \pm 0.02$	1.08
$0,0,2$	2.12	$r_{002} = 0.95 \pm 0.02$	0.95
$n+m=2, l=1$	2.24	$r_{021} = 1.08 \pm 0.03$ $r_{111} = 1.10 \pm 0.03$	1.09
$n+m=4, l=0$	2.36	$r_{040} = 1.12 \pm 0.03$ $r_{130} = 1.17 \pm 0.03$ $r_{220} = 1.19 \pm 0.03$	1.15
$n+m=1, l=2$	2.43	$r_{012} = 1.06 \pm 0.03$	1.06
$m+n=3, l=1$	2.55	$r_{031} = 1.15 \pm 0.03$ $r_{121} = 1.19 \pm 0.03$	1.17
$0,0,3$	2.62	$r_{003} = 1.01 \pm 0.02$	1.01
$m+n=6, l=0$	2.98	$r_{060} = 1.24 \pm 0.03$ $r_{150} = 1.30 \pm 0.03$ $r_{240} = 1.33 \pm 0.03$ $r_{330} = 1.33 \pm 0.03$	1.30
$m+n=5, l=1$	3.17	$r_{051} = 1.28 \pm 0.03$ $r_{141} = 1.33 \pm 0.03$ $r_{231} = 1.35 \pm 0.03$	1.32

3.1. Baryons N, Δ^-, Δ^{++}

As shown above $E_{nml} = 0.31(n+m+l+3)$, and thus, as calculated above

$$r_{nml} = \frac{1}{3}\sqrt{\frac{h\upsilon}{k}}\left(\sqrt{n+1}+\sqrt{m+1}+\sqrt{l+1}\right) \quad (7)$$

Using the value $h\upsilon = 0.31$ GeV and the experimental value $r_{000} = r_0 = (0.83 \pm 0.02)$ fm[14] for the proton radius, we obtain $k = (0.45 \pm 0.03)$ GeV/fm^2. Therefore, we have

$$r_{nml} = \frac{0.83 \pm 0.02}{3}\left(\sqrt{n+1}+\sqrt{m+1}+\sqrt{l+1}\right) \quad (8)$$

in fermis. The application of this formula to the experimentally observed levels (in terms of energy) produce Table 1 below.

3.2. Baryons Σ and Λ

As shown above $E_{nml} = 0.31(n+m+2) + 0.50(l+1)$, and thus, according to Eq. (6) we have

$$r_{nml} = \frac{1}{3}\left(\sqrt{\frac{h\upsilon_1}{k_1}}\left(\sqrt{n+1}+\sqrt{m+1}\right)+\sqrt{\frac{h\upsilon_3}{k_3}}\sqrt{l+1}\right) \quad (9)$$

Using the experimental value $r_{000} = (0.78 \pm 0.02)$ fm for the ground state of Σ^- [14] and the above value of $k_1 = (0.45 \pm 0.03)$ GeV/fm^2, we obtain from Eq. (9) $k_3 = (1.08 \pm 0.03)$ GeV/fm^2. Therefore, the equation for the radii of these baryons is given by

$$r_{nml} = \frac{1}{3}\Big[\left(0.83 \pm 0.02\right)\left(\sqrt{n+1} + \sqrt{m+1}\right) + \left(0.68 \pm 0.02\right)\sqrt{l+1}\Big] \quad (10)$$

Applying this formula to the experimentally observed levels (in terms of energy) we obtain the values displayed on Table 2 below.

3.3. Baryons Ξ

Using Eq. (6) and what was calculated above, we have that the radii of these baryons are described by

$$r_{nml} = \frac{1}{3}\Big[\left(0.83 \pm 0.02\right)\sqrt{n+1} + \left(0.66 \pm 0.02\right)\left(\sqrt{m+1} + \sqrt{l+1}\right)\Big]. \quad (11)$$

For the ground state (0,0,0) which has the energy $E_{nml} = 1.31$ GeV we obtain from Eq. (11) $r_{000} = \left(0.73 \pm 0.02\right)$ fm which is very close to the experimental value of $\left(0.74 \pm 0.02\right)$ fm[6]. This result confirms the validity of the general formula

$$r_{nml} = \frac{1}{3}\left(\sqrt{\frac{h\upsilon_1(n+1)}{k_1}} + \sqrt{\frac{h\upsilon_2(m+1)}{k_2}} + \sqrt{\frac{h\upsilon_3(l+1)}{k_3}}\right).$$

Using this formula to the experimentally observed levels (in terms of energy) we obtain the values displayed on Table 3 below.

3.4. Baryons Ω

From what was calculated above, and with the use of Eq. (6), we obtain that the radii are described by

$$r_{nml} = \frac{1}{3}\Big[\left(0.68 \pm 0.02\right)\left(\sqrt{n+1} + \sqrt{m+1} + \sqrt{l+1}\right)\Big]. \quad (12)$$

We can, thus, predict that the ground state has a radius of about $r_{000} = \left(0.68 \pm 0.02\right)$ fm. The calculation of the other levels produce Table 4 below.

Table 3. Radii for baryons Ξ according to Eq. (11). $< r_{nml} >$ and r_{nml} have the same error bars

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)	$< r_{nml} >$
$0,0,0$	1.31	$r_{000} = 0.73 \pm 0.02$	0.73
$1,0,0$	1.62	$r_{100} = 0.85 \pm 0.02$	0.85
$n=0, m+l=1$	1.81	$r_{001} = 0.82 \pm 0.02$	0.82
$2,0,0$	1.93	$r_{200} = 0.93 \pm 0.02$	0.93
$n=1, m+l=1$	2.12	$r_{101} = 0.94 \pm 0.02$	0.94
$3,0,0$	2.24	$r_{300} = 1.01 \pm 0.02$	1.01
$n=0, m+l=2$	2.31	$r_{002} = 0.90 \pm 0.02$ $r_{011} = 0.92 \pm 0.02$	0.91
$4,0,0$	2.55	$r_{400} = 1.07 \pm 0.02$	1.07

Table 4. Predicted radii for baryons Ω according to Rq. (12). $< r_{nml} >$ and r_{nml} have the same error bars

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)	$< r_{nml} >$
$0,0,0$	1.50	$r_{000} = 0.68 \pm 0.02$	0.68
$n+m+l=1$	2.00	$r_{001} = 0.77 \pm 0.02$	0.77
$n+m+l=2$	2.50	$r_{002} = 0.85 \pm 0.02$ $r_{011} = 0.87 \pm 0.02$	0.86

4. Conclusions

Considering that the effective interaction between any two quarks of a baryon can be approximately described by a simple harmonic oscillator, we derive a general formula that describes the energy levels of baryons and using it we can obtain a general formula for the description of the radii of baryons. The calculation is very consistent and agrees very well with the experimental value for the ground state of Ξ^-. Since the formula for the radius was deduced from the expression for the energy in Cartesian coordinates there is not a way at the moment of identifying the radii in terms of J and L. On the other hand we observe that within the same (n,m,l) level the radii do not change much and, thus, within the same energy level, the radii should not depend much on L. Of course, high values of the energy allow high values of L and J as can be seen on Table 2 and Table 3 of reference[16], and thus the radius tend to increase with L and J, but there is not a simple relation between the radius and the value of L or the value of J.

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