Set-theoretic Operators On Degenerated Neutrosophic Set

Haibin Wang [a,*], Yanqinq Zhang [a], Rajshekhar Sunderraman [a], Feijun Song [b]

[a] *Computer Science Department, Georgia State Univeristy, Atlanta, Georgia 30303, USA*

[b] *Ocean Engineering Department, Florida Atlantic University, Dania, Florida 33004, USA*

Abstract

Neutrosophic set is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework. From technical point of view, neutrosophic set should be specified. In this paper, we define the set-theoretic operators on an instance of neutrosophic set called degenerated neutrosophic set (DNS). Various properties of DNS are proved, which are connected to the operations and relations over DNS. Finally, we introduce the convexity of degenerated neutrosophic sets.

Key words: neutrosophic set, degenerated neutrosophic set, set-theorectic operator, convexity

* Corresponding author.

Email addresses: `hwang17@student.gsu.edu` (Haibin Wang), `yzhang@cs.gsu.edu` (Yanqinq Zhang), `raj@cs.gsu.edu` (Rajshekhar Sunderraman), `fsong@oe.fau.edu` (Feijun Song).

1 Introduction

Neutrosophy was introduced by Florentin Smarandache in 1980. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" [1]. Neutrosophic set is a powerful general formal framework which generalizes the concept of the classic set, fuzzy set [2], interval valued fuzzy set [3], intuitionistic fuzzy set [4], interval valued intuitionistic fuzzy set [5], paraconsistent set [1], dialetheist set [1], paradoxist set [1], tautological set [1]. A neutrosophic set A defined on universe U. $x = x(T, I, F) \in A$ with T, I and F being the real standard or non-standard subsets of $]0^-, 1^+[$. T is the degree of true membership function in the set A, I is the degree of indeterminate membership function in the set A and F is the degree of false membership function in the set A. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators must be specified. Otherwise, it will be difficult to apply in the real applications. In this paper, we define the set-theoretic operators on an instance of neutrosophic set called degenerated neutrosophic set (DNS). A degenerated neutrosophic set A defined on universe X. $x = x(T, I, F) \in A$ with T, I and F being the subinterval of $[0, 1]$. Degenerated neutrosophic set can represent uncertainty, imprecise, incomplete and inconsistent information which exist in real world. The degenerated neutrosophic set generalizes the following sets:

(1) the *classical set*, $I = \emptyset$, $\inf T = \sup T = 0$ or 1, $\inf F = \sup F = 0$ or 1 and $\sup T + \sup F = 1$.
(2) the *fuzzy set*, $I = \emptyset$, $\inf T = \sup T \in [0, 1]$, $\inf F = \sup F \in [0, 1]$ and $\sup T + \sup F = 1$.
(3) the *interval valued fuzzy set*, $I = \emptyset$, $\inf T, \sup T, \inf F, \sup F \in [0, 1]$, $\sup T + \inf F = 1$ and $\inf T + \sup F = 1$.
(4) the *intuitionistic fuzzy set*, $I = \emptyset$, $\inf T = \sup T \in [0, 1]$, $\inf F = \sup F \in [0, 1]$ and $\sup T + \sup F \leq 1$.
(5) the *interval valued intuitionistic fuzzy set*, $I = \emptyset$, $\inf T, \sup T, \inf F, \inf T \in [0, 1]$ and $\sup T + \sup F \leq 1$.
(6) the *paraconsistent set*, $I = \emptyset$, $\inf T = \sup T \in [0, 1]$, $\inf F = \sup F \in [0, 1]$ and $\sup T + \sup F > 1$.
(7) the *interval valued paraconsistent set*, $I = \emptyset$, $\inf T, \sup T, \inf F, \sup F \in [0, 1]$ and $\inf T + \inf F > 1$.

The relationship among degenerated neutrosophic set and other sets is illustrated in the Fig 1.

We define the set-theoretic operators on degenerated neutrosophic set (DNS). various properties of DNS are proved, which are connected to the operations and relations over DNS.

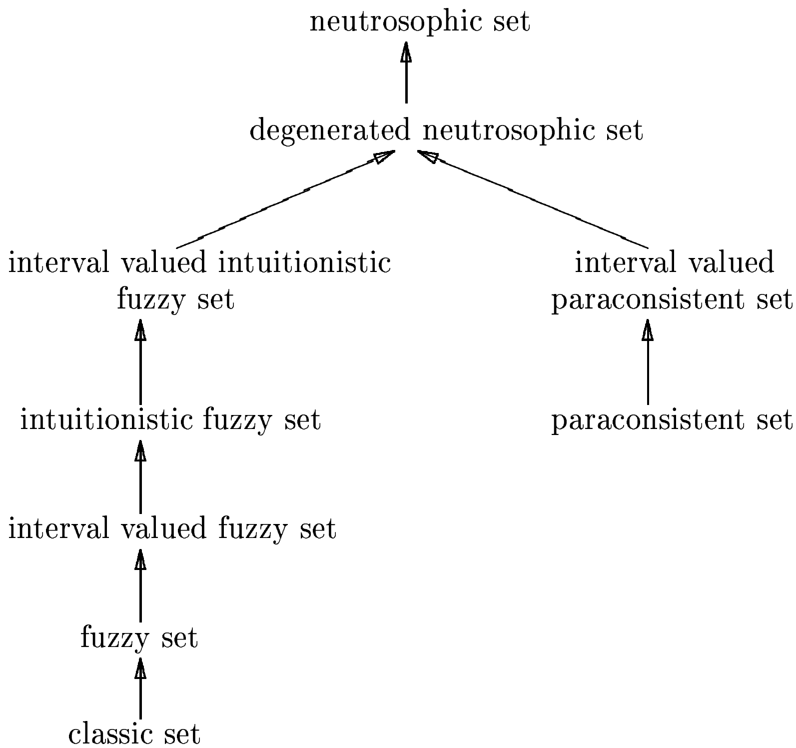

Fig. 1. Relationship among degenerated neutrosophic set and other sets

The rest of paper is organized as follows. Section 2 gives a brief overview of neutrosophic set. Section 3 gives the definition of degenerated neutrosophic set and set-theoretic operations. Section 4 proves some properties of set-theoretic operations. Section 5 gives the definition of convexity of degenerated neutrosophic sets and prove some properties of convexity. Section 6 concludes the paper.

2 Neutrosophic Set

This section gives the brief overview of concepts of neutrosophic set defined in [1]. Let S_1 and S_2 be two real standard or non-standard subsets. Then $S_1 \oplus S_2 = \{x|x = s_1 + s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\} \oplus S_2 = \{x|x = 1^+ + s_2, s_2 \in S_2\}$. $S_1 \ominus S_2 = \{x|x = s_1 - s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\} \ominus S_2 = \{x|x = 1^+ - s_2, s_2 \in S_2\}$. $S_1 \odot S_2 = \{x|x = s_1 \cdot s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$.

Definition 1 (Neutrosophic Set) *Let X be a space of points (objects), with a generic element in X denoted by x.*
A neutrosophic set A in X is characterized by a truth-membership function T_A, *a* indeterminancy-membership function I_A *and a* false-membership *function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is*

$$T_A : X \to]0^-, 1^+[, \tag{1}$$
$$I_A : X \to]0^-, 1^+[, \tag{2}$$
$$F_A : X \to]0^-, 1^+[. \tag{3}$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2 *The* complement *of a neutrosophic set A is denoted by $\bar{A}$ and is defined by*

$$T_{\bar{A}}(x) = \{1^+\} \ominus T_A(x), \tag{4}$$
$$I_{\bar{A}}(x) = \{1^+\} \ominus I_A(x), \tag{5}$$
$$F_{\bar{A}}(x) = \{1^+\} \ominus F_A(x), \tag{6}$$

for all x in X.

Definition 3 (Containment) *A neutrosophic set A is* contained *in the other neutrosophic set B, $A \subseteq B$, if and only if*

$$\inf T_A(x) \leq \inf T_B(x)\,, \sup T_A(x) \leq \sup T_B(x), \tag{7}$$
$$\inf F_A(x) \geq \inf F_B(x)\,, \sup F_A(x) \geq \sup F_B(x), \tag{8}$$

for all x in X.

Definition 4 (Union) *The* union *of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminancy-membership and false-membership functions are related to those of A and B by*

$$T_C(x) = T_A(x) \oplus T_B(x) \ominus T_A(x) \odot T_B(x), \tag{9}$$
$$I_C(x) = I_A(x) \oplus I_B(x) \ominus I_A(x) \odot I_B(x), \tag{10}$$
$$F_C(x) = F_A(x) \oplus F_B(x) \ominus F_A(x) \odot F_B(x), \tag{11}$$

for all x in X.

Definition 5 (Intersection) *The* intersection *of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminancy-membership and false-membership functions are related to those of A and B by*

$$T_C(x) = T_A(x) \odot T_B(x), \tag{12}$$
$$I_C(x) = I_A(x) \odot I_B(x), \tag{13}$$
$$F_C(x) = F_A(x) \odot F_B(x), \tag{14}$$

for all x in X.

Definition 6 (Difference) *The* difference *of two neutrosophic sets A and B is a neutrosophic set C, writeen as $C = A \setminus B$, whose truth-membership, indeterminancy-membership and false-membership functions are related to those of A and B by*

$$\begin{aligned} T_C(x) &= T_A(x) \ominus T_A(x) \odot T_B(x), & (15)\\ I_C(x) &= I_A(x) \ominus I_A(x) \odot I_B(x), & (16)\\ F_C(x) &= F_A(x) \ominus F_A(x) \odot F_B(x), & (17) \end{aligned}$$

for all x in X.

3 Degenerated Neutrosophic Set

In this section, we present the notion of *degenerated neutrosophic set* (DNS). Degenerated neutrosophic set (DNS) is an instance of neutrosophic set which can be used in real scientific and engineering applications.

Definition 7 (Degenerated Neutrosophic Set) *Let X be a space of points (objects), with a generic element in X denoted by x.*
A degenerated neutrosophic set (DNS) A in X is characterized by truth-membership function T_A, indeterminancy-membership function I_A and false-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$.

A degenerated neutrosophic set (DNS) in R^1 is illustrated in Fig. 2.

When X is continuous, a DNS A can be written as

$$A = \int_X \langle T(x), I(x), F(x)\rangle / x, \;\; x \in X \tag{18}$$

When X is discrete, a DNS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i)\rangle / x_i, \;\; x_i \in X \tag{19}$$

Example 1 *Assume that $X = [1, 2, \cdots, 10]$. SMALL is a degenerated neutrosophic set of X defined by*
$SMALL = \langle[1, 1], [0, 0], [0, 0]\rangle/1 + \langle[0.9, 1], [0, 0], [0, 0]\rangle/2 +$
$\langle[0.6, 0.8], [0, 0], [0.2, 0.3]\rangle/3 + \langle[0.3, 0.4], [0, 0], [0.5, 0.7]\rangle/4 +$
$\langle[0.1, 0.3], [0.1, 0.2], [0.8, 1]\rangle/5 + \langle[0.1, 0.2], [0, 1, 0.2], [0.9, 1]\rangle/6 +$

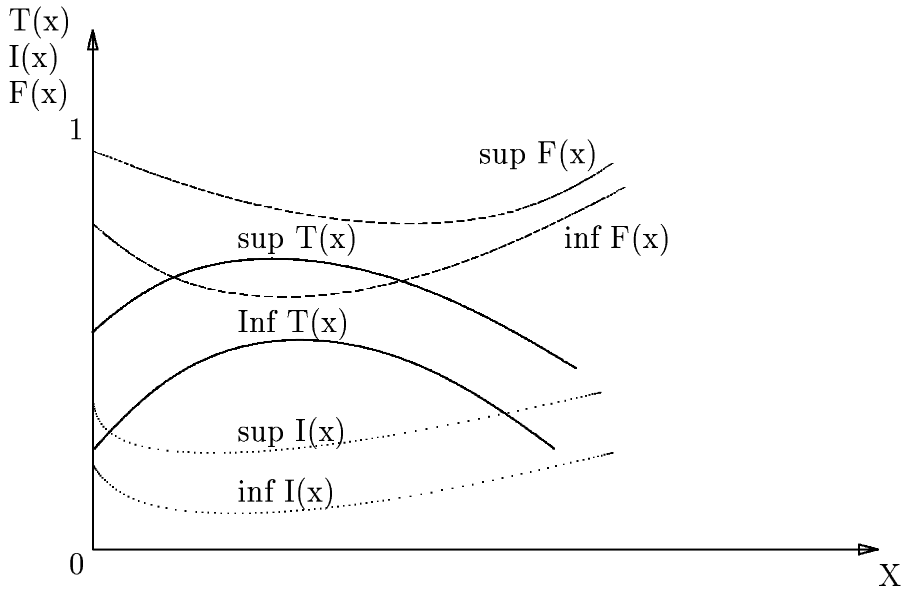

Fig. 2. Illustration of degenerated neutrosophic set in R^1

$\langle [0,0.1],[0.1,0.2],[0.9,1]\rangle/7 + \langle [0,0],[0,0.1],[0.9,1]\rangle/8 +$
$\langle [0,0],[0,0],[1,1]\rangle/9 + \langle [0,0],[0,0],[1,1]\rangle/10.$

Definition 8 *A degenerated neutrosophic set A is* empty *if and only if its* $\inf T_A(x) = \sup T_A(x) = 0$*,* $\inf I_A(x) = \sup I_A(x) = 1$ *and* $\inf F_A(x) = \sup T_A(x) = 0$*, for all x in X.*

We now present the set-theoretic operators on degenerated neutrosophic set.

Definition 9 (Complement) *The* complement *of a degenerated set A is denoted by $\bar{A}$ and is defined by*

$$\begin{aligned} T_{\bar{A}}(x) &= F_A(x), & (20)\\ \inf I_{\bar{A}}(x) &= 1 - \sup I_A(x), & (21)\\ \sup I_{\bar{A}}(x) &= 1 - \inf I_A(x), & (22)\\ F_{\bar{A}}(x) &= T_A(x), & (23)\end{aligned}$$

for all x in X.

Example 2 *Let the SMALL be the degenerated neutrosophic set E defined in Example 1. Then,* $\bar{E} = \langle [0,0],[1,1],[1,1]\rangle/1 + \langle [0,0],[1,1],[0.9,1]\rangle/2 +$
$\langle [0.2,0.3],[1,1],[0.6,0.8]\rangle/3 + \langle [0.5,0.7],[1,1],[0.3,0.4]\rangle/4 +$
$\langle [0.8,1],[0.8,0.9],[0.1,0.3]\rangle/5 + \langle [0.9,1],[0.8,0.9],[0.1,0.2]\rangle/6 +$
$\langle [0.9,1],[0.8,0.9],[0,0.1]\rangle/7 + \langle [0.9,1],[0.9,1],[0,0]\rangle/8 +$
$\langle [1,1],[1,1],[0,0]\rangle/9 + \langle [1,1],[1,1],[0,0]\rangle/10.$

Definition 10 (Containment) *A degenerated neutrosophic set A is* contained *in the other degenerated neutrosophic set B, $A \subseteq B$, if and only if*

$$\inf T_A(x) \le \inf T_B(x)\,, \sup T_A(x) \le \sup T_B(x), \tag{24}$$
$$\inf I_A(x) \ge \inf I_B(x)\,, \sup I_A(x) \ge \sup I_B(x), \tag{25}$$
$$\inf F_A(x) \ge \inf F_B(x)\,, \sup F_A(x) \ge \sup F_B(x), \tag{26}$$

for all x in X.

Definition 11 *Two degenerated sets A and B are* equal, *written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$*

Definition 12 (Union) *The* union *of two degenerated neutrosophic sets A and B is a degenerated neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminancy-membership and false-membership functions are related to those of A and B by*

$$\inf T_C(x) = \max(\inf T_A(x), \inf T_B(x)), \tag{27}$$
$$\sup T_C(x) = \max(\sup T_A(x), \sup T_B(x)), \tag{28}$$
$$\inf I_C(x) = \min(\inf I_A(x), \inf I_B(x)), \tag{29}$$
$$\sup I_C(x) = \min(\sup I_A(x), \sup I_B(x)), \tag{30}$$
$$\inf F_C(x) = \min(\inf F_A(x), \inf F_B(x)), \tag{31}$$
$$\sup F_C(x) = \min(\sup F_A(x), \sup F_B(x)), \tag{32}$$

for all x in X.

Theorem 1 *$A \cup B$ is the smallest degenerated neutrosophic set containing both A and B.*

Proof Let $C = A \cup B$. $\inf T_C = \max(\inf T_A, \inf T_B)$, $\inf T_C \ge \inf T_A$, $\inf T_C \ge \inf T_B$. $\sup T_C = \max(\sup T_A, \sup T_B$, $\sup T_C \ge \sup T_A$, $\sup T_C \ge \sup T_B$. $\inf I_C = \min(\inf I_A, \inf I_B)$, $\inf I_C \le \inf I_A$, $\inf I_C \le \inf I_B$,
$\sup I_C = \min(\sup I_A, \sup I_B)$, $\sup I_C \le \sup I_A$, $\sup I_C \le \sup I_B$, $\inf F_C = \min(\inf F_A, \inf F_B)$, $\inf F_C \le \inf F_A$, $\inf F_C \le \inf F_B$.
$\sup F_C = \min(\sup F_A, \sup F_B)$, $\sup F_C \le \sup F_A$, $\sup F_C \le \sup F_B$. That means C contains both A and B.
Furthermore, if D is any extended vague set containing both A and B, then $\inf T_D \ge \inf T_A$, $\inf T_D \ge \inf T_B$, so $\inf T_D \ge \max(\inf T_A, \inf T_B) = \inf T_C$. $\sup T_D \ge \sup T_A$, $\sup T_D \ge \sup T_B$, so $\sup T_D \ge \max(\sup T_A, \sup T_B) = \sup T_C$. $\inf I_D \le \inf I_A$, $\inf I_D \le \inf I_B$, so $\inf I_D \le \min(\inf I_A, \inf I_B) = \inf I_C$. $\sup I_D \le \sup I_A$, $\sup I_D \le \sup I_B$, so $\sup I_D \le \min(\sup I_A, \sup I_B) = \sup I_C$. $\inf F_D \le \inf F_A$, $\inf F_D \le \inf F_B$, so $\inf F_D \le \min(\inf F_A, \inf F_B) = \inf F_C$. $\sup F_D \le \sup F_A$, $\sup F_D \le \sup F_B$, so $\sup F_D \le \min(\sup F_A, \sup F_B) = \sup F_C$. That implies $C \subseteq D$.

Definition 13 (Intersection) *The* intersection *of two degenerated neutrosophic sets A and B is a degenerated neutrosophic set C, written as $C =$*

$A \cap B$, whose truth-membership, indeterminancy-membership functions and false-membership functions are related to those of A and B by

$$\inf T_C(x) = \min(\inf T_A(x), \inf T_B(x)), \tag{33}$$
$$\sup T_C(x) = \min(\sup T_A(x), \sup T_B(x)), \tag{34}$$
$$\inf I_C(x) = \max(\inf I_A(x), \inf I_B(x)), \tag{35}$$
$$\sup I_C(x) = \max(\sup I_A(x), \sup I_B(x)), \tag{36}$$
$$\inf F_C(x) = \max(\inf F_A(x), \inf F_B(x)), \tag{37}$$
$$\sup F_C(x) = \max(\sup F_A(x), \sup F_B(x)), \tag{38}$$

for all x in X.

Theorem 2 *$A \cap B$ is the largest degenerated neutrosophic set contained in both A and B.*

Proof The proof is analogous to the proof of theorem 1.

Definition 14 (Difference) *The* difference *of two degenerated neutrosophic sets A and B is a degenerated neutrosophic set C, writeen as $C = A \setminus B$, whose truth-membership, indeterminancy-membership and false-membership functions are related to those of A and B by*

$$\inf T_C(x) = \min(\inf T_A(x), \inf F_B(x)), \tag{39}$$
$$\sup T_C(x) = \min(\sup T_A(x), \sup F_B(x)), \tag{40}$$
$$\inf I_C(x) = \max(\inf I_A(x), 1 - \sup I_B(x)), \tag{41}$$
$$\sup I_C(x) = \max(\sup I_A(x), 1 - \inf I_B(x)), \tag{42}$$
$$\inf F_C(x) = \max(\inf F_A(x), \inf T_B(x)), \tag{43}$$
$$\sup F_C(x) = \max(\sup F_A(x), \sup T_B(x)), \tag{44}$$

for all x in X.

Theorem 3 $A \subseteq B \leftrightarrow \bar{B} \subseteq \bar{A}$

Proof $A \subseteq B \Leftrightarrow \inf T_A \leq \inf T_B, \sup T_A \leq \sup T_B, \inf I_A \geq \inf I_B, \sup I_A \geq \sup I_B, \inf F_A \geq \inf F_B, \sup F_A \geq \sup F_B \Leftrightarrow \inf F_B \leq \inf F_A, \sup F_B \leq \sup F_A, 1 - \sup I_B \geq 1 - \sup I_A, 1 - \inf I_B \geq 1 - \inf I_A, \inf T_B \geq \inf T_A, \sup T_B \geq \sup T_A \Leftrightarrow \bar{B} \subseteq \bar{A}$.

4 Properties of Complement, Union and Intersection

In this section, we will prove some properties of union, intersection, complement defined on degenerated neutrosophic sets as in Section 3.

Property 1 (Commutativity) $A \cup B = B \cup A$, $A \cap B = B \cap A$

Proof Here, we prove the first identity.
$\inf T_{A \cup B} = \max(\inf T_A, \inf T_B) = \max(\inf T_B, \inf T_A) = \inf T_{B \cup A}$, $\sup T_{A \cup B} = \max(\sup T_A, \sup T_B) = \max(\sup T_B, \sup T_A) = \sup T_{B \cup A}$,
$\inf I_{A \cup B} = \min(\inf I_A, \inf I_B) = \min(\inf I_B, \inf I_A) = \inf T_{B \cup A}$, $\sup I_{A \cup B} = \min(\sup I_A, \sup I_B) = \min(\sup I_B, \sup I_A) = \sup T_{B \cup A}$,
$\inf F_{A \cup B} = \min(\inf F_A, \inf F_B) = \min(\inf F_B, \inf F_A) = \inf F_{B \cup A}$,
$\sup F_{A \cup B} = \min(\sup F_A, \sup F_B) = \min(\sup F_B, \sup F_A) = \sup F_{B \cup A}$, That is $A \cup B = B \cup A$.

Property 2 (Associativity) $A \cup (B \cup C) = (A \cup B) \cup C$,
$A \cap (B \cap C) = (A \cap B) \cap C$.

Proof Follows quickly from associativity of min and max.

Property 3 (Distributivity) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Proof Follows quickly from distributivity of min and max.

Property 4 (Idempotency) $A \cup A = A$, $A \cap A = A$.

Proof Here, we proof the first identity.
$\inf T_{A \cup A} = \max(\inf T_A, \inf T_A) = \inf T_A$,
$\sup T_{A \cup A} = \max(\sup T_A, \sup T_A) = \sup T_A$,
$\inf I_{A \cup A} = \min(\inf I_A, \inf I_A) = \inf I_A$,
$\sup I_{A \cup A} = \min(\sup I_A, \sup I_A) = \sup I_A$,
$\inf F_{A \cup A} = \min(\inf F_A, \inf F_A) = \inf F_A$,
$\sup F_{A \cup A} = \min(\sup F_A, \sup F_A) = \sup F_A$,
That is $A \cup A = A$.

Property 5 $A \cap \Phi = \Phi$, $A \cup X = X$, *where* $\inf T_\Phi = \sup T_\Phi = 0$, $\inf I_\Phi = \sup I_\Phi = \inf F_\Phi = \sup F_\Phi = 1$ *and* $\inf T_X = \sup T_X = 1$, $\inf I_X = \sup I_X = \inf F_X = \sup F_X = 0$.

Proof

(1) $\inf T_{A \cap \Phi} = \min(\inf T_A, 0) = 0$,
$\sup T_{A \cap \Phi} = \min(\sup T_A, 0) = 0$,
$\inf I_{A \cap \Phi} = \max(\inf I_A, 1) = 1$,
$\sup I_{A \cap \Phi} = \max(\sup I_A, 1) = 1$,
$\inf F_{A \cap \Phi} = \max(\inf F_A, 1) = 1$,
$\sup F_{A \cap \Phi} = \max(\sup F_A, 1) = 1$,
that is $A \cap \Phi = \Phi$.

(2) $\inf T_{A \cup X} = \max(\inf T_A, 1) = 1$,

$\sup T_{A\cup X} = \max(\sup T_A, 1) = 1$,
$\inf I_{A\cup X} = \min(\inf I_A, 0) = 0$,
$\sup I_{A\cup X} = \min(\sup I_A, 0) = 0$,
$\inf F_{A\cup X} = \min(\inf F_A, 0) = 0$,
$\sup F_{A\cup X} = \min(\sup F_A, 0) = 0$,
that is $A \cup X = X$.

Property 6 $A \cup \Psi = A$, $A \cap X = A$, *where* $\inf T_\Phi = \sup T_\Phi = 0$, $\inf I_\Phi = \sup I_\Phi = \inf F_\Phi = \sup F_\Phi = 1$ *and* $\inf T_X = \sup T_X = 1$, $\inf I_X = \sup I_X = \inf F_X = \sup F_X = 0$.

Proof

(1) $\inf T_{A\cup\Psi} = \max(\inf T_A, 0) = \inf T_A$,
$\sup T_{A\cup\Psi} = \max(\sup T_A, 0) = \sup T_A$,
$\inf I_{A\cup\Psi} = \min(\inf I_A, 1) = \inf I_A$,
$\sup I_{A\cup\Psi} = \min(\sup I_A, 1) = \sup I_A$,
$\inf F_{A\cup\Psi} = \min(\inf F_A, 1) = \inf F_A$,
$\sup F_{A\cup\Psi} = \min(\sup F_A, 1) = \sup F_A$,
that is $A \cup \Psi = A$.

(2) $\inf T_{A\cap X} = \min(\inf T_A, 1) = \inf T_A$,
$\sup T_{A\cap X} = \min(\sup T_A, 1) = \sup T_A$,
$\inf I_{A\cap X} = \max(\inf I_A, 0) = \inf I_A$,
$\sup I_{A\cap X} = \max(\sup I_A, 0) = \sup I_A$,
$\inf F_{A\cap X} = \max(\inf F_A, 0) = \inf F_A$,
$\sup F_{A\cap X} = \max(\sup F_A, 0) = \sup F_A$,
that is $A \cap X = A$.

Property 7 (Absorption) $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$

Proof

(1) $\inf T_{A\cup(A\cap B)} = \max(\inf T_A, \min(\inf T_A, \inf T_B)) = \inf T_A$, $\sup T_{A\cup(A\cap B)} = \max(\sup T_A, \min(\sup T_A, \sup T_B)) = \sup T_A$,
$\inf I_{A\cup(A\cap B)} = \min(\inf I_A, \max(\inf I_A, \inf I_B)) = \inf I_A$, $\sup I_{A\cup(A\cap B)} = \min(\sup I_A, \max(\sup I_A, \sup I_B)) = \sup I_A$,
$\inf F_{A\cup(A\cap B)} = \min(\inf F_A, \max(\inf F_A, \inf F_B)) = \inf F_A$,
$\sup F_{A\cup(A\cap B)} = \min(\sup F_A, \max(\sup F_A, \sup F_B)) = \sup F_A$, that is $A \cup (A \cap B) = A$.

(2) $\inf T_{A\cap(A\cup B)} = \min(\inf T_A, \max(\inf T_A, \inf T_B)) = \inf T_A$, $\sup T_{A\cap(A\cup B)} = \min(\sup T_A, \max(\sup T_A, \sup T_B)) = \sup T_A$,
$\inf I_{A\cap(A\cup B)} = \max(\inf I_A, \min(\inf I_A, \inf I_B)) = \inf I_A$, $\sup I_{A\cap(A\cup B)} = \max(\sup I_A, \min(\sup I_A, \sup I_B)) = \sup I_A$,
$\inf F_{A\cap(A\cup B)} = \max(\inf F_A, \min(\inf F_A, \inf F_B)) = \inf F_A$,
$\sup F_{A\cap(A\cup B)} = \max(\sup F_A, \min(\sup F_A, \sup F_B)) = \sup F_A$, that is $A \cap (A \cup B) = A$.

Property 8 (DeMorgan's Laws) $\overline{A \cup B} = \bar{A} \cap \bar{B}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

Proof

(1) $\inf T_{\overline{A\cup B}} = \min(\inf F_A, \inf F_B) = \min(\inf T_{\bar{A}}, \inf T_{\bar{B}})$, $\sup T_{\overline{A\cup B}} = \min(\sup F_A, \sup F_B) = \min(\sup T_{\bar{A}}, \sup T_{\bar{B}})$, $\inf I_{\overline{A\cup B}} = 1 - \min(\sup I_A, \sup I_B) = \max(1 - \sup I_A, 1 - \sup I_B) = \max(\inf I_{\bar{A}}, \inf I_{\bar{B}})$, $\sup I_{\overline{A\cup B}} = 1 - \min(\inf I_A, \inf I_B) = \max(1 - \inf I_A, 1 - \inf I_B) = \max(\inf I_{\bar{A}}, \inf I_{\bar{B}})$, $\inf F_{\overline{A\cup B}} = \max(\inf T_A, \inf T_B) = \max(\inf F_{\bar{A}}, \inf F_{\bar{B}})$, $\sup F_{\overline{A\cup B}} = \max(\sup T_A, \sup T_B) = \max(\sup F_{\bar{A}}, \sup F_{\bar{B}})$, that is $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

(2) $\inf T_{\overline{A\cap B}} = \max(\inf F_A, \inf F_B) = \max(\inf T_{\ bar A}, \inf T_{\bar{B}})$, $\sup T_{\overline{A\cap B}} = \max(\sup F_A, \sup F_B) = \max(\sup T_{\bar{A}}, \sup T_{\bar{B}})$, $\inf I_{\overline{A\cap B}} = 1 - \max(\sup I_A, \sup I_B) = \min(1 - \sup I_A, 1 - \sup I_B) = \min(\inf I_{\bar{A}}, \inf I_{\bar{B}})$, $\sup I_{\overline{A\cap B}} = 1 - \max(\inf I_A, \inf I_B) = \min(1 - \inf I_A, 1 - \inf I_B) = \min(\sup I_{\bar{A}}, \sup I_{\bar{B}})$, $\inf F_{\overline{A\cap B}} = \min(\inf T_A, \inf T_B) = \min(\inf F_{\bar{A}}, \inf F_{\bar{B}})$, $\sup F_{\overline{A\cap B}} = \min(\sup T_A, \sup T_A) = \min(\sup F_{\bar{A}}, \sup F_{\bar{B}})$, that is $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

Property 9 (Involution) $\overline{\overline{A}} = A$

Proof $\inf T_{\bar{A}} = \inf F_A$, $\sup T_{\bar{A}} = \sup F_A$, $\inf I_{\bar{A}} = 1 - \sup I_A$, $\sup I_{\bar{A}} = 1 - \inf I_A$, $\inf F_{\bar{A}} = \inf T_A$, $\sup F_{\bar{A}} = \sup T_A$. $\inf T_{\overline{\overline{A}}} = \inf T_A$, $\sup T_{\overline{\overline{A}}} = \sup T_A$, $\inf I_{\overline{\overline{A}}} = \inf I_A$, $\sup I_{\overline{\overline{A}}} = \sup I_A$, $\inf F_{\overline{\overline{A}}} = \inf F_A$, $\sup F_{\overline{\overline{A}}} = \sup F_A$, that is $\overline{\overline{A}} = A$.

5 Convexity of Degenerated Neutrosophic Set

We assume that X is a real Euclidean space E^n for correctness.

Definition 15 (Convexity) *A degenerated neutrosophic set A is convex if and only if*

$$\inf T_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\inf T_A(x_1), \inf T_A(x_2)), \tag{45}$$
$$\sup T_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\sup T_A(x_1), \sup T_A(x_2)), \tag{46}$$
$$\inf I_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf I_A(x_1), \inf I_A(x_2)), \tag{47}$$
$$\sup I_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup I_A(x_1), \sup I_A(x_2)), \tag{48}$$
$$\inf F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf F_A(x_1), \inf F_A(x_2)), \tag{49}$$
$$\sup F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup F_A(x_1), \sup F_A(x_2)), \tag{50}$$

for all x_1 and x_2 in X and all λ in $[0, 1]$.

Fig. 2 is an illustration of convex degenerated neutrosophic set.

Theorem 4 *If A and B are convex, so is their intersection.*

Proof Let $C = A \cap B$, then
$\inf T_C(\lambda x_1+(1-\lambda)x_2) \geq \min(\inf T_A(\lambda x_1+(1-\lambda)x_2), \inf T_B(\lambda x_1+(1-\lambda)x_2))$,
$\sup T_C(\lambda x_1+(1-\lambda)x_2) \geq \min(\sup T_A(\lambda x_1+(1-\lambda)x_2), \sup T_B(\lambda x_1+(1-\lambda)x_2))$,
$\inf I_C(\lambda x_1+(1-\lambda)x_2) \leq \max(\inf I_A(\lambda x_1+(1-\lambda)x_2), \inf I_B(\lambda x_1+(1-\lambda)x_2))$,
$\sup I_C(\lambda x_1+(1-\lambda)x_2) \leq \max(\sup I_A(\lambda x_1+(1-\lambda)x_2), \sup I_B(\lambda x_1+(1-\lambda)x_2))$,
$\inf F_C(\lambda x_1+(1-\lambda)x_2) \leq \max(\inf F_A(\lambda x_1+(1-\lambda)x_2), \inf F_B(\lambda x_1+(1-\lambda)x_2))$,
$\sup F_C(\lambda x_1+(1-\lambda)x_2) \leq \max(\inf F_A(\lambda x_1+(1-\lambda)x_2), \inf F_B(\lambda x_1+(1-\lambda)x_2))$,
Since A and B are convex: $\inf T_A(\lambda x_1+(1-\lambda)x_2) \geq \min(\inf T_A(x1), \inf T_A(x2))$, $\sup T_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\sup T_A(x1), \sup T_A(x2))$, $\inf I_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf I_A(x1), \inf I_A(x2))$, $\sup I_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup I_A(x1), \sup I_A(x2))$, $\inf F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf F_A(x1), \inf F_A(x2))$, $\sup F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup F_A(x1), \sup F_A(x2))$,
$\inf T_B(\lambda x_1+(1-\lambda)x_2) \geq \min(\inf T_B(x1), \inf T_A(x2))$, $\sup T_B(\lambda x_1+(1-\lambda)x_2) \geq \min(\sup T_B(x1), \sup T_A(x2))$, $\inf I_B(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf I_B(x1), \inf I_A(x2))$, $\sup I_B(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup I_B(x1), \sup I_A(x2))$,
$\inf F_B(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf F_B(x1), \inf F_A(x2))$,
$\sup F_B(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup F_B(x1), \sup F_A(x2))$,
Hence,
$\inf T_C(\lambda x_1 + (1-\lambda)x_2) \geq \min(\min(\inf T_A(x_1), \inf T_A(x_2))$
$, \min(\inf T_B(x_1), \inf T_B(x_2))) = \min(\min(\inf T_A(x_1), \inf T_B(x_1)),$
$\min(\inf T_A(x_2), \inf T_B(x_2))) = \min(\inf T_C(x_1), \inf T_C(x_2))$, $\sup T_C(\lambda x_1 + (1-\lambda)x_2) \geq \min(\min(\sup T_A(x_1), \sup T_A(x_2)),$
$\min(\sup T_B(x_1), \sup T_B(x_2))) = \min(\min(\sup T_A(x_1), \sup T_B(x_1)),$
$\min(\sup T_A(x_2), \sup T_B(x_2))) = \min(\sup T_C(x_1), \sup T_C(x_2))$,
$\inf I_C(\lambda x_1 + (1-\lambda)x_2) \leq \max(\max(\inf I_A(x_1),$
$\inf I_A(x_2)), \max(\inf I_B(x_1), \inf I_B(x_2))) = \max(\max(\inf I_A(x_1),$
$\inf I_B(x_1)), \max(\inf I_A(x_2), \inf I_B(x_2))) = \max(\inf I_C(x_1), \inf I_C(x_2))$,
$\sup I_C(\lambda x_1 + (1-\lambda)x_2) \leq \max(\max(\sup I_A(x_1), \sup I_A(x_2))$
$, \max(\sup I_B(x_1), \sup I_B(x_2))) = \max(\max(\sup I_A(x_1), \sup I_B(x_1)),$
$\max(\sup I_A(x_2), \sup I_B(x_2))) = \max(\sup I_C(x_1), \sup I_C(x_2))$,
$\inf F_C(\lambda x_1 + (1-\lambda)x_2) \leq \max(\max(\inf F_A(x_1), \inf F_A(x_2)),$
$\max(\inf F_B(x_1), \inf F_B(x_2))) = \max(\max(\inf F_A(x_1), \inf F_B(x_1)),$
$\max(\inf F_A(x_2), \inf F_B(x_2))) =$
$\max(\inf F_C(x_1), \inf F_C(x_2))$, $\sup F_C(\lambda x_1 + (1-\lambda)x_2)$
$\leq \max(\max(\sup F_A(x_1), \sup F_A(x_2)),$
$\max(\sup F_B(x_1), \sup F_B(x_2))) = \max(\max(\sup F_A(x_1), \sup F_B(x_1)),$
$\max(\sup F_A(x_2), \sup F_B(x_2))) = \max(\sup F_C(x_1), \sup F_C(x_2))$.

Definition 16 (Strongly Convex) *A degenerated neutrosophic set A is* *strongly convex if for any two distinct points x_1 and x_2, and any λ in the open interval* $(0,1)$,

$$\inf T_A(\lambda x_1 + (1-\lambda)x_2) > \min(\inf T_A(x_1), \inf T_A(x_2)), \tag{51}$$
$$\sup T_A(\lambda x_1 + (1-\lambda)x_2) > \min(\sup T_A(x_1), \sup T_A(x_2)), \tag{52}$$
$$\inf I_A(\lambda x_1 + (1-\lambda)x_2) < \max(\inf I_A(x_1), \inf I_A(x_2)), \tag{53}$$
$$\sup I_A(\lambda x_1 + (1-\lambda)x_2) < \max(\sup I_A(x_1), \sup I_A(x_2)), \tag{54}$$
$$\inf F_A(\lambda x_1 + (1-\lambda)x_2) < \max(\inf F_A(x_1), \inf F_A(x_2)), \tag{55}$$
$$\sup F_A(\lambda x_1 + (1-\lambda)x_2) < \max(\sup F_A(x_1), \sup F_A(x_2)), \tag{56}$$

for all x_1 and x_2 in X and all λ in $[0,1]$.

Theorem 5 *If A and B are strongly convex, so is their instersection.*

Proof The proof is analogus to the proof of Theorem 4.

6 Conclusions and Future Works

In this paper, we have presented an instance of neutrosophic set called degenerated neutrosophic set (DNS). The degenerated neutrosophic set is a generalization of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy sets, interval valued intuitionistic fuzzy set, interval type-2 fuzzy set [6] and paraconsistent set. The notions of inclusion, union, intersection, complement, relation, and composition have been defined on degenerated neutrosophic set. Various properties of set-theoretic operators have been proved. In the future, we will create the logic inference system based on degenerated neutrosophic set and find the appropriate applications such as expert system, data mining, question-answering system, bioinformatics and database, etc.

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