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NEUTROSOPHIC FILTERS

A. A. SALAMA & H. ALAGAMY

Department of Mathematics and Computer Sciences, Faculty of Sciences, Port Said University, Egypt

ABSTRACT

In this paper we introduce the notion of filters on neutrosophic set which is considered as a generalization of fuzzy filters studies in [6], the important neutrosophic filters has been given. Several relations between different neutrosophic filters and neutrosophic topologies are also studied here. Possible applications to computer sciences are touched upon.

KEYWORDS: Fuzzy Filters, Neutrosophic Sets, Neutrosophic Filters, Neutrosophic Topology, Neutrosophic Ultrafilters

INTRODUCTION

The fuzzy set was introduced by Zadeh [11] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. After the introduction of the neutrosophic set concept [8, 9, 10].

The fundamental concepts of neutrosophic set, introduced by Smarandache in 2002 [7, 8] and Salama in 2012[10], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts[1, 2, 3, 4, 5, 7, 11], such as a neutrosophic set theory

PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [8, 9], Atanassov in [1, 2, 3], Salama [10] and Kul Hur at el [6]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is nonstandard unit interval.

Definition 2.1. [10]

Let T, I, F be real standard or nonstandard subsets of $[^{-}0,\ 1^{+}]$, with

Sup_T=t_sup, inf_T=t_inf

Sup_I=i_sup, inf_I=i_inf

Sup_F=f_sup, inf_F=f_inf

n-sup=t_sup+i_sup+f_sup

n-inf=t_inf+i_inf+f_inf,

T, I, F are called neutrosophic components

Definition 2.2. [10]

Let X be a non-empty fixed set. A neutrosophic set (NS for short or ($(A \in N^X)$) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A .

Definition 2.3 [10] The NSs 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

1_N may be defined as:

(1_1) $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

BASIC PROPERTIES OF NEUTROSOPHIC FILTERS

Definition 3.1. Let N be a neutrosophic subsets in a set X. Then N is called a neutrosophic filter on X, if it satisfies the following conditions:

(N_1) Every neutrosophic set in X containing a member of N belongs to N .

(N_2) Every finite intersection of members of N belongs to N .

(N_3) O_N not in N .

In this case, the pair (X, N) is called neutrosophic filtered by N .

It follows from (N_2) and (N_3) that every finite intersection of members of N is not O_N . Furthermore, there is no neutrosophic set. We obtain the following results.

Proposition 3.1. The condition (N_2) is equivalent to the following two conditions

(N_{2a}) The intersection of two members of N belongs to N .

$(N_{2b})\, 1_N$ belongs to N

Proposition 3.2. Le N be a non-empty neutrosophic subsets in X satisfying (N_1).Then,

- $1_N \in N$ iff $N \neq O_N$
- $O_N \notin N$ iff $N \neq$ all neutrosophic subsets of X.

From the above Propositions (3.1) and (3.2), we can characterize the concept of neutrosophic filter:

Theorem 3.1. Let N be a neutrosophic subsets in a set X. Then N is neutrosophic filter on X, if and only if it is satisfies the following conditions

- Every neutrosophic set in X containing a member of N belongs to N.
- If $A, B \in N$, then $A \cap B \in N$.
- $N^X \neq N \neq O_N$.

Proof: It's clear.

Theorem 3.2. Let $X \neq \phi$. Then the set $\{1_N\}$ is a neutrosophic filter on X. Moreover if A is a non-empty neutrosophic set in X, then $\{B \in N^X : A \subseteq B\}$ is a neutrosophic filter on X

Proof: Let. $N = \{B \in N^X : A \subseteq B\}$. Since $1_N \in N$ and $O_N \notin N$, $O_N \neq N \neq N^X$. Suppose $U, V \in N$, then $A \subseteq U, A \subseteq V$. Thus $\mu_A(x) \leq \min(\mu_U(x), \mu_V(x))$, $\sigma_A(x) \leq \min(\sigma_U(x), \sigma_V(x))$ or $\sigma_A(x) \leq \max(\sigma_U(x), \sigma_V(x))$ and $\gamma_A(x) \leq \max(\gamma_U(x), \gamma_V(x))$ for all $x \in X$. So $A \subseteq U \cap V$ and hence $U \cap V \in N$.

COMPARISON OF NEUTROSOPHIC FILTERS

Definition 4.1. Let N_1 and N_2 be two neutrosophic filters on a set X. Then N_2 is said to be finer than N_1 or N_1 coarser than N_2 if $N_1 \subset N_2$

If also $N_1 \neq N_2$, then N_2 is said to be strictly finer than N_1 or N_1 is strictly coarser than N_2.

Two neutrosophic filters are said to be comparable, if one is finer than the other. The set of all neutrosophic filters on X is ordered by the relation N_1 is coarser than N_2, this relation is induced the inclusion relation in N^X.

Proposition 4.1. Let $(N_j)_{j \in J}$ be any non-empty family of neutrosophic filters on X. Then $N = \cap_{j \in J} N_j$ is a neutrosophic filter on X. In fact N is the greatest lower bound of the neutrosophic set $(N_j)_{j \in J}$ in the ordered set of all neutrosophic filters on X.

Remark 4.1.The neutrosophic filter by the single neutrosophic set 1_N is the smallest element of the ordered set of all neutrosophic filters on X.

Theorem 4.1. Let A be a neutrosophic sets in X. Then there exists a nutrosophic filter $N(A)$ on X containing A iff for any finite subset $\{S_1, S_2, ..., S_n\}$ of A, $\cap_{i=1} S_i \neq O_N$. In fact $N(A)$ is the coarest neutrosophic filter containing A.

Proof $(\Rightarrow)$ Suppose there exists a nutrosophic filter $N(A)$ on X containing A. Let B be the set of all the finite intersections of members of A. Then by (N_2), $B \subset N(A)$. By (N_3), $O_N \notin N(A)$. Thus for each member B of B, Hence the necessary condition holds

$(\Leftarrow)$ Suppose the necessary condition holds. Let $N(A) = \{A \in N^X : A \text{ contains a member of B}\}$. Where B is the family of all the finite intersections of members of A. Then we can easily check that $N(A)$ satisfies the conditions in Definition3.1

The neutrosophic filter $N(A)$ defined above is said to be generated by A and A is called a sub - base of $N(A)$.

Corollary 4.1. Let N be a neutrosophic filter in a set X and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than N and such that $A \in N^{/}$ iff and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than N and such that $A \in N^{/}$ iff $A \cap U \neq O_N$ for each $U \in N$.

Corollary 4.2 A set φ of a neutrosophic filter on a non-empty set X, has a least upper bound in the set of all neutrosophic filters on X iff for all finite sequence $(N_j)_{j \in J}, 0 \le j \le n$ of elements of φ and all $A_j \in N_j (1 \le j \le n)$, $\cap_{j=1} A_j \neq O_N$

Corollary 4.3. The ordered set of all neutrosophic filters on a non-empty set X inductive.

If Λ is a sub base of a neutrosophic filter N on X, then N is not in general the set of neutrosophic sets in X containing an element of Λ; for Λ to have this property it is necessary and sufficient that every finite intersection of members of Λ should contain an element of Λ. Hence we have the following result:

Theorem 4.2. Let β is a set of neutrosophic sets on a set X. Then the set of neutrosophic sets in X containing an element of β is a neutrosophic filter on X iff β has the following two conditions

(β_1) The intersection of two members of β contain a member of β.

(β_2) $\beta \neq O_N$ and $O_N \notin \beta$.

Definition 4.2. Let Λ and β are neutrosophic sets on X satisfying conditions (β_1) and (β_2) is called a base of neutrosophic filter it generates. Two neutrosophic bases are said to be equivalent, if they generate the same neutrosophic filter.

Remark 4.2. Let Λ be a subbase of neutrosophic filter N. Then the set β of finite intersections of members of Λ is a base of filter N.

Proposition 4.2. A subset β of a neutrosophic filter N on X is a base of N iff every member of N contains a member of β.

Proof $(\Rightarrow)$ Suppose β is a base of N. Then clearly, every member of N contains an element of β. $(\Leftarrow)$ Suppose the necessary condition holds. Then the set of neutrosophic sets in X containing a member of β coincides with N by reason of $(N_j)_{j \in J}$.

Proposition 4.3. On a set X, a neutrosophic filter $N^{/}$ with base $\beta^{/}$ is finer than a neutrosophic filter N with base β iff every member of β contains a member of $\beta^{/}$.

Proof This is an immediate consequence of Definitions 4.2 and 4.4.

Proposition 4.4. Two neutrosophic filters bases β and $\beta^{/}$ on a set X are equivalent iff every member of β contains a member of $\beta^{/}$ and every member of $\beta^{/}$ and every member of $\beta^{/}$ contains a member of β.

NEUTROSOPHIC ULTRAFILTERS

Definition 5.1. A neutrosophic ultrafilter on a set X is a neutrosophic filter N such that there is no neutrosophic filter on X which is strictly finer than N (in other words, a maximal element in the ordered set of all neutrosophic filters on X).

Since the ordered set of all the neutrosophic filters on X inductive, Zorn's lemma shows that

Theorem 5.1. If N be any neutrosophic ultrafilter on a set X, then there is a neutrosophic ultrafilter than N.

Propostition 5.1. Let N be a neutrosophic ultrafilter on a set X. If A and B are two neutrosophic subsets such that $A \cup B \in N$, then $A \in N$ or $B \in N$.

Proof: Suppose not. Then there exist neutrosophic sets A and B in X such that $A \notin N, B \notin N$ and $A \cup B \in N$. Let $\Lambda = \{M \in N^{X} : A \cup M \in N\}$. It is straightforward to check that Λ is a neutrosophic filter on X, and Λ is strictly finer than N, since $B \in \Lambda$.This contradiction the hypothesis that N is a neutrosophic ultrafilter.

Corollary 5.1. Let N be a neutrosophic ultrafilter on a set X and let $(N_j)_{1 \le j \le n}$ be a finite sequence of neutrosophic sets in X. If $\bigcup_{j=1} N_j \in N$, then at least one of the N_j belongs to N.

Definition 5.2. Let A be a neutrosophic set in a set X. If U is any neutrosophic set in X, then the neutrosophic set $A \cap U$ is called trace of U an A and denoted by U_A. For all neutrosophic sets U and V in X, we have $(U \cap V)_A = U_A \cap V_A$.

Definition 5.3. Let A be a neutrosophic set in a set X. Then the set Λ_A of traces an $A \in N^X$ of member of Λ is called the trace of Λ an A.

Propsition 5.2. Let N be a neutrosophic filter on a set X and $A \in N^X$. Then the trace of N_A of N an A is a neutrosophic filter iff each member of N meets A.

Proof. From the result in Definition5.3, we see that N_A satisfies (N_2).If $M \cap A \subset P \subset A$, then $P = (M \cup P) \cap A$. Thus N_A satisfies (N_1). Hence N_A is a neutrosophic filter iff it satisfies (N_3). i.e. iff each member of N meets A.

Definition 5.4. Let N be a neutrosophic filter on a set X and $A \in N^X$. If the trace N_A of N an A, then N_A is said to be induced by N an A.

Propsition 5.3. Let N be a neutrosophic filter on a set X induced a neutrosophic filter N_A on $A \in N^X$. Then trace β_A on A of a base β of N is a base of N_A.

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