Cosmology in Context:Current studies of the early universe through astronomy and particle physics, experiments, observations, and theories.

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2013

Contents

List of Figures

Acknowledgements

It must be acknowledged that this work began its life as a thesis project under Professors Dr Jesús Pando, and Dr. Anuj Sarma at DePaul University in Chicago. Two years ago towards the completion of this project a falling out was had with one of them and this thesis was abandoned. Now after refreshing the manuscript a little I present this monograph. The word "thesis" appears in the text many times to remind the reader of how this paper started its life. I thank both of them for their help with this work, and with subsequent work at DePaul university (such as my new thesis project.)

I must acknowledge my father Dr. Hontas F. Farmer, and mother Mrs. Barbara A. Farmer. Without their help and support I could not have done this research.

I must also say that without the Quantum Field Theory I learned from Dr. Mikhail Stephanov , the Particle Physics I learned from Dr. Thomas Imbo , and the General Relativity I learned from Dr. Arthur Licht , at the University of Illinois at Chicago, I could not have done the work presented here. I want to thank Dr. João Magueijo for permission to use his figure in this thesis. I also must thank the science teams at NASA and the European Space Agency for their work on the Cosmic Microwave Background. Last, I want to thank anyone who I asked to read this work for clarity of composition for their helpful comments.

Finally, the researchers cited in this thesis for their contributions to the overall body of knowledge that my work is built on. Finally, the researchers whose work is cited in this book. Without those who came before me I could not travel as far.

Foreword

This is a comprehensive review of the published research in cosmology focusing on the time period from the big bang to the last scattering of cosmic microwave background radiation. This is a period of approximately 380,000 years. Theoretical, observational, and experimental research with a bearing on cosmology will be covered. First, a time line of events from the big bang to last scattering of CMB photons will be provided. Then, a review of theoretical research related to the big bang, cosmic inflation, and baryogenesis will be covered. Next, a review of observational as well as experimental work on the cosmic microwave background, big bang nucleosynthesis, and efforts to directly detect gravitational waves. After that, a look at research on the edge of accepted cosmology such as loop quantum cosmology, and the possible time variation of fundamental constants. Last but not least this author will present a tiny, and novel theoretical idea, a Lagrangian which captures all of the physics of the standard model of cosmology.

Part I

A timeline from the Big Bang to the last scattering of the Cosmic Microwave Background.

The subject of this thesis is a topical review of published research literature concerning the first 380,000 years of the the universes existence. This is a collection of research which deals with many varied types of physics. To make some sense out of the whole menagerie I have written chapter one as a timeline of events and placed the various pieces of research in their temporal context.

Chapter 1

The first few hundred thousand years of existence.

This chapter will simply be a timeline of events from the Big Bang to the emission of the Cosmic Microwave Background Radiation about 380,000 years later. The research that will be detailed later on in this thesis will be mentioned in its temporal context in this chapter. Each subject being mentioned in order of the period of time it concerns and not its importance or level of acceptance by the cosmological community.

The following diagram is a graphic representation of this timeline. In essence every sentence of this chapter is about what is represented in this figure. Every stage of universal evolution shown on this figure is discussed in this chapter, and detailed in subsequent chapters.

As figure 1.1 shows the universe started out very small and dense before time $t < 10^{-36}\,sec.$. Then expanded rapidly, at the same time matter was created from energy. Then elements heavier than hydrogen were created within the first three minutes of the Big Bang. Then for a long long time, from three minutes to three hundred and eighty thousand years, and the emission of the Cosmic Microwave Background, the universe was filled with a fog of mostly protons, electrons, and photons. These events did not begin and end at the same exact moment everywhere in the universe. Small fluctuations at these times would eventually evolve into the large scale structures, and clusters of galaxies we see today.

Details of the research that informs this timeline will be given in subsequent parts, chapters, and sections of this thesis.

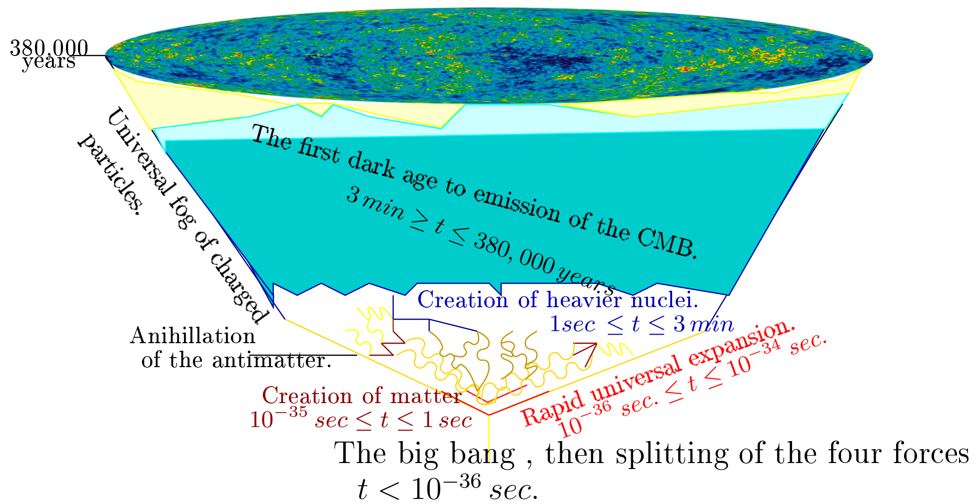


Figure 1.1: This is a rendering of what the Big Bang, and history of the universe would have looked like up to the emission of the Cosmic Microwave Background radiation (CMB).

1.1 The Big Bang $t < 10^{-36}\,sec.$

The Big Bang occurred at some time $t < 10^{-36}\,sec.$. The classical Big Bang starts at time equals zero, in a singularity where known physics breaks down. Then for a reason we do not know the universe began to expand and that expansion is what we call the big bang. There is no agreement on what the Big Bang was beyond saying that it was something that occurred at a point where the universe was so small, dense, and energetic that classical physics does not apply. This is *not* an explosion in any physical sense. An explosion is a sudden free expansion of hot gases. The Big Bang was the expansion of space-time itself from a singular point. The explosion metaphor is not physically correct in any sense.

There are theories which attempt to probe the time of the Big Bang itself. They involve quantization of gravity and or the unification of the fundamental forces of nature. These are not observationally supported at the moment. However theoretical physicist find them interesting for their mathematical consistency even when and where classical physics breaks down.

1.2 Rapid universal expansion. $10^{-36}\, sec. \leq t \leq 10^{-34}\, sec.$

Directly following the Big Bang from $10^{-36}\, sec. \leq t \leq 10^{-34}\, sec.$ the universe expanded exponentially. The exact mechanism of this expansion is a matter of intense theoretical, observational, and even experimental research. Most of this research is done under the heading of “inflation". inflationary theory was proposed to explain how the contents of the universe could be very uniform on the cosmic scale as observed in the cosmic microwave background radiation.

1.3 Creation of matter.$10^{-35}\, sec \leq t \leq 1\, sec$

Happening at the same time as the last phases of the rapid universal expansion the first matter was created. As the expansion of the universe came to an end, the very field the caused its rapid expansion reheated the universe. In the process, the creation of matter and anti-matter was thrown in to just enough thermal disequilibrium to create more matter than anti-matter. This resulted in a universe visibly filled with matter.

This initial matter was in the form of electrons, and quarks. The quarks would very quickly combine to form protons and neutrons. Atoms, however, could not yet persist, only ions of hydrogen and free electrons. The theories and the evidence that backs up these theories will be discussed in part two chapter four of this thesis.

1.4 Creation of heavier nuclei. $1 sec \leq t \leq 3\, min$

The period $1 sec \leq t \leq 3\, min$ is when heavier nuclei than that of simple hydrogen were produced. The nucleus of hydrogen in its simplest form is just a proton. The universe was at the right temperature and density during these minutes to allow the fusion of hydrogen into heavier elements. During this period, heavy isotopes of hydrogen were produced as well as helium, lithium, and beryllium. The ratio’s of these elements are one of the tightest constraints on theories about the early universe. The research that informs our view of this period will be discussed in part three chapter 6 of this thesis.

1.5 The first dark age and the last scattering of the Cosmic Microwave Background radiation. $3\, min \leq t \leq 380,000\, years$

After the creation of the first heavy nuclei the universe was too hot for stable atoms to exist. It was in a sense a universe of plasma. Vast clouds of ionized gas were all that would exist during this period. This is because a photon could not travel far before combining with a proton and electron to from a true hydrogen atom. At the same time the universe was so hot and dense that any atom that did form would become excited and lose all of its electrons. Those electrons would be captured, and photons emitted only to be reabsorbed almost instantly. This was all that there was in the universe for hundreds of thousands of years.

At a point about 7,000 years or so into this period the universes energy density was no longer dominated by particles moving at relativistic speeds. This marked the transition from a universe dominated by radiation to one dominated by matter. This changed the mathematical law governing the expansion of the universe with time from $t^{\frac{1}{2}}$ to $t^{\frac{2}{3}}$.

About 380,000 years after the Big Bang the universe became cool enough, and of low enough density to allow the propagation of light. This first light would be the only light until the first stars and galaxies formed. This first light is what we now detect as the Cosmic Microwave Background radiation. Encoded in its hot spots, and warm spots, and polarization is information on the density, temperature, and composition of the entire universe.

Observations of the Cosmic Microwave Background (CMB), along with measurements of the mass of the universe, and other theoretical and observational considerations have allowed cosmologists to build up a model for the universe. This model has a universe filled with mostly dark energy (symbolized as Λ) and cold dark matter (CDM). Observations of the CMB back up this model primarily via its ability to fit data gathered on the angular power spectrum of the CMB.

The ΛCDM model fits the data we have very well. The details of this model are still in question. For example, there are a number of specific models for why the universe expanded rapidly. There are a number of possible forms of cold dark matter. Observations which will answer many questions, and reveal new ones, are covered in chapter five of this thesis.

1.6 Organization of this thesis.

This thesis is organized into five parts. Each part focuses on a broad type of research. Part one is a timeline meant to place each area of research into temporal context.

Part two focuses on the mathematical foundations of theoretical cosmology starting with a brief but thorough review of General Relativity and then inflationary cosmology. This part finishes with a review of Quantum Field Theory and particle physics which leads to a discussion of theories on the creation of matter. This part describes in detail the current standard model of cosmology known as the concordance or ΛCDM model (Λ dark energy, CDM Cold Dark matter). This model fits all the observations made to date very well, and has great flexibility.

Part three concentrates on observational and experimental particle cosmology. The work covered here focuses on observations of the Cosmic Microwave Background radiation. The CMB is a rich source of data on the earliest evolution of the universe. In particular, observations underway right now may reveal a Cosmic Gravitational Background. A strong gravitational wave background is predicted by inflationary cosmology and the ΛCDM model. The first evidence of a gravitational background may be found in the polarization of the CMB. Next observations which will improve our measurements of the ratios of light elements created after the Big Bang will be reviewed. Last, experimental work at the Large Hadron Collider will be discussed in relation to its bearing on cosmology.

Part four focuses on theories and observations on the frontier of cosmological research. This research focuses on various theories and observations which are controversial and less well tested than the standard models. These models often seek to extend adjust, supersede and/or supplant the current standard models. This part includes Loop Quantum Cosmology, and M-theory which give mathematical insight into the nature of the Big Bang that the standard models do not. This part also includes observations which suggest that certain quantities which seem to be constant in space, have varied with time. An alternative model for the universes rapid expansion will be discussed. Last but not least a Lagrangian for the standard model of cosmology proposed by this author, and submitted for publication to peer reviewed journals will be outlined.

Part five is an executive summary of the first four parts of the thesis.

Part II

Research related to and the mathematical foundations of the standard model of theoretical cosmology.

This section explains the current standard model of cosmology known as the concordance or ΛCDM model. This is a model where the universe is filled primarily with dark energy (Λ), and cold dark matter (CDM). The deep reasons why so many cosmologist, and physicist prefer this model will be made clear by examination of the fundamentals. Those fundamentals are General Relativity, inflation, and theoretical particle Physics.

Next Quantum Field Theory (QFT) and elementary particle physics will be explained. Then the standard model of particle physics. Last but not least the extension of the standard model which includes possible candidate dark matter particles will be discussed.

> Es ist immer angenehm, über strenge Lösungen einfacher Form zu verfügen. (It is always pleasant to have exact solutions in simple form at your disposal.) So said Karl Schwarzschild in "On the Gravitational Field of a Mass Point according to EinsteinâĂŹs Theory," 1916.[5]

Chapter 2

The Friedman-Lemaître-Robertson-Walker metric.

The theory which will concern us most is General Relativity by way of a specific solution to the Einstein field equations. This solution is the one which gave us the mathematical theory of the big bang. The other important component of modern cosmology is known as inflation. The rapid universal expansion, proposed by inflation, addresses certain issues of the previously mentioned solution to Einstein's field equations.

2.1 A brief introduction to Einsteins field equations of General Relativity.

To understand theoretical cosmology one must understand the Einstein field equations of General-Relativity and one particular solution to those equations, the Friedman-Lemaître-Robertson-Walker metric. For now let it suffice to say that a solution to the Einstein field equations is a metric and since this thesis discusses no other kind of metric that the converse is also true in this particular context. To give a more detailed definition would require a number of mathematical tools, and would distract from the topic of this chapter.

To keep this thesis uncluttered with an abundance of mathematical derivation an informal discussion of these points of mathematics will be in the main body text. A more detailed and mathematical account of General Relativity will be given in Appendix A, and a good book to refer to on this topic is [6].

The Einstein field equations are in tensor form.

There are a few ways to define a tensor. A simple and intuitive definition is that a tensor of rank "n" is a quantity that has magnitude and "n" directions. Thus a tensor of rank one has a magnitude and one direction, a tensor of rank one is just a vector. The definition a dictionary of mathematics would give is "An abstract object having a definitely specified system of components in every coordinate system under consideration such that, under transformation of coordinates, the components of the object undergo a transformation of a certain nature." Which while mathematically correct is not very useful for the purposes of this thesis.

A mathematical yet immediately applicable definition would be the following:

A tensor of rank n in a m dimensional space, over the field of real numbers, is a function which is linear in n variables with m^ncomponents which, under transformation of coordinates, the components of the object undergo a transformation of a certain nature and it maps n vectors to the real numbers.

$$M_{\mu\nu}V^{\mu}V^{\nu} \to m \tag{2.1}$$

, $m \in \mathbb{R}$.

The most important example of a tensor for our purposes would be the metric tensor $g_{\mu\nu}$ which, *by definition*, is a solution to the Einstein field equations, and maps vectors in spacetime to the real numbers in such a way that the output is a "distance" between the vectors. Tensors will be defined over a field of real numbers never complex numbers unless explicitly stated otherwise.

There is also the Ricci curvature tensor $R_{\mu\nu}$which measures how curved the space-time manifold (a vector space with the property that it is locally homeomorphic to the flat Minkowski space of Special Relativity) is. The Ricci scalar which is a product of the Ricci tensor, and the metric expresses this curvature in the form of a tensor of rank zero known as the Ricci scalar.

The third important tensor in the Einstein field equations is the stress energy tensor $T_{\mu\nu}$. This tensor represents the distribution of mass-energy-momentum in the spacetime manifold. In cosmology another term is added which represents the vacuum energy or "dark energy" Λ. This is known as the cosmological constant. The vacuum energyΛ along with the cold dark matter which is thought to make up most of the universes mass give their name to the concordance model of cosmology, ΛCDM. This model provides a good fit with all of the data gathered to date.

With the quantities that make up the Einstein field equations described the equation(s) are.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2.2}$$

$G_{\mu\nu}$ is known as the Einstein tensor. It is a tensor of rank two in a four dimensional space. It therefore has 16 components. So the Einstein Field equation is really as many as 16 coupled partial differential equations. All of the exact solutions to these equations have been found by assuming one type of symmetry or the other.

2.2 Deriving the Friedman-Lemaître-Robertson-Walker metric.

The solution to the Einstein field equations that will concern us the most is due to Alexander Friedman, Georges Lemaître, Howard Percy Robertson and Arthur Geoffrey Walker. Friedman and Lemaître derived this metric from the Einstein field equations, Robertson and Walker proved that this metric is the only one that fits two assumptions about the nature of space, isotropy and homogeneity. This derivation will draw on the work of Robertson and Walker as found in Caroll [6].

To derive this metric it will be assumed that the space-time of the universe has the following properties.

The space manifold of the universe will be invariant under translations or *homogeneous*. In more mathematical terms this means that the metric will be the same throughout the manifold. Given the manifold M and two points $p, q \in M$ there exist an isometry that maps p into q.

The space manifold of the universe will be invariant under rotations or *isotropic*. In mathematical terms this means around some point p on the manifold M there exist a space that is tangent to the manifold (T_p). For any two vectors $V, W \in T_p$ there exist a isometry that will map V into W.

Observations of the cosmic microwave background back these assumptions up. The homogeneity and isotropy of space is necessary for the isotropy of the CMB but *not* sufficient to explain it. The isotropy of the CMB is a property of the contents of the universe not of the space-time manifold of the universe itself.

To visualize this consider the surface of a ideal three dimensional sphere. Under any rotation the sphere looks the same. At any two points the metric on the sphere will be the same. The surface of the sphere is homogeneous and isotropic.

The assumption of isotropy and homogeneity of space-time is valid on the cosmological length scale of clusters of clusters of galaxy's. On the smaller scale of solar systems and planets these assumptions do not hold. On this scale each object distorts the space-time in such a way that the metric is not the same. For example compare gravity on Earths surface and in Earth orbit. The difference in gravity is due to the difference in the metric at those two points thus the metric cannot be the same throughout the space near Earth. It is in fact subject to a very different metric from FLRW.

With the assumptions of a homogeneous and isotropic space the metric can almost be written down without solving an equation.

$$ds^2 = -dt^2 + R^2(t)d\Omega^2 \tag{2.3}$$

The function $R(t)$scales the space part of the metric with time and hence is known as the scale factor and carries the dimension of length. The spatial part of the metric can be written in a general form as follows.

$$d\Omega^2 = \omega_{ij} dw^i dw^j \tag{2.4}$$

The coordinates w^i are to be chosen in such a way that any cross terms in the metric cancel out. These are known as comoving coordinates. ω_{ij} is a metric tensor for the three dimensional space part of the manifold. In a space the the isotropy and homogeneity that has been assumed the Ricci tensor on the spatial part of this manifold will be[6].

$$R_{ij} = \frac{R}{3}\omega_{ij} \tag{2.5}$$

To get a more specific form for this metric, we can guess that it will have spherical symmetry. Spherical symmetry is maximal symmetry as well. To see this, again, consider a perfect three dimensional sphere. Rotate the sphere and no matter the perspective it looks the same, translate from one point to the other it still looks the same. It is a homogeneous and isotropic manifold. For such a space the most general form for the space part of the metric is

$$d\Omega^2 = e^{2\beta(\bar{r})}d\bar{r}^2 + \bar{r}^2 d\theta^2 + sin^2(\theta)\bar{r}^2 d\phi^2 \tag{2.6}$$

Where $\bar{r}$ is a radial coordinate with no units since length is encoded in the scale factor up to this point. The next step is to solve e for β. This would be done by finding the components of the Ricci tensor, and setting them equal to the metric. Then solving the resulting system of equations for beta. This has all been done before and the answer is[6].

$$\beta = -\frac{1}{2} ln(1 - k\bar{r}^2) \tag{2.7}$$

Now substitute beta into the equation for the metric.

$$d\Omega^2 = e^{-ln(1-k\bar{r}^2)} d\bar{r}^2 + \bar{r}^2 d\theta^2 + sin^2(\theta)\bar{r}^2 d\phi^2 \tag{2.8}$$

$$d\Omega^2 = \frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 d\theta^2 + sin^2(\theta)\bar{r}^2 d\phi^2 \tag{2.9}$$

k in the above is normalized to take on the values $k \in \{-1, 0, +1\}$. These values relate to a open, flat, and closed universe respectively. The FLRW metric looks like this.

$$ds^2 = -dt^2 + R^2(t) \left(\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 d\theta^2 + sin^2(\theta)\bar{r}^2 d\phi^2 \right) \tag{2.10}$$

Following the lead of Sean Caroll's book let us make the scale factor dimensionless and the radial coordinate dimensionful with the unit of length. This will be done by dividing R by a constant fundamental length. The only length that fits is the Planck length $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616252(81) \times 10^{-35}$. This length enters and is defined in terms of fundamental constants and as such should not vary. This particular length also plays a role in theories of quantum gravity in which it defines a smallest possible physical length.

$$a(t) = \frac{R(t)}{\ell_P} \tag{2.11}$$

$$\ell_P \bar{r} = r \tag{2.12}$$

With these substitutions the Friedman-Lemaître-Robertson-Walker metric is in the following form....

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + sin^2(\theta) r^2 d\phi^2 \right) \tag{2.13}$$

In terms of its metric tensor the FLRW solution is [6].

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a(t)^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & a(t)^2 r^2 & 0 \\ 0 & 0 & 0 & a(t)^2 r^2 \sin^2\theta \end{pmatrix} \tag{2.14}$$

$a(t)$ is found by solving the Friedman equations, which while non-linear have simple and physically informative solutions. What solution is valid depends on weather the universe is filled with mostly radiation, matter or as it currently is dark energy. More details are given in section 2.3.

2.3 Friedman's equations and their solutions.

To solve Friedman's equations we will start with the Friedman-Lemaître-Robertson-Walker metric (FLRW)[6].

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2 \right) \right) \tag{2.15}$$

We want to solve for the scale factor $a(t)$.

Given the FLRW metric, a positive cosmological constant, and that the stress energy tensor is equal to zero (a vacuum state) solve for the scale factor $a(t)$.

The scale factor in the FLRW metric is a function of time which controls how space will expand (or contract) with time. It is the evolution of this scale factor which gives us the current expansion of the universe, as well as its past expansion. The object of this problem is not to find a metric, we have that. The object is not to solve for the stress energy tensor, since we have chosen that to be zero. In its place is a positive cosmological constant Λ. The solution to be derived will be valid for the universe as it exist now dominated by the dark energy Λ.

These are the Einstein Field equations to be solved for $a(t)$.

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = R_{\mu\nu} - Rg_{\mu\nu} + g_{\mu\nu}\Lambda = 0 \tag{2.16}$$

This simplifies to an equation involving the Ricci tensor, the Ricci scalar, and *Lambda*.

$$\Rightarrow R_{\mu\nu} + (\Lambda - R)g_{\mu\nu} = 0 \tag{2.17}$$

The next step is to find the Ricci tensor and Ricci scalar for the FLRW metric. These are well known for this metric and are given in[6, p.333]. The Ricci tensor for the FLRW metric is.

$$R_{\mu\nu} = \begin{pmatrix} -3\frac{\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & \frac{a\ddot{a}+2\dot{a}^2+2k}{1-kr^2} & 0 & 0 \\ 0 & 0 & r^2(a\ddot{a}+2\dot{a}^2+2k) & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta(a\ddot{a}+2\dot{a}^2+2k) \end{pmatrix} \tag{2.18}$$

The Ricci scalar for the FLRW metric is.

$$R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \tag{2.19}$$

With these the Einstein field equations can be written explicitly. For compactness substitute... $A = a\ddot{a} + 2\dot{a}^2 + 2k$. The resulting Einstein field equations represented with matrices are.

$$\begin{pmatrix} -3\frac{\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & \frac{A}{1-kr^2} & 0 & 0 \\ 0 & 0 & r^2 A & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta A \end{pmatrix}$$

$$+\left(\Lambda - 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\right)\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2r^2 & 0 \\ 0 & 0 & 0 & a^2r^2 sin^2\theta \end{pmatrix} = 0$$

At this point the space like components are clearly common to all terms and can be simply canceled out. To do so multiply by the following matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-kr^2 & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2} sin^{-2}\theta \end{pmatrix} \tag{2.20}$$

The simplification that results is dramatic. This problem started out with as many as 16 coupled, and non-linear differential equations. With the assumptions and simplifications that have been made, we are left with only two independent equations.

$$\begin{pmatrix} -3\frac{\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{pmatrix} + \left(\Lambda - 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\right) \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{pmatrix} = 0 \tag{2.21}$$

The two independent equations are.

$$\left\{ \begin{matrix} -3\frac{\ddot{a}}{a} - \Lambda + 6\frac{\ddot{a}}{a} + 6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{k}{a^2} = 0 \\ a\ddot{a} + 2\dot{a}^2 + 2k + \Lambda a^2 - 6\ddot{a}a^2 - 6\dot{a}^2 - 6k = 0 \end{matrix} \right\} \tag{2.22}$$

To further simplify the problem add one equation to the other, which results in a number of cancellations. The result leaves one equation in terms of the scale factor and its time derivatives, and a single constant k. k depends on the geometry of the universe. $k = 1, 0, -1$ gives results appropriate for a closed universe, a flat universe, or a hyperboloidal open universe respectively. In addition as shown in table 2.1 there are the possibilities of a matter dominated, radiation dominated, and Λ dominated universe. Right now we observe a nearly flat universe in a Λ dominated phase of its evolution.

Every observation we have points to us living in a very flat universe. For such a universe $k = 0$ Setting $k = 0$ Which leads to the following simplification.

$$-a\ddot{a} + \dot{a}^2 + k = 0 \tag{2.23}$$

$$-a\ddot{a} + \dot{a}^2 = 0 \tag{2.24}$$

Equation 2.24 can be solved by the the elementary technique of letting $a = Me^{Nt}$. Then taking derivatives to get $\dot{a} = MNe^{Nt}$, and $\ddot{a} = MN^2e^{Nt}$ where M and N are constants. Now to check this candidate solution satisfies the equation.

$$-M^2N^2e^{2Nt} + M^2N^2e^{2Nt} = 0 \tag{2.25}$$

As it turns out this solution will satisfy the equation for any constants M and N. With that the constants can be set equal to whatever values make physical sense. Commonly M is set equal to one in this case, and N equal to the Hubble constant. $H_0 = 100h(km/sec/Mpc)$ where $h \simeq 0.7$ [6]. The result is the solution for a positive cosmological constant.

$$a(t) = e^{H_0 t} \tag{2.26}$$

Table 2.1: Solutions to the Friedman Equations

	ω	$\rho(a)$	$a(t)$	$a(\tau)$	τ_i
Matter Dominated	0	a^{-3}	$t^{\frac{2}{3}}$	τ^2	0
Radiation Dominated	$\frac{1}{3}$	a^{-4}	$t^{\frac{1}{2}}$	τ	0
Λ Dominated	-1	1	$e^{H_0 t}$	$-\tau^{-1}$	$-\infty$

This is physically valid because it aligns with all of our observations of a expanding and accelerating universe. The expansion is driven by the positivity of the cosmological constant causing a negative pressure thus expanding the universe. The table 2.1 is similar to one in [1].

Lurking inside the above derivation are two fundamental equations of modern cosmology. These are the Friedman equations. [6]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{2.27}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2.28}$$

The Hubble parameter also appears.

$$\frac{\dot{a}}{a} = H \tag{2.29}$$

With these equations in hand the density parameter can be defined. [6]

$$\Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_{crit}} \tag{2.30}$$

With these definitions, the Friedman equation can be written in terms of these various parameters in a very simple looking form. [6]

$$\Omega - 1 = \frac{k}{H^2 a^2} \tag{2.31}$$

For k=0 $\Omega = 1$ Which is very close to the observed value of 0.7 meaning the universe is nearly perfectly flat. The density of the universe is very close to the critical density[7, 6]. The other two options are open universe if $k < 0$, and a closed universe if $k > 0$.

2.4 Successes of Friedman-Lemaître-Robertson-Walker cosmology.

FLRW theory has to its credit good agreement with many observations. It predicts correctly that every galaxy would be observed to move away from ours. It predicts that the universe was at a finite time in the past in a hot, dense state, which is in accord with the existence of the cosmic microwave background. The CMB is a relic of that past hot dense era. FLRW theory predicts the growth of the universe through the past radiation and matter dominated era's, as well as in the current dark energy dominated era. These successes are of great importance to modern cosmology and form the basis of all accepted theoretical cosmology. FLRW theory is the big bang theory and thanks to it we have a scientific answer for what happened in the beginning of time.

The pure General Relativity which will be discussed in the body of this thesis is complete. Appendix A is a brief review of General Relativity in which enough of the basic theory is described to understand this thesis for anyone who wants/needs to be reminded. A text book which has many more details on General Relativity is "An Introduction to General Relativity Spacetime and Geometry" by Sean M. Carroll [6].

2.5 Problems posed by the Cosmic Microwave Background given the Friedman-Lemaître-Robertson-Walker metric.

The Cosmic Microwave Background (CMB) is very smooth and even in temperature (at 2.725 kelvin). The universe is filled with mass-energy at a density about 70 percent of the critical density which would make it flat.[6]

These very specific values are observed in widely separated regions of the sky. This is so even though widely separated regions are outside each others past event horizons and could not communicate at the speed of light. These widely separated regions should not have been able to become so uniform if they were outside each others horizons. These are the isotropy, flatness and horizon problems presented by our observations of the cosmic microwave background.

All of these problems are related (see figure2.1) . The isotropy and flatness problems are both manifestations of the observed large scale

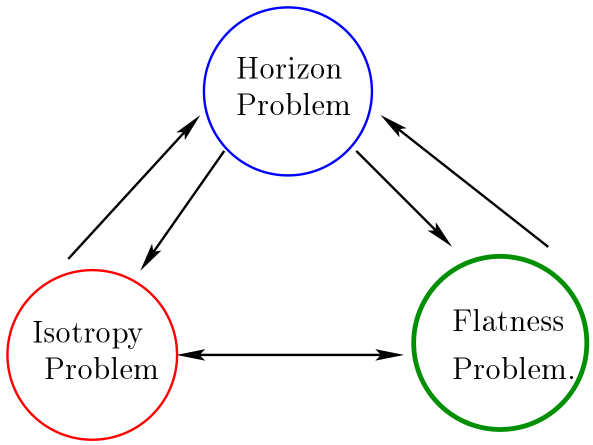


Figure 2.1: All of these problems are related and can be thought of as one problem of large scale unexplained, within standard big bang theory, uniformity.

uniformity of the universe. (Bear in mind this isotropy is related to but not the same as the isotropy of spacetime described previously. This deals with the contents of the universe.) In a sense those two are the same problem, viewed from two different perspectives. They both suggest the horizon problem, because widely separated regions of the universe were not causally connected in standard big bang theory. The fact that in standard big bang theory widely separated regions are not causally connected makes the observed isotropy of the CMB and flatness of spacetime problematic.

2.5.1 The isotropy of the cosmic microwave background.

The most careful measurements of the late 70's and early 80's had found little or no difference in the temperature of the cosmic microwave background (CMB) from one point in the sky to the other ($T = 2.725K$). The problem is that widely separated regions of the observable universe would not have been able to communicate with each other even at the speed of light. If these regions could not communicate then the CMB should vary wildly in temperature.

This isotropy is not assumed or expected by the Friedman-Lemaître-Robertson-Walker metric. This is the isotropy of the contents of the universe at the time of last scattering 380,000 years after the big bang. There is no reason to assume that the contents of the universe needed to start out with a smooth distribution. This is in contrast to the isotropy of the spacetime on the cosmological scale which is assumed by the FLRW metric.

The most natural assumption is to assume nothing about the distribution of the contents of the universe after the big bang. How would such a chaotic and easily perturbed, or alternatively, a truly random initial state, evolve into the very smooth state we observe in the form of the CMB, when widely separated regions could not even communicate at light speed? Those are the problems presented by the isotropy of the CMB.

2.5.2 The apparent flatness of the universe.

The flatness problem can also be thought of in the same terms as the isotropy problem. The curvature of the whole of the universe can be thought of as a sort of gravitational background. The FLRW metric only gives a flat space-time for a specific and critical density. It just so happens that on the cosmological scale the universe is very nearly that density.

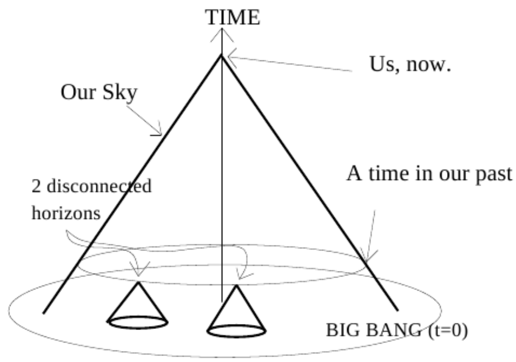
This flatness is an average over the whole universe, locally around massive bodies the spacetime is curved. However on the large scale of hundreds of parsecs the spacetime of the universe is flat. The curvature of the universe was determined by measuring the density of the universe. This density was seen to be nearly equal to the critical density of the universe for which the Friedman–Robertson–Walker-Lemaître (FRWL) metric gives a flat space-time.

It bears mentioning that the critical density's value, as it is now given, is dependent on the existence of dark matter. Searches for dark matter such as the Cryogenic Dark Matter Search (CDMS) have so far yielded little. They have found two *possible* detections which have a 23 percent chance of being background noise[8]. By the standards of particle physics this is not enough to say that a new particle has been detected. In particle physics a signal needs to be clean and have no noise out to six standard deviations. In spite of the lack of a direct detection cosmologists generally believe that dark matter of some kind exist. It has been used to explain the flatness of the universe, and the shape of galaxy's and clusters of galaxy's without introducing a theory of gravity more complex than General Relativity.

2.5.3 The horizon problem.

Both of the above contain within them and suggest the horizon problem. To see this problem physically consider Special Relativity. We are sure that at all times light was the fastest thing in the universe.

Figure 2.2: The Horizon Problem. The points on the second circle would be points on the “surface of last scattering” of the Cosmic Microwave Background radiation. Points on that surface would not have been in contact with each other. Yet the CMB is of basically the same temperature, the universe of of the same curvature etc. This homogeneity presents a problem. The horizon problem to be specific.



This defines a cone in spacetime outside of which a particular event cannot effect the future, or be effected by the past. Latter events can only depend on events within the past light cone of those events. The past light cone is matched by a similar cone which leads into the future. In General Relativity these become “event horizon’s”. Much like the boundaries between a black hole, and the universe. Light has not yet reached us from outside these horizons. Figure 2.2 found in [9] and used with permission illustrates this nicely.

The points on the second circle in figure 2.2 would be points on the “surface of last scattering” of the Cosmic Microwave Background radiation. That surface exist because just before the CMB was emitted the universe was filled with an optically thick, opaque, cloud of ionized gas in which photons were always being scattered. Their last scattering is the last time those photons interacted with that ionized gas. Widely separated points on that surface would not have been in contact with each other. In spite of the impossibility of that contact in standard big bang theory the CMB is of basically the same temperature, the universe of of the same curvature etc. This homogeneity presents a problem the horizon problem to be specific.

This diagram uses conformal time. In conformal time the FLRW metric reduces to the following.[1]

$$ds^2 = a^2(\tau)[-d\tau^2 + dx^2] \tag{2.32}$$

This is just a Minkowski metric multiplied by a conformal factor which depends on conformal time. For this reason diagrams like figures 2.2 and 2.3 and 2.4 can be drawn, and intuition about these problems can be drawn from said diagrams. Figures 2.4 and 2.3 are conformal diagrams for FLRW which also demonstrate the horizon problem. Notice that each point on the surface of last scattering in each diagram has its own past light cone. Given this no two points right next to each other would be of a similar temperature let alone points that are on opposite sides of the universe.

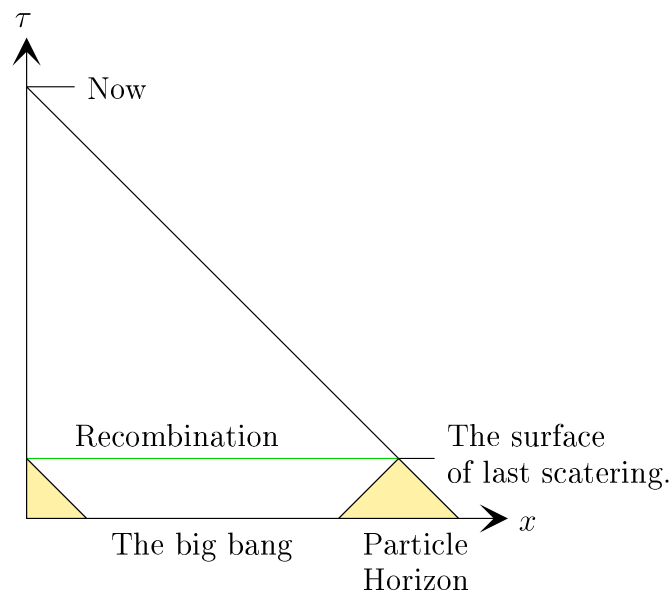


Figure 2.3: On this conformal diagram one can see the problem with the standard big bang. No two points on the surface of recombination share the same past light cone. Yet they are of remarkably uniform CMB temperature. This is so in spite of those points not sharing any of their past. [1]

On the conformal diagram in figure 2.3 one can see the problem with the standard big bang. Notice how the period before the line representing recombination, the last scattering of the CMB photons is separated into many regions which could not share the same past light cone. Thus causing a problem for the standard big bang given of the observed uniformity of the CMB, and flatness of the universe. Regions which do not share the same past could not have the same temperature to the degree seen in the CMB.[1]

2.5.4 In summary the problem is uniformity of the Cosmic Microwave Background.

The three problems can really be thought of as aspects of just one, uniformity. As shown clearly by figures 2.2 and 2.3 this means we have a multitude of causally disconnected regions which are never the less of a nearly uniform density, and temperature at last scattering of the CMB. This requires an explanation.

The hot and cold spots in the CMB are related to the presence or absence of matter. Therefore the isotropy of the CMB implies a overall isotropy of the distribution of matter and energy in the universe. This relates the isotropy problem directly to the flatness problem. The fact that the CMB is so even in temperature implies that all parts of the universe were causally connected early on. This relates the isotropy of the CMB directly to the Horizon problem. They are all interconnected in a sense they are all parts of the same problem.

Their appeared to have been two alternatives to solve this problem. One was that the initial conditions of the big bang were finely tuned to result in the universe we observe. The second was a dynamical process that would take a variety of initial conditions and result in the uniformity we observe. The first alternative leads to the question why should the initial conditions have been so finely tuned? The answer for that question is not very clear. A dynamical process is philosophically more appealing. The dynamical process that most cosmologists believe is responsible for ensuring the uniformity of the universe is known as inflation.

2.6 Inflation proposed as the explanation.

In modern inflationary theory a scalar ($\phi(x)$), or vector ($A_\mu(x)$) field of unknown origin is introduced. This field rolls "slowly" down a potential hill, and in doing so drives an exponential expansion of the universe. The universe expands by 60 e-folds in the period from 10^{-36} to 10^{-34} seconds after the big bang. In doing so the problems of the standard inflation-less big bang are solved. [1, 10]

In 1979 the concept of inflation was first enunciated by Alan Guth, then published in 1981[11]. Guth's concept of inflation was based in part on theories found in particle physics. His particular model did not fit the observed isotropy of the CMB [1]. Simpler models of inflation than that of Guth were soon proposed almost simultaneously by by Andrei Linde, Paul Steinhardt, and Andreas Albrecht.[12] These models are known as slow roll inflation. These models solved certain problems that the early models of Guth and others had[1].

The simple single scalar field model illustrates all the important features of inflation. Figure 2.4 sums up what happened very nicely. It is inspired by a figure in [13]. This is a comparison of the FLRW evolution of the scale factor with inflation theory. The solid line is the inflationary curve. As you can see the scale factor, which is part of the FLRW metric, in inflationary theory grows exponentially in a period of about 10^{-35} seconds. The two dashed lines show the evolution of the scale factor that one would expect without inflation, which correspond to the radiation dominated solution to the Friedman equations.

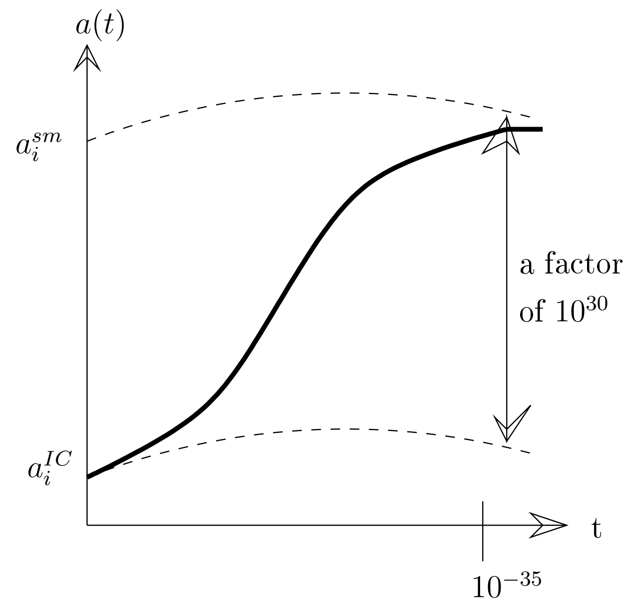


Figure 2.4: A comparison of the FLRW evolution of the scale factor with inflation theory. The solid line is the inflationary curve.

2.6.1 How inflation Solves the problems.

Inflation solves the horizon problem and thus the isotropy and flatness problems by giving the whole universe, as it existed 380,000 years after the big bang, the same past light cone. This allows the universe to attain the degree of thermal equilibrium observed in the CMB, as well as reaching the critical density. This inflationary period lasted to about 10^{-35} seconds after the big bang.

Figure 2.5 is a conformal diagram which shows how inflation modifies the FLRW metric. The big bang appears at negative infinity, and

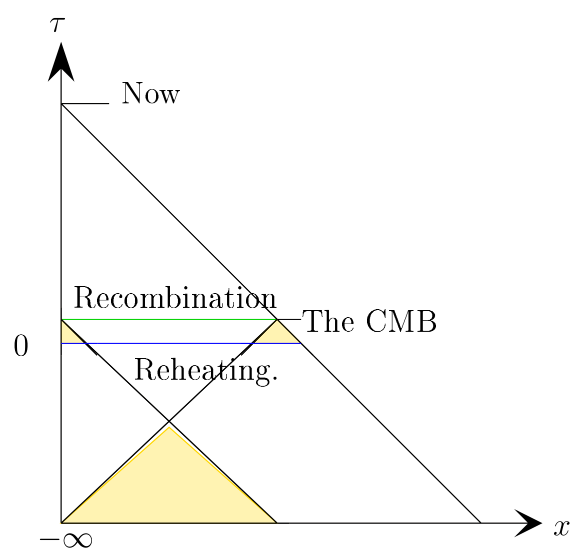


Figure 2.5: This conformal diagram shows how inflation modifies the FLRW metric. The big bang appears at negative infinity, and reheating is at zero. Instead of each point on the surface of last scattering having a different past light cone, the whole surface has the same past. With all points sharing the same past light cone equilibrium on the scale observed in the CMB is no longer a problem.[1]

reheating is at zero[1]. All points on the surface of last scattering share the same past light cone. This allows the CMB to be of a nearly uniform temperature. This also allows the universe to assume the critical density and flatten out. All of the problems are related to and intertwined with each other. By solving the horizon problem inflation solves the isotropy and flatness problems.

Indeed *any* theory that has the last figure as its conformal diagram will solve the mysterious uniformity problems of the big bang. Just so long as it gives the entire surface of last scattering of the CMB the same past light cone thus allowing the equilibrium and cosmological uniformities already discussed to appear. If the conformal diagram is different from the above it cannot solve these problems because the past light cone of the surface of last scattering will not be unified. It is for this reason that more than one alternative theory of inflation can be proposed to meet the various physical constraints imposed by other observations. Observations such as the power spectrum of the CMB being almost perfectly Gaussian, as far as could be determined from WMAP data. [14]

Modifying the FLRW metric space with inflation solves the horizon flatness and isotropy problems because it gives the entire surface of last scattering the same past light cone as shown by the conformal diagram figure 2.5.

2.7 Models of inflation: single scalar field "Slow-Roll" inflation.

Single scalar field "Slow-Roll" inflation is perhaps the simplest model of inflation. This type of inflation, is called slow-Roll due to the dynamics being mathematically similar to a particle slowly rolling down hill in a classical potential. [1] Figure 2.6 illustrates a slow roll potential. When the potential energy $V(\phi)$ dominates acceleration occurs. Inflation stops when the kinetic and potential energy are of comparable magnitude. During reheating the energy of the scalar field is converted into radiation.

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] \tag{2.33}$$

Notice that this is just the Einstein Hilbert action plus terms appropriate for a generalized scalar field. The term $\frac{1}{2}\sqrt{-g}R$ is the standard Einstein-Hilbert term. The term $\frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$ is the kinetic energy of the scalar field. $V(\phi)$ is as always the potential energy of the scalar field ϕ. With this quantity in hand one can

Figure 2.6: A Slow Roll Potential. This is an example of a slow roll inflation potential. When the potential energy $V(\phi)$ dominates the potential energy acceleration occurs. Inflation stops when the kinetic and potential energy are of comparable magnitude. During reheating the energy of the scalar field is converted into radiation. This figure is taken from [1]
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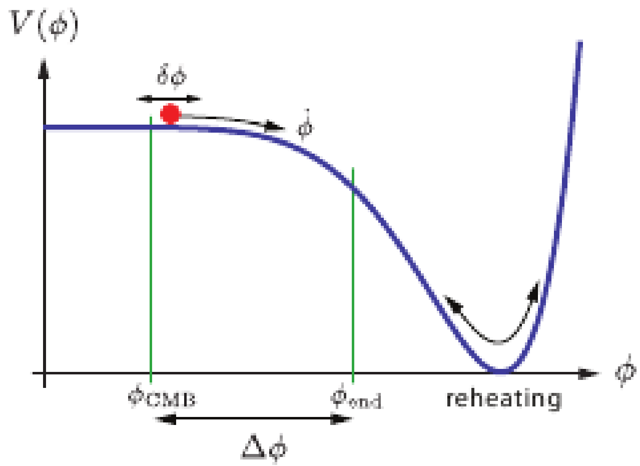


derive the Stress-Energy tensor and find the Hubble parameter as is done in [1]. Using those quantities one can then write down the rate of acceleration of the scale factor, which gives inflation its name.

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_\phi + 3p_\phi) = H^2(1 - \varepsilon) \tag{2.34}$$

ϵ is termed the slow roll parameter, and can be written in terms of the evolution of the Hubble parameter.[1]

$$\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} \tag{2.35}$$

This is not enough. The second time derivative of the scalar field needs to be small enough to sustain inflation for a long enough time to ensure the flatness and isotropy of the universe can set in. For this reason a second slow roll parameter is introduced. η expressed in terms of the potential this parameter is.

$$\eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{1}{V}\frac{\partial^2 V}{\partial \phi^2} \tag{2.36}$$

The first slow roll parameter can be similarly stated in terms of the potential of the scalar field.

$$\epsilon_{\mathrm{v}}(\phi) \equiv \frac{M_{\mathrm{pl}}^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \phi}\right)^2 \tag{2.37}$$

The Planck mass M_{pl} has been introduced by Bauman "to make the parameters manifestly dimensionless". The fact is any mass would have done this. Why not the mass of a Proton, or the mass of the planet Earth, or any other random mass? The answer is connected to the problem of quantum gravity. The Planck mass would seem to be a fundamental quantity in such theories along with the Planck length and Planck area. In all such theories the Planck scale is important. Let us not forget that inflation occurs when the universe is very young, small and dense. Gravity will be strong in that very early period, hence quantum gravitational effects will play a bit role.

In the slow roll regime the Hubble parameter H is approximately constant and

$$\dot{\phi} \approx -\frac{1}{3H}\frac{\partial V}{\partial \phi} \tag{2.38}$$

The time evolution of the scale factor $a(t)$ is then the same as that of a universe dominated by a cosmological constant.

$$a(t) = e^{Ht} \tag{2.39}$$

Inflation then ends when the slow roll conditions are violated. For the isotropy and flatness of the universe to set in would require at least 60 e folds of inflation. Various potentials have been tried and all work to varying degrees. At first many assumed that the Higgs potential could have played a role. After all the Higgs field, if it is as real as the standard model needs it to be, should have played a big role in the physics of the early universe, and is a scalar field. The Higgs field does not work the best of all [1]. A popular slow roll potential according to Bauman is the Coleman-Weinberg potential equation 2.40 which was originally derived for a proposed SU(5) grand unification quantum field theory [1]. (Symmetry groups and quantum field theory are covered in chapter four and appendix B.)

$$V(\phi) = V_0\left[\left(\frac{\phi}{\mu}\right)^4\left(\ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\right) + \frac{1}{4}\right] \tag{2.40}$$

Single scalar field slow roll inflation is just one example of an inflationary theory. There are others which have various combinations of scalar or even vector fields.

2.7.1 Double scalar field slow roll inflation.

As the name implies, double scalar field slow roll inflation is a version of inflation which involves two scalar fields. The difference between this theory and single scalar are in the action of this theory. A simple model would have an action of the form.

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\,\partial_\nu\psi - V(\phi) - U(\psi)\right] \tag{2.41}$$

In his review, Bauman says that this theory looses its predictive power due to having more free parameters. [1]

2.7.2 Chaotic inflation.

Chaotic inflation model is also one of the simpler ones. The form of the action is the same as for single field inflation however the potential is specifically of the form of equation 2.42 [1, 15].

$$V(\phi) = \lambda_p\phi^p \tag{2.42}$$

Einstein summation is not indicated here. "p" is an exponent on the ϕ and simply a subscript on the λ.

This is a very non linear equation. Such non linear equations often lead to chaos. This model would depend sensitively on initial conditions. In other words this model still has one of the problems of the big bang theory. This chaotic model.

In the single or dual field slow roll models the finely tuned initial conditions, are the shape of the potential and the slow roll parameters. Even in this model with one parameter λ_p, the subsequent evolution of the system would follow a rather unpredictable path. Even a slight difference in this initial condition would lead to a different final state. Unless, in this model all trajectories settle on a attractor which has the characteristics of the final state we observe. One other possibility found in the literature is that every possible final state of inflation that could happen does happen. The result would be a fractal structure of universes separated by false vacuums. This is still an improvement over standard big bang theory without inflation, as that theory has dozens of free parameters, this only has one.

2.8 Vector inflation

Models of inflation that depend on scalar fields are very appealing. Scalar fields are as simple as they come. Scalar fields have no preferred direction. Scalar fields have a problem, they have never been observed in any particle physics experiment. This is a huge problem with inflationary theories that depend on scalar fields. Let a alone the problem of detecting the inflation field itself.

For that one reason alone an inflationary model based on a vector field is desirable. There have been notable attempts at such a model. [16, 17] The action in the model of Golovnev et al [16] is equation 2.43

$$S = \int dx^4 \sqrt{-g} \left(-\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left(m^2 + \frac{R}{6} \right) A_\mu A^\mu \right) \tag{2.43}$$

The units in use are Planck units with $\hbar = c = G = 1$. Notice that like the Einstein-Hilbert action it contains the Ricci scalar. It also contains a field tensor $F_{\mu\nu}$ and field vector potential A_μ. Where $F_{\mu\nu}$ is an antisymmetric tensor field similar to the electromagnetic field tensor. Notice that this field has a mass associated so it can couple into gravity.

While this vector model is much more complex than the scalar models, it has the big advantage of relying on a type of field, a vector field, we know does exist in nature. It is also possible for vector inflation to explain the tiny degree of anisotropy in the CMB. If we assume that the fields were randomly oriented as inflation progressed, that randomness in the field would have resulted in the great degree of isotropy, as the vectors would on average cancel each other. As inflation progressed on smaller scales the vectors would not cancel and combined with initial quantum fluctuations could explain the slight anisotropies in the CMB.

The main reason this theory is not as popular as theories which rely on scalar fields is due to the great increase in mathematical complexity, for limited benefit to cosmology. There are simply easier ways to model what we see.

2.9 Summary

In chapters two and three what has been presented are the theories behind big bang and inflationary cosmology. Inflationary cosmology was motivated by the problems of the Friedman-Lemaître-Robertson-Walker (FLRW) metric. Those problems are rooted in uniformities, the nearly uniform temperature of the Cosmic Microwave Background (CMB), and the flatness of spacetime. These uniformities were problems because the FLRW metric by itself does not give the whole CMB one single past light cone this is known as the horizon problem.

The inflation field's effect is to cause the universe to grow exponentially for a tiny fraction of a second about $10^{-36}\,sec. \leq t \leq 10^{-34}\,sec.$. this growth alters the FLRW metric in such a way that the entire CMB shares the same past light cone. In the process solving the horizon problem, and other problems.

There are a number of specific models for inflation. The simplest model involves a single scalar field coupled indirectly to gravity which drives inflation. One of the more complicated models involves a vector field coupled directly to the curvature of spacetime in its Lagrangian. All of these models fit within the overall ΛCDM model.

Chapter 3

Quantum Field Theory and Particle Physics for Cosmology.

The visible matter in the universe is overwhelmingly composed of matter and little to no antimatter. This simple observational fact presents a huge problem for cosmologist since the standard model of particle physics predicts a universe made of equal parts matter and anti matter. This chapter will address research on this problem which is at the heart of the origin of all visible matter in the universe.

The origin of the visible matter is known as baryogenesis, meaning the creation of baryons, particles which are made up of three quarks such as protons and neutrons. The creation of other particles made of two quarks called mesons, and leptons which are not made of quarks would have occurred at the same time an under the same conditions as baryogenesis via related processes.

In the literature this issue is often simply referred to as the problem of baryon asymmetry. Theories which try to solve the problem include terms which break the various symmetries of the standard model of particle physics. A brief review of these theories has been provided in this thesis in section B. For more in depth coverage please see [18][19].

3.1 Elements of Theoretical Particle Physics

Symmetries are built into each and every quantum field theory, and a theorem due to Emmy Noether plays a huge role. Noether's theorem states that " Every continuous symmetry group of the action has an associated conserved current". In her words...

If the integral I is invariant with respect to a $\mathfrak{G}_\rho$, then ρ linearly independent combinations of the Lagrange expressions become divergences — and from this, conversely, invariance of I with respect to a $\mathfrak{G}_\rho$ will follow. The theorem holds good even in the limiting case of infinitely many parameters[20].

In the way we physicist now think of this the integral in the theorem is identified as the action S. The linearly independent combinations of the Lagrange expressions are continuity equations. It is notable that the symmetry group $\mathfrak{G}_\rho$ being continuous means that any Lie group, and its associated Lie algebra, will be of importance.

The three Lie groups, and their associated Lie algebras that are most important to particle physics combine to form the symmetry group of the standard model of particle physics are $U(1) \times SU(2) \times SU(3)$. These groups are as follows.

- U(1) is a group of one by one matrices which are unitary. In short it is the complex scalars of magnitude 1.
- SU(2) is the group of 2 by 2 unitary matrices with determinant one. The associated Lie algebra of this one, as with the one below, is just that of complex matrices under a commutator.
- SU(3) is the group of 3 by 3 unitary matrices with determinant one. This group has the property of being Non-Abelian, and as such it was used to model the strong force in the theory of Quantum Chromodynamics.

3.2 The Standard Model of Particle Physics.

The standard model of particle physics is often presented as figure 3.1. Figure 3.1 shows all the known particle fields that have been discovered. Every kind of matter we *know* exist is comprised of these particle fields. In the column on the right are the particle fields associated with electromagnetism γ the photon, the Strong force g the gluon, and the Weak force W and Z bosons. The other three columns are the particle fields for the Up, Strange, Down, Charmed, Top, and Bottom quarks. The quarks interact with each other via the strong force. The bottom row is for the leptons and their associated neutrinos which interact via the weak force, and electromagnetism.

In a nutshell, the above is the standard model of particle physics. This is a huge oversimplification. The next section is going to explain the real theory behind the model.

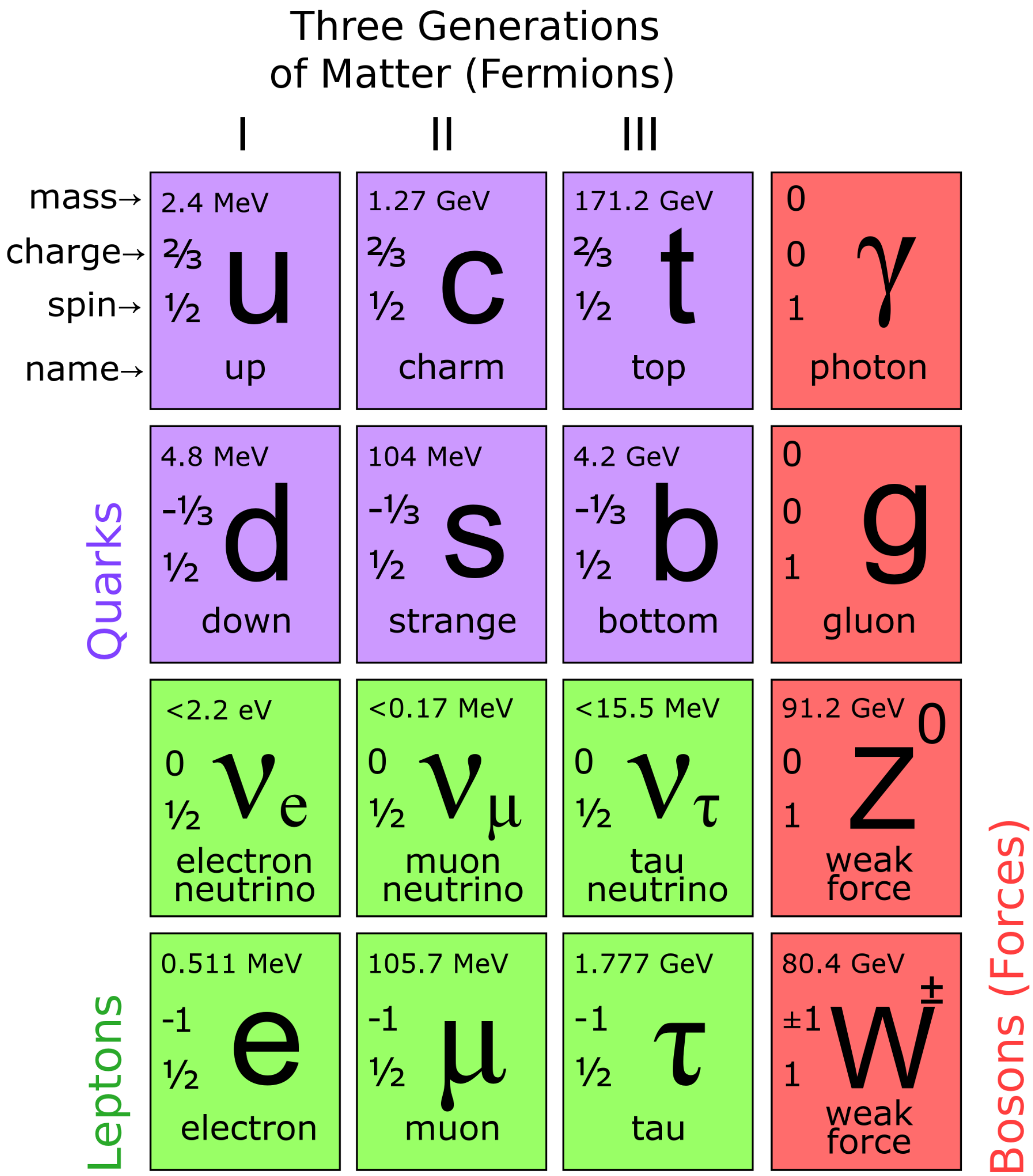


Figure 3.1: The Standard Model of Particle Physics.

3.2.1 The Mathematical Details of the Standard Model.

The standard model of particle physics is made up of three basic parts which have been unified into one innocent looking Lagrangian. The parts are Quantum Electro Dynamics, Electroweak theory, and Quantum Chromodynamics. These parts of the standard model deal with the electromagnetic interaction, the electroweak interaction, and the strong atomic interaction respectively. Quantum electrodynamics is the simplest part of the model and so will be detailed here.

Quantum Electro Dynamics (QED) is the simplest of the quantum field Theories which describes a actual force, electromagnetism. QED is the QFT which governs much of the world we see. When a ray of light bounces off a mirror, that's QED at work. Of course that can be well understood without it, never the less the micro physics of that simple event is QED. On the quantum level a mirror is a plane of atoms, which absorb then re-emit light. The same basic physical theory can explain something as unusual as electron-positron scattering, or pair production of an electron and positron. The following is the Lagrangian for QED.

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \imath e\gamma^{\mu}\bar{\psi}A_{\mu}\psi \tag{3.1}$$

Note that the D_{μ} in the above is just a gage covariant derivative, not unlike that found in General Relativity. For QED it is given by. $D_{\mu} = \partial_{\mu} + ieA_{\mu}$. Notice that this complex theory is made largely of the same basic parts that appeared in classical electro dynamics. The ones which are not found in classical electro dynamics are, Dirac spinors ψ and $\bar{\psi}$, and the gamma matrices γ^{μ}. The gamma matrices are known as pseudo-vectors. Written out they are matrices, but they behave under Lorentz transformation as vectors.

The defining property of the Dirac gamma matrices is how they behave under an anti commutator in the following way.

$$\{\gamma^{\mu}, \gamma^{\nu}\} = \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu} I \tag{3.2}$$

The Dirac spinors are defined as being solutions to the Dirac Equation. The precise form of the Dirac Spinor to use differs for particles and anti particles etc. More details can be found in. [19]

The electroweak force, the marriage of electromagnetism and the weak force has the symmetry $U(1) \times SU(2)$. Quantum Chromodynamics which holds hadrons, particles composed of quarks, together has the precise symmetry of $SU(3)$. SU(3) is a non-abelian field theory, a field theory built up from a non-commutative symmetry group. The whole standard model has the symmetry of the Cartesian (direct) product of those three groups and is non-abelian. This was built from the bottom up from a huge number of particle physics experiments to that mathematical symmetry. With the following definitions in place the Lagrangian for the Standard model of particle physics. It is the sum of the Lagrangian densities for the strong force and the electroweak force.

$$\mathcal{L}_{SM} = \imath\bar{\psi}_k\gamma_\mu D^\mu\psi_k - \frac{1}{4}F^{j\mu\nu}F_{j\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \imath\bar{q}_j\gamma^\mu D_\mu q^j - \frac{1}{2}tr\left(G_{\mu\nu}G^{\mu\nu}\right) \tag{3.3}$$

Where the G's are the Gluon fields. This Lagrangian is the real standard model of particle physics.

3.3 Baryogenesis in the standard model.

Baryogenesis is a word for the generation of matter from the fields of the standard model. A mystery of baryogenesis is that matter is created in the standard model by pair production. Pair production creates particles and anti particles in equal numbers. This symmetry between particles and anti particles arises from the symmetry groups discussed previously. Preserving overall symmetry ensures pair production. However the universe we see appears to be composed completely of normal matter.

Theories which seek to explain this observed asymmetry of matter and anti matter do so by introducing terms which break the symmetry of the standard model at high enough energies. This is often done by introducing new particles and fields.

3.3.1 Sakharov's conditions for baryogenesis

The conditions under which this could occur were first enunciated by Andrei Sakharov in 1967. Sakharov's conditions are [18].

- Violation of baryon number B.
- Violation of Charge and Parity CP symmetry.
- A (local) loss of thermal equilibrium.

The short reasons for all of these are that violation of conservation of baryon number or, B violation is needed due to the fact that we begin with B=0 and end with B=(all the baryons that will ever exist in the form of normal matter). The baryon number of a particle is +1 and of an anti particle is -1. Clearly if there was a perfect symmetry then all the matter would have been eliminated by all the anti matter.

B violation, can be found within the confines of the unmodified standard model. This is so because of the details of the vacuum states of SU(2) gage theory. There exist N possible vaccua. These are states of the quantum field which contain various numbers of particles. The fields can be excited by thermal energy and were during the period of reheating after inflation. These vacua are known as sphalerons. Each time the vacuum goes through a sphaleron transition the B symmetry is broken. When this happens nine quarks and three leptons will be produced. All of this would driven by the heat provided by the reheating after inflation in the standard model of cosmology. [18]

Charge and Parity symmetry need to be violated, otherwise the observed asymmetry between matter and anti matter would not be observed. Conjugation of the charges and parity symmetry would have guaranteed a universe composed of equal numbers of particles and anti-particles. Simple observation shows this is not the case, therefore this symmetry must have been broken. However the standard model needs to be modified in order to contain a strong source of CP violation. The simplest modification is known as the minimally super symmetric standard model (MSSM). To go into this in any detail would be beyond the scope of this thesis. However it bears mentioning that while this model provides strong enough CP symmetry breaking to produce the matter we see it comes at the cost of more particles we haven't observed. Each particle in the SM has a super symmetric partner in the MSSM. None of these hypothetical particles have been observed. On the other hand some of those particles may be the dark matter which is predicted on cosmological grounds. For this and other reasons physicist are very hopeful that super symmetry will be observed in future particle physics experiments. [18]

Lastly there has to be a local loss of thermal equilibrium in order for these reactions to go anywhere. The point is made by Sakharov that in thermal equilibrium that annihilations of particles and particle creations will be just as likely. One proposed mechanism for this is known as electroweak phase transition. To understand and visualize this treat the contents of the early universe before baryogenesis (quantum fields) as if they were fluids obeying the laws of thermodynamics. The reheating of the universe after inflation would energize this "fluid" to a certain temperature and it begins to "boil". This "boiling" is referred to in the literature as bubble nucleation. These "bubbles" are regions of space where the energy of the fields are slightly lower. Near the walls of these "bubbles" there is a interface between the bubble and the rest of the "fluid". Across the interface there is a difference in the rates of creation of particles by the sphaleron processes. This lack of thermal equilibrium would allow baryogenesis to occur and result in the observed baryon number asymmetry.[18]

3.4 Summary

To understand the creation of matter from fields of energy requires quantum field theory. One way quantum field theories are built by deducing the underlying symmetries of nature from experimental data. These symmetries are represented mathematically by symmetry groups. According to Noether's theorem these symmetries in the action of the field model conserved currents of charges. The problem is that a universe in which there are no particles would only be able to produce particles, and anti particles, in equal numbers unless those very symmetries were broken.

Breaking these symmetries, along with a lack of thermal equilibrium allows the production of particles by the quantum fields of the standard model. As it turns out there exist mechanisms within the standard model which will allow the breaking of the symmetry that causes conservation of baryon number. With the addition of super symmetry the conservation of charge and parity can be circumvented. The leading way this is done leads to the theory of super symmetry which contains particles that could be the dark matter that is predicted on cosmological grounds.

Part III

On observational and experimental particle cosmology.

This part deals with observational and experimental particle cosmology. The first area which will be covered is research on the Cosmic Microwave Background Radiation (CMB). This area is extremely important since very careful measurements of the CMB could inform diverse area's of physics and cosmology.

The second main area of focus, for this part, is the synthesis of elements heavier than hydrogen in the first minutes after the big bang. The steady improvement of observations in this area will inform theories of cosmology by setting a tolerance that any new models must meet.

Chapter 4

Research on the Cosmic Microwave Background Radiation and the Cosmic Gravitational Background.

The Cosmic Microwave Background Radiation (CMB) is the earliest directly observable phenomena we can detect. About 380,000 years after the big bang the universe finally expanded and cooled to the point where neutral atoms could persist. This event is known as recombination or last scattering. The last light to scatter off these first true atoms was also the first light to propagate in to a transparent optically thin universe. The CMB is that first light.

The surface defined by the last interaction of the CMB photons with the primordial gas cloud is called the surface of last scattering. The CMB photon light was initially gamma radiation ($\lambda \approx 0.9753\mu m$). That light has been Doppler shifted down to the microwave end of the spectrum (to $\lambda \approx 1.063mm$). Since the universe was imagined to have began with a hot dense state, not unlike a star, it was expected that the radiation from the resulting hot gas would have a black body spectrum. This is precisely what was observed. Today the CMB appears as a black body with a very nearly uniform temperature of 2.725 K.

Small deviations from this temperature, on the order of micro Kelvin, have been measured by space based experiments such as the Cosmic Background Explorer (COBE), and the Wilkinson Microwave Anisotropy Probe (WMAP). These are the data that have produced the now famous all sky maps of the CMB. These small deviations from the uniformity of the CMB encode within them data about the universe as it existed before last scattering. These small deviations, warm spots and cold spots correspond to clusters of clusters of galaxy's, and great voids in the universe at latter dates.

The importance of CMB physics cannot be overstated. Humanity has performed particle physics experiments at the energies and densities that existed at earlier times than the CMB, but not the overall conditions. As such it is crucial that we glean as much information from it as possible. The CMB is our best evidence for the big bang, and inflation. The CMB may even provide evidence of gravitational waves which would confirm one more prediction of General Relativity, and depending on the form of those waves could confirm inflation. Several large experiments are underway and searching for this Cosmic Gravitational Background.

4.1 Historical Background

In the 40's and 50's a number of astrophysicists made a physical prediction based on the big bang. They predicted that the whole sky should be filled with a relic radiation as if the sky was filled with a black body. This radiation would be the red shifted light and infrared emission of the big bang. The first to compute a temperature for the ambient radiation in space was George Gamow who computed a temperature of 50 k[21]. Gamow did not state that there would be a universal and pervasive black body spectrum background. Robert Dickie of Princeton on the other hand did predict just such a black body spectrum[22]. For that reason it can be said Dickie was the first to predict a Cosmic Microwave Background. Dickie made a range of predictions from 50K to 6K for the temperature of this radiation. [22]

Then in the 1960's technology caught up to theory. Two American teams of physicist would end up working on the problem of detecting the relic radiation. One led by Robert Dickie of Princeton did so on purpose, and wanted to detect the radiation. The other team of Arno Penzias, and Robert Wilson did not want to detect the radiation.

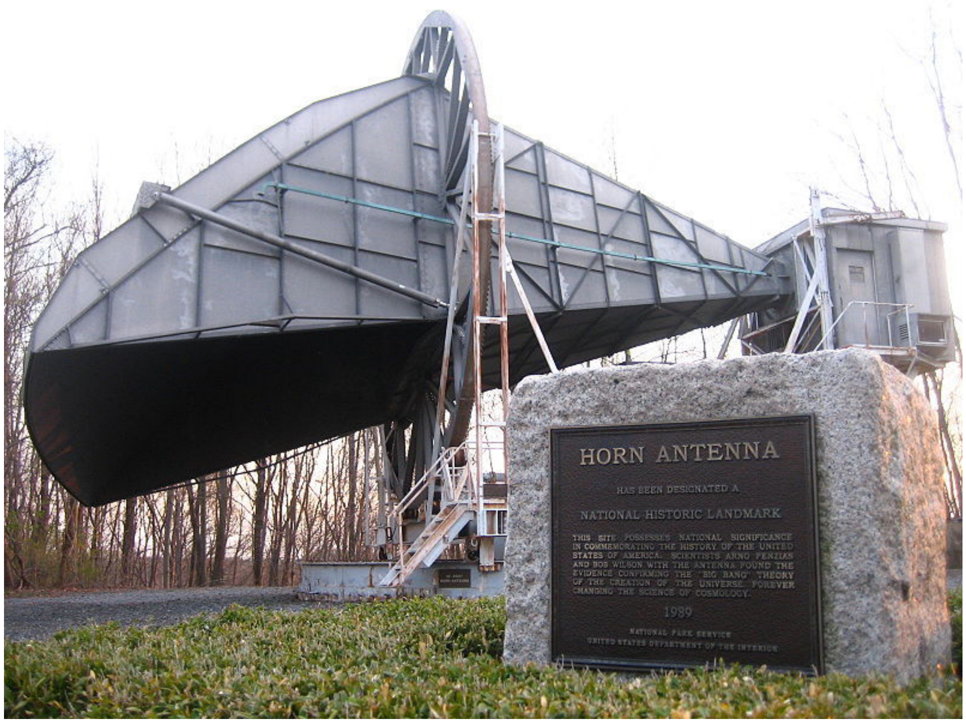
Penzias and Wilson had wanted to detect radio waves bounced of early eco balloon satellites, which simply reflected a radio beam aimed at them. To do this they had to eliminate as much background noise as possible. Their receiver was known as the Horn (as seen in figure 4.1). When Penzias and Wilson did their work at Bell labs their antenna was the cutting edge of Microwave technology. They switched on their receiver and found that from all directions they heard a microwave hiss. They eliminated all possible sources of noise, from nesting birds to the heat of their own equipment[23]. Then they contacted Dickie. In this way the Cosmic Microwave Background radiation was found. A major prediction of big bang theory was confirmed. By 1978 when Penzias and Wilson were awarded the Nobel Prize for their discovery the big bang, not yet with inflation it came latter, was the standard model of cosmology.

The next step was the measurement of the CMB by the Cosmic (microwave) Background Explorer collaboration (COBE). What they sought was a precise measurement of the spectrum of the CMB. Penzias and Wilson had shown that the CMB emanated from all directions, was very homogeneous etc. COBE was designed to go a step further and tell us if the spectrum was truly a black body and detect any anisotropy[24]. The COBE probe had a angular resolution of 7 degrees and was able to detect temperature fluctuations of +/- 100 micro Kelvin[3]. Anisotropy is exactly what was found by COBE producing the image in figure4.3. This image shows the hot and cold regions of the sky in a relatively low angular resolution compared to latter data. For this work George F. Smoot and John C. Mather were awarded the Nobel Prize in physics in 2006. The citation read "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation".[25]

In the last number of years high quality data was gathered on it by the Wilkinson Microwave Anisotropy Probe (WMAP). This is what produced the famous all sky maps we have all become accustomed to. (One such image has been used in the first figure in this thesis.) However WMAP's data left a few things to be desired. For example it did not show us the greatest possible detail given how technology has advanced since WMAP was first launched. That is where the next generation probe launched by the European Space Agency comes in.

The Planck explorer is the probe which will confirm the observations of WMAP and go a number of steps farther. Consider that for the WMAP at 30 Ghz the angular resolution is 40 arcminutes and for Planck the angular resolution at that frequency is 33 arcminutes. (The Planck probe and WMAP cover different frequencies in general 30 Ghz is a frequency common to both and therefore most comparable.) It will measure the anisotropy of the CMB to much greater detail, and angular resolution than WMAP. Planck will measure the polarization modes of the CMB as well. Planck will also look for non Gaussianity in the power spectrum (a plot of CMB temperature variance on the sky VS frequency as in figure 4.4) of the CMB, which

Figure 4.1: The Horn, used by Penzias and Wilson.In the 1960's when Penzias and Wilson did their work at Bell labs this was the cutting edge of Microwave technology.



To put these sucessive measurements of the CMB in perspective take a look at a simmulated image of the microwave sky as seen by the horn (figure 4.2). According to the WMAP science team this shows how the microwave sky would have appeared to The Horn had it been able to scan the whole sky [2]. A basically featureless black body radiating at 2.725 K. This is the data that was available until the Cosmic Background Explorer (COBE) experiment. The CMB appears completely uniform and featurless with the level of technology available to Penzias and Wilson.

Figure 4.2: The Microwave Sky as seen by the horn. According to the WMAP science team this shows how the microwave sky would have appeared to The Horn had it been able to scan the whole sky. A basically featureless black body radiating at 2.725 K. This is the data that was available until the COBE experiment. [2]

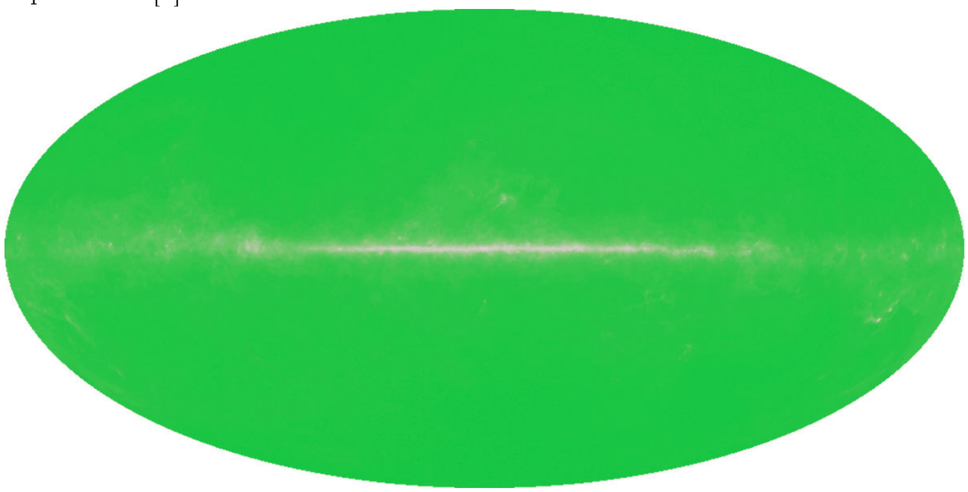
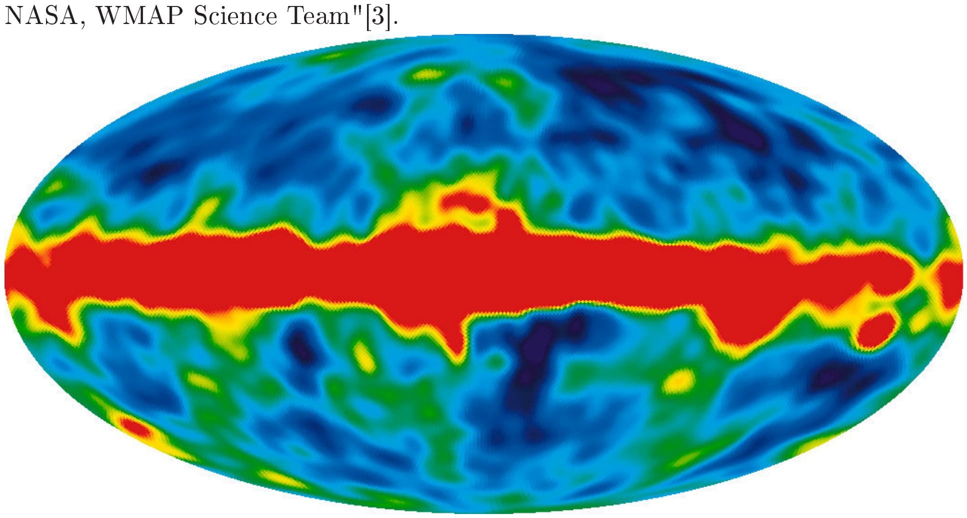


Figure 4.3: “The all-sky image produced by the COBE Satellite. It is a low resolution image of the sky (7 degree resolution), but obvious cold and hot regions are apparent in the image. The large red band is the microwave emissions from our own galaxy. This image shows a temperature range of Âś 100 micro Kelvin. It was processed through the same data pipe as the first year WMAP data. The largest version of the image has a scale added. Courtesy of the NASA, WMAP Science Team"[3].



if found would indicate new physics. With this data our theories of the early universe will be put to the test.

4.2 Theoretical motivations

Figure 4.4 shows the power spectrum(a plot of CMB temperature variance on the sky VS frequency) of the CMB as measured by WMAP, with a best fit line provided by the standard ΛCDM model of cosmology. (Part two of this thesis presented the essential components of the ΛCDM model.) The Planck collaboration hopes to improve on this by measuring slight non-Gaussianity which would indicate new physics and that a more complicated model of inflation than single field slow roll is called for.[4]

Better observations of the CMB than we have ever had before will allow cosmologist to throw out certain models of inflation and determine which one is correct. The up coming and on going observations may also detect evidence of gravitational waves via a particular mode of polarization in the CMB. The Planck mission may even take results so and fine that they eliminate the simplest model of inflation, single scalar field slow-roll. These are the chief scientific goals of the Planck mission, and a number of other planned ground and balloon borne missions.

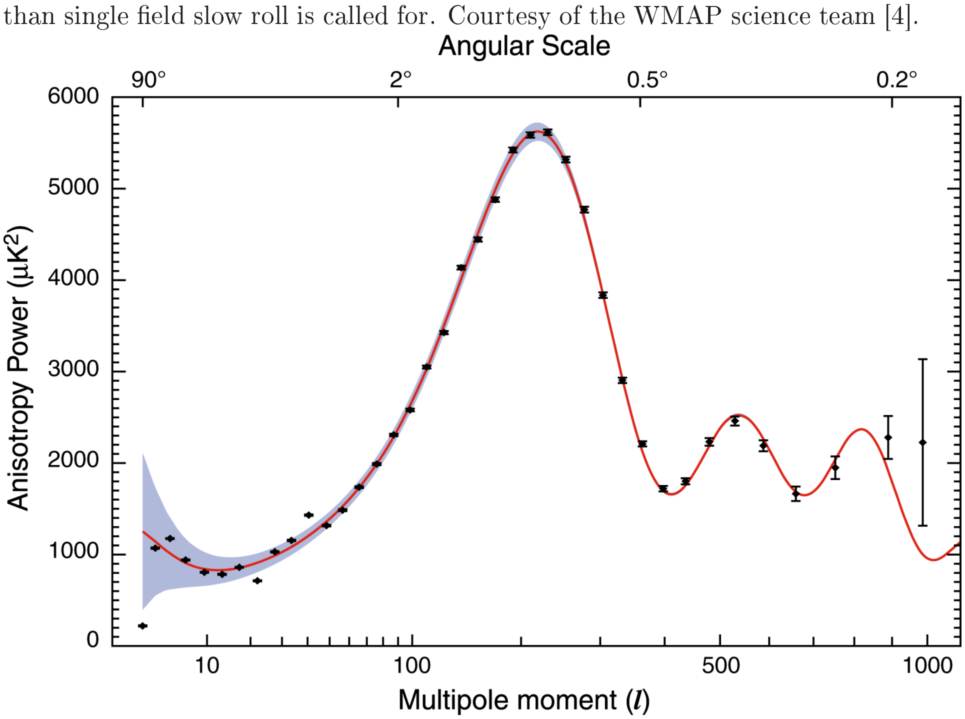
These are models of inflation in which the field (s) ϕ are multiplied by each other in the Lagrangian. Such terms imply a strong "self interaction" of the inflation field with itself in such a theory. The behavior of such fields is strongly non-linear as compared to theories such as single field slow roll inflation. [15, 1]

The reason that single field slow roll inflation predicts a gaussian power spectrum (a plot of CMB temperature variance on the sky VS frequency) is because of the solutions to the equations of motion of that particular field which are gaussian functions. Such a Gaussian spectrum is approximately what WMAP observed (figure 4.4), but the Planck probe would be able to detect small deviations predicted by theories of chaotic, or vector field inflation among others.[26]

4.2.1 Gravity waves and the B-Mode polarization of the CMB.

The motions of large celestial bodies, and violent cosmic events are thought to produce gravitational waves due to the nature of Einstein's General theory of Relativity. It is possible to linearize General Relativity, decompose it into a part which behaves like an electric field, and a part which behaves like a magnetic field. This linearizion

Figure 4.4: The power spectrum of the CMB as measured by WMAP. This figure shows the power spectrum of the CMB as measured by WMAP, with a best fit line provided by the standard ΛCDM model of cosmology. The Planck collaboration hopes to improve on this by measuring slight non-Gaussianity which would indicate new physics, and that a more complicated model of inflation than single field slow roll is called for. Courtesy of the WMAP science team [4].



of General Relativity leads to a wave equation. This procedure is only valid in the weak field limit, so no where near a black hole for example. It is from such mathematics that gravitational waves are predicted. What these waves are is ripples in space-time. As these ripples travel the geodesics will be distorted. It is this distortion of the “shortest distances from point A to point B" which the following projects depend on.

Inflation just like every other major celestial event is thought to have a gravitational wave signature. It can be thought of as a gravitational analogue of the CMB. Inflation, indeed the different models of inflation, would have distinct gravitational backgrounds.

The Planck explorer has the primary objective of observing the cosmic microwave background in greater detail than the WMAP project. A big goal of Planck is to measure what is known as the B mode polarization of the CMB. By doing so it would be able to indirectly detect gravitational waves.[26]

The polarization of the CMB is caused by Thompson scattering of the primordial radiation by electrons during the very last phase of the matter-radiation coupled era. These polarizations, known as E mode and B mode are linearly proportional to the Stokes Parameters Q and U. It is these stokes parameters which can be observed, and from them the polarizations calculated. [27]

The best measured multipole moment so far is the vector E mode. The B mode is a tensor mode which if it is present and of a certain magnitude can constitute a indirect detection of gravitational waves [26]. This is important for the future of gravitational wave astronomy which right now is still speculative (but based on everything we know it should be possible.) Detection of this B mode polarization and gravitational waves would lend more support to General Relativity, and inflationary cosmology.

4.3 The Planck Mission.

The European Space Agencies Planck mission is a deep space probe which will orbit the sun earth Lagrange point L2, facing away from the Earth and facilitate a more detailed study of the CMB than was possible during the WMAP mission. The Planck probe has better sensors, and better electronics than WMAP. Both technologies have improved since WMAP was launched. Figure 4.5 shows a comparison of early Planck data to comparable WMAP data. The two images in figure 4.5 show the plane of our galaxy and its emission in the foreground.

Table 4.1: Planck Instruments and Capabilities

	LFI	HFI
Detector Technology	HEMT arrays	Bolometer arrays
Center Frequency (GHz)	30 44 70	100 143 217 353 545 857
Angular Resolution (arcmin)	33 24 14	10 7.1 5.0 5.0 5.0 5.0
$\Delta T/T$ per pixel ($Stokes\, I$)	2.0 2.7 4.7	2.5 2.2 4.8 14.7 147 6700
$\Delta T/T$ per pixel ($Stokes\, Q\, \&\, U$)	2.8 3.9 6.7	4.0 4.2 9.8 29.8 - -

4.3.1 Objectives

As discussed above, the polarization of the CMB is one aspect of the CMB of which Planck will make a detailed study. The two specific polarizations that will be examined are the E- Mode and B-Mode polarizations. With the WMAP only the E-Mode polarization could be read. The B-mode polarization happens to be the type which we think would be induced by gravitational waves. This is so because gravity is a tensor field, and the B-mode would only be induced by such a field. If the B-mode polarization exist, and has the correct signature that would provide justification for further gravity wave observations.

The Planck mission will also be able to observe the CMB in greater detail than the WMAP mission did. Specifically it will be able to detect and characterize the non-gaussianity of the CMB power spectrum. This data could rule out the simpler models of inflation. [26, 10, 28]

The Planck mission also has the objective of trying to observe new and unexpected physics. Physics which could either lend support to or rule out certain speculative, and/or non standard models.[26, 28]

Last but not least the Planck probe will to locate and map galaxy clusters via its observations of the CMB. Current theory and observations indicate that warm spots in the CMB correspond to galaxy clusters. The Planck mission will be able to make detailed enough observations to allow us to map the location of clusters so far away, that the only light we see which indicates their existence is in the form of the CMB. [26, 28]

4.3.2 Instruments

Table 4.1 which is similar to one in [26] summarizes the instruments and capabilities of the Planck probe.

The Low Frequency Instrument(s) (LFI) consist of arrays of high electron mobility transistors (HEMT arrays). The High Frequency

Instrument consist of Bolometers cooled to a temperature of 0.1K. Both of these instruments are integrated into the focal plane of Planck's 1.5 m microwave telescope. [28]

These instruments will be able to fix the value of various cosmological parameters to within 1% of their actual values [26]. In particular the Planck mission will be able to reduce the uncertainty in the baryon abundance parameter by more than one half [29]. From the WMAP value of about 0.145 to 0.063[29]. While the uncertainty in the number of equivalent neutrino's will be reduced by a factor of 1/3 over the WMAP data[29].

4.3.3 Early Planck Data

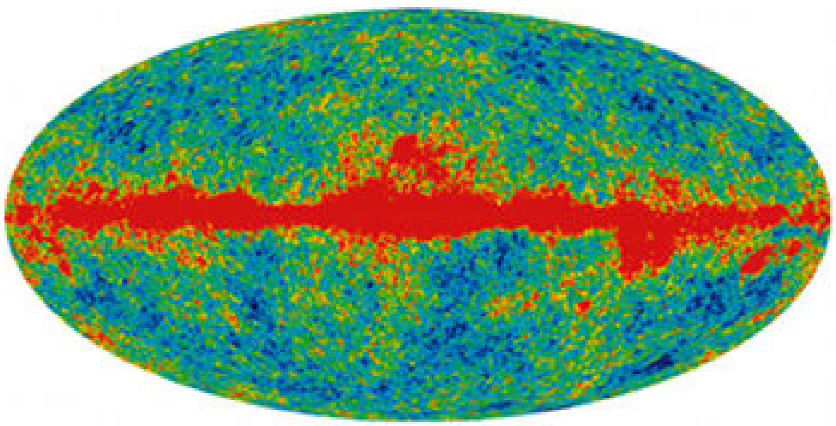
On July 5th 2010 the Planck collaboration released their first all sky map. A side by side comparison with WMAP is figure 4.5. The bottom image is the latest Planck data before the foreground is removed.

The Planck data shows much more detail than WMAP data and should produce great new insights.

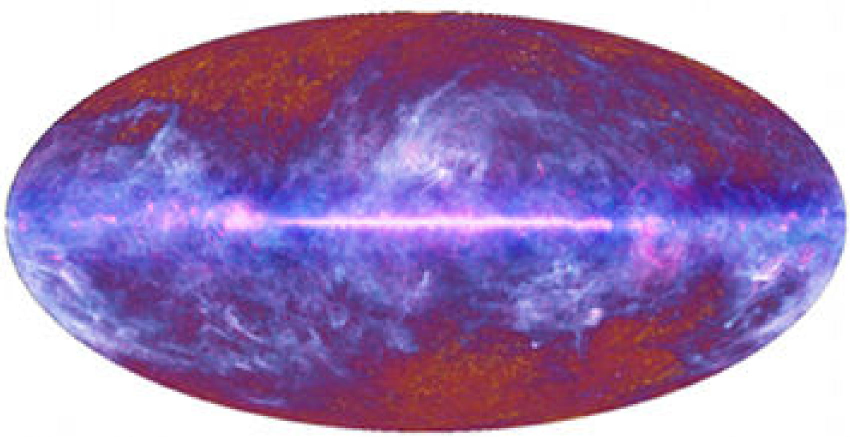
The Planck data has not yet had the foreground sources removed to reveal only the CMB. The background is somewhat visible in the high latitudes of the image, so says the Planck team. Their are still point sources visible in those regions too. The Planck team is now working on careful analysis of the foreground [30]. This analysis uses the multiple frequency bands in which Planck can detect microwaves to discriminate between the foreground and the background. Then the foreground can be digitally removed. Work on removing the background is currently underway and is being performed using the Franklin supercomputer at the National Energy Research Scientific Computing Center in Berkeley, Calif. This computer center will handle the most computationally intensive task for the Planck team world wide[31].

4.4 Ground Based Observations

Concurrently and complementary to Planck are a variety of competing ground and balloon based experiments which will all attempt to detect any primordial B-Mode Polarization of the CMB. This paper will focus on projects which are scheduled to begin in 2010 or latter. Table 2.2 is derived from a similar one in [26]. Looking at the table, note that the angular resolutions could potentially be as good as that of Planck. Thanks to Planck, and the following ground based



WMAP Q-Band map



PLANCK all sky survey.

Figure 4.5: The top image is the WMAP data for seven years of collection before the foreground is removed. The bottom image is the Planck data for one year without the background having been removed. Note the superior detail of the Planck image. Images courtesy of the WMAP science team, and the Planck science team respectively.

Table 4.2: Ground based observations planned for the near future. With comparison to Planck.

Experiment	Angular resolution (arcmin)	Frequency (GHz)	Goal (r)	Starting Year
Ground Based				
ABS [32]	30	145	0.1	2010
BRAIN [33]	~ 60	90, 150, 220	0.01	2010
Keck Array	60 - 30	100, 150, 220	0.01	2010
Balloon Borne				
PAPPA [34]	30	90, 210, 300	0.01	2010
PIPER	~ 15	200, 270, 350, 600	0.007	2013
SPIDER [27]	58 - 21	100, 145, 225, 275	0.01	2010
Satellite				
Planck	33 - 5	30 - 353	0.05	

observations, in the coming decade the cosmological community is going to have plenty of high definition data to digest.

The Atacama B-Mode search (ABS) is an experiment which will be situated in the Atacama desert of Chile[32]. It will be constructed in the US inside a standard shipping container, then shipped to Chile. The detector itself is composed of cryogenicly cooled transition edge sensor bolometers and is very compact at only one meter tall[32]. Atmospheric interference will be filtered out with the aid of a half wave plate very near the aperture and before the beam forming optics. For far more details please see [35].

The B-mode RAdiation INterferometer (BRAIN) experiment is very similar to the ABS and Planck in that it too uses a bolometric instrument. This project is different in that the detectors are cross linked to form an interferometer. The BRAIN collaboration intends to install their instrument in Antarctica at the Concordia research station operated by France and Italy (at a 3233 meter altitude). The instrument is in the process of being fully installed now and should be completed by 2011 [?]. More details are available in [33].

The balloon based Primordial Anisotropy Polarization Pathfinder Array (PAPPA) is a balloon based experiment which will employ a innovative array of polarimeters on a chip. It will have the capability of simultaneously measuring the stokes parameters I , Q and U and hence the polarization of the CMB. [34]

SPIDER employs cryogenically cooled bolometers just like those above. It will be flown from Alice Springs Australia. It's stated science goal is to measure the B-mode polarization of the CMB. [27]

4.5 Efforts to directly detect gravitational waves.

Laser Interferometer Gravitational (Wave) Observatory (LIGO) is a ground based observatory which is actively searching for gravitational waves. It uses three widely placed, massive laser interferometers in an attempt to detect gravitational waves.

The laser beams are split, then travel along the arms of this massive interferometer for a few miles. A passing gravitational wave would cause one beam to travel a different distance than the other. It would distort the metric differently in one direction than the other. When the light is recombined, and the interference pattern carefully measured , then a difference in phase can be interpreted as the effects of a gravitational wave.

One detector by itself would not be a very sensitive instrument. For that reason LIGO is composed of three massive laser interferometers. Two of which are located in Hanford, Washington, USA, and one in Livingston, Louisiana, USA. This number of interferometers is the minimum needed for the proper error checking, or coincidence in the data. The data is then analyzed for coincident events. These coincident events are then taken as being the possible signals. The more detectors the better one can distinguish the actual signal from the noise. Recent work done by the LIGO collaboration has been in cooperation with the European Virgo and Geo projects, which are also based on laser interferometry and so similar to LIGO as to be nearly identical. This collaboration allows a greater degree of coincidence, which turn allows more sensitivity to the true signals, and less noise.

One search looked at compact binary systems with stars of masses between 2 and 35 solar masses[36]. Such systems have been observed to loose energy as the stars orbit each other. This energy would presumably be radiated away in the form of gravitational waves. However this search did not detect any waves. A more recent search tried to detect gravitational wave burst from violent cosmological events[37]. Events such as supernovae. Theoretically these events should produce a gravitational waves, none were detected[37].

The great difficulty in working with LIGO, Virgo and Geo, is the noise induced by ground vibrations. This is an inherent problem with ground based observation of gravitational waves. The next step is the Laser Interferometer Space Antenna (LISA) mission, a joint NASA ESA mission. This will be three satellites in a orbit distant from the Earth or any other bodies. These satellites will form a laser interferometer by flying in formation. The noise level will be far lower in this environment.

The culmination of this work on direct gravitational wave detection will be a space based gravitational wave observatory known as the Big Bang Observer (BBO). BBO is billed as allowing the direct imaging of events that are father back in time than the Cosmic Microwave Background radiation (CMB). It is the BBO which will answer the question of which inflationary model is correct by way of its gravitational echo, a sort of gravitational background signal. BBO will not only detect these waves but will allow humanity to image the cosmic gravitational background. Doing so will answer deep questions about the universes earliest evolution almost back to the big bang itself.

4.6 Summary

The subject of this chapter was research on the Cosmic Microwave Background (CMB) and the Cosmic Gravitational Background (CGB). Research being done on the CMB will inform research being done on the CGB. The primary research projects underway in these two area's are the European Space Agency's Planck explorer satellite, and the Laser Interferometer Gravitational (Wave) Observatory (LIGO).

The Planck satellite orbiting the Sun- Earth Lagrange point L2 has imaged the whole sky. Once the foreground sources are removed a sharper picture of the CMB than has ever been seen before should be obtained. Among the goals of Planck are the detection of a B-mode polarization of the CMB which could indicate gravitational waves acting when the CMB was emitted. Another goal is to detect non Gaussian in the power spectrum of the CMB. Small non-guassianity in the fit is one prediction of more complicated theories of inflation such as vector inflation and chaotic inflation.

The LIGO collaboration's work on gravitational wave detection, along with their colleagues efforts, will lay the groundwork for future direct detection of gravitational waves. Detecting such waves would confirm a major prediction of General Relativity and open a new window on the universe. Future projects based on this technology such as the Big Bang Observer will reveal information on the earliest evolution of the universe.

Chapter 5

Atomic and Particle Cosmology.

This chapter will focus on observations and experimentation in particle physics which have a bearing on cosmology and vice versa. Specifically observations of the ratios of the light elements, and work being done at the Large Hadron Collider (LHC). Careful measurement of the relative abundances of light elements which resulted from the primordial nucelosynthesis following the big bang would be of interest to particle theorist. Any grand unified theory would have to be able to predict these ratios. Experimental work done at the LHC will also have a bearing on cosmology as more is learned about the fundamental constituents of the universe.

5.1 Big Bang Nucelosynthesis

Nuclear fusion is any nuclear reaction in which two lighter nuclei combine to form a heavier nuclei. It is by that process that elements heavier than hydrogen were formed in the early universe. The reactions that will concern us involve hydrogen fusing to form helium, lithium and beryllium. Heavier elements were not produced. This is because unlike the core of a star which is compressed for millions or billions of years the period of nucelosynthesis after the big bang only lasted a matter of minutes. It takes great pressure and thermal energy to cause fusion. The interaction cross sections for fusion of heavier elements are smaller. As the universe expanded it cooled and this cooling made fusion into heavier elements less likely and so they were not formed in astrophysically interesting amounts. This process is known as Big Bang Nucelosynthesis (BBN)

The timing of this period of nucelosynthesis is one of the tightest constraints on the possible theories of cosmology. From many experiments we know nuclear physics very well. What can be calculated are pressures temperatures and times will produce the relative abundances we have observed. The first attempt at this was done by Ralph Alpher, Hanse Bethe, and George Gammow in 1949. [38]

All of the complexity of these processes can be reduced to one parameter, the ratio of baryons to photons. The ratio of any two primordial abundances should equal the ratio of baryons to photons. If a cosmological model predicts a different ratio then it can be ruled out. Depending on the studies these ratios are known to within 10 to 0.1 percent. [39]

These ratios of light elements, in particular the primordial Deuterium/Hydrogen (D/H) are well known from observations of distant quasars, and other low metallicity sources[39, 29]. These galaxy's existed at a time where the first generations of stars had not had a chance to modify the ratio's by their fusion, and are referred to as having low metallicity[39]. The ratio's of D/H at those locations and times are consistent with the D/H measured in the interstellar medium (ISM) via absorption of light of the wavelength corresponding to the first transition in the Lyman series (Lyman α) absorption ($D/H \approx 3.4 \times 10^{-5}$)[39]. There are many technical reasons for this, which are the province of observational astrophysicist, something I am not. These details are provided in [39, 29].

5.1.1 Improving observations of the relative abundances of light elements.

Observations of the of ratios of light elements due to Big Bang Nucelosynthesis (BBN) could constrain how much gravity could vary from its general relativistic description (as required by some species of string theory among other theories) [40]. Variation in certain fundamental constants can also be constrained by observations of BBN (which is again required by some types of string theory among other theories) [40]. "Report by the ESA-ESO Working Group on Fundamental Cosmology" identified as key questions [41] .

> Is standard cosmology based on the correct physical principles? Are features such as dark energy artifacts of a different law of gravity, perhaps associated with extra dimensions? Could fundamental constants actually vary? [41]

BBN observations can answer some of these questions[40]. Only certain alternatives will be compatible with the constraints imposed

Table 5.1: Near Future Telescopes in Comparison to Keck and VLT

Telescope	Main Mirror Diameter	Key Instruments	completion
Keck[45]	Two , 10 meter	High Res Echelle Spec. Keck-Keck Interferometer Near Infrared Camera	
VLT[46]	Four, 8.2 meter	Nasmyth Adaptive Optics System UV and Visual Echelle Spect. X-shooter	
Giant Magellan[43]	24.5 meters	Near-IR multi-object spec. GMT Wide-Field Optical Spec.	2018
TMT[44]	30 meters	Near-IR echelle spec. (NIRES) High-Res. Optical Spec. (HROS) InfraRed Multislit Spec. (IRMS)	2018
E-ELT[42]	42 meters	Nine stations for fixed instruments	2018

by improved observations. The ESA-ESO report also claims that a larger telescope a “European Extremely Large Telescope” E-ELT would be necessary to make observations accurate enough to yield such constraints [41]. This telescope is due to being operations in 2018 [42]. To put its capabilities in perspective, it is planned to have a 42 meter in diameter main mirror for 1300 square meters of collecting area[42]. The Keck telescopes in Hawaii have 10 meter diameter main mirrors. The E-ELT is only the largest of planned future ELT’s. The others being the 21 meter diameter Giant Magellan telescope[43], and The 30 Meter Telescope (TMT)[44].

In short there is still much to be learned from good old fashioned direct observation of the universe.

5.2 Large Hadron Collider

The Large Hadron Collider (LHC) is where both nuclear and particle physics, that could have occurred during both baryogenesis, and nucelosynthesis will be studied in detail. The conditions which will be created for a split second in each collision event will mimic the environment at that time (save for the strength of gravity). It is these facts which allow the often repeated claim that the LHC will “recreate the big bang”.

The main science objectives of the LHC can be stated briefly as detection of the Higgs Boson, testing of the standard model, revelation of physics beyond the standard model, and experimentation in high energy nuclear physics.

Foremost among the science objectives of the LHC is detection of the Higgs boson. The Higgs field was proposed in the theory which united electromagnetism, and the weak atomic force. Through interaction with the Higgs field particles gain their masses. If the Higgs (scalar) boson is not detected then we all will have to think again about our standard model of particle physics, and the origin of mass.

The LHC will also look for certain physics beyond the standard model. Specifically scientist working on it will look for what is called large extra dimensions, and super symmetric particles. Large extra dimensions stems from M-Theory. String/M theory needs more than four dimensions, the extra dimensions are thought to be compacted into a length no longer than the Planck length. Large extra dimensions would be evidence in support of very speculative theories such as M theory.

In the process of testing the standard model (based on $U(1) \times SU(2) \times SU(3)$ gage symmetry) other theories will also be tested. There is no shortage of proposals and counter proposals for grand unification. Aside from super symmetric theories such as M-theory, SU(5) grand unification theory theory still has supporters in the form of $SU(5) \times U(1)$ theory[47]. These grand unification schemes contain candidate inflation fields. So if one of them is supported with the detection of a particle an “inflation" at the LHC, then their particular candidate inflation field(s)/mechanism would become the preferred choice in cosmology.

Last but not least the LHC will perform experiments in nuclear physics by collisions of lead nuclei with each other. In doing so states of matter which would have existed during baryogenesis, and nucelosynthesis could exist for a split second. The LHC will be performing controlled experiments in nuclear physics at energies which humanity has never reached before [48].

5.3 Summary

Precise theoretical predictions of the relative abundances of light elements in the early universe depends on the underlying models of particle physics in subtle ways. At high energies interactions that have low probability can take place which don’t happen outside of particle accelerators at this time. Observations of big bang nucelosynthesis will have a bearing on theoretical particle physics by fixing the ratios of light elements. This will eliminate various alternatives to the standard model from consideration and support others. Conversely work done at the worlds accelerator laboratories could effect the field of astrophysics and cosmology through the production of a Higgs or even an inflaton. A inflaton is a particle of the quantized inflation field. Detection of a certain kind or number of distinct inflaton fields would dictate which model of inflation could be correct .

A detection of the Higgs boson would give a boost to scalar inflation theory as so far no fundamental scalar fields have been detected. Very serious scientist still question the details of the standard ΛCDM model of cosmology while recognizing its good agreement with observations. The details are still being explored and there is much work to be done in cosmology. In particular details of inflation, the nature of dark energy and dark matter. The research just described will go a long way to solving these problems.

Part IV

Theories and observations of the frontier of cosmology.

In this section theories and observations at the frontier of cosmology will be reviewed. These theories have been proposed to fill in gaps in the standard models, or explain controversial observations that the standard models don't.

The standard theory of gravity, General Relativity, is a classical theory. However the universe is fundamentally Quantum Field Theoretical / Quantum Mechanical in nature. This is a short coming of the theory of General Relativity which keeps it from being regarded as the truly fundamental theory of gravity. A number of quantization's of General Relativity have been proposed. The one which will be hilited in this thesis is Loop Quantum Gravity and the cosmology that takes it into account.

The standard model of cosmology addresses all the well received observations that have been made adequately. However, one observation of a time variance in the fine structure constant cannot be explained easily by that model. For this reason a model in which the speed of light varies in the earliest moments has been proposed.

Some of these ideas will prove to be wrong, some will prove to be right, and some will be modified by new observations that they perhaps partially fit. That said a complete education in cosmology cannot be had without knowing of these theories.

Chapter 6

Quantum Cosmology

Quantum Cosmology is the application of quantum mechanical ideas to problems in cosmology. In particular the application of models of quantum gravity to cosmology in particular the big bang and the instants just after it. The goal of these cosmologies is the resolution of the singularity found in standard classical cosmology. These theories lack experimental or observational support however they are mathematically quite rigorous. Here will be presented a brief overview of this field and some of its more active branches. Those are Loop Quantum Cosmology and M-Theory cosmology.

Loop Quantum Cosmology is the cosmology that results when a theory of quantum gravity known as Loop Quantum Gravity is applied to the earliest phase of the universes evolution. The main focus of this chapter is the cosmology and not the details of the mathematical physics. For this reason, the deep details of Loop Quantum Gravity are not important. This chapter will introduce the basics of Loop Quantum Gravity. Then this chapter will discuss the cosmological implications of Loop Quantum Gravity.

M theory cosmology is the result of the application of M theory to the problems of cosmology. It's major claim is that the universe we live in is simply a four dimensional subspace, within a higher dimensional bulk (of 11 dimensions). Furthermore this universe is only one in a landscape of $10^{5}00$ possible universes. One theory holds that the big bang was simply a collision between two parallel subspaces.

6.1 Loop Quantum Gravity

Loop Quantum Gravity is an approach to quantum gravity which does not require any more than the four dimensions of space-time in which we really live. Unlike string/M theory it does not try to be a unified field theory or predict particle masses or anything else. This is a theory of gravity and nothing else. However like any theory of quantum gravity it will have much to say about the earliest moments of existence [49].

The key idea of LQG is to not try to define any sort of fixed static background metric. Instead the theory is to be defined on a abstract differentiable manifold. No metric is to be specified but it is to be solved for from a constraint equation. The "Loop" in Loop Quantum Gravity is the technique for defining a gage invariant operator in quantum field theory called a Wilson Loop. The loop being a closed path on the manifold around which a parallel transport would take place. Of course this formulation is diffeomorphism covariant [49]. This method of quantization was conducive to quantizing the reformulation of classical General Relativity due to Abhay Ashtekar. These new variables are defined by the following. Using the Vielbien formalism described in Appendix A. Fix a three dimensional manifold M with the SU(2) connection $A_i^a(x)$ and vector density $E_i^a(x)$ which transform in the vector representation of SU(2). That is they have SU(2) gage symmetry. The indices's i,j,... are for internal SU(2), and a,b,... are spatial. The connection and vector fields are equal to...

$$A_a^i \equiv \Gamma_a^i + \gamma k_a^i \tag{6.1}$$

$$gg^{ab} \equiv 8\pi G E_i^a E_i^b \tag{6.2}$$

In the above g is the determinant of the metric g^{ab} for a specific constant time and γ is a real number the Barbero-Immirzi parameter. An alternative and convenient form for those equations is in terms of the Pauli matrices.

$$E^a = -\imath E_i^a \sigma_i \tag{6.3}$$

and

$$A_a^i = -\frac{\imath}{2} A_a^i \sigma_i \tag{6.4}$$

.

For most this is much easier to visualize and work with. Note that these obey the commutator relationship:

$$\left[E_a^i, A_j^b\right] \simeq \imath\hbar\delta_a^b\delta_j^i \tag{6.5}$$

The Lie algebra of this theory follows logically from this commutator and it is in fact just a representation of the SU(2) gage symmetry group.

The dynamics of this theory are determined from a number of constraint equations (or in one proposal just one "master constraint") [50, 49]. Of course, one of them involves the Hamiltonian, not the Lagrangian, which is very different from quantum field theory in flat space-time in which the Lagrangian is used [49]. This difference stems from the choice of using a differentiable abstract manifold instead of a metric space. The equation that is used is a variation on the Wheeler Dewitt equation. To proceed father we need to understand the concept of a spin network. A spin network is a mathematical and graphical representation of a set of states with each link having a spin value. Stated plainly in LQG these spin networks *represent* space-time.

Let $|s\rangle$ be a "spin network" state, $E^{\epsilon\epsilon'}$ E has the action of creating new vertices's. Not unlike the raising and lowering operators in standard quantum mechanics. The Hamiltonian in this theory is then

$$\hat{H}|s\rangle = A_{\epsilon\epsilon'}E^{\epsilon\epsilon'}\,|s\rangle = \frac{1}{2}A^a_{\epsilon\epsilon'}\sigma_a E_b^{\epsilon\epsilon'}\sigma^b\,|s\rangle \tag{6.6}$$

This is known as the Hamiltonian constraint and it defines all of the energy dynamics of Loop quantum gravity and fully specifies the dynamics of the theory.

The prime result from Loop Quantum gravity for cosmology is the quantization of area. Area is an operator in the Hilbert space of LQG. There are eigenvalues and eigenstates of area. Due to the Heisenberg uncertainty principle there is a smallest physical area. Up to a linear parameter the Barbero-Immirzi parameter,this minimal area is the same order of magnitude as the reduced Planck area. The same goes for length and volume as well. The prime result of Loop Quantum Gravity is a derivation of the fundamental discreteness of space-time from first principles.[49]

6.1.1 Application of LQG to Cosmology, Loop Quantum Cosmology.

Loop Quantum Cosmology (LQC) is the application of Loop Quantum Gravity to the universe as a whole. It makes definite predictions about the nature of the big bang itself. It also predicts the same evolution of the universe a short time after the big bang as standard big bang theory.[51, 52, 50]

In Loop Quantum Gravity the universe was never pressed into a singular point. The singularity is resolved because space-time is discrete on the order of the reduced Planck length-time. Furthermore the universe would have expanded in increments of the Planck length-time, area, and volume eigenvalues. So at the very beginning the expansion of the universe was not a smooth expansion by any means. Weather the universe was still discrete enough to effect the process of inflation when it started is an open question. It is likely that this theory will supply small order corrections to the Friedman equations which would effect inflation on the large scale.

In this way LQG will have a direct astronomically observable effects on inflation. Thus, even this theory which seems so strange, connects to mainstream astronomical observations.

Chapter 7

Research on possible time variation in fundamental constants.

Inflation is the standard and accepted theory for solving the horizon, flatness, and isotropy problems of the standard big bang theory. Varying Speed of light cosmology is an alternative theory to inflation proposed to solve the same problems as inflation by many of the theorist who developed specific models of inflation, to take into account a time varying fine structure constant α. This chapter will discuss the objections to VSL cosmology then how the proponents of this variant model rebut those objections. Then details of one of the more simple and promising variants of VSL theory.

In this model from $10^{-36}\,sec. \leq t \leq 10^{-34}\,sec.$ The speed of light was 60 orders of magnitude higher than it is now then dropped exponentially to very *very* nearly the same value it has today. The only evidence of any change in α being a possible difference in the fine splitting of spectral lines in distant dust clouds illuminated by quasars.

The proposal of a time varying speed of light (VSL) is due in large part to the controversial observation of a varying α. Inflation is a very good explanation for the problems it was meant to solve, however it does not speak to something like a varying fine structure constant at all. VSL was proposed to address this very possibility.

There are actually a number of theories which include a varying speed of light (VSL) under certain circumstances [9]. Not all of those theories are explicitly cosmological in nature. For example the standard model of particle physics allows for the propogation of virtual particles at speeds faster than c for very brief periods. Without the inclusion of these virtual particles the theory would not work as well as it does.

7.1 Variation of the fine structure constant.

A recent and controversial observation by John Webb et. al. of a small variation of the value of the fine structure constant from its current value is what motivates these ideas [53, 54]. This observation was taken by observing the fine splitting in the absorption lines of clouds of gas and dust illuminated in the visible by quasars in the background. This question has been considered very important by the European Space Agency and European Southern Observatory (ESO) [41] .

In spectroscopy what one does is use a device which spreads out light into its spectrum. The most familiar example of this would be a prism. Then one observes the bright lines of emission, or the dark lines of absorption by the light source. These lines occur at characteristic wavelengths for every element. Fine splitting of spectral lines is due to the angular momentum of electrons in the atoms. It is this fine splitting which gives the fine structure constant its name.

The method sounds simple. The splitting between a doublet of spectral lines is proportional to α^2. If this splitting is different for the same wavelength of light, then it signifies a variation in alpha. The problem is the value of alpha depends on three fundamental constants which according to established physics either cannot, should not, or do not vary at all.

$$\alpha = \frac{e^2}{\hbar c \; 4\pi\varepsilon_0} = \frac{e^2 c\mu_0}{2h} = 7.2973525376 \times 10^{-3} \pm 6.8 \times 10^{-10} \quad (7.1)$$

Written with the dimensionful quantities that make up this dimensionless constant In CGS units.

$$\alpha = \frac{e^2}{\hbar c} \quad (7.2)$$

For alpha to vary one of those “constants” must vary. Variation in electric charge (e) has been investigated in a cosmological context [55, 56, 57, 53, 54]. Furthermore, allowing coupling constants such as the electric charge to vary with momentum is a standard feature of quantum field theory.

Since those publications by Webb, studies by Chand and Srianand claimed to find that alpha does not vary and were critical of Webb et al [58, 59]. Murphy and Webb replied by claiming that Chand and Srianand made fundamental systematic errors in their analysis of their data.[60] Webb et. al. then go after Chand and Srianand's own study of the fine structure constant. So on and so on, back and forth.

From a totally different direction MacGibbon conducted a theoretical study of a varying fine structure constant on the thermodynamics of black holes [55]. She found that a varying electric charge, hence varying fine structure constant as reported by Webb, would not violate the generalized second law of thermodynamics as applied to black holes [55]. The only objection raised to MacGibbon's paper were raised by Flaumbaum, *who worked with Webb* [56]. Flaumbaum claimed that MacGibbon missed a term in her computation [56]. An accusation to which MacGibbon has replied and shown to not be true [57]. MacGibbon also mentions in her reply that Flaumbaum has proposed physics beyond the standard model to explain the variation in e, which MacGibbon's paper shows is not necessary [57].

Unlike the observational data I can make some comments about MacGibbon's approach based on my own study of this matter for my self published book [61]. This is a very straight forward argument. It would not surprise me that varying the fine structure constant would not effect the thermodynamics of black holes in the regime MacGibbon studied. Consider the accepted standard formula for the Entropy of a black hole as found in numerous sources.

$$S_{BH} = \frac{kA}{4\ell_{\mathrm{P}}^2} \tag{7.3}$$

Where $\ell_{\mathrm{P}} = \sqrt{G\hbar/c^3}$ is the Planck length.

MacGibbon studied a varying electric charge not a varying speed of light. The electric charge is not a obvious factor in this equation so intuitively I would not expect it to effect S of a black hole. If however the speed of light were higher in the early universe, it would have lead to a different Planck length, and a different value for the entropy of a black hole. This fact would have physical implication if the universe, as some speculate, created numerous primordial black holes.

7.2 Proposal of a varying speed of light as a solution to this problem.

In light of this controversy, and the problem posed by a varying fine structure constant, one of the objectives of the planned "extremely large telescopes" which are discussed in this thesis is to study this phenomena and confirm or refute its authenticity [42, 43, 44]. Varying speed of light cosmology (VSL) was inspired in part by this observation. VSL replaces classic Inflation with a varying speed of light during the Inflationary period.

7.2.1 Objections to varying speed of light cosmology.

VSL cosmology is not a widely accepted theory by any means. For all the observations that the cosmological community has confidence in its just not needed. Further there are a number of common and elementary objections to the notion that the speed of light could vary.

Ellis in a comment on varying speed of light cosmology raises many of these elementary issues [62]. He comments that the variance of a fundamental constant of nature is only of physical importance, if that fundamental constant is dimensionless. This is because fundamental constants that are dimensionful can be set to any numerical value by a choice of units. Given that fact, how can varying the speed of light solve the horizon problem, or any of the the problems?

Along these same basic lines Ellis argues that varying the speed of light amounts to a change of coordinates in the metric. Such a coordinate change could not be physically significant and would not solve the horizon problem.

Ellis objects to VSL on the grounds that varying the speed of light, would break Lorentz invariance and causality. Ellis goes on to argue that a break in Lorentz invariance would lead to a break in Maxwell's equations, as well as the Einstein field equations.

The same exact issues were all also raised and refuted by Albrecht and Magueijo in their first papers on VSL theory. They were also addressed by Magueijo and Moffat in a response to Ellis.[63, 64]

Albrecht and Magueijo point out, correctly, that the same objections that are made for varying the speed of light can be made about any theory which would propose variation in any dimensionful "constant"[64] (or more correctly parameter.) For that reason only dimensionless ratio's of parameters are fundamental to physics. The

Hubble parameter is such a quantity yet in standard cosmology it is allowed to vary with time as part of inflationary cosmology. [64]

Ellis's objections which are based on field theory and Lorentz invariance are addressed by the fact that Lorentz invariance is not a global symmetry. The Maxwell equations only relate to the Lorentz transformations at the local level, not the global level.[64] The reason that the Einstein field equations need not be altered to deal with a varying speed of light, is because Einstein did not assume global Lorentz invariance when he formulated them. He assumed...

For infinitely small four-dimensional regions the theory of relativity in the restricted sense is appropriate, if the coordinates are suitably chosen.[65, p. 118]

Which in a nutshell is what it means for a symmetry like Lorentz symmetry to be local. Lorentz symmetry really only applies from point to point, at each point, yet at the same time, not over a larger region. If Lorentz symmetry was global then General Relativity could not be correct. Lorentz symmetry implies a truly flat spacetime metric every where. While on the cosmological scale the metric, FLRW, is very nearly flat, it is not perfectly flat. The metric near planets stars and black holes is not flat at all. Objections about conformal diagrams ignore the fact that metrics such as FRLW have to be transformed from their Lorentz breaking versions to their conformal versions in order to draw a conformal diagram with nice straight lines.

In short their are no purely theoretical reasons to object to VSL cosmology.

Observations are another matter. The tolerances on what could have happened in the inflationary/VSL epoch are still very tight due to measurements of relative abundances of light elements, and observations of the CMB's power spectrum and anisotropy. However like inflation there exist more than one theory of VSL cosmology so it has the same wiggle room as inflationary cosmology, for the time being, and it can fit the data presented to date by projects such as WMAP.

Future observations, such as those planned at the European Extremely Large Telescope[42, 43, 44], and gravitational wave background observations[26] could falsify VSL cosmology with a high degree of confidence. If alpha is found to not vary with time then VSL need not even be considered. If a gravitational wave background of the proper form ,predicted by inflation, is observed then VSL could be ruled out. Conversely we could have alpha varying with a observed gravitational wave background that indicates inflation. Then we would all need to think again!

7.2.2 Locally Lorentz invariant varying speed of light cosmology.

With the purely theoretical objections to varying speed of light cosmology dealt with we can move onto a simple model of VSL cosmology.

The theory of locally Lorentz invariant VSL was proposed by Magueijo in [66]. In this theory Magueijo required that at all times $\alpha_i \propto g_i \propto \hbar c \propto c^q$ where q was a dimensionless parameter. This implies minimal coupling to matter. A scalar field is then defined $\psi = \log(c/c_0)$ (In other words the field is parameterized as one over the index of refraction of the space-time that existed during the inflationary/VSL epoc.) The action of the theory is...

$$S = \int d^4x\sqrt{-g}(e^{a\psi}(R - 2\Lambda + \mathcal{L}_\psi) + \frac{16\pi G}{c_0^4}e^{b\psi}\mathcal{L}_m) \tag{7.4}$$

Where $\mathcal{L}_\psi$ is a term in the Lagrangian which encodes the dynamics of the scalar field.

$$\mathcal{L}_\psi = -\kappa(\psi)\nabla_\mu\psi\nabla^\mu\psi \tag{7.5}$$

It is then postulated that at all times Λ is proportion to the n'th power of c/c_0.

$$\Lambda \propto (c/c_0)^n = e^{n\psi} \tag{7.6}$$

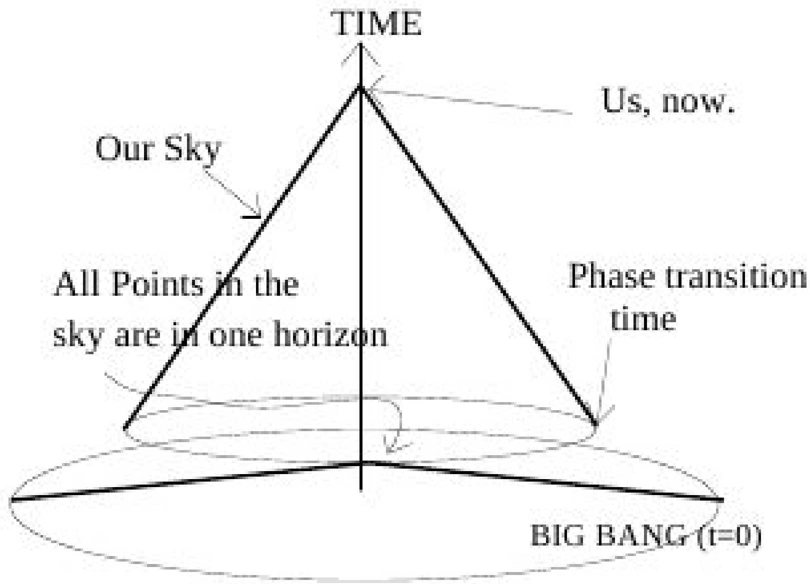
This gives a general equation for ψ in which Λ produces a potential which drives the variation of ψ and therefore c.

$$\Box\psi = \frac{32\pi G}{c^4\kappa}\mathcal{L}_m + \frac{1}{\kappa}n\Lambda \tag{7.7}$$

Lorentz invariant VSL looks very similar to inflation theory in many ways.[9, 66] The main physical difference is that in inflation the scalar field ϕ has nothing to do with light. The scalar fieldϕ is eventually related to the Hubble parameter instead of to light. The scalar field is also treated as a fundamental field, or no consideration is given to its origins what so ever. It will also be noted that what is denoted here as L_m is the same as what was denoted in the section on General Relativity as $\mathcal{L}_{\text{matter}}$.

There are a plethora of other VSL theories which will now be listed here. Theories where the speed of light is dependent on wavelength. String/M-theory based attempts where the speed of light is different on the membrane and in the bulk. Quantum Field theory in curved

Figure 7.1: The VSL solution to the horizon problem. This may look odd, however, compare this to the conformal diagram of figure 2.5. They are drawn differently yet they are in deed the same.



space times which allows photon propagation off the light cone, in other words photons may travel faster or slower than light. Alternative theories of gravity which have two metrics one for gravity and the other for matter in which c may vary.

7.2.3 Varying speed of light Cosmology

The basic idea of varying speed of light cosmology (VSL) is that the speed of light was high enough, long enough to allow the entire universe to reach thermal equilibrium. All points on the sky were within each others light cone long enough to explain the spatial flatness and isotropy of the CMB and solve the horizon problem.

How high a speed of light is high enough? Roughly 60 orders of magnitude higher than it is now. How long is long enough? From $10^{-36}\,sec. \leq t \leq 10^{-34}\,sec.$ or just as long as inflation would have lasted.

Figure 7.1 shows the result of VSL as a conformal diagram. This is a three dimensional variant of figure 2.5. The much higher speed of light casually connects all of the universe for briefest of moment. Thus solving the horizon problem. The other problems are also solved since the universe is dominated by radiation at this stage. All parts of the universe can communicate with all other parts at the increased speed of light. Radiation can zip from one end of the universe to the other during this brief period. The isotropy that would eventually tell tale on the CMB, as well as the near critical density flatness can all set in before the speed of light drops.

As for the anisotropy of the CMB; In inflation theory this anisotropy can be explained by inflation having lasted slightly longer or shorter in different regions. This leads to density fluctuations, which show up as warm spots in the CMB, and clusters of galaxy's which we look up and see. In VSL cosmology the speed of light would depend on local conditions, as indicated by equation 8.4 therefore VSL can also accommodate density fluctuations, etc. etc. The difference in time that the speed of light varying in one place or the other would be almost immeasurably short, just enough to allow for the slight differences in temperature of the CMB, and observed density to allow for large scale structure formation.

Like the vectorial variant of inflation, VSL has the advantage of not having to rely on a type of field that has never been observed in nature, or in particle physics experiments. The field which varies in VSL is simply the EM field as it interacts with space-time. This is a huge advantage over the non vectorial theories of inflation. Further what would drive Lorentz invariant VSL is the cosmological constant, which we have already observed in the form of dark energy.

VSL cosmology, was proposed in order to explain an anomalous observation as well as solving the same problems as inflation. There are certain common objections all of which have been overcome by clever theoretical formulation of this theory. Their are observational test for this theory in the works. This theory is compatible with the same data as inflation when it comes to the slight anisotropy of the CMB.

7.3 VSL and inflation as equivalent models.

Inflation and varying speed of light cosmology are on a certain level completely equivalent models. That level being their effect on the FLRW space-time as shown by their conformal diagrams. The other was pointed out by Avelino and Martins and will now be reviewed here.

Avelino and Martins point out that there is one fundamental and dimensionless ratio which can tell us if a cosmological model solves the horizon, flatness and isotropy problems of the big bang. They called it “the expansion number”. They noticed that nature provides cosmology with natural units of length and time. The unit of length being the curvature scale $\ell_c \equiv a|k|^{-1/2}$ and the unit of time being the Hubble time $H^{-1} \equiv a/(da/dt)$. The a and k are both just as found in the FLRW metric. The expansion number is the ratio of these [14].

$$C_e = \frac{c|k|^{1/2}}{aH} \tag{7.8}$$

This definition shows that all varying cosmic speed theories, which includes every model of inflation, and VSL cosmology are generic. That is they are fundamentally the same [14]. Assuming only the cosmological principle one can see that the resolution of the horizon problem results from having a period in the history of the universe where the scale factor grows faster than the Hubble radius [14]. This condition causes a decreasing cosmic speed, mathematically

$$\frac{d}{dt}C_e < 0 \tag{7.9}$$

Any cosmological model for which that is true can solve the horizon problem and hence the other problems of the big bang. The various models of inflation and VSL meet this criterion. Any future proposed cosmological model must also satisfy this criterion as a necessary but not sufficient condition.[14]

The unassuming nature of Avelino and Martins's paper has been part of why it has only been referenced three times. This lack of fanfare mask what I am sure will prove to be a very important test for future cosmological models.

The same point made by Avelino and Martins can be seen in the way that the FLRW metric was written in this thesis and in Sean Caroll's book [6]. In section **??** the scale factor $a(t)$ is made dimensionless, and for reasons argued by critics of VSL much more physically meaningful through the Planck length. This was done by dividing R by $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616252(81) \times 10^{-35}$. This length was assumed in that instance to be a constant and fundamental length. Suppose c varies in that formula dropping exponentially after the big bang from a very high value, to the value of c we observe today?

$$a(t) = \frac{R(t)}{\ell_P} \tag{7.10}$$

$$\ell_P \bar{r} = r \tag{7.11}$$

Figure 7.2: The black line in this figure is the scale factor, the red line is the speed of light c. c drops to its current value very quickly but its variance drives an exponential expansion in the scale factor much the same as is found in models of inflation.

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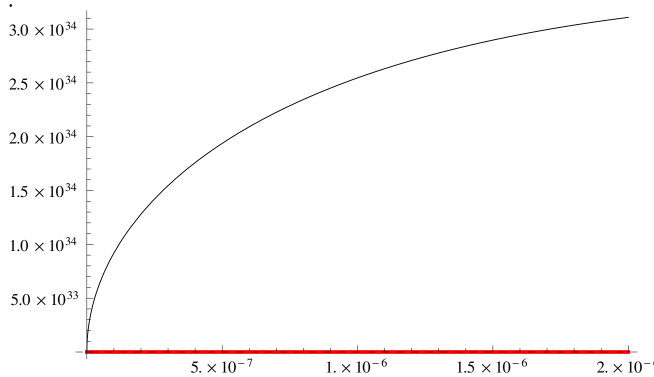


What will the scale factor do? The following plot,7.2 should illuminate this subject.

As figure 7.2 shows, the scale factor increases exponentially as the speed of light drops exponentially. This exponential increase in the scale factor for this brief period is, in effect, the same as inflation. At this point this thesis has shown several different lines of reasoning which lead to the conclusion that inflation and varying speed of light cosmology are just different models for the very same physics up to the apparent lack of a varying fine structure constant in inflation. If observations of a varying fine structure constant are ever confirmed VSL will be able to take the place of inflation. In such a new standard model of cosmology that would be only minimally modified from the current concordance model by its presence.

Part V

Summation

The summation of this thesis will condense all of the most salient points covered and integrate everything into one unified picture of the field of cosmology.

Chapter 8

Summary

The universe began from a state of high density, and energy and expanded rapidly cooling in the process. This has been termed the big bang. According to classical theories the big bang started from a point of infinite density and zero volume. According to theories of Quantum Gravity and Quantum Cosmology the big bang began from a point of very high but finite density. According to these theories the universe was on the order of the Planck volume $10^{-105}m^3$, and the time was about $10^{-44}sec$.

After the big bang at $10^{-36}sec$. the universe began to expand rapidly. The forces of nature known to physics began to differentiate themselves. Gravity was the first to go its own way as space-time expanded. Then the three remaining quantum fields. As this occurred space time went through a period of rapid expansion or , inflation. This inflation would, according the standard model of cosmology lead to a universe which is flat, and uniform or isotropic on the cosmological scale.

While space time was rapidly expanding and for the next second after it, all the normal matter in the universe was created. The symmetries that keep matter from being created, in most circumstances, were broken. Matter and anti matter were created and most of the matter was annihilated by the anti matter. This process is known as baryogenesis. Shortly after this, and for the next few minutes of existence the universe was hot and dense enough to synthesize heavy ions such as helium. The process lead to the creation of a universe that was dominated by hydrogen ions, free photons, and free electrons. No light could travel far as photons would be constantly reabsorbed and emitted as hydrogen atoms would form then fly apart.

Finally the universe expanded and cooled to the point that light could freely propagate. This light has came to us in the form of microwaves. These microwaves are the Cosmic Microwave background radiation. They last interacted with matter 380,000 years after the big bang.

Research on this period from the big bang to the emission of the Cosmic Microwave background was the topic selected for and covered by this masters thesis.

8.1 The concordance model of cosmology and its evidence.

Section one was a review of theoretical research related to the concordance model of cosmology. This models is also known by the name ΛCDM for the two major components of the universe according to this model. The dark energy component or cosmological constant Λ, and cold dark matter CDM. First the thesis covered the basics of General Relativity. This theory of gravity is at the heart of modern cosmology. From this theory we get a set of equations which can postdict the evolution of the universe as time is reversed. The universe shrinks to a point, a classical singularity.

The second theory that needs to be understood as part of the concordance model is cosmic Inflation. Inflation is a model for the rapid expansion of the universe following the big bang. This expansion is driven by a unknown, field or fields which has decayed away.

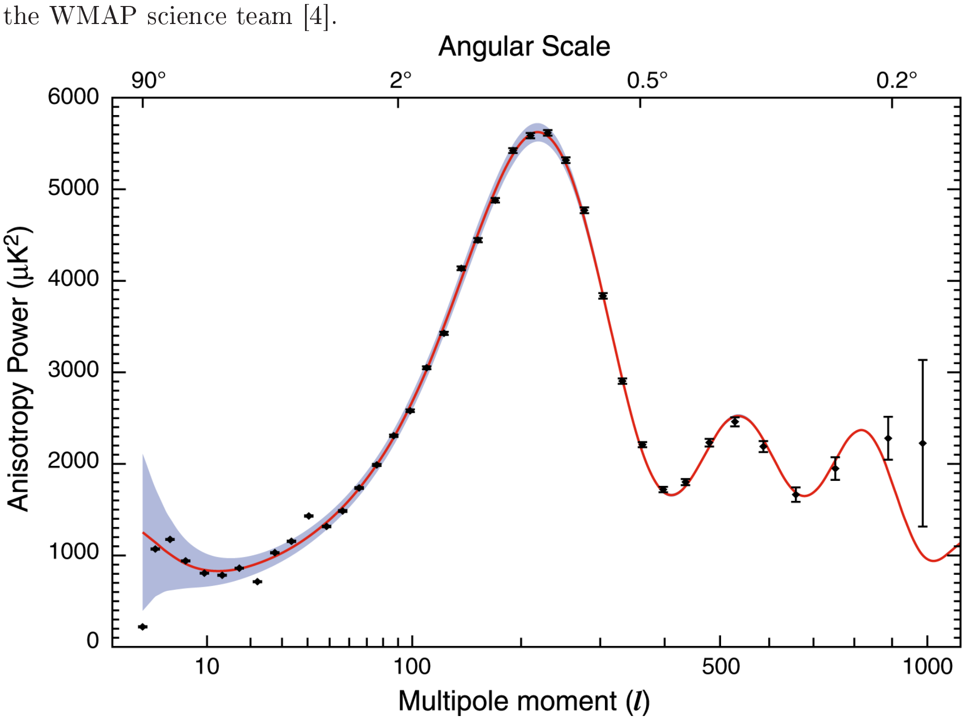
The single best evidence to date for this model is the observed angular power spectrum of the Cosmic Microwave Background as measured by the Wilkinson Microwave Anisotropy Probe (WMAP) and how well it fits this data.

8.2 Observational and experimental studies.

Section two was a review of research literature on observational and experimental studies which deal with the time period in question. The most important projects , the largest projects being the European Space Agency's Planck probe, and the CERN's Large Hadron Collider.

The Planck probe is a satellite which orbits the Sun-Earth Lagrange point L2, facing away from the Earth and Sun. It's sensors have a greater angular resolution than those of the WMAP probe. It's sensors also cover a wider range of frequencies. This probe will also

Figure 8.1: The power spectrum of the CMB as measured by WMAP. This figure shows the power spectrum of the CMB as measured by WMAP, with a best fit line provided by the standard ΛCDM model of cosmology. Courtesy of the WMAP science team [4].



be able to collect more detailed data on the polarization of the CMB. Encoded in this data could be the first hard evidence of gravitational waves. These waves would be a gravitational analog of the CMB and depending on their features would support the concordance model. The Planck probes data will allow us to refine our understanding of the universe's early evolution in many many ways. For example take the data from the WMAP probe gathered over seven years and compare it to the data from Planck after just one year as in figure 9.2. The fore grounds have not yet been removed, however the greater angular resolution of the Planck data is obvious to the casual observer.

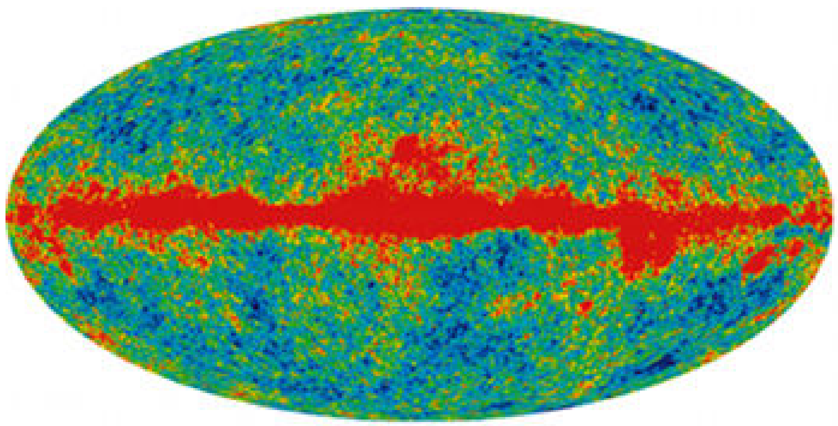
The research being done at the Large Hadron collider will also touch on cosmological issues. As part of its overall research program a search will be conducted for any signs of particles that could be identified as dark matter, or particles of the proposed inflation field. The main objective of the LHC is the detection of a Higgs boson. This impacts on cosmology because the simplest models of inflation call for the existence of a quantum field which is similar to the Higgs in terms of its spin angular momentum. If the Higgs is not found then it cast doubt on such models of inflation, and favors more complex models such as vector inflation.

In addition to the above huge new telescopes are in the works which will allow humanity to see farther back in time, and in greater detail than has heretofore be available. In particular these will allow more careful observations of distant primordial gas clouds. The spectra of these clouds, their composition and fine structure will rule out particular cosmological models and raise new questions.

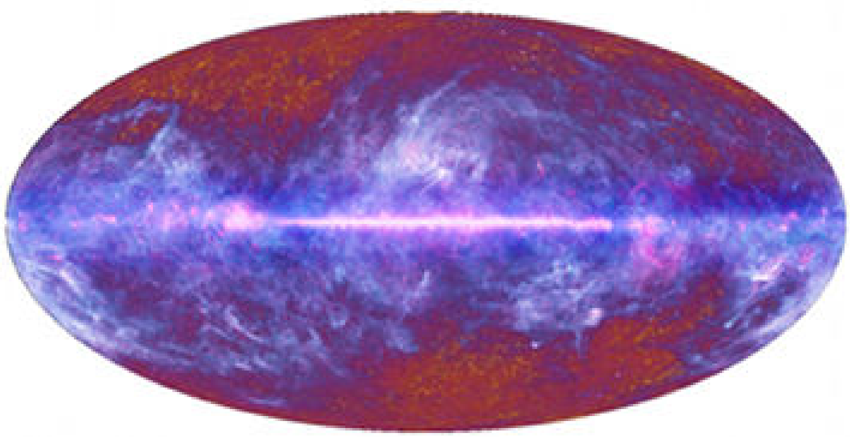
8.3 Speculative yet promising new theories.

In the very last section models which seek to extend, improve, and in some cases replace portions of the concordance model were covered. These include quantum cosmological models such as loop quantum gravity, and M theory cosmology to theories of a time varying speed of light.

Loop quantum gravity is a highly speculative but mathematically well founded model of quantum gravity. This model extends General Relativity to the quantum domain. In the process it has shown mathematical results such quantization of area and volume. LQG agrees well with semi classical work done on the thermodynamics of black holes. In application to the big bang LQG makes predictions in cosmology which are very interesting. In this quantum cosmology



WMAP Q-Band map



PLANCK all sky survey.

Figure 8.2: The top image is the WMAP data for seven years of collection before the foreground is removed. The bottom image is the Planck data for one year without the background having been removed. Note the superior detail of the Planck image. Images courtesy of the WMAP science team, and the Planck science team respectively.

the big bang does not begin from a singularity of infinitesimal extent and infinite density but from a tiny region of very small volume, on the order of 10^{-105} meters, and very high density.

WRITE HERE EXECUTIVE SUMMARY OF M THEORY COSMOLOGY.

In addition to quantum theories of cosmology there is also an alternative to cosmological inflation. This is known as varying speed of light cosmology. The motivation for proposing this theory was to explain the unconfirmed observation of a varying fine structure constant. The fine structure constant depends on the speed of light, Planck's constant, and the charge of an electron. This cosmology makes the same predictions as inflation but is able to account for a momentarily much higher speed of light, which would have decayed rapidly to very nearly the speed of light we observe today. As this author's meta analysis of this model showed the speed of light varying in the way suggested by VSL theory results in a exponential increase in the scale factor which controls the rate of expansion of the universe. This is essentially the same prediction made by inflationary models.

Last this author did write and submit for peer reviewed publication a Lagrangian which if validated by observations and experimentation could provide a simple and elegant mathematical framework for the standard Lambda CDM, (inflation, dark energy and dark matter) model of cosmology.

8.4 Conclusions

From my review of the current research the following conclusions can be drawn. The most important being that there is still much work to be done in theoretical, and observational cosmology. The current models of cosmology has the broad outlines of what occurred after the big bang mostly right. The alternative models mentioned even agree on these broad details and would represent only minor changes to the concordance model. However the details are lacking.

The biggest detail being a specific model for the rapid universal expansion. There is more than one model. Each model makes ever so slightly different predictions that can and will soon be tested. While the overall cosmological model does not depend on weather the inflation field was a scalar or a vector our overall understanding of physics does.

The second biggest detail that needs to be filled in is the identity of the dark matter that comprises most of the matter in the universe according to the concordance picture. Research on this area is outside the scope of this thesis. However research in this area will impact the field of cosmology. More work remains to be done in solving the problem of dark matter.

The third biggest detail is the fact that the current model is built on a classical theory of gravity. Which is a problem since at the density and energy of the big bang a quantum mechanical model is needed to understand what happens. Research on this is at the frontier of cosmology and theoretical physics and is highly speculative right now. However such work is where some new physics may or may not be revealed.

While this model is not perfect by any means it fits all of the agreed upon data collected to date and is flexible enough to accommodate minor changes such as various models of inflation or even VSL (if the observations of a time varying fine structure constant reported on in section 7.1 are confirmed) Filling out these blanks in the standard model of cosmology will occupy cosmologist and particle physicist for a great while.

Part VI

Appendices

In these appendices I have written summaries of the basics of General Relativity and Quantum Field Theory. Those two theories are the foundation stones of the study of the cosmology of the early universe. To these I add my own feeble attempt to explain the difficulties humanity has had in detecting dark matter in earth bound experiments, while explaining the practically irrefutable astronomical evidence for it.

Appendix A

The Principles of Relativity.

For those who read this thesis in the future and do not already know General Relativity. This is for you. If you already know General Relativity then this is just a review.

The principles of General Relativity must be understood in order to understand classical and quantum cosmology. To that end, this chapter will lay the foundations necessary to understand the research which will be presented in this thesis. I know that General Relativity is not part of the core courses at most instiutions. So it is understandable that a *student* who may read this in the future may not know these things.

I would encourage as many people as possible to study General Relativity. Sadly it seems to be a dying art.

Topics to be covered in this chapter will include the mathematical concepts which define spacetime. Those concepts being manifolds, a specific type of mathematical space. Tensors, which are functions on a manifold which obey certain transformation rules. Metrics, are in practice are a type of tensor, which define distance for that manifold. Diffeomorphisms, which are a large family of transformations on manifolds, which in General Relativity the laws of physics must be covariant with respect to. Curvature of spacetime will be defined with some rigor. From there the Einstein field equations will be built up. The formulation of General relativity in terms of non-coordinate bases which are used in formulations of quantum gravity and quantum cosmology will be presented. Next the theory of conformal diagrams will be presented. Conformal diagramming provides a useful method for understanding problems in General Relativity without needing to solve complex equations. After that, two important solutions to these equations, known as the Schwarzchild (black hole) metric, and the Friedman-Lemaître-Robertson-Walker (big bang FLRW) metric will be discussed.

With the mathematical machinery of General Relativity in place, the discussion of the FLRW metric and classic big bang theory will be undertaken. This metric is known as the big bang metric due to it's property of having spacetime grow or shrink with time due to it's containing a scale factor. This scale factor will be discussed at length. This metric, when combined with observations of a very uniform universe, on the cosmic scale, leads to a problem. How can the universe be this uniform is it a coincidence, or was something else happening in those first moments which drove the universe to uniformity? The answer to that question is yes there was something else that drove the universe to uniformity. That something else is thought to be very rapid expansion of spacetime. In the next chapter this theory will be discussed in detail.

A.1 Mathematical Concepts of General Relativity.

General Relativity is the theory of gravity due to Einstein which states that curved spacetime is the true source of gravity[65]. Spacetime is curved by the presence of mass, energy and momentum currents.

The best and simplest way to think of General Relativity is to think of it in reference to Newtons Law of inertia. Where that law has been generalized to take account of curved non-Euclidean spacetimes. Newtons law of inertia says:

Objects at rest will tend to stay at rest, and objects in motion will tend to stay in motion along straight lines *unless acted upon by an outside force.*

The important part to consider is "along straight lines". To Newton the only geometry there was had been settled thousands of years before by Euclid and others. Space and time were separate things. Straight lines were just straight lines, nothing could change that.

To see this more clearly look at the mathematics of newtons second law of motion. *The net force on a particle is equal to the time rate of change of the particles net momentum.* Show here expanded out as it's total derivative.

$$\overrightarrow{F} = \frac{d}{dt}\overrightarrow{P} = m\frac{d\overrightarrow{V}}{dt} + \overrightarrow{V}\frac{dm}{dt} \tag{A.1}$$

Next apply Newtons law of inertia and set the force equal to zero and assume that the mass is constant to get the following equation.

$$\overrightarrow{0} = m\frac{d^2}{dt^2}\overrightarrow{X} \tag{A.2}$$

The solution to this equation is...

$$\overrightarrow{X} = \overrightarrow{v_0}t + \overrightarrow{x_0} \tag{A.3}$$

...just the equation for a straight line in Euclidean space. In what will follow how to determine the equation for the equivalent of a "straight line" will be determined in a spacetime that is curved and a geometry which is not Euclidean. In Einstein's universe of General Relativity we all live in a differentiable vector space which reacts to the motions of momentum and energy currents through it. Space is no longer a fixed background in which more interesting things occur, but it is a physical entity which acts on and reacts to everything in it.

In these curved spaces "straight lines" are now geodesics. A geodesic is the shortest path from point A to point B in any space. In the flat spacetime of Special Relativity that means a straight line much as it does in Euclidean space. In General Relativity the gravitational field can be thought of as the change from flat space with Euclidean straight lines, to a curved space with geodesics. The spaces of General Relativity locally, on a small enough scale, resemble truly flat Minkowski space. Minkowski space in turn resembles Euclidean space on a small enough scale. This is true no matter how contorted the space becomes. Just what this means will be given more rigor latter in the chapter.

All of this said Newton's law of inertia still survives, after a fashion, in General Relativity. *Objects in motion tend to stay in motion along geodesics unless acted upon by an outside force.* Hence when you are falling from a high place you are not being pulled by the Earth. In fact you are simply following the geodesic from one point to another as if no force was acting. Due to the variation in the curvature with position in space acceleration is felt called gravity.

In the following sections the General theory of Relativity will be built up from it's mathematical foundations up.

A.2 Prior knowledge which will be assumed.

In building up Einstein's General theory of Relativity it is necessary to assume a certain level of prior mathematical knowledge. It will be assumed that an interested student reader is knowledgeable of basic terms of linear algebra, linear transformations , and maps. It will be assumed that the student reader is knowledgeable of vector spaces and inner products. Furthermore it will be assumed that the student reader is familiar with Special Relativity and classical field theory (Lagrangian, and Hamiltonian dynamics stress energy tensors etc). These terms will be assumed as known to a student reader and will not be explained in detail. Any good textbook on the subjects of linear algebra or electromagnetism will explain these points.

A.3 Tensors

The language of General relativity is the language of tensors, and tensor fields acting over certain vector spaces. There are several equivalent definitions of a tensor.

A tensor of rank n in a m dimensional space, over the field of real numbers, is a function which is linear in n variables with m^n components which, under transformation of coordinates, the components of the object undergo a transformation of a certain nature and it maps n vectors to the real numbers.

$$M_{\mu\nu}V^{\mu}V^{\nu} \to m \tag{A.4}$$

, $m \in \mathbb{R}$.

The simplest way to think of this definition of a tensor is by way of it's representation as a matrix. For example a tensor of rank zero in a four dimensional space would be a single number denoted as q. Such tensors are referred to as *scalars*. The next example would be a tensor of rank one in four dimensional space. This would be an array with four elements and one index. These are notated like so q^{μ} in the index notation common to Special Relativity. Alternatively, these will be written as a column matrix. Tensors of this rank are known as vectors. Last, but not least, are tensors of rank two in a four dimensional space. These tensors are denoted as $Q^{\mu\nu}$ and represented as a four by four matrix. Tensors of rank two and above are simply referred to as tensors.

The most familiar tensor to anyone who has studied Special Relativity would be the Lorentz transformation tensor Λ^{μ}_{ν}

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.5}$$

The math of General Relativity is a tensor field theory. For that reason a definition for a tensor field is necessary.

A tensor field of rank n in a m dimensional space, over the field of real numbers, is a functional which is linear in n functions of n variables with m^n components which, under transformation of coordinates, the components of the object undergo a transformation of a certain nature and it maps n vector valued functions to a function of the real numbers.

$$M_{\mu\nu}(x^\mu)V^\mu(x^\mu)V^\nu(x^\mu) \to m(x^\mu) \tag{A.6}$$

, $m \in \mathbb{R}$.

The way these are presented is often using the familiar function notation $A_\mu(x^\mu)$, or alternatively they will use the partial derivative. Other than the elements of a tensor field being functions everything about the first definition applies to them. The most familiar example of a tensor field would be the electromagnetic field tensor.

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix} \tag{A.7}$$

The difference between the electromagnetic field tensor, and the Lorentz transformation tensor, is that the Lorentz transformation tensor is not an explicit function of the coordinates.

The Lorentz transformation tensor is an example of a tensor acting on the spacetime itself. Tensors can be thought of as stretching or expanding space, even a flat space like the space of Special Relativity.

The space of Special Relativity is known as Minkowski space, and it has associated with it, the Minkowski metric. In most Special Relativity text, just what a metric is, in general terms, is never defined. *A metric is a function in a given space which defines distances in that space.* In Special Relativity the metric tensor is used in just that way in defining the separation between points in Minkowski spacetime. In this way, the Minkowski metric is used to define a “inner product". In standard form the Minkowski metric tensor is as follows.

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{A.8}$$

The Minkowski inner product of two vectors X and Y is.

$$\langle x, y \rangle = \eta_{\mu\nu} x^\mu y^\nu = -x^0y^0 + x^1y^1 + x^2y^2 + x^3y^3 \tag{A.9}$$

Note that this metric is symmetric which is reflected in the matrix being diagonal of the Minkowski metric tensor. Like all such matrices it is bilinear, in that this function takes a pair of vectors to produce it's output. Last but not least it produces a output that is a real an nonzero number as long as both of the inputs are not zero. In other words it is non-degenerate. The Minkowski metric, and Minkowski space show all of the basic properties that a spacetime in General Relativity needs to have. These spacetimes are in mathematics terms known as manifolds.

A.3.1 Basic Tensor Operations

There are some basic operations that can be performed on general tensors which will show up in this thesis. In the literature and in this thesis there are two convenient ways of presenting these operations. One is the old and familiar index notation. The other is a more modern and cleaner index free notation. Which one will be used depends on the context, further in some places a odd combination of both will come in handy. In all cases the notation presented will have been found in literature and is a standard for discussing the particular topic.

The most basic operations are addition and subtraction. The general rule is that *Tensors of differing rank cannot be added or subtracted from each other.* Think of this in terms of tensors being presented as matrices. What does it mean to subtract a column vector from a matrix? It means nothing, it is undefined and non-sensical. Provided that the tensors are of the same rank (or in matrix form they are of the same dimensions) they can be added and subtracted element by element. For example.

$$a^{\mu\nu} - b^{\mu\nu} = a^{00} - b^{00}, a^{11} - b^{11}, a^{01} - b^{01}, ... \tag{A.10}$$

The next tensor operation to be concerned with is that of the product. Tensors have more than one kind of multiplication. Each with a different notation. The most familiar by now would be the inner product as defined above in Minkowski spacetime.

The next most familiar, to anyone who has had advanced Quantum Mechanics, would be the outer product. In Quantum Mechanics one may have seen expressions such as $|\phi><\psi|$. These same expressions exist in General Relativity. However in General Relativity the notation is different. The common representation of an outer product in index notation is two vectors next to each other like so $A^\mu B^\nu$. Similarly in index free notation this will often be presented as two vectors next to each other with no symbol in between. In terms of matrices a simple example of a outer product would be...

Let V be a vector space with vectors $A, B \in V$. $A = (a1, a2)$ and $B = (b1, b2)$ The outer product of these two vectors would be.

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} \begin{pmatrix} b1 \\ b2 \end{pmatrix} = \begin{pmatrix} a1b1 & a1b2 \\ a2b1 & a2b2 \end{pmatrix} \tag{A.11}$$

Taking a derivative of a tensor is another operation which will show up again and again, and again in this thesis. This will often be denoted like so.

$$\partial_\mu A^{\mu\nu} = \left(\partial_0 A^{0\nu} + \partial_1 A^{1\nu} + \partial_2 A^{2\nu} + \partial_3 A^{3\nu}\right) \tag{A.12}$$

Where a notation has been used that is commonly found in the study of Special Relativity, and Quantum Field Theory. Another common notation is known as comma notation for taking a derivative with respect to a particular index. The last expression would be denoted as $A^{\mu\nu}, \mu$ That notation will also appear in this thesis.

The reason for all of these different ways of denoting a derivative is because of the various ways of denoting the other operations on a tensor. Their are many context in which they appear and in each one there is a different standard of notation. For this reason no single notation can be chosen which will work in all cases. This author will mention which notation is in effect if it is not clear from the context.

A.4 Manifolds

In the most informal sense a manifold is a vector space which is similar enough to Euclidean space. To define a manifold mathematically we first need to define a specific type of linear transformation known as a homeomorphism. If a given vector space is at least locally homeomorphic to Euclidean space then it is similar enough. *A function H is a homeomorphism if it has the following properties.*

- H is a map from one vector space M to a vector space N.
- H:M $\rightarrow$ N is onto.
- H:M $\rightarrow$ N is one to one.
- H^{-1} exist.

With the notion of a Homeomorphism defined a manifold can be defined as a vector space M which for which there exist a map $H : M \rightarrow E$ (where E represents Euclidean space.) If that map H exist then M is a manifold. That map H does not need to be global it can be local. A local homeomorphism simply means that each point in the manifold has a small neighborhood around it which is homeomorphic to to Euclidean space.

In formulating General Relativity Einstein applies this idea as he himself stated it.

For infinitely small four-dimensional regions the theory of relativity in the restricted sense is appropriate, if the coordinates are suitably chosen.[65, p. 118]

In so doing he defines the type of manifold that is used in General Relativity a manifold which is locally homeomorphic to Minkowski space is known as a pseudo Riemannian manifold. Such a manifold is equipped with a metric which like the Minkowski metric is bilinear, symmetric and non-degenerate, as previously discussed. The difference will be that in General Relativity the metric tensor is now a tensor field. This has to be the case so that the flat spacetime of Special Relativity will be one solution to the field equations of General Relativity.

A.5 General Covariance

General covariance is the generalization of Lorentz covariance as seen in Special Relativity. In special Relativity the laws of physics need to be written in such a way that they are covariant with respect to Lorentz transformations. Lorentz covariance ensures that the laws of physics will be of the same form in any inertial frame of reference. To deal with this a more robust mathematical framework is needed which will extend Lorentz covariance to handle reference frames which are not in relative inertial motion.

A.5.1 Why is General Covariance important?

Why is general covariance important? Why isn't Lorentz Covariance enough for a theory of gravity? In a nutshell Lorentz covariance is not flexible enough to handle accelerated frames of reference, so general covariance and it's richer mathematical structure are needed to handle any relative motion what so ever.

Lorentz covariance is not enough for a theory that incorporates gravity because gravity causes acceleration and Lorentz transformations can't handle a relative acceleration. In the presence of a gravitational field everything is being accelerated. Hence the frame of reference is not inertial. So a more general form of covariance is needed. Covariance that can handle any kind of accelerated reference frame what so ever. That is what general covariance is. Lorentz covariance is valid when the acceleration is zero, General covariance applies when the acceleration is not zero.

The concept of general covariance also connects to the principle of equivalence. This is often thought of in terms of gravitational mass being equal to inertial mass. A more illuminating way to think of it is in terms of the equivalence of the laws of physics across different frames of reference no matter their relative accelerations.

The classic thought experiment is considering a astronaut in a box in orbit, and another astronaut in a box in free fall. The box in orbit experiences no or very little gravitational acceleration. Physics experiments performed in this frame of reference confirm that there is no gravitational field. For example if the astronaut in orbit takes out a tennis ball and places it next to his head, it will not fall. Now consider what happens when the astronaut in a box in free fall would see if she placed a tennis ball next to her head. Would the ball fall, or would the ball float just like it would in zero gravity. The answer is of course that the ball would float.

This leads us to another more useful statement of equivalence. That experiments performed in reference frames with the same acceleration are equivalent. From this general statement it follows that the gravitational and inertial masses are equal. If these masses were not equal, then the astronaut in the freely falling reference frame would see a different result than the one in zero gravity.

A.5.2 Diffeomorphism Covariance

Since Lorentz transformations are not enough what should they be replaced with in General Relativity? Them answer is diffeomorphisms. *A diffeomorhpism D from one manifold M to another manifold N is defined as a map with the following properties.*

- D is a map from one differentiable manifold M to another differentiable manifold N.
- D:M $\rightarrow$ N is onto.
- D:M $\rightarrow$ N is one to one.
- The map D is smoothly differentiable (at least to some degree).
- The map D has an inverse D^{-1}:N $\rightarrow$ M which has all of the above properties.

This all means that any map which has all of the above properties will be valid in General Relativity. Notice that a Diffeomorphism is a Homeomorphism with the added requirements of differentiability of the map and the manifolds. This differentiability is needed because the field equations of General Relativity will be differential equations much like every other law of physics. For those reasons diffeomorphism covariance is required.

A law of physics is generally covariant if it is diffeomorphism covariant This statement can be taken as proven by the definition of a diffeomorphism. It is general enough to accommodate any possible acceleration. A Lorentz transformation is one type of diffeomorphism. In an older notation a diffeomorphism would be written as a tensor transformation like so.

$$T'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\,\mu}} \frac{\partial x^{\beta}}{\partial x'^{\,\nu}} T_{\alpha\beta} \tag{A.13}$$

That is a specific kind of diffeomorphism, a covariant transformation. The more modern formalism is more general and it is what will be used in this thesis.

A good example of two spaces that are related by a diffeomorphism would be the flat Minkowski spacetime of Special Relativity, and the curved spacetime around a star like our sun. Think about it in a physical way. Imagine spacetime with no matter in it. It will be flat Minkowski spacetime. Then imagine a star floating into that spacetime. That space time will smoothly transform into a curved one due to the mass of the star. This thought experiment is an example of a diffeomorphism in action. Just what does it mean for a spacetime to be flat or curved?

A.6 Curvature of spacetime, Einstein's Field Equations, and two important solutions.

To understand what it means for a spacetime to be curved we need to think of what it means to take a derivative on a curved spacetime. First a derivative in curved spacetime will be defined and from there the concept of a curved spacetime will be clarified mathematically. In the process the machinery of Einstein's field equations will be exposed.

A.6.1 Derivatives in Curved Space-Time; Christoffel Symbols

A common statement about General Relativity is that it is a theory in which gravity is not a force, but a artifact of a curved spacetime. Just what is a curved spacetime? How does one determine if a given spacetime (pesudo-Riemannian) manifold is curved? The answer to both of these can be found by formulating the covariant derivative ∇. The covariant derivative will have to obey two fundamental rules.

It must be linear.

$$\nabla(Y+X) = \nabla Y + \nabla X \tag{A.14}$$

It must also obey the product rule.

$$\nabla(Y \otimes X) = (\nabla Y) \otimes X + Y \otimes (\nabla X) \tag{A.15}$$

In which $\otimes$ in the above is any multiplication like product (scalar,inner, outer, or tensor products). If these two rules are not followed then much of our usual skills in solving differential equations would be rendered useless. Linearity, and the product rule are very important algebraic rules used again and again in solving differential equations. These two requirements mean that we can write this covariant derivative as the standard partial derivative plus a corrective term. The result is the following.

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \tag{A.16}$$

Where the $\Gamma^\nu_{\mu\lambda}$ is known as the Christoffel connection coefficients. They are found by taking several derivatives of the metric tensor. [6]

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right) \tag{A.17}$$

This allows a conceptually simple, if mathematically tedious test of curvature. Any space will be flat if the Christoffel connection is zero in all it's components. It is clear that for the Minkowski metric which is composed of 1's on the diagonal, this will be the case. Hence flat spacetime is the spacetime of Special Relativity up to a scaling factor.

A.6.2 The Geodesic Equation

It was written earlier that General Relativity can be thought of as a generalization of Newton's law of inertia to accommodate non Euclidean geometries. Specifically to accommodate the notion of a space where straight lines were not simply straight lines but were replaced with the concept of a geodesic. Here is the equation for finding the geodesics of a given geometry. [6]

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0 \tag{A.18}$$

.

λ in the above equation is a parameter, the most common choice in practice is the proper time. Note that if the Christoffel connection coefficients are zero and λ is replaced with time t, then the equation for a straight line, in flat spacetime is recovered. Thus Newtonian physics is clearly a special case of General Relativistic physics.

This equation is very important in General Relativity, since it is this equation which will give the paths followed by an unaccelerated test particle in General Relativity. In other words if you want to know the path of a planet about a star, or stars about the center of a galaxy using General Relativity this is the equation that needs to be solved.

The most important special case of a geodesic would be a null or "light-like" geodesic.

$$g_{\mu\nu}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\nu}}{d\lambda} = 0 \tag{A.19}$$

The geodesics which satisfy the previous equation are the paths that would be followed by a ray of light in a given geometry. This is the equation that one would use to analyze something like gravitational lensing for example.

These equations will come up again and again in this thesis in a number of context. In a sense finding these geodesics is one of the main objectives of applied General Relativity.

A.6.3 Riemann and Ricci curvature tensors

The Christoffel symbols tell if a spacetime is curved, and provide a correction to the derivatives on that spacetime. However they don't tell just how curved a spacetime is To do that we need to take the second derivative of the metric. The result is the Riemann curvature tensor. The Riemann curvature tensor is by definition...

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \tag{A.20}$$

This is a tensor of rank four, hence it has four indices's. To represent this tensor with a matrix one would need a four dimensional hyper-cubic array of elements. This is very unwieldy to work with. In practice a simplification of this tensor is used. Using the metric to contract two indices's the Riemann curvature can be put into a simpler form. This simpler form is known as the Ricci curvature tensor.

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} = g^{\lambda\delta} R_{\lambda\mu\delta\nu} \tag{A.21}$$

This curvature tensor is the one which is used in the Einstein field equations. It is a tensor of rank two and can therefore be represented by a four by four square matrix and handled in the familiar way. Furthermore at this dimension this tensor can fit into the Einstein field equation which as we shall soon see requires subtraction of one tensor from another. The other tensors being of rank two means contraction of the Riemann tensor into a tensor of rank two, the Ricci tensor was the most sensible step.

The Ricci tensor can be contracted once more to arrive at the Ricci curvature scalar as follows... using whichever metric tensor is appropriate.

$$R_{\mu\nu} g^{\mu\nu} = R \tag{A.22}$$

That looks easy... if the metric tensor is already known. However in General Relativity the classic problem is to find the metric tensor given a particular distribution of mass energy. The way to find out which metric tensor to use is to set up and solve the Einstein field equations.

A.6.4 Einstein's Field Equations of Gravity.

The Einstein field equations are the result of about a decade of intense calculations and trial and error by Albert Einstein. The are the result of his initial problem which was finding a way to incorporate gravity into relativity. The conclusion he arrived at was as we now know to generalize relativity to account for reference frames in relative states of acceleration. Thus generalizing relativity into the theory presented in this paper. He came to the conclusion that gravity was the result of a curving of spacetime. The left side of the equation he arrived at is as follows.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{A.23}$$

The Einstein tensor $G_{\mu\nu}$ is equal to the difference between the Ricci curvature minus one half of the Ricci scalar curvature times the Metric tensor.

The right hand side of the Einstein equations consist mainly of the stress energy tensor of the system. $T_{\mu\nu}$ This will be different from system to system. The General formula for it due to David Hilbert is.

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{\text{matter}}\sqrt{-g})}{\delta g_{\mu\nu}} = 2\frac{\delta \mathcal{L}_{\text{matter}}}{\delta g_{\mu\nu}} + g^{\mu\nu}\mathcal{L}_{\text{matter}} \qquad \text{(A.24)}$$

Where $\mathcal{L}_{\text{matter}}$ is the Lagrangian excluding gravitational terms of any kind. To compute the Hilbert stress energy one needs to compute the Lagrangian for a system while *ignoring* gravity. In classical physics that would be all the kinetic energy and electromagnetic energies but not gravity. It cannot be stressed enough that $\mathcal{L}_{\text{matter}}$ does not include any form of gravity. To include any form of gravity in that Lagrangian would be to assume the very thing we are trying to figure out.

With all of the tensors that are part of the Einstein field equations defined and explained the equations themselves can be written.

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu} \qquad \text{(A.25)}$$

Where Λ is the cosmological constant. This is the most common and canonical form of the Einstein field equations. These equations can also be re written with the stress energy tensor taking a more prominent role. In the following form solving for the metric tensor is far more straight forward if the stress energy tensor is a given.

$$R_{\mu\nu} - g_{\mu\nu}\Lambda = 8\pi G(T_{\mu\nu} - \frac{1}{2}T\,g_{\mu\nu})\,. \qquad \text{(A.26)}$$

These two forms of the Einstein field equations complement each other. These allow one to choose a configuration of stress-energy and solve for the metric it would generate about itself. Alternatively one can begin with a given metric, and solve for the stress-energy that would generate that metric. For the last many years cosmologist have taken the observed universe's uniformity, which will be discussed at length latter, and used a metric that represents that uniform universe, to solve for the needed stress energy. It is from such a calculation that the amount of dark energy has been divined[6].

It is also worth noting that the dynamics of General Relativity can be written in terms of an invariant action integral. This is known as the Einstein-Hilbert action.

$$S_H = \int \sqrt{-g} R d^n x \qquad \text{(A.27)}$$

This was arrived at by first realizing that a Lagrangian in a curved spacetime would have two factors. One would depend on the metric and would be of the form $\sqrt{-g}$. The other would be a scalar.

The only scalar in the theory of General Relativity that contains second derivatives of the metric tensor is the Ricci scalar. David Hilbert deduced this form of the action for General Relativity as would Einstein.

Discussion of the Einstein Field Equations.

Notice that the way in which these laws of physics are written is 100% diffeomorphism covariant. They make no reference to any particular manifold, or coordinate system what so ever. Hence they would be valid at all places and times in any frame of reference in any state of motion. This background independence of the laws of physics is, for a theoretical and mathematical physicist, one of the great lessons of General Relativity. Well constructed theories will be background independent or in other words diffeomorphism covariant.

The other, and in Sean Caroll's opinion more profound lesson of General Relativity is that gravitation is merely a consequence of the curvature of the spacetime metric. Gravity is in that sense not a real “force" there is no pushing or pulling or particle to transfer momentum. Everything merely follows along a geodesic just as if it were in inertial motion along a straight line in a flat spacetime.

Given that the metrics are so important in General Relativity what are some solutions to Einstein's equations? There are a infinite number of possible solutions. Most of which are found by computational means and are not exact. There are however a few exact solutions to Einstein's equations. Two of these will be discussed in detail.

A.7 Metrics, or Solutions to Einstein's Equations.

Terminologically a solution to the Einstein field equations is referred to as a metric, because that's what one is solving for. There are several such solutions to the Einstein field equations which are exact and have no approximations. There are also several techniques which are not exact, grid based computational methods for example. For the purposes of this thesis only two metrics will be useful or important.

Without proof or derivation here are the two most cosmologically useful solutions to Einstein's Field Equations. The first being the Schwarzschild metric, the second being the Friedman-Lemaître-Robertson-Walker metric. The Schwarzschild metric is historically significant because it was the first exact solution to Einstein's Field Equations due to Karl Schwarzschild[5]. More importantly the Schwarzschild metric gives us the basic physics of the simplest possible black hole.

The Friedman-Lemaître-Robertson-Walker (FLRW) metric gives us a solution for a expanding universe, which was once concentrated into a tiny point, or the big bang theory[1].

The Schwarzschild metric due to Karl Schwarzchild was derived from considering Einstein's equations as they existed when Schwarzschild obtained them in a spacetime near a spherically symmetric non rotating uncharged mass. This is a very artificial situation since any body of any appreciable mass that has been observed so far has been seen to rotate about some axis or the other. The assumption is further made that the metric will exhibit azimuthal symmetry. These assumptions simplified the problem and allowed Schwarzschild to derive the first exact solution to Einstein's field equations as they existed in 1916[5].

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2GM}{r}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \tag{A.28}$$

Alternatively these solutions will be presented in terms of their associated line element. This formula will give the length of a geodesic in a curved space, which has Schwarzschild geometry.

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2 \tag{A.29}$$

The Schwarzschild metric is the simplest metric which will give a black hole. The quantity 2GM is known as the Schwarzschild Radius. For any given mass M there will be a nonzero radius to which if it were compressed it would become a black hole. There is no classical limit to the size M has to be for this collapse to be possible. In nature as far as we know it takes a supernova compressing the core of a star to create black holes.

This is also the metric which was used to test General Relativity early on. From it a effective potential which can model the orbit of Mercury can be obtained. From this metric the amount that a mass such as the sun would bend a light ray can be found. In all but the most exotic environments close to charged spinning black holes this metric will work as a good approximation.

A.8 General Relativity in non-coordinate bases.

This formalism is known as "tetrad formalism". In all of the above formalism we have assumed that there exist a natural basis for the space we are working in. In this formalism no such assumption is made, and the result is actually a simplification. In this formalism a sort of "square root or factorization of the metric is taken" in terms of what are known as differential forms. What is known as the spin connection instead of the Christoffel connection is used in this formalism. This formalism has even been used, after a fashion, in the theory of Loop Quantum Gravity, which extends gravity into a quantum theory. This may sound more complex than using straight calculus. This author has in practice found this formalism to be far more useful for solving actual problems. For a in depth review of this technique which includes exercises see Appendix J of [6].

Before delving into the abstract algebra some basic terms need to be defined. New operations need to be defined in order for any of this to make sense. First there is the antisymmetric or wedge product$\wedge$. For this brief review the wedge product of two differential forms is all that is needed. [6]

$$(A \wedge B)_{\mu\nu} = 2A_{[\mu} B_{\nu]} = A_\mu B_\nu - A_\nu B_\mu \tag{A.30}$$

Next there is the derivative operator as shown above called a exterior derivative. If ω is a P form (where P denotes the tensor order of the form. i.e. a vector is a one form, a tensor is a two form etc) and η is a q form[6].

$$d\left(\omega \wedge \eta\right) = \left(d\omega\right) \wedge \eta + \left(-1\right)^p \omega \wedge \left(d\eta\right) \tag{A.31}$$

Two interesting and useful results of differential forms would be the gradient $d(\phi)_\mu = \partial_\mu \phi$. The next is the fact that $d\left(dA\right) = 0$. These two results are very useful when solving problems in General Relativity using differential forms. Equations that had second derivatives now only have first exterior derivatives. Equations that used to be calculus are now just algebra, as promised. Now how to write a metric tensor in terms of these forms? In terms of tetrads and differential forms the metric tensor is.[6]

$$g_{\mu\nu} = e^\mu_a e^\nu_b \eta^{ab} \tag{A.32}$$

The quantities e^μ_a represent a $n \times n$ invertible matrix. With their inverses defined by the equations. $e^\mu_a e^a_\nu = \delta^\mu_\nu \; e^a_\mu e^\mu_b = \delta^a_b$ [6]

η^{ab} is the usual or canonical form of the spatial Minkowski metric which can be two three or four dimensional. This formalism so far is not specific to the number of dimensions.

The next key equation to know is the equation for the spin connection[6].

$$\omega_b^a \wedge e^b = -de^a \tag{A.33}$$

There are three new symbols that need to be considered. ω_b^a is called the spin connection. This replaces the normal Christoffel connection. Both of these are related by the following equation.

$$\omega_{\mu b}^a = e_\nu^a e_b^\lambda \Gamma_{\mu\lambda}^\nu - e_b^\lambda \partial_\mu e_\lambda^a \tag{A.34}$$

The utility of this technique becomes apparent when one considers the form that quantities such as the Ricci tensor take in this formalism. It is much simpler in terms of the operations that have to be carried out.[6]

$$R_b^a = d\omega_b^a + \omega_c^a \wedge \omega_b^c \tag{A.35}$$

In this formalism the Einstein field equation can be written as follows.

$$G_b^a = R_b^a - \frac{1}{2} g_b^a R + g_b^a \Lambda = 8\pi G T_b^a \tag{A.36}$$

This does not look simpler on the face of it. However looking at the underlying math. One can do algebra in terms of the antisymmetric product, and calculus in terms of the exterior derivative. Effectively reducing the order of the differential equations by one. Or one can solve second order hyperbolic-elliptic partial differential equations in the traditional form. The choice of which technique is simpler depends on the problem. In most cases the formulation just shown is the way to go.

Furthermore this formulation of General Relativity will become useful in the discussion of Quantum Gravity and Quantum cosmology. In particular the theory of Loop Quantum Gravity which will be discussed at length latter in this thesis uses some of this formalism. The first step in the formulation of Loop Quantum Gravity is to reformulate classical General Relativity using what are referred to as “new variables" or Astekar variables, which are rooted in the approach just described.

A.9 Conformal Transformations and Conformal Diagrams.

Conformal transformations and the related Conformal diagrams can be used to analyze and illustrate problems in cosmology in a simple and beautiful way. For that reason a discussion of these topics is warranted. In a nutshell a conformal transformation is a scaling of the metric which leaves null geodesics invariant.

A transformation such as ...

$$\tilde{g}_{\mu\nu} = \omega^2(x) g_{\mu\nu} \tag{A.37}$$

... is a conformal transformation if it satisfies.

$$\tilde{g}_{\mu\nu} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = \omega^2(x) g_{\mu\nu} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0 \tag{A.38}$$

A conformal transformation multiplies the metric by a spacetime dependent function and leaves the null geodesics invariant.

Related to this idea is the construction of conformal diagrams. A conformal diagram is a ordinary spacetime diagram in which a curved spacetime has been transformed in such a way that radial light cones are portrayed at 45 degree angels. The advantage to such a construction is that the resulting diagram is visualization. Many problems that would require complex mathematics to address can be solved almost by inspection once a conformal diagram has been constructed. For example for Schwarzschild spacetime the conformal diagram looks like. [6]

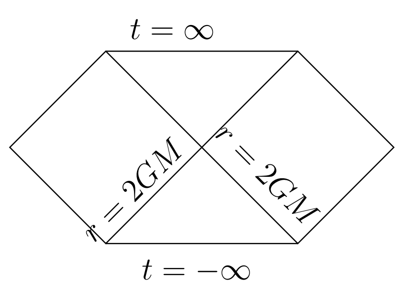


Figure A.1: This is the conformal diagram for the Schwarzchild solution (or metric) to Einstein's field equations. This geometry is essentially the one in which we live.

Figure A.1 is the conformal diagram for the Schwarzchild solution (or metric) to Einstein's field equations. The 45 degree lines are null geodesics, which take on the form of a "light cone". The lines labeled $r = 2GM$ are the event Horizon's of the black hole.Using a diagram like this it is a easy matter to see how a particle will behave in this spacetime geometry given it's position. One can see that there are path's from time minus infinity, to time plus infinity which do not encounter the singularity. The conformal diagram makes it possible to realize this without having to solve for a number of path's using the Schwarzschild metric.

For more details on this please see [6] and [1].

Applying this technique to FLRW spacetime requires one more key concept, conformal time. This is needed due to the scale factor in the FLRW metric and it's time evolution. Conformal time τ is used figures 2.2 and 2.3. It is defined by the following equation.

$$\tau = \int \frac{dt}{a\left(t\right)} \tag{A.39}$$

$a\left(t\right)$ is the scale factor which appears in the Friedman–Robertson–Walker-Lemaître (FRWL).

Another name for the conformal time is the co-moving particle horizon. For matter and radiation dominated universes this works out to

$$\tau \equiv \int_0^t \frac{dt'}{a(t')} = \int_0^a d\ln a \left(\frac{1}{aH}\right) \propto \left\{ \begin{array}{c} a \\ a^{1/2} \end{array} \right. \tag{A.40}$$

[1]

Using conformal time and constructing a conformal diagram for the FLRW metric the problems of cosmology can be approached in a rigorous yet intuitive way without resorting to solving nonlinear differential equations for the geodesics.

Conformal transformations and conformal diagrams are useful in visualization of the spacetime geometries encountered in General Relativity. Questions which depend on the causal structure of the spacetime, such as weather or not a particle at a given point could be effected by an event at another point, can be easily answered. Just such an issue will prove critical to the examination of certain issues with the standard big bang theory and the FLRW metric.

Appendix B

Basics of quantum field theory.

B.1 Lagrangians in Quantum Field Theory.

One of the most important things to note is that in QFT x and t are both just parameters not fields or operators. In a Lorentz invariant mechanics x and t are both part of the same four vector. They either had to both be operators, or both parameters. As it happens they are parameters of the quantum fields.

There are basically two ways to approach quantum field theory. Start from quantum mechanics and make it Lorentz invariant. The other option is to start with a Lorentz invariant theory then quantize it. The most common approach is to start from a Lorentz invariant field theory and make it quantum. Take for example the Lagrangian of Lorentz invariant electrodynamics.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) \tag{B.1}$$

The field in this theory isn't really the tensor $F^{\mu\nu}$, it's the potential A_μ. The operator is the partial derivative ∂_μ. As it will turn out, this is already a quantum field theory.ics. This gives the field of a freely propagating photon.

Spin also plays a very interesting part in Quantum Field Theory. Spin one fields are always represented by vectors in QFT. They are sometimes referred to as "vector bosons". In keeping with this pattern spin two fields have been most naturally represented using tensors. Accordingly the are referred to often as tensor bosons. The theorized graviton would be such a particle. Then there are spin zero fields which would be represented as scalar quantities. The Higgs particle would be such a particle. Suppose we wanted to write a Lagrangian, which was Lorentz invariant for a freely propagating scalar field ϕ. Why not propose a Lagrangian as simple as...

$$\mathcal{L} = \frac{\hbar^2}{2m}\phi \tag{B.2}$$

What's wrong with this Lagrangian is that it is utterly trivial. This can be seen by computing the stress energy tensor, which involves taking derivatives with respect to $(\partial_\mu \phi)$. The stress energy tensor of this Lagrangian would go to zero everywhere.

How about a slightly more complicated Lagrangian? The next most complicated Lorentz invariant Lagrangian would be.

$$\mathcal{L} = \frac{\hbar^2}{2m}\left(\partial_\mu \phi\right)^2 \tag{B.3}$$

This Lagrangian represents only the kinetic term for a scalar field. This field would propagate freely and not interact with anything. The next simplest non-trivial Lagrangian could almost be guessed from knowing that it is the Lagrangian for a massive scalar field. This is the Klein-Gordon Field.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}\phi^2 \tag{B.4}$$

This is the Lagrangian for the Klien-Gordon field. In practice the field is usually modeled as being a complex scalar field.

B.2 Calculations in Quantum Field Theory.

One good way to see the utility of this theory is to do some calculations. One way to look at calculations in QFT is in terms of Feynman diagrams. The skill of working with these takes time to develop. An easy way to explain it is in terms of money. In communities where people are illiterate, money is counted in terms of the faces on it. Usually only bills are considered. Tell some people \$543 and they won't understand that. However the same people will understand five Benjamins, two Jacksons and three Washingtons. Working with Feynman diagrams is kind of like that. One learns to look at a mathematical expression which is essentially a number. Then express that number in terms of these graphical diagrams. One also learns how to look at the diagrams and encode them into mathematics.

Consider a simple example. A scalar field with a ϕ^4 interaction term.

$$\mathcal{L} = (\partial_\mu \phi)^2 - m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{B.5}$$

This is a Klien Gordon field with a self interaction term. One can begin to make meaningful calculations by simply remembering the following correspondences.

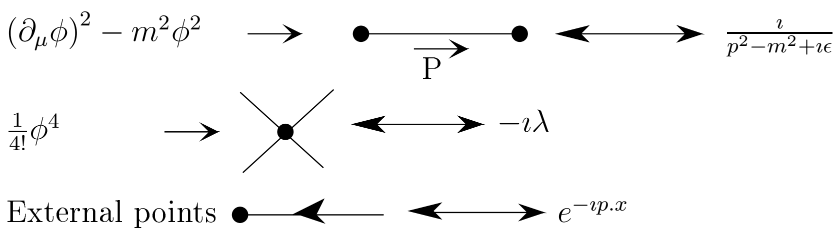


Figure B.1: Relating Lagrangians to Feynman diagrams.

In this way just by looking at it, a interaction in field theory, can be decomposed into diagrams. Then those diagrams can be reduced back into terms which appear in the Feynman diagram expansion. Likewise one can start from a term expand it in terms of Feynman diagrams and work our way to finding the interaction cross section of this theory.

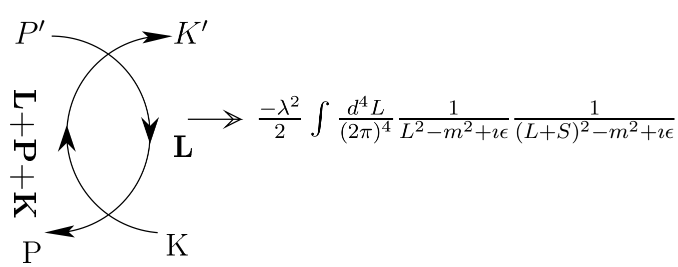


Figure B.2: Translating Feynman Diagrams to Mathematics. A simple diagram can be mathematically complex.

Where in figure B.2 shows a simple Feynman diagram with one loop in the momentum. Integration over this momentum will eventually give the probability amplitude for the interaction, which can be squared to give the cross-section of the interaction. As you can see above the mathematics can become very convoluted from a simple looking diagram. This is a large part of why Feynman diagrams are used.

What are these "groups" "Lie groups" and Lie algebra's just written of? To start let us look at the definition of a group for the purpose of mathematics and build from there.

Definition: A Group ($\boldsymbol{G}$) is a set G along with a binary operation $\circ$ which has the following properties. Denote a group as $\boldsymbol{G} = \{G, \circ\}$.

- Let $g_1, g_2 \in \boldsymbol{G}$ then $g_1 \circ g_2 \in \boldsymbol{G}$ $\forall$ $g_1, g_2 \in \boldsymbol{G}$.
- $\forall$ $g_1, g_2, g_3 \in \boldsymbol{G}$. $(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3)$
- $\exists$ some $I \in G$ such that $I \circ g_i = g_i \in \boldsymbol{G}$
- For each and every element g_i of $\{G, \circ\}$ there exist another element g_j such that $g_i \circ g_j = I \in \boldsymbol{G}$

Example: Consider integers under multiplication. $\{\mathbb{Z}, \times\}$

Clearly any integer times any integer is an integer. Multiplication is obviously associative. The identity element is obviously going to be 1. But how about the inverse element. While those exist they are not integers. Therefore $\{\mathbb{Z}, \times\}$ is not a group

Instead consider $\{\mathbb{Z}, +\}$. For this proposed group it is obvious that any two integers added is an integer, addition is associative, the identity element would be zero. For the last requirement each and every integer has an additive inverse which is also an integer, the negative integers. Therefore $\{\mathbb{Z}, +\}$ is a group.

Now how about Lie groups? What makes a Lie group different is continuity or countability. *A Lie group is a group which is built from an uncountable and continuous set. Along with a binary operation that is smooth and invertible.* In other words the set that the group is also a manifold. A manifold as, defined in chapter two, is a space which is locally similar enough to Euclidian space. Consider this simple example.

Example: Consider the real number line as a set along with addition. So the proposed Lie group is $\left\{\mathbb{R}^1, +\right\}$.

For the same reasons that the integers are a group this is a group.

Now to define a Lie algebra?

Definition: A Lie Algebra $\boldsymbol{L}$ defined on a manifold which is in turn defined over a field of scalars(often the same as a particular Lie group though it does not have to be, in QFT the scalars are always the complex numbers.) along with an operation traditionally denoted with a bracket [,] , known as a Lie bracket with the following properties... let $v, w, x \in \boldsymbol{L}$,

- $[v, v] = 0$
- $[v, w] = -[w, v]$
- $[v, [w, x]] + [w, [x, v]] + [x, [v, w]] = 0$
- Let a, b be scalars $[av+bw, x] = [av, x]+[bw, x]$ likewise $[v, aw+bx] = [v, aw] + [v, bx]$

The Lie Algebra associated with the group is really what we end up working with most of the time. Theoretical physicist often refer to them just by referring to a Lie group without specifying a group operation. This is done because in practice *the Lie group we deal with are often represented by matrices* for whom the group operation is always matrix multiplication.

Example: Hermitian Operators in Quantum mechanics under the commutator are familiar example of a Lie Algebra.

When one is trying to find out the full set of operators for a physical system they are trying to figure out it's Lie algebra.

Appendix C

A Lagrangian formulation of the Lambda CDM model with predictions relating to particle astrophysics.

The Lambda CDM model or "concordance model" is the standard model of modern cosmology. This model contains a number of separate theories with different mathematical formulations. The subject of this paper is a proposed Lagrangian which would provide a unified mathematical framework for the concordance model of cosmology. This unification is achieved by a combination of the f(R) approach, with the standard LCDM approach. It is postulated that Dark matter-energy fields depend on the Ricci curvature R, and dark energy fields weaken as the Ricci scalar (R) increases or strengthen as R decreases. The utility of this is a great simplification compared to the currently accepted formulation. One Lagrangian plus one constraint can model the same physics as the three Lagrangian's found in the standard formulations. The unexpected degree of difficulties in observing the fermion like WIMPS of dark matter in Earth based observatories are also explained

C.1 Introduction

The ΛCDM model or "concordance model" is the standard model of modern cosmology. This model contains a number of separate theories with different mathematical formulations. $f(R)$ gravity is an actively researched alternative in which gravity is modeled with functions of the Ricci curvature R in the action. The f(R) program, and the inflation with dark matter plus dark energy program both have desirable traits. Suppose they were both combined, by parameterizing the scalar and vector fields of inflation using the Ricci curvature. This unification would in effect make the scalar and vector inflationary models into f(R) models. What would be the consequences of such a unification? Can a unified model explain the negative results of searches for dark matter particles on earth[8, 67], or the halos of dark matter around galaxy's[68], or the apparent lack of dark matter within 13,000 light years of the sun [69]?

The subject of this paper is a proposed Lagrangian which would provide a unified mathematical framework for the concordance model of cosmology. In the process new insight will be gained into the nature of dark matter and dark energy which the separate formulations do not provide. The motivation for writing this paper is to provide a unified mathematical basis for Lambda CDM.

There are certain mysteries to the standard model of cosmology. It contains vast amounts of matter and energy of a mysterious type described as "dark". Dark matter which we cannot detect in spite of massive efforts such as the cryogenic dark matter search II (CDMS II) and XENON100[8, 67]. Energy which we can only detect by it's effect on the acceleration of the expansion of the universe. Energy which is then modeled with a simple constant Λ. This simple model makes very good predictions and matches observations.

There has to be a mathematically more elegant, informative, and dynamic formulation than the current collection of no less than three very different parts (depending on how one counts). The following outlines an attempt at a unified and ultimately simpler model.

C.2 The Lagrangian

We have not observed any dark matter particles on Earth to date. The best results available are signals indistinguishable from noise[8, 67]. It has also been observed that dark matter halo's form at a characteristic distance from galaxies[68]. One way to explain these observations would be to have dark matter decay as the Ricci curvature increases. Based on those observations I postulate the following:

Dark matter-energy fields depend on the Ricci curvature R, dark energy fields weaken as (R) increases or strengthen as R decreases.

The fields precise behavior will depend on which metric and hence which R is in effect. In the case of a galaxy the Ricci curvature corresponding to Schwarzchild's metric would be used, in the case of the universe the Friedman-Lemaitre-Robertson-Walker R would be used.

To realize the postulate mathematically first write the fields with R as a parameter.

$$A^{\mu} = A^{\mu}(R), \phi = \phi(R) \tag{C.1}$$

Upon review of the published literature one finds Lagrangian's for inflation, dark matter, dark energy, etc[15, 17, 16, 70, 1, 11, 71]. The standard formulation of Lambda CDM would consist of Einsteins field equation, a Lagrangian for inflation, another one for dark matter, and another one for dark energy. These all model the universe very well. So, it makes sense to use these theories as a starting point. To realize this postulate mathematically let us write the Lagrangian for a scalar and vector field, parameterized with and dependent upon Ricci curvature R, in curved space time. Each field has a mass which is at least an effective mass that has no assumed dependence on any dynamical variables. The resulting action is....

$$\begin{aligned} s = \int \sqrt{-g}\,\big(-\tfrac{R}{16\pi} - k(\phi)\nabla^{\mu}\phi\nabla_{\mu}\phi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left(D_{\mu}\gamma^{\mu} - m_{\psi}\right)\psi \\ -\tfrac{1}{2}\left(m_{\phi}^{2}\phi^{2} + m_{A}^{2}A^{\mu}A_{\mu}\right) - \tfrac{R}{6}\left(\phi^{2} + A^{\mu}A_{\mu}\right) - \beta\bar{\psi}\gamma^{\mu}\psi A_{\mu}\Big)\, d^{4}x \end{aligned} \tag{C.2}$$

Using functions of the Ricci curvature has been done before in a program known as f of R gravity. Here the functions f(R) are identified with the scalar and vector fields of inflation. It is assumed that said fields have at least an effective mass. This mass is not assumed to depend on any variables at the outset, however it will be shown that a value for this effective mass is derivable and can depend on both R and Λ. Mass dependence on Ricci curvature is a feature of many published models of f(R) gravity[72, 73].

These fields are similar, yet not identical, to those found in theories of inflation in which they drive the rapid expansion [15, 17, 16, 70, 1, 11, 71]. To see how inflation arises in this theory the equations of motion need to be derived and solved.

C.3 Equations of Motion.

Following all the elementary steps of classical field theory the Euler-Lagrange equations for this theory can be derived. One of those equations is for R itself. That is none other than the Einstein field equation. Then there are two more equations one for the scalar and one for the vector fields. One more constraint is desirable. The Stress energy tensor of this field must be proportional to the cosmological constant. This ensures agreement with known observations. The result is a set of three equations, derived from the above action.

$$\left\{ \begin{array}{c} R^{\mu\nu} - Rg^{\mu\nu} = \frac{8\pi G}{c^2} T^{\mu\nu} \\ \nabla_\alpha F^{\alpha\mu} - \left(\frac{m_A^2}{2} + \frac{R}{6}\right) A^\mu = 0 \\ \nabla_\mu \nabla^\mu \phi - \left(\frac{m_\phi^2}{2} + \frac{R}{6}\right) \phi = 0 \\ \left(\nabla_\mu \gamma^\mu - m_\psi\right) \psi = 0 \end{array} \right\} \tag{C.3}$$

In which the stress energy tensor has the following form.

$$\begin{array}{c} T^{\mu\nu} = -2k(\phi)\nabla^\mu\phi\nabla^\nu\phi - F^{\mu\nu} g_{\lambda\delta} F^{\lambda\delta} - \left(m_A^2 + \frac{R}{3}\right) A^\mu A^\nu + \frac{i}{2}\bar{\psi}\gamma^\mu\nabla^\nu\psi \\ -g^{\mu\nu}\left(k(\phi)\nabla_\mu\phi\nabla^\mu\phi + F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\left(m_\phi^2\phi^2 + m_A A^\mu A_\mu\right)\right. \\ \left. -\frac{R}{6}\left(\phi^2 + A^\mu A_\mu\right) + \bar{\psi}\left(D_\mu\gamma^\mu - m_\psi\right)\psi\right) \end{array} \tag{C.4}$$

The stress energy needs to be at least proportional to the cosmological constant times the metric. This results in the following equation of constraint, which is not derivable from the Lagrangian. In the following $\lambda$is simply a constant of proportionality. This constraint is introduced in the same spirit as the cosmological constant. Λ is an important part of most any viable cosmological model. This equation of constraint ensures that the proposed model can match observations which have already been made.

$$T^{\mu\nu} = \lambda g^{\mu\nu}\Lambda \tag{C.5}$$

C.3.1 Solutions

The next task is to solve these equations for the scalar and vector fields. First the scalar fields solution.

$$\phi(R) = \phi_0 Exp\left(\int_1^R \frac{\left(\frac{m_\phi^2}{2} + \frac{R'}{6}\right)}{\Box R'} dR'\right) \tag{C.6}$$

Next we will solve for the vector field. The A^0 component must be zero in order to satisfy the equation of motion. The derivatives which make up F^{00} work out that way just as one would expect for an electromagnetism like field. In the process of solving for A^0 the effective mass of the A field can be calculated.

$$\frac{m_A^2}{2} + \frac{R}{6} = 0 \to m_A = \sqrt{\frac{-R}{3}} \tag{C.7}$$

On the cosmic scale space time is very nearly flat. In fact the curvature of space time observed to date it slightly negative. Therefore this effective mass would be small but at a characteristic distance from concentrations of luminous matter such as galaxies. This is in accord with the observations reported in[68]. In a positively curved space time the mass of this field is imaginary. This would appear to be a problem, but for the fact that so far no dark matter particles have been detected on Earth in spite of very concerted efforts [8, 67]. This theory predicts that no WIMP corresponding to the type of vector field described here will ever be detected near a concentration of luminous matter such as the Earth.

For the space like components the solution is almost identical to that for the scalar field.

$$A^\mu = \left(0,\ A_0^i Exp\left(\int_1^R \frac{\left(\frac{m_A^2}{2} + \frac{R'}{6}\right)}{\Box R'} dR'\right)\right) \tag{C.8}$$

Where $i \in \{1, 2, 3\}$.

The effective masses of these fields are fixed theoretically by the constraint that the stress energy tensor $T^{\mu\nu}$ needs to be proportional to the cosmological constant. It is possible to determine the effective mass m_ϕ from that constraint. To find an expression for this mass note that the T^{00} component of the stress energy tensor will be of a simple form. Terms which depend on the vector field drop out as it's zero in that component. Terms which depend on the velocity $\nabla^0\phi$ can be set to zero to ensure the resulting effective mass acts as a rest mass of the particle. The resulting equation is

$$T^{00} = g^{00}\left(\frac{1}{2}m_\phi^2 + \frac{R}{6}\right)\phi^2 = \lambda g^{00}\Lambda \tag{C.9}$$

Which simplifies to...

$$m_\phi = \sqrt{\frac{6\lambda\Lambda - R\phi}{3\phi}} \tag{C.10}$$

The effective mass of this scalar field cannot be zero unless the following equation holds true.

$$R\phi(R) = 6\lambda\Lambda \tag{C.11}$$

This equation determines a characteristic radius at which a dark matter halo would be observed from a galaxy. This is a point at which the Schwarzchild curvature due to the galaxy gives way to the large scale FLRW space time. This is in accordance with the observations in [68]. Within this radius the space time curvature would be large enough to make the mass of the scalar field imaginary, meaning no particles. Only outside of this radius can particles associated with this field exist.

This effective mass was not a priori assumed to depend on explicitly on the Ricci curvature R. However in the f of R gravity regime implicit dependence of effective mass m on R is a standard feature found in many publications [72, 73]. The bare rest masses of these particles would be found by setting R equal to zero. When R equals zero m_A is zero. The vector field is then fundamentally massless much like an EM field. The scalar fields effective massm_ϕ wold be not be zero at that point. The scalar field has a bare rest mass the vector field only has an effective mass. The fields would still contribute stress energy to the stress-energy tensor regardless of their effective mass.

C.3.2 Probability of fermion fermion annihilation to curvature.

In terrestrial experiments which search for dark matter we have assumed that the dark matter will be fermionic. The way that fermion like dark matter particles behave in this theory, in terms of their effective masses, will be the same as for the above particles. However there is an even more interesting interaction in this theory. Let us consider the amplitude and cross section for the annihilation of four of these fermions into R.

$$< R|\bar{\psi}\psi\bar{\psi}\psi >=< R|A^\mu A_\mu >< A^\mu A_\mu|\bar{\psi}\psi\bar{\psi}\psi > \tag{C.12}$$

After some computation the answer works out to the following.

$$< R|\bar{\psi}\psi\bar{\psi}\psi >= \frac{(A_0^\mu A_{0\mu})\left(\bar{\psi}_{dir}\psi_{dir}\right)e^{G[R]}}{s'[R]e^{S[R]}}\left(\frac{R}{G'[R]} - 1\right) \tag{C.13}$$

In equation C.13 the term $G[R]$ is a functional of the Ricci curvature scalar R which results from multiplying these fields together,$S[R]$ is the action as a functional of the Ricci curvature scalar R . The terms A_0^μ is constant., and $\bar{\psi}_{dir}$ is the standard solution for the Dirac fields. G and S will oscillate about. The interesting part of the squared probability will look like.

$$\left| < R|\bar{\psi}\psi\bar{\psi}\psi > \right|^2 \approx (R-1)^2 = R^2 - 2R + 1 \tag{C.14}$$

Equation C.14 shows us that the cross section for these particles simply annihilating increases in area as the curvature of space time increases, and decreases as the curvature of space time decreases. Therefore as gravity becomes stronger, the particles lifetime becomes shorter. This behavior would explain why we have had so much trouble observing dark matter fermions in experiments on earth, while their astronomical existence is beyond question.

C.3.3 Inflation

Inflation is in this model. To see it consider the effective masses shown in equations seven and ten. The physics of standard big bang theory is modeled using the FLRW metric. In this metric at time equals zero the curvature of space time is infinite. At that point the effective masses of these fields would be imaginary infinity. At the same time the strength of the fields would be zero. When the universe begins to expand the curvature begins to decrease, this in turn causes the mass of the field to roll towards zero. As the mass rolls it drives the inflationary expansion of the universe. All the while the dark mass of the particles is converted into dark energy of the associated fields.

Thus the story of the universe is the story of two massive fields transforming one form of energy into another, along with some other stuff we call ordinary matter.

C.4 Conclusions

The proposed Lagrangian contains all the physics needed to represent the Lambda CDM model. There is a source of dark matter, dark energy, and inflation. The behavior of the fields is in agreement with our overall observations . This Lagrangian also provides a minimal explanation for why dark matter has been so hard to observe in experiments such as CDMS II and XENON100. The dark matter simply decays into dark energy when the curvature R is too high. Thus there are not “particles” to detect in a region of high space time curvature, like on Earth. This would provide an explanation for why it would be harder than expected to detect these particles in a ground based experiment.

This model also explains observations of a dark matter halo around galaxies at a characteristic distance in a simple and natural way. The dark matter's effective mass is imaginary when the curvature is positive. Which means it physically and classically cannot exist.

The dark matter mass in this theory is simply the effective mass of the fields and their associated bosonic particles. There may well be other fermionic and super symmetric types of dark matter. Certainly numerous particles which will be discovered at accelerator laboratories in the future which may or may not be dark matter candidates exist. I have no hypothesis about such dark matter, or how the hypothesized particles could be produced via accelerator based experiments in this model at this time. Their is a disputed observation by Moni Bidin et. al. which may support this theory[74]. They found indications that the density of dark matter relatively near earth may be less than the standard models predict. Bovy and Tremaine's analysis found more dark matter consistent with the standard estimates[69]. The problem with those analyses is that they contain the implicit assumption that the density of dark matter will be uniform and spherically symmetric.

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