Sargent plot, weak interactions, and the geometry of the flat space-time

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ABSTRACT

Sargent plot, summarizes part of one of the pioneers experiments on nuclear beta decay. Here it is proposed to be used as a motivation, to teach undergraduate students some basic features of the weak interactions. This is done by considering only elementary aspects of special relativity and quantum theories.

1 – Introduction

It has been elapsed a time interval of eighty years, since B. W. Sargent [1,2] published in the prestigious periodic: "Proceedings of the Royal Society of London", a study of the beta-decay from Uranium and other radioactive nuclei. In figure 2 of his paper, Sargent makes a plot of the logarithms of the disintegration constant, λ, versus logarithms of the maximum energies, E_m, of the beta-particles (electrons or positrons). These plots could be approximately interpolated by two straight lines of different inclinations. If each disintegration curve (not the interpolations) is represented by a power law in the maximum energy, the exponent lies between 3 and 7. In this note we intend to use this remarkable discovery as a means to motivate undergraduate students in the learning of the weak interactions.

2. Statement of the problem

It is well known from the study of particle physics that the great majority of elementary particles are short-lived, exceptions being done to the proton and the electron for instance. If there is not a conservation law which forbids a given decaying mode, this process will occur. With respect to the fundamental interactions participating in the game, we have by order of intensity, the strong force associated to quarks and gluons, the electromagnetic force between electric charges and intermediated by the photons, and the weak force intermediated by the massive W and Z bosons [3,4]. If the two first forces are prohibited to work, due to some conservation law, may be the weak force plays its role. This is just what happens in the nuclear transmutation (the alchemist dream?), which was investigated by Sargent.

One of the most important properties of the weak interaction, is that it is the only force which could change a quark flavor in the process. For instance, in the nuclear beta decay there is a change of the atomic number, therefore in the reaction there is a conversion of a neutron in a proton or vice-versa. The proton is constituted by two ups, and one down quarks, and the neutron by two downs and one up quarks.

Schematically the reactions studied by Sargent can be represented as

$$N_F \rightarrow N_S + \beta + \upsilon, \qquad (1)$$

where N_F , N_S , β and υ, stand for father nucleus, son nucleus, electron or positron, and neutrino or anti-neutrino. Naturally, reaction (1) must satisfy all the conservation laws as : energy conservation, lepton-number conservation, electric charge conservation, and so on…The first successful theory of the weak interactions was introduced by Fermi [5], and in his theory the four intervenient particles interact at a point. However, according to the modern viewpoint, the weak force is intermediated by very massive bosons, the $W^{\pm}$ and the Z^0 bosons.

3. Calculating the disintegration constant

Now, if we call λ the disintegration constant, E_m the maximum energy of the beta rays, and k another constant, Sargent's law, if the exponent is taken as the arithmetic average between the highest one equal to 7(seven) and the lowest one equal to 3(three), can be written

$$\lambda = k\, E_m^{5}. \qquad (2)$$

Next we intend to propose a simple derivation of the Sargent's law given by (2). In quantum mechanics, or in quantum field theory, the Heisenberg uncertainty principle it is always at work. In particular, this happens in the process of particle decaying thinking in terms of the time-energy uncertainty relation. Let us write

$$\Delta E\, \Delta t = h, \qquad (3)$$

and taking $\Delta E = E_m$, and $1 / (\Delta t) = \upsilon$, we have

$$\upsilon = h/E_m . \tag{4}$$

Pursuing further, the maximum energy of the beta particle corresponds to the energy difference among the mass-energy of the father nucleus minus the sum of mass-energies of the son nucleus and that of the beta particle. Besides this, at certain instant, the reaction has an amount E_m of energy at its disposal and looks at the possibility of creating a virtual boson of mass-energy equal to $M_W c^2$. Meanwhile, special relativity theory, puts space and time at equal footing, and as a consequence we have the energy-momentum four-vector.

Now let us think about a four-dimensional hypercube in the momentum space, with an edge equal to $M_W c$, a characteristic momentum associated to the W-boson. Besides this, we may imagine a small hypercube which size in the momentum space is E_m/c . Therefore the fraction f of the great hypercube occupied by the small one is given by

$$f = [E_m/(M_W c^2)]^4. \tag{5}$$

The decay width Γ then reads

$$\Gamma = \upsilon\, f\, \alpha_*^2 = \alpha_*^2\ \{ E_m^5/[h\, (M_W c^2)^4]\}. \tag{6}$$

The α_* appearing in relation (6) is a coupling constant and the energy-dependent weak coupling constant α_W is related to it through the relation

$$\alpha_W = \alpha_*\ [E/(M_W c^2)]^2. \tag{7}$$

When the energy E equals to the mass-energy of the W-boson, the weak coupling constant α_W coincides with α_*, which has a magnitude comparable to the electromagnetic coupling α. Relation (7) customs to be interpreted as the weak coupling being “weak”, due to the fact that in the low-energy process, the particles have a small fraction of the W- boson mass-energy.

In order to “see” Sargent’s law in equation (6), we must interpret the disintegration constant λ as being proportional to the decay width Γ.

4. Concluding remarks

By concluding we would like to point out that, we have made before some considerations analogous to the ones of this work, in a paper dealing with the muon and (speculated) proton decays [6]. On the other hand, a deeper treatment of the weak interactions can be found in some well known books in particle physics, such those by Griffiths [7], Halzen and Martin [8], and Kane [9]. There the decay rates and scattering cross sections are calculated by using the Feynman diagrams techniques in conjunction with the Fermi's Golden Rule. Finally the difference found in the inclinations interpolated in the Sargent plot, perhaps can be explained by the fact that some beta-rays are slowed-down by the electronic shells of the atomic structure (and this will be more remarkable for the less energetic ones), while others are speeded-up.

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