How Much Degrees Of Temperature A Warp Drive Achieves When At Superluminal Speeds?? The Analysis Of Gonzalez-Diaz Applied To The Natario Warp Drive Spacetime.

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity.There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001.The warp drive seems to be very attractive because allows interstellar space travel at arbitrarily large speeds avoiding the time dilatation and mass increase paradoxes of Special Relativity. However it suffers from a very serious drawback:
In order to travel significant interstellar distances in reasonable amounts of time a ship would need to attain at least 200 times the speed of light in order to visit a star like Gliese 581 at 20 light-years with potential habitable exoplanets.
Some years ago Gonzalez-Diaz discovered that a warp drive at luminal speed develops an Horizon similar to the Schwarzschild Event Horizon that appears in the ∞ and at superluminal speeds this Horizon approaches the position of the spaceship.This means to say that as fast the ship goes by then as near to the ship the Horizon is formed.Plus as fast the ship goes by as hotter the Horizon becomes placing any ship and its astronauts in a dangerous thermal bath.
Gonzalez-Diaz applied its conclusions to the Alcubierre warp drive.
In this work we apply the analysis of Gonzalez-Diaz to the Natario warp drive and we arrive at an interesting result:When at luminal speed the Horizon is formed not in the ∞ but in the end of the Natario warped region and as fast the ship goes by superluminally this Horizon do not approaches the ship remaining inside the Natario warped region and keeping a constant temperature.This makes the Natario warp drive a better candidate for interstellar space travel when compared to its Alcubierre counterpart.

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1 Introduction

The Warp Drive as a solution of the Einstein field equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However one serious drawback that affects the warp drive is the quest of the temperature a warp drive achieves when at superluminal speeds.

In order to travel significant interstellar distances in reasonable amounts of time a ship would need to attain 200 times the speed of light in order to visit a star like Gliese 581 at 20 light-years with potential habitable exoplanets.[2]

Some years ago in 2007 Gonzalez-Diaz discovered that a warp drive at luminal speed develops an Horizon similar to the Schwarzschild Event Horizon that appears in the ∞ and at superluminal speeds this Horizon approaches the position of the spaceship.This means to say that as fast the ship goes by then as near to the ship the Horizon is formed.Plus as fast the ship goes by as hotter the Horizon becomes placing any ship and its astronauts in a dangerous thermal bath.(see eq 6 pg 2 and eq 34 pg 4 in [3]).

Gonzalez-Diaz applied its conclusions to the Alcubierre warp drive.

We apply the analysis of Gonzalez-Diaz to the Natario warp drive spacetime and we arrive at an interesting result:When at luminal speed the Horizon is formed not in the ∞ but in the end of the Natario warped region and as fast the ship goes by superluminally this Horizon do not approaches the ship remaining inside the Natario warped region and keeping a constant temperature.This makes the Natario warp drive a better candidate for interstellar space travel when compared to its Alcubierre counterpart.

In this work we use the Geometrized System of Units in which $c = G = 1$

In order to completely understand the ideas behind this work fully acquaintance with the Natario geometry is required.While many readers of warp drive works have acquaintance or are familiarized with the Alcubierre geometry,the Natario geometry is not familiar to the major part of warp drive readers and even some readers demonstrated difficulties to fully understand the mathematics behind [2].

[1] do not violates Relativity
[2] see Wikipedia.The Free Encyclopedia

In order to get familiarized with the Natario geometry we recommend a start-up with sections 1 to 4 and 6 in [6](skip section 5) followed with sections 1 and 2 in [5] and terminating with sections 1 and 2 in [4].

For those readers familiarized with the Alcubierre geometry but not familiarized with Gonzalez-Diaz ideas we would recommend a start-up with [3].

2 How Much Degrees Of Temperature A Warp Drive Achieves When At Superluminal Speeds?? The Analysis Of Gonzalez-Diaz Applied To The Natario Warp Drive Spacetime.

The warp drive spacetime according to Natario for the coordinates rs and θ is defined by the following equation:(see Appendix E in [4] for details)

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs}drs + X^{\theta}rsd\theta]dt - drs^2 - rs^2 d\theta^2 \tag{1}$$

The expressions for X^{rs} and X^{θ} are given by:(see pg 5 in [2],see also Appendix D in [4] for details)[3]

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{2}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{3}$$

$n(rs)$ is the Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $n(rs) = 0$ for small rs(inside the warp bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the Natario warped region(pg 5 in [2])

For simplicity we consider only the motion in the equatorial plane parallel to the $x - axis$.Note that this is a two-dimensional approach in the dimensions of rs and t similar to the one used by Gonzalez-Diaz in pg 2 of [3].Then $\theta = 0$ and $cos(\theta) = 1$. The Natario warp drive now becomes:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{4}$$

But since we are considering the motion in the $x - axis$ only or parallel to the $x - axis$ where $\theta = 0$ or $cos(\theta) = 1$ then we are left with:

$$X^{rs} = 2v_s n(rs) \tag{5}$$

We have the following results for the g_{00} of the metric given above:

$$g_{00} = 1 - (X^{rs})^2 \tag{6}$$

The term $g_{00} = 1 - (X^{rs})^2$ is very important:This is the g_{00} components of the Natario warp drive metric that will generate the Gonzalez-Diaz Horizon(pg 2 of [3]).

According to Gonzalez-Diaz an Horizon occurs when $g_{00} = 0$.Then we have the following expressions:

$$1 - (X^{rs})^2 = 0 \tag{7}$$

Giving :

$$(X^{rs})^2 = 1 \dashrightarrow X^{rs} = 1 \tag{8}$$

Rearranging the terms in g_{00} for the Natario warp drive we have:

$$2v_s n(rs) = 1 \dashrightarrow n(rs) = \frac{1}{2v_s} \tag{9}$$

[3]We consider here the Natario Vector $nX = vsdx$ and not $nX = -vsdx$

Note that when $v_s = 1$(luminal speed $c = 1$) we have the following result:

$$n(rs) = \frac{1}{2v_s} = \frac{1}{2} \tag{10}$$

The Horizon is formed in the intersection between the end of the Natario warped region and the regions outside the warp bubble and not in the ∞ as stated by Gonzalez-Diaz.(see eq 6 pg 2 in [3])

Note that when v_s is superluminal $v_s >> 1$ the $n(rs)$ and the Horizon remains always inside the Natario warped region leaving the warped region and approaching the region inside the bubble only when $v_s \to \infty$.

$$n(rs) = \frac{1}{2v_s} << \frac{1}{2} \tag{11}$$

$n(rs)$ is the Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $n(rs) = 0$ for small rs(inside the warp bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the Natario warped region(pg 5 in [2])

Since rs is the distance travelled by an Eulerian observer with respect to the center of the warp bubble($rs = 0$)we must find the distance between the Horizon and the center of the bubble.Then we need to find the corresponding rs for the Horizon.To do so we need to compute the Natario shape function $n(rs)$.

$f(rs)$ is the Alcubierre shape function defined as:(see eq 6 pg 4 in [1])

$$f(rs) = \frac{\tanh[@(rs + R)] - \tanh[@(rs - R)]}{2\tanh(@R)} \tag{12}$$

In the expression above R is the radius of the warp bubble and $@$ is the Alcubierre parameter related to the thickness of the warp bubble.According to Alcubierre we have 3 possible values for $f(rs)$:(see eq 7 pg 4 in [1])

- 1)- inside the warp bubble where the ship resides $f(rs) = 1$
- 2)- outside the warp bubble where Earth resides $f(rs) = 0$
- 3)- in the Alcubierre warped region (warp bubble walls) $1 > f(rs) > 0$

The Natario shape function $n(rs)$ that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble written in function of the Alcubierre shape function $f(rs)$ is given by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{13}$$

- 1)- inside the warp bubble where the ship resides $n(rs) = 0$
- 2)- outside the warp bubble where Earth resides $n(rs) = \frac{1}{2}$
- 3)- in the Natario warped region (warp bubble walls) $\frac{1}{2} > n(rs) > 0$

Considering a warp bubble with 100 meters of radius($R = 100$) and an Alcubierre parameter $@ = 5000$ from $rs = 0$ to $rs <= 99,9$ meters the value of $f(rs) = 1$ and $n(rs) = 0$ as expected for the behavior of a region inside the warp bubble. From $rs >= 100,1$ meters to $rs = \infty$ the value of $f(rs) = 0$ and the value of $n(rs) = \frac{1}{2}$ as expected for the behavior of a region outside the warp bubble.In the region between $rs > 99,9$ meters to $rs < 100,1$ meters $f(rs)$ starts to drop from 1 to 0 and $n(rs)$ starts to grow from 0 to $\frac{1}{2}$ as expected for the behavior of a region inside the Alcubierre or Natario warped regions.

When $rs = 100$ meters $f(rs) = \frac{1}{2}$ and $n(rs) = \frac{1}{4}$.This points depicts the center of both Alcubierre and Natario warped regions.The thickness Δ of this bubble is $\Delta <= 0,2$ meters.

Plotting these values in Microsoft Excel we can observe that the term $\tanh[@(rs + R) = 1$ and also the term $\tanh(@R) = 1$.Then the Alcubierre shape function can be written as:

$$f(rs) = \frac{1 - \tanh[@(rs - R)]}{2} \tag{14}$$

$$f(rs) = \frac{1}{2}(1 - \tanh[@(rs - R)]) \tag{15}$$

The Natario shape function is then given by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{16}$$

$$n(rs) = \frac{1}{2}[1 - \frac{1}{2}(1 - \tanh[@(rs - R)])] \tag{17}$$

In order to simplify we define the term $\tanh[@(rs - R)] = P$ and the Natario shape function now becomes:

$$n(rs) = \frac{1}{2}[1 - \frac{1}{2}(1 - P)] \tag{18}$$

Then we have:

$$n(rs) = \frac{1}{2}[1 - \frac{1}{2}(1 - P)] = \frac{1}{2}[1 - \frac{1}{2}(1 - P)] = \frac{1}{2} - \frac{1}{4}(1 - P) \tag{19}$$

The Horizon occurs when

$$n(rs) = \frac{1}{2v_s} \tag{20}$$

Hence we should expect for:

$$n(rs) = \frac{1}{2v_s} = \frac{1}{2} - \frac{1}{4}(1 - P) \tag{21}$$

Solving for P we have:

$$\frac{1}{2} - \frac{1}{4}(1 - P) = \frac{1}{2v_s} \tag{22}$$

$$\frac{1}{2} = \frac{1}{4}(1 - P) + \frac{1}{2v_s} \tag{23}$$

$$\frac{1}{4}(1-P) = \frac{1}{2} - \frac{1}{2v_s} \tag{24}$$

$$(1-P) = \frac{4}{2} - \frac{4}{2v_s} = 2 - \frac{2}{v_s} = 2(1 - \frac{1}{v_s}) \tag{25}$$

$$(1-P) = 2(1 - \frac{1}{v_s}) \tag{26}$$

$$1 = P + 2(1 - \frac{1}{v_s}) \tag{27}$$

$$P = 1 - 2(1 - \frac{1}{v_s}) \tag{28}$$

But we defined $\tanh[@(rs-R)] = P$ .Then we should expect for:

$$\tanh[@(rs-R)] = 1 - 2(1 - \frac{1}{v_s}) \tag{29}$$

$$@(rs-R) = arctanh(1 - 2(1 - \frac{1}{v_s})) \tag{30}$$

$$(rs-R) = \frac{1}{@} arctanh(1 - 2(1 - \frac{1}{v_s})) \tag{31}$$

$$rs = R + \frac{1}{@} arctanh(1 - 2(1 - \frac{1}{v_s})) \tag{32}$$

$$r_{horizon} = R + \frac{1}{@} arctanh(1 - 2(1 - \frac{1}{v_s})) \tag{33}$$

Above is given the expression for the distance between the center of the warp bubble($rs = 0$) and the point where the Horizon occurs $r_{horizon}$ in the Natario warp drive spacetime according to Gonzalez-Diaz.(see eq 6 pg 2 in [3]).

Plotting this equation in Microsoft Excel using the previous given values for R and @ we can see that at $vs = 1$ the Horizon occurs at $1,0000015707962 \times 10^2$ meters.At $vs = 200$ the Horizon occurs at $9,999984387730 \times 10^1$ meters.At $vs = 2000$ the Horizon occurs at $9,999984302042 \times 10^1$ meters.At $vs = 6000$ the Horizon occurs at $9,999984295371 \times 10^1$ meters.

Note that from $vs = 1$ light speed to $vs = 200$ 200 times faster than light and from $vs = 200$ to $vs = 6000$ 6000 times faster than light the Horizon approached the ship in the center of the bubble by a very small distance and remained inside the Natario warped region at a faraway safe distance not threatening the ship or the astronauts inside it.

This is very different from the result of Gonzalez-Diaz obtained for the Alcubierre warp drive that states as fast the ship goes by as closer the Horizon forms near the ship position.(see eq 6 pg 2 in [3]).

Inserting our values of $r_{horizon}$ in the Gonzalez-Diaz equation of the temperature (see eq 34 pg 4 in [3])).[4]

$$T = \frac{1}{2\pi r_{horizon}} \tag{34}$$

We can see that from 200 times faster than light to 6000 times faster than light the temperature remains constant since $r_{horizon}$ have little variations.Then in the Natario warp drive spacetime as fast the ship goes by the temperature remains approximately constant.

This is very different from the result of Gonzalez-Diaz obtained for the Alcubierre warp drive that states as fast the ship goes by as hotter the Horizon becomes threatening the ship and the astronauts inside.(see eq 34 pg 4 in [3]).

[4]we inserted the values of the $r_{horizon}$ in an ad-hoc manner in the Gonzalez-Diaz equation obtained for the Alcubierre warp drive.A detailed study would imply in the derivation of the temperature equation for the Natario warp drive.Our point is to underline the fact that the Horizon do not approaches the ship

3 Conclusion

In this work we performed the Gonzalez-Diaz analysis to the Natario warp drive and we arrived at an interesting result:When at luminal speed the Horizon is formed not in the ∞ but in the end of the Natario warped region and as fast the ship goes by superluminally this Horizon do not approaches the ship remaining inside the Natario warped region and keeping a constant temperature.This makes the Natario warp drive a better candidate for interstellar space travel when compared to its Alcubierre counterpart.

4 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[5]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[6]

5 Remarks

Some references of this work came from scientific web-servers available to consultation by the general public(eg:arXiv,HAL). We can provide the other references in PDF Acrobat Reader for those interested.

[5] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[6] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

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