

# **The spherical solution of the quantum gravity and the revised gravity field equation**

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## **ABSTRACT**

In the general relativity theory, using Einstein's revised gravity field equation (add the cosmological term), discover the spherical solution of the quantum gravity.

**PACS Number:04,04.90.+e,04.60**

**Key words:The general relativity theory,**

**The revised gravity field equation**

**The quantum gravity**

**The spherical solution**

**The cosmological term**

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## I. Introduction

This theory is that it discovers the spherical solution of the quantum gravity using the revised gravity field equation (add the cosmological term).

Think that use following the formula.

$$\alpha = \frac{hc}{GM^2} \text{ is non-Dimension number. } \alpha \text{'s Dimension is } \frac{J \cdot s \cdot m / s}{N \cdot m^2 \cdot kg^2 / kg^2} = \frac{J \cdot m}{J \cdot m} = 1$$

$h$  is the plank constant,  $c$  is the light speed,  $G$  is the gravity constant,  $M$  is the matter's mass.

The spherical solution (The Schwarzschild solution) of the general relativity is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad (1)$$

## II. Additional chapter - I

In this theory, the general relativity theory's revised field equation (add the cosmological term) is written completely.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

Eq (2) multiply  $g^{\mu\nu}$  and does contraction,

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R + \Lambda g^{\mu\nu} g_{\mu\nu} \\ = -R + 4\Lambda = -\frac{8\pi G}{c^4} T^{\lambda}_{\lambda} \end{aligned} \quad (3)$$

Therefore, Eq (2) is

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(4\Lambda + \frac{8\pi G}{c^4} T^{\lambda}_{\lambda}\right) + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \\ R_{\mu\nu} = -\frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}\right) + \Lambda g_{\mu\nu} \end{aligned} \quad (4)$$

In this time, the spherical coordinate system's vacuum solution is by  $T_{\mu\nu} = 0$

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \quad (5)$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t, r) dt^2 - \frac{1}{c^2} [B(t, r) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (6)$$

Using Eq(6)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = -\Lambda A \quad (7)$$

$$R_{rr} = \frac{A'}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = \Lambda B \quad (8)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = \Lambda r^2 \quad (9)$$

$$R_{\phi\phi} = \sin^2 \theta R_{\theta\theta} \quad (10)$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0$$

$$R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \quad (11)$$

In this time,  $' = \frac{\partial}{\partial r}$ ,  $\dot{\phantom{x}} = \frac{1}{c} \frac{\partial}{\partial t}$

By Eq(11),

$$\dot{B} = 0 \quad (12)$$

By Eq(7) and Eq(8),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br} \left( \frac{A'}{A} + \frac{B'}{B} \right) = -\frac{(AB)'}{rAB^2} = 0 \quad (13)$$

Therefore,

$$A = \frac{1}{B} \quad (14)$$

If Eq(9) is inserted by Eq(14),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left( \frac{r}{B} \right)' = \Lambda r^2 \quad (15)$$

If solve Eq(15)

$$\frac{r}{B} = r + C + \frac{1}{3} \Lambda r^3 \rightarrow \frac{1}{B} = 1 + \frac{C}{r} + \frac{1}{3} \Lambda r^2 \quad (16)$$

In this time, be able to think following the formula.

$$C = -\frac{2GM}{c^2}, \quad \Lambda = \Sigma(M) = \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp \left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right]$$

$\alpha_1 > 0, \beta_1 > 0$ ,  $\alpha_0, \alpha_1, \beta_1$  are real numbers. (17)

$$\frac{1}{B} = 1 - \frac{2GM}{rc^2} + \frac{1}{3} \Sigma(M) r^2$$

$$\Sigma(M) = \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp \left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right]$$

$\alpha_1 > 0, \beta_1 > 0$ ,  $\alpha_0, \alpha_1, \beta_1$  are real numbers.

(18)

Therefore, Eq(18) is

$$A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} + \frac{1}{3}\Sigma(M)r^2$$

$$\Sigma(M) = \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left[-\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1}\right]$$

$\alpha_1 > 0, \beta_1 > 0, \alpha_0, \alpha_1, \beta_1$  are real numbers.

(19)

To know Eq(19)'s third term, does Newton's limitation

$$\frac{d^2 x^\lambda}{d\tau^2} = -\Gamma^{\lambda}_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$$\frac{d^2 r}{dt^2} \approx \frac{1}{2} c^2 \frac{\partial(-A)}{\partial r} = -\frac{GM}{r^2} - \frac{\alpha_0}{3} \left( \frac{c^6}{G^2 M^2} \right) \exp\left[-\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1}\right] r \quad (20)$$

To know Eq(20)'s second term, use Hubble's constant  $H_0$ ,

$$V_{galaxy} = \frac{r_{galaxy}}{T_0} = H_0 r_{galaxy},$$

$$a_{galaxy} = \frac{V_{galaxy}}{T_0} = \frac{r_{galaxy}}{T_0^2} = H_0 \frac{r_{galaxy}}{T_0} = H_0^2 r_{galaxy}$$

$V_{galaxy}$  is the galaxy's velocity,  $r_{galaxy}$  is the distant of the galaxy and the other galaxy,  $a_{galaxy}$  is the galaxy's acceleration,  $T_0$  is the present universe's time.

(21)

Therefore, Eq(20)'s second term is concerned about the universe's inflation. Therefore,

$$\text{If } -\frac{GM_{galaxy}}{r_{galaxy}^2} \sim 0, 0 < \frac{1}{T_0^2} = H_0^2 = -\frac{\alpha_0}{3} \left( \frac{c^6}{G^2 M_{galaxy}^2} \right) \exp\left[-\alpha_1 \left( \frac{hc}{GM_{galaxy}^2} \right)^{\beta_1}\right], \alpha_0 < 0 \quad (22)$$

Therefore, the universe's time  $T$  is concerned about the galaxy's mass  $M_{galaxy}$ .

### III. Additional chapter-II

About the spherical solution,

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2} + \frac{1}{3}\Sigma(M)r^2\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2} + \frac{1}{3}\Sigma(M)r^2\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

(23)

In Eq(23), if  $h \rightarrow 0$ ,

$$\begin{aligned}\Sigma(M) &= \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left\{ \alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right\} \rightarrow \alpha_0 \frac{c^4}{G^2 M^2} \\ d\tau^2 &= \left( 1 - \frac{2GM}{rc^2} + \frac{\alpha_0}{3} \frac{c^4}{G^2 M^2} r^2 \right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left( 1 - \frac{2GM}{rc^2} + \frac{\alpha_0}{3} \frac{c^4}{G^2 M^2} r^2 \right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]\end{aligned}\quad (24)$$

In Eq(23), if  $M \rightarrow 0$ ,

$$\begin{aligned}\Sigma(M) &= \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left\{ \alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right\} \rightarrow 0 \\ d\tau^2 &= dt^2 - \frac{1}{c^2} [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]\end{aligned}\quad (25)$$

In Eq(23), if  $c \rightarrow \infty$ ,

$$\begin{aligned}\Sigma(M) &= \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right] \rightarrow 0 \\ d\tau^2 &= dt^2\end{aligned}\quad (26)$$

In Eq(23), if  $G \rightarrow 0$ ,

$$\begin{aligned}\Sigma(M) &= \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right] \rightarrow 0 \\ d\tau^2 &= dt^2 - \frac{1}{c^2} [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]\end{aligned}\quad (27)$$

The spherical solution (the vacuum solution) of the revised gravity field equation (add the cosmological term) is

$$d\tau^2 = \left( 1 - \frac{2GM}{rc^2} + \frac{1}{3} \Sigma(M) r^2 \right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left( 1 - \frac{2GM}{rc^2} + \frac{1}{3} \Sigma(M) r^2 \right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

$$\Sigma(M) = \alpha_0 \left( \frac{c^4}{G^2 M^2} \right) \exp\left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} \right]$$

$$\alpha_1 > 0, \beta_1 > 0, \alpha_0, \alpha_1, \beta_1 \text{ are real numbers.}$$

(28)

#### IV. Conclusion

It found the spherical solution of the quantum gravity.

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