Equilibrium and Stability of the Upright Human Body

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Abstract. We propose a new parameterization for describing forward and backward leanings of the straight human body with the feet fixed on the ground. During this motion stability of static equilibrium of the upright human body depends on the basic parameters such as coordinates X_{CG} and Y_{CG} of the center of gravity of the body in the upright position and the radius R of the base of support. We introduce two coefficients $k_1= Y_{CG}/R$ and $k_2= X_{CG}/R$ which are enough to calculate critical values of the angle α between the vertical axis OY and the axis of the cylinder the body is fitted into. We have calculated critical values α_{cr}' and α_{cr}'' when the body becomes unstable during leanings back and forward correspondingly. It is shown that stability of the upright body may be characterized by the angle $\alpha_{cr}= |\alpha_{cr}'|+|\alpha_{cr}''|$ which strongly depends on k_1 and weakly depends on k_2 and α_{opt} – the value of α for the most stable position when the body's center of gravity is exactly above the geometrical center of the base of support. Stable equilibrium will be achieved for larger α_{cr} and smaller $|\alpha_{opt}|$.

Introduction

Application of principles of balance and stability when performing specified sports skills is necessary to success. Many coaches would be wise to spend more time studying sport mechanics like balance and stability in order to improve the performance of their players [1].

To perform balance skills, athletes must have adequate strength to support the body, and they must be able to shift the weight quickly into the correct position at the right time. They must also know their position in space, called kinesthetic awareness, as well as possess quick reactions, coordination, agility, and flexibility.

Within "mechanics" there are two sub-fields of study: statics, which is the study of systems that are in a state of constant motion either at rest (with no motion) or moving with a constant velocity; and dynamics, which is the study of systems in motion in which acceleration is present, which may involve kinematics [2].

In this work we discuss static equilibrium of the upright human body and propose a theoretical biomechanical model how to estimate stability of that equilibrium.

General Theory

Equilibrium is a state of zero acceleration where there is no change in the speed or direction of the body. **Balance** is the ability to control equilibrium (either static or dynamic). **Stability** is the resistance to a change in the body's acceleration, or the resistance to a disturbance of the body's equilibrium.

Balance within muscle groups and alignment of the skeletal system affect body equilibrium and balance. Small shifts of bones can affect the whole skeletal system.

The most important factors for achieving balance are the following [3, 4]:

•A person has balance when the COG falls within the base of support (BOS) (the upright body is only stable when the line of gravity lies within the foot base);

•A person has balance in direct proportion to the size of the BOS (the larger the base of support, the more balance);

•A person has balance depending on mass (the greater the mass, the more balance).

A unique point is associated with every body, around which the body's mass is equally distributed in all directions. This point is known as the center of mass or the mass centroid of the body. In the analysis of bodies subject to gravitational force, the center of mass may also be referred to as the center of gravity (CG), the point about which a body's weight is equally balanced in all directions or the point about which the sum of the moments produced by the weights of the body segments is equal to zero (Fig. 1).

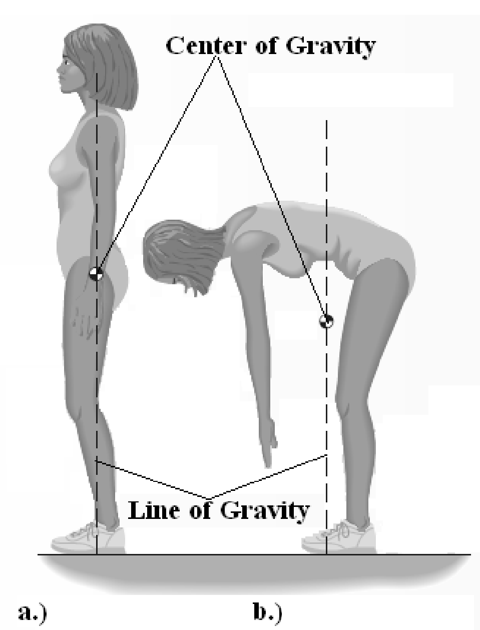


FIGURE 1. Center of gravity and line of gravity of a human body.

The CG of a perfectly symmetrical object of homogeneous density, and therefore homogeneous mass and weight distribution, is at the geometric center of the body. However, when mass distribution within an object is not constant, the CG shifts in the direction of greater mass. It is also possible for an object's CG to be located physically outside of the object (Fig. 1 b.)). **Line of gravity** is gravity's action line which is visualized as a vertical line projecting downwards from the center of gravity (Fig. 1).

A commonly used procedure for estimating the location of the total body CG from projected film images of the human body is known as the segmented method. This procedure is based on the concept that since the body is composed of individual segments (each with its own CG), the location of the total body CG is a function of the location of the respective segmental CG's. Some body segments, however, are more massive than others and have a larger influence on the location of the total body CG. When the products of each body segment's CG location and its mass are summed and then divided by the sum of all the segmental masses (total body mass), the result is the location of the total body CG. The segmental method uses data for average locations of individual body segment CG's as related to a percentage of segment length. Thus,

$$X_{CG} = \frac{\sum_{i=1}^{i=n} x_i \cdot m_i}{m}, \quad Y_{CG} = \frac{\sum_{i=1}^{i=n} y_i \cdot m_i}{m}, \quad Z_{CG} = \frac{\sum_{i=1}^{i=n} z_i \cdot m_i}{m} \qquad (1)$$

where X_{CG}, Y_{CG}, Z_{CG} are the coordinates of the total body CG, x_i, y_i, z_i are the coordinates of the individual segment CG's, and m_i is the individual segment mass.

The masses of body segments can be count if you know the total height and weight of a person. One of frequently used methods was described by Zatsiorskji and Selujanov (1979), who determined the parameters B_0, B_1 a B_2 for each body segment. The equation for body mass is following

$$m_i = B_0 + B_1 m + B_2 h, \qquad (2)$$

where m (kg) is total mass of a person and h (in cm!) is height of a person.. The parameters B_0 , B_1 a B_2 see [5].

Centre of gravity of body segments [5] was determined experimentally: hand (manus) 39:61 %, forearm 43:57 %, upper arm 44:56 %, head+neck 50:50 %, trunk 42:58% (from shoulder), thigh 43:57%, shank 41:59 %, foot 40:60 % of total length of each segment (from proximal end).

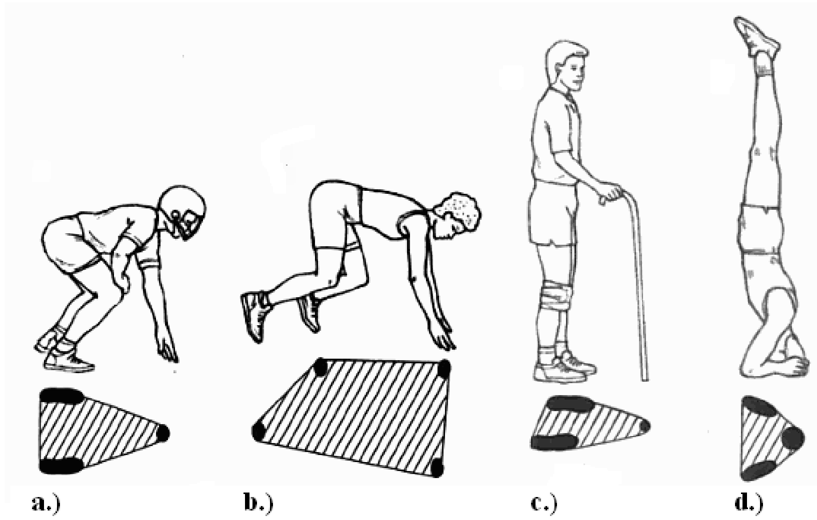


FIGURE 2. Slices and bases of support.

Base of support is the supporting area beneath the body, it includes the points of contact with the supporting surface and the area between them; these points may be body parts (such as feet), or extensions of body parts (such as crutches or other walking aids) (Fig. 2).

Consider an arbitrary area A in the x-y plane. Let A be divided into parts and denote the positions of the parts by The centroid, or average position of the area, is given by:

$$\bar{x} = \frac{\sum_{i=1}^{i=n} x_i \cdot A_i}{\sum_{i=1}^{i=n} A_i}, \quad \bar{y} = \frac{\sum_{i=1}^{i=n} y_i \cdot A_i}{\sum_{i=1}^{i=n} A_i} \tag{3}$$

In summary, high stability (low mobility) is characterized by a large base of support, a low center of gravity, a centralized center of gravity projection within the base of support, a large body mass, and high friction at the ground interface. Low stability (high mobility), in contrast, occurs with a small base of support, a high center of gravity, a center of gravity projection near the edge of the base of support, a small body mass, and low friction.

Biomechanical Model of the Upright Human Body

Let's consider first two tilted cylinders. The base of support for the cylinders is the area where they are in contact with the floor. For the two cylinders below, the left cylinder's CG is above the base of support so the upward support force from the base is aligned with the downward force of gravity (Fig. 3).

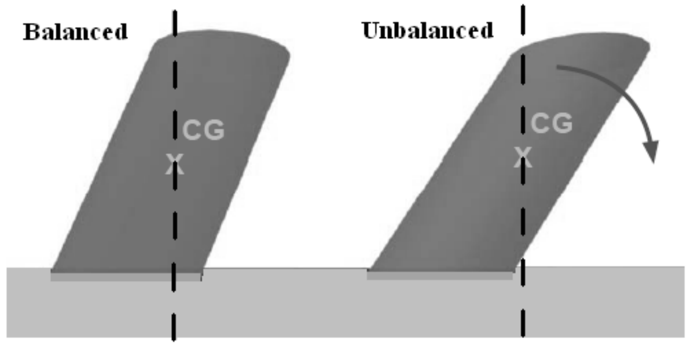
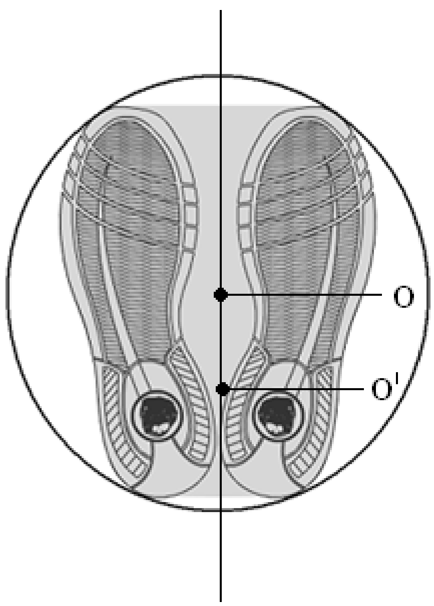
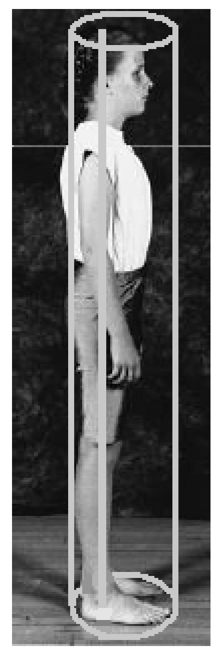


FIGURE 3. Two tilted cylinders and their lines of gravity.

For the cylinder on the right the CG is not above the base of support so these two forces cannot align and instead create a torque that rotates the object, tipping it over.



a.)



b.)

FIGURE 4. a.) Sportsman's base of support, b.) Upright human body fitted into the imaginary cylinder.

A similar approach may be used for modeling the upright human body's static equilibrium. In Fig. 4 a.) you see the base of support of a sportsman. We can imagine the sportsman as if he has been placed into a cylinder (Fig. 4 b.)). Let's consider the sportsman's leanings back and forward keeping his body straight and feet still on the ground. During the motion the cylinder also will change its form as it is shown in Fig. 5. The sportsman's center of gravity will rotate around O′, where O′ is the projection of the sportsman's center of gravity in the upright position and the position of the sportsman's body will depend on the angle α between the axis of the cylinder and the OY axis.

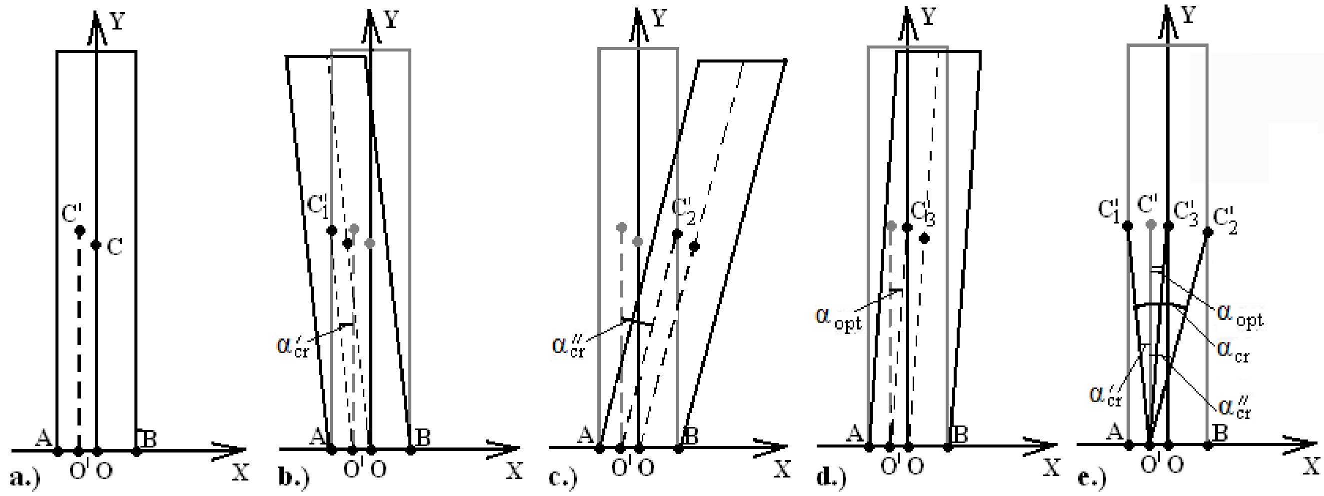


FIGURE 5. Different positions of a straight human body: a.) upright position, b.) critical position during backward leaning, c.) critical position during forward leaning, d.) the most stable position of the body, e.) comparative diagram for the positions shown in Figures a.) - d.).

There are three main positions:

1.) The sportsman's CG is exactly above the back edge of the base of support A (Fig. 5 b.));

2.) The sportsman's CG is exactly above the front edge of the base of support B (Fig. 5 c.));

3.) The sportsman's CG is exactly above the geometrical center of the base of support O (Fig. 5 d.));

In the first two positions increasing of α will lead to the transition from balanced equilibrium to unbalanced equilibrium. In the third position the sportsman's body will have maximal stability. Let α_{cr}', α_{cr}'' and α_{opt} be the corresponding values of α. These three positions will be determined by the corresponding values of α: α_{cr}', α_{cr}'' and α_{opt}, which can be calculated using the following formulas:

$$\alpha_{cr}' = \arcsin\left(\frac{1}{k_1}\cdot(1+k_2)\right) \tag{4}$$

$$\alpha_{cr}'' = \arcsin\left(\frac{1}{k_1}\cdot(1-k_2)\right) \tag{5}$$

$$\alpha_{opt} = \arcsin(k_3) \tag{6}$$

Where k_1, k_2 and k_3 are coefficients determined by the following equations:

$$k_1 = \frac{|O'M|}{|OA|} = \frac{|O'M_1|}{|OA|} = \frac{|O'M_2|}{|OA|} = \frac{|O'M_3|}{|OA|} = \frac{Y_{CG}}{R} = \frac{l}{R} \tag{7}$$

$$k_2 = \frac{X_{CG}}{|OA|} = \frac{X_{CG}}{R} \tag{8}$$

$$k_3 = \frac{X_{CG}}{Y_{CG}} = \frac{X_{CG}}{l} = \frac{k_2}{k_1} \tag{9}$$

R is the radius of the base of support, X_{CG} and Y_{CG} – are the initial coordinates of the sportsman's center of gravity in the upright position, l= Y_{CG} is the length of the segment connecting CG of the sportsman and O′.

Formulas (4)-(9) allow us to estimate how stable the upright position of the sportsman is. High stability is characterized by large value of the angle $\alpha_{cr}=|\alpha_{cr}'|+|\alpha_{cr}''|$ and small value of α_{opt}. The model may be successfully applied not only to the upright position of the human body but to all positions where the motion of the sportsman's body can be described using tilted cylinders as well.

Discussion

In this section we will show how values of α_{cr}', α_{cr}'', α_{cr} and α_{opt} depend on the basic initial parameters of the sportsman: X_{CG} and Y_{CG} - the initial coordinates of the sportsman's center of gravity and R – the radius of the base of support. Using formulas (4-9) we have calculated α_{cr}', α_{cr}'' and α_{cr} as functions of k_2 where k_2 changes from -1 to 1. In Fig. 6 a.), b.) are shown graphs of α_{cr}', α_{cr}'' calculated for k_1=2, 4, 6, 8, 10. We see that α_{cr}' increases when k_2 increases and α_{cr}'' decreases when k_2 increases and dependencies have almost linear character for k_1=4, 6, 8, 10.

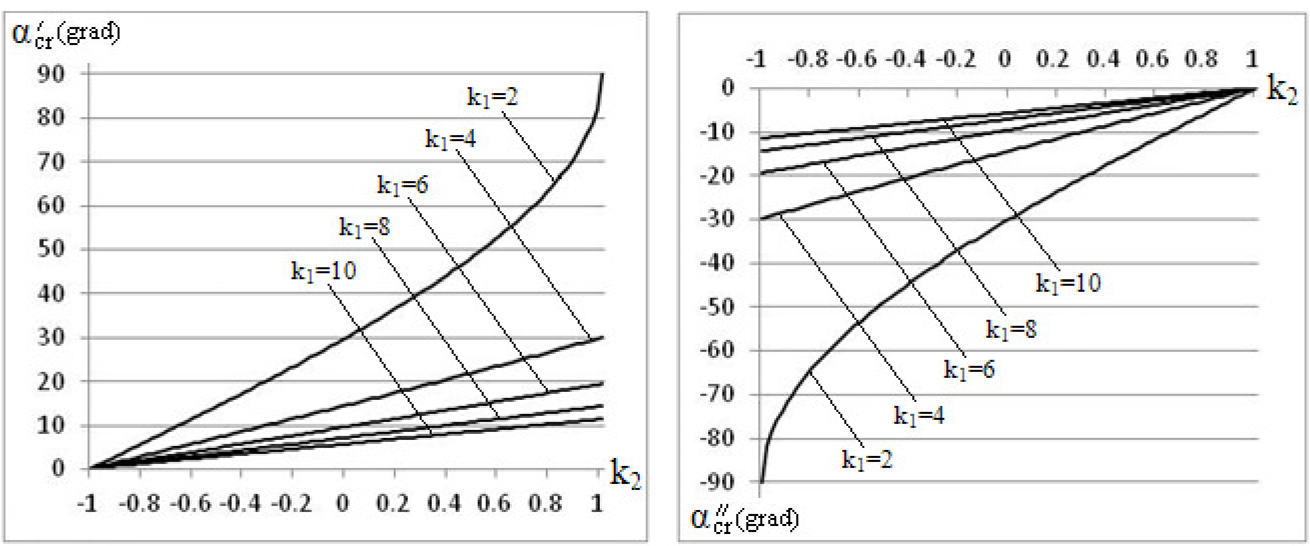


FIGURE 6. Dependencies of $\alpha_{cr}'$and α_{cr}'' on the ratio of X_{CG} to R.

In Fig. 6 a.) are shown graphs of α_{cr} calculated for k_1=2, 4, 6, 8, 10. We see that for k_1=4, 6, 8, 10 α_{cr} weakly depends on k_2 and dependencies have almost linear character: $\alpha_{cr}(k_1, k_2)\approx$const if k_1 is fixed. In Fig. 6 b.) we calculated how $\alpha_{cr}(k_1, 0)$ depends on k_1. If $k_1\geq4$ when we can find $\alpha_{cr}(k_1, 0)$ from Fig 6. b.) and for all $-1\leq k_2\leq1$we will have good approximation $\alpha_{cr}(k_1, k_2)\approx \alpha_{cr}(k_1, 0)$. For $\alpha_{cr}(k_1, 0)$ we can use a simple formula $\alpha_{cr}(k_1, 0)\approx120/k_1$, which fits very well into the graph of $\alpha_{cr}(k_1, 0)$ shown in Fig. 6 b.).

The coefficient $k_1=l/R$ depends mainly on the proportions of the sportsman: his height and feet size. Lower sportsmen with larger base of support will be more stable. Obviously, α_{cr} decreases from 60° to 0° while the value of k_1 increases from 2 to $+\infty$.

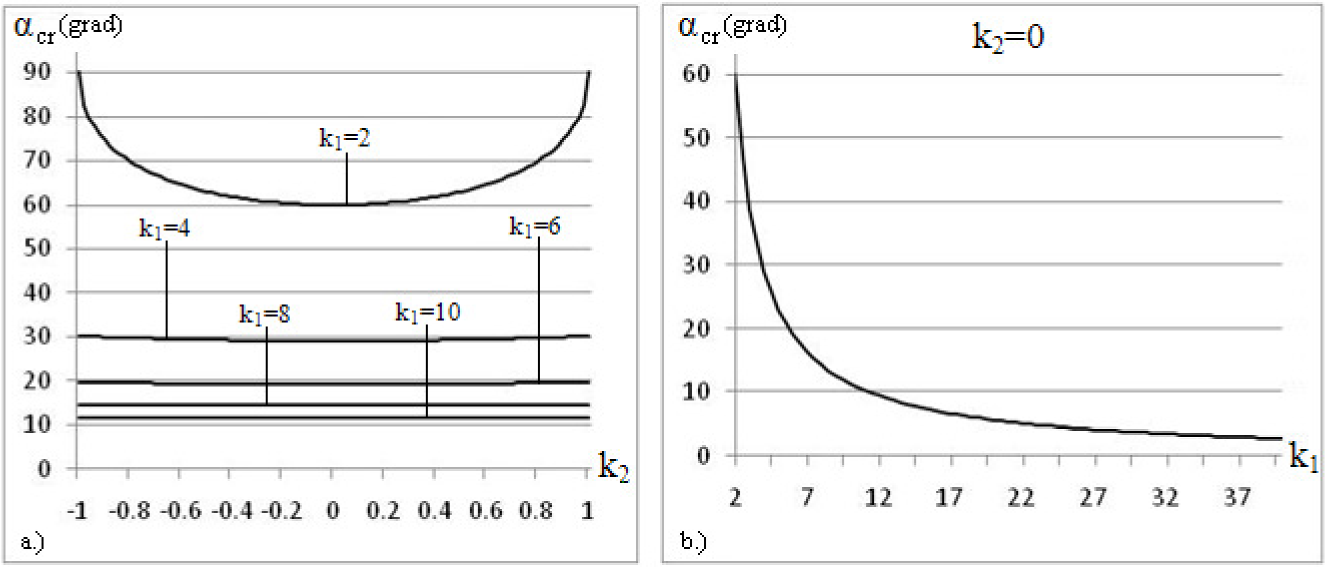


FIGURE 7. a.) Dependence of α_{cr} on the ratio of X_{CG} to R for k_1=2, 4, 6, 8, 10. b.) Dependence of $\alpha_{cr}(k_1, 0)$ on k_1.

Using our formula (6) we can calculate the dependence of α_{opt} on $k_3=X_{GC}/Y_{GC}$ (see Fig. 7). Here negative values of α_{opt} correspond to the negative values of X_{GC} and mean that the sportsman must lean forward to reach the optimal position of equilibrium. If $\alpha_{opt}>0$ the sportsman must lean backward in order to reach the optimal position. The dependence is almost linear in the range $-0.6 \le k_3 \le 0.6$ and can be described by the formula $\alpha_{opt} \approx 58 \cdot k_3 = 58 \cdot X_{CG}/Y_{CG}$.

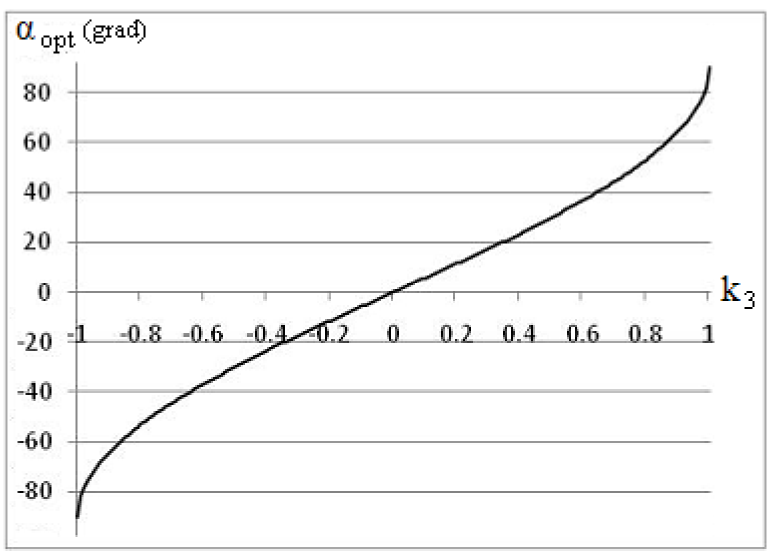


FIGURE 7. Dependence of α_{opt} on k_2.

Taking into account that the average ratio of y coordinate of center of mass to height in males is approximately 0.560 and the average ratio of center of mass to height in females is approximately 0.543 we can calculate the average values of α_{cr}', α_{cr}'', α_{cr} and α_{opt} for males and females.

For male sportsmen standing upright CG has coordinates (-0.5·R, 0.56·h), where h is the sportsman's height and R/h≈1/12. In this case $k_{1man}=l/R\approx 0.56\cdot 12=6.72$. For female sportsmen CG has coordinates (-0.5·R, 0.543·h) and $k_{1woman}=l/R\approx 0.543\cdot 12=6.516$.

sex	k_1	k_2	α_{cr}'(grad)	α_{cr}''(grad)	α_{cr}	α_{opt}(grad)
male	**6.72**	**-1**	**0.00**	**-17.32**	**17.32**	**-8.56**
female	**6.52**	**-1**	**0.00**	**-17.88**	**17.88**	**-8.83**
male	**6.72**	**-0.7**	**2.56**	**-14.65**	**17.21**	**-5.98**
female	**6.52**	**-0.7**	**2.64**	**-15.12**	**17.76**	**-6.17**
male	**6.72**	**-0.5**	**4.27**	**-12.90**	**17.17**	**-4.27**
female	**6.52**	**-0.5**	**4.40**	**-13.31**	**17.71**	**-4.40**
male	**6.72**	**0**	**8.56**	**-8.56**	**17.12**	**0.00**
female	**6.52**	**0**	**8.83**	**-8.83**	**17.66**	**0.00**
male	**6.72**	**0.5**	**12.90**	**-4.27**	**17.17**	**4.27**
female	**6.52**	**0.5**	**13.31**	**-4.40**	**17.71**	**4.40**
male	**6.72**	**0.7**	**14.65**	**-2.56**	**17.21**	**5.98**
female	**6.52**	**0.7**	**15.12**	**-2.64**	**17.76**	**6.17**
male	**6.72**	**1**	**17.32**	**0.00**	**17.32**	**8.56**
female	**6.52**	**1**	**17.88**	**0.00**	**17.88**	**8.83**

Table 1. Values of α_{cr}', α_{cr}'', α_{cr} and α_{opt} for males (k_1=6.72) and females (k_1=6.52) for k_2= -1.0, -0.7, -0.5, 0.0, 0.5, 0.7, 1.0.

We have calculated α_{cr}', α_{cr}'', α_{cr} and α_{opt} for males (k_1=6.72) and females (k_1=6.52) for different values of k_2=-1.0, -0.7, -0.5, 0.0, 0.5, 0.7, 1.0. The results are displayed in Table 1. We see that $\alpha_{cr}\approx 17.25°$ for males and $\alpha_{cr}\approx 17.80°$ for females. For the typical value of k_2= -0.5 (see Fig. 4 a.)) $\alpha_{opt}\approx$-4.27° for males and $\alpha_{opt}\approx$-4.40° for females. This means that females are slightly more balanced than males but in order to achieve the most stable position they have to lean their bodies slightly more forward than males.

Conclusion

In this work we investigate how stability of equilibrium of the upright human body depends on the basic parameters such as coordinates X_{CG} and Y_{CG} of the center of gravity of the body in the upright position and the radius R of the base of support. We introduce two coefficients $k_1= Y_{CG}/R$ and $k_2= X_{CG}/R$ which are enough to calculate critical values of the angle α between the vertical axis OY and the axis of the cylinder the body is fitted into. We have calculated critical values of α - α_{cr}' and α_{cr}'' when the body becomes unstable during leanings back and leaning forward correspondingly. It is shown that stability of the upright body may be characterized by the angle $\alpha_{cr}= |\alpha_{cr}'| + |\alpha_{cr}''|$ which strongly depends on k_1 and weakly depends on k_2 and α_{opt} – the value of α for the most stable position when the body's center of gravity is exactly above the geometrical center of the base of support.

In summary, high stability will require larger values of α_{cr} and smaller values of α_{opt}. We have derived approximate formulas for α_{cr} : $\alpha_{cr}(k_1, k_2)\approx 120/k_1=120\cdot R/Y_{CG}$ if $k_2\geq 4$ and α_{opt}: $\alpha_{opt} \approx 58\cdot k_3=58\cdot X_{CG}/Y_{CG}$ if $-0.6\leq k_3\leq 0.6$.

We have calculated values of α_{cr}', α_{cr}'', α_{cr} and α_{opt} for males (the average value of k_1 is 6.72) and females (the average value of k_1 is 6.52) and different values of k_2= -1.0, -0.7, -0.5, 0.0, 0.5, 0.7, 1.0. For k_2= -0.5 (the average value of k_2) $\alpha_{cr}=17.17°$ and $\alpha_{opt}=-4.27°$ in men and $\alpha_{cr}=17.71°$ and $\alpha_{opt}=-4.40°$ in women. We see that women are slightly more stable than men.

The proposed model may be successfully applied to all body positions which can be described using tilted cylinders.

References:

1. P. Grimshaw and A. Burden, Sport and Exercise Biomechanics, 2007, ISBN 0-203-48830-X, pp. 156-163.
2. Diana Cotoros, Biomechanical Analyzes of Human Body Stability and Equilibrium, Proceedings of the World Congress on Engineering 2010, Vol II, WCE 2010, June 30 - July 2, 2010, London, U.K.
3. Duane Knudson, Fundamentals of Biomechanics, 2003 Kluwer Academics/Plenum Publishers, New York, ISBN 0-306-47474-3, pp.178-188.
4. Zatsiorskyi, V.M., Fundamentals of sports metrology, PHC&S, 1979, Moscow, Russian Federation.
5. Mc Ginnis, Peter Merton, Biomechanics of Sports and Exercise, 2nd edition, 2005, ISBN 0-7360-5101-5, pp. 137-143.