A Note on the Classical Vector Fields

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Using the density of the lines of force, we have obtained the laws of Coulomb, Biot-Savart and Newton.

Key words: density of the lines of force.

In geometry, we have that *angle* = *arc/radius* or $\theta = l/r$, and therefore *solid angle* = *susrface/radius*^{2} or $\Omega = S/r^2$. Now, we define the (classical) electric vector field produced by the source electric charge, q_1, as proportional, k_e, to the number of lines of force per unit area, $n = N/S$, per solid angle; then

$$\vec{E}_1 = k_e \frac{N}{S}\frac{S}{r_1^2}\vec{u}_r = k_e \frac{N}{r_1^2}\vec{u}_r = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\vec{u}_r \tag{1}$$

ε_0 being the electric permittivity of the vacuum, with

$$k_e N = \frac{1}{4\pi\varepsilon_0} q_1 \tag{2}$$

The electric force on a test electric charge, q_2, would be

$$\vec{F}_{e12} = q_2\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\vec{u}_r \tag{3}$$

For the magnetic vector field, it would be the same:

$$\vec{B}_1 = -k_m \frac{N}{S}\frac{S}{r_1^2}\vec{u}_r = -k_m \frac{N}{r_1^2}\vec{u}_r = -\frac{\mu_0}{4\pi}\frac{i_1 l_1}{r_1^2}\vec{u}_r \tag{4}$$

μ_0 being the magnetic susceptibility of the vacuum, i_1 the source electric current and l_1 the conductor length, with

$$k_m N = \frac{\mu_0}{4\pi} i_1 l_1 \tag{5}$$

The magnetic force on a test electric current, i_2, of conductor length, l_2, would be

$$\vec{F}_{m12} = i_2 l_2 \vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{i_1 l_1 i_2 l_2}{r_{12}^2}\vec{u}_r \tag{6}$$

And also for the gravitational vector field:

$$\vec{\Gamma}_1 = -k_g \frac{N}{S}\frac{S}{r_1^2}\vec{u}_r = -k_g \frac{N}{r_1^2}\vec{u}_r = -G\frac{m_1}{r_1^2}\vec{u}_r \tag{7}$$

G being the Newton's gravitational constant and m_1 the source mass, with

$$k_g N = G m_1 \tag{8}$$

The gravitational force on a test mass, m_2, would be

$$\vec{F}_{g12} = m_2\vec{\Gamma}_1 = -G\frac{m_1 m_2}{r_{12}^2}\vec{u}_r \tag{9}$$

By convention, the electric lines of force are directed outward for positive electric charges and inward for negative electric charges. Then, two electric charges of the same sign repel each other because their lines of force go in opposite directions, and two electric charges of different sign attract each other because their lines of force go in the same direction. In two parallel conductors, the conductors attract if the currents go in the same direction and repels otherwise.

In the gravitational field, however, the gravitational lines of force would not drag the bodies in the same form as the electric lines of force, because so whether we consider those outgoing or incoming, the force would always be repulsive instead of attractive. We assume that the gravitational lines of force produce the gravitational attraction bending the space (as stipulates the general relativity of Einstein) and that this curvature decreases with the distance to the source field.

In summary, using the density of the lines of force, we have obtained the laws of Coulomb, Biot-Savart and Newton.