The Natario Warp Drive using Lorentz Boosts according to the Harold White Spacetime Metric potential θ.

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity.There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001.The warp drive seems to be very attractive because allows interstellar space travel at arbitrarily large speeds avoiding the time dilatation and mass increase paradoxes of Special Relativity. However it suffers from a very serious drawback:Interstellar space is not empty:It is fulfilled with photons and particle dusts and a ship at superluminal speeds would impact these obstacles in highly energetic collisions disrupting the warp field and placing the astronauts in danger.This was pointed out by a great number of authors like Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati.
In order to travel significant interstellar distances in reasonable amounts of time a ship would need to attain 200 times the speed of light but according to Clark,Hiscock and Larson the impact between the ship and a single photon of Cosmic Background Radiation(COBE)would release an amount of energy equal to the photosphere of a star like the Sun.And how many photons of COBE we have per cubic centimeter of space between Earth and Gliese 581 a star at 20 light-years with potential habitable exo-planets?This serious problem seems to have no solution at first sight.
However some years ago Harold White from NASA appeared with an idea that may well solve this problem:According to him the ship never surpass the speed of light but the warp field generates a Lorentz Boost resulting in an apparent superluminal speed as seen by the astronauts on-board the ship and by observers on the Earth while the warp bubble is always below the light speed with the ability to manoeuvre against these obstacles avoiding the lethal collisions.
Harold White applied its conclusions for the Alcubierre warp drive.
In this work we examine the feasibility of the White idea for the Natario warp drive using clear mathematical arguments and we arrived at the conclusion that the line of reason of Harold White is correct and can be applied to the Natario geometry.

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1 Introduction

The Warp Drive as a solution of the Einstein field equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons of Cosmic Background Radiation(COBE).

In order to travel significant interstellar distances in reasonable amounts of time a ship would need to attain 200 times the speed of light but according to Clark,Hiscock and Larson a single collision between a ship at 200 times light speed and a COBE photon would release an amount of energy equal to the photosphere of a star like the Sun.(see pg 11 in [3]).And how many photons of COBE we have per cubic centimeter of space between Earth and Gliese 581 a star at 20 light-years with potential habitable exoplanets??

These highly energetic collisions would pose a very serious threat to the astronauts as pointed out by McMonigal,Lewis and O'Byrne (see pg 10 in [4]).

Another problem:these highly energetic collisions would raise the temperature of the warp bubble reaching the Hawking temperature as pointed out by Barcelo,Finazzi and Liberati.(see pg 6 in in [5]).At pg 9 they postulate that all future spaceships cannot bypass 99 percent of the light speed.

However some years ago in 2003 Harold "Sonny" White from NASA Lyndon B.Johnson Space Center Houston Texas proposed a different idea. According to him a ship leaves the Earth and achieves a subluminal velocity of $0,1c$.The spacetime metric is then engineered to produce a Lorentz Boost resulting in an apparent speed of 10c as seen by observers on Earth and observers on-board the ship.However the warp bubble moves always with $0,1c$ allowing manoeuvres against interstellar obstacles.(see pg 6 in [8],pg 5 in [9])).

Without the Lorentz Boost the ship would take 43 years to reach Proxima Centauri however according to White the Lorentz Boost is like a film played in "fast forwarding mode".It "accelerates" the clocks on Earth and on-board the ship so both observers "sees" an apparent speed of 10c and the completion of the journey in $4,3$ months and not in 43 years.The Lorentz Boost is like a "jump into the Future".Both

[1]do not violates Relativity

Earth and ship "sees" 43 years passing in $4,3$ months.This may sounds unbelievable but the mathematics employed by Harold White is entirely correct although accessible only to advanced readers.[2]

Harold White wrote its works for the Alcubierre warp drive.

The purpose of this work is to demonstrate the idea of White applied to the Natario warp drive proving that the "fast-forwarding" is entirely correct and giving all the needed mathematical demonstrations in a clear formalism.

In this work we use the Geometrized System of Units in which $c = G = 1$

In order to completely understand the ideas behind this work fully acquaintance with the White ideas and the Natario geometry is required.While many readers of warp drive works have acquaintance or are familiarized with the Alcubierre geometry,the Natario geometry is not familiar to the major part of warp drive readers and even some readers demonstrated difficulties to fully understand the mathematics behind [2].

In order to get familiarized with the Natario geometry we recommend a start-up with sections 1 to 4 and 6 in [12](skip section 5) followed with sections 1 and 2 in [11] and terminating with sections 1 and 2 in [10].

For those readers familiarized with the Alcubierre geometry but not familiarized with the White concept of Lorentz Boosts and "fast forwarding" we would recommend a start-up with [8] first being followed by [9].A step-by-step description of the calculations used by White for the Alcubierre geometry can be found in [7].

This work will look very similar to [7] because it is intended to be also a step-by-step description of the White ideas applied to the Natario warp drive.We would recommend a reading of [7] before starting to read this work

[2]He presented his results in a resumed way.A beginner or intermediate student would not figure out the White idea at first sight.

2 The Natario Warp Drive using Lorentz Boosts according to the Harold White Spacetime Metric potential θ: Boost observed by a Ship Frame Coordinates System

Spacetime metrics in General Relativity are often written in the following form:
(see eq 7.13 pg 175 in [6])(signature (-,+,+,+))

$$ds^2 = -e^{-2a(t,r)}dt^2 + e^{2b(t,r)}dr^2 + r^2 d\Omega^2 \tag{1}$$

changing the signature to (+,-,-,-) we have:

$$ds^2 = e^{-2a(t,r)}dt^2 - e^{2b(t,r)}dr^2 - r^2 d\Omega^2 \tag{2}$$

defining $\theta = -2a$ and $\vartheta = 2b$ we have:

$$ds^2 = e^{\theta(t,r)}dt^2 - e^{\vartheta(t,r)}dr^2 - r^2 d\Omega^2 \tag{3}$$

The expression above is very important as we will shall see later.

The central idea of White is the apparent velocity vs defined as (see pg 6 in [8],pg 5 and 17 in [9]).

$$vs = v_{eff} = v_i \times \gamma \tag{4}$$

In the expression above $vs = v_{eff}$ is the apparent speed seen by the Earth and observers on-board the spaceship (10c) while v_i is the real speed of the warp bubble$(0,1c)$.
γ is the Lorentz Boost generated by the g_{00} component of spacetime metrics with the form similar or resembling eq 3[3].
From above we can easily see that the real velocity $v_i << 1$ but due to the Lorentz Boost γ we can have $vs >> 1$ or $v_{eff} >> 1$.

The warp drive spacetime according to Natario for the coordinates rs and θ is defined by the following equation:(see Appendix E in [10] for details)

$$ds^2 = [1 - (X^{rs})^2 - (X^\theta)^2]dt^2 + 2[X^{rs}drs + X^\theta rsd\theta]dt - drs^2 - rs^2 d\theta^2 \tag{5}$$

The expressions for X^{rs} and X^θ are given by:(see pg 5 in [2],see also Appendix D in [10] for details)

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{6}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{7}$$

$$X^\theta = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{8}$$

$$X^\theta = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{9}$$

[3]see Lorentz Transformations in Wikipedia:The Free Encyclopedia

$n(rs)$ is the Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $n(rs) = 0$ for small rs(inside the warp bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the Natario warped region(pg 5 in [2])

For simplicity we consider only the motion in the equatorial plane parallel to the $x - axis$.
Then $\theta = 0$ and $cos(\theta) = 1$. The Natario warp drive now becomes:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{10}$$

But since we are considering the motion in the $x - axis$ only or parallel to the $x - axis$ where $\theta = 0$ or $cos(\theta) = 1$ then we are left with:

$$X^{rs} = -2v_s n(rs) \tag{11}$$

$$X^{rs} = 2v_s n(rs) \tag{12}$$

Placing the following Natario warp drive equations together in order to extract the spacetime metric tensor component g_{00}:

-)-generic case:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs}drs + X^{\theta}rsd\theta]dt - drs^2 - rs^2 d\theta^2 \tag{13}$$

-)-equatorial plane motion:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{14}$$

Using the g_{00} of the exponential metric in eq 3

$$ds^2 = e^{\theta(t,r)}dt^2 - e^{\vartheta(t,r)}dr^2 - r^2 d\Omega^2 \tag{15}$$

Which is :

$$g_{00} = e^{\theta(t,r)} \tag{16}$$

We have the following results for the combined g_{00}:

-)-generic case:

$$g_{00} = 1 - (X^{rs})^2 - (X^{\theta})^2 = e^{\theta(t,r)} \tag{17}$$

-)-equatorial plane motion:

$$g_{00} = 1 - (X^{rs})^2 = e^{\theta(t,r)} \tag{18}$$

The terms $g_{00} = 1 - (X^{rs})^2 - (X^{\theta})^2$ and $g_{00} = 1 - (X^{rs})^2$ are very important:These are the g_{00} components of the Natario metric that will generate the Lorentz Boost essential to the "fast-forwarding" concept.

Rearranging the terms in g_{00} for the Natario warp drive we have:

$$e^{\theta(t,r)} = 1 - (X^{rs})^2 - (X^{\theta})^2 \tag{19}$$

$$e^{\theta(t,r)} = 1 - (X^{rs})^2 \tag{20}$$

The White spacetime metric potential θ for the Natario warp drive and its respective Lorentz Boost γ are given by:[4].

-)-generic case:

$$\theta = \ln[|1 - (X^{rs})^2 - (X^{\theta})^2|] \tag{21}$$

-)-equatorial plane motion:

$$\theta = \ln[|1 - (X^{rs})^2|] \tag{22}$$

-)-Lorentz Boost:

$$\gamma = \cosh(\theta) \tag{23}$$

$$\gamma = \cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|]) \tag{24}$$

$$\gamma = \cosh(\ln[|1 - (X^{rs})^2|]) \tag{25}$$

Note the fact that we used $|1 - (X^{rs})^2 - (X^{\theta})^2|$ and not $1 - (X^{rs})^2 - (X^{\theta})^2$.This is due to the chosen signature (+,-,-,-).With a signature (-,+,+,+) the term would be $(X^{rs})^2 + (X^{\theta})^2 - 1$.[5] Since the apparent speed $vs >> 1$ we cannot have a logarithm of a negative number.The modulus fills the gap between both signatures.

Again the expressions for X^{rs} and X^{θ} are given by:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{26}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{27}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{28}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{29}$$

[4] again see Lorentz Transformations in Wikipedia:The Free Encyclopedia
[5] again see Appendix E in [10]

We derived the Lorentz Boost using the apparent velocity vs contained in the terms X^{rs} and X^{θ}.In order to find the real velocity v_i of the warp bubble we must use(see pg 6 in [8],pg 5 and 17 in [9]):

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} \tag{30}$$

-)-generic case:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|])} \tag{31}$$

-)-equatorial plane motion:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\ln[|1 - (X^{rs})^2|])} \tag{32}$$

Now we must examine the most important point of view of the White idea:The concept that allows a warp bubble with a real velocity of $v_i = 0,1$c being seen with an apparent speed of $vs = 10$c by observers on-board the ship and on Earth: The concept of the "fast-forwarding film".In order to do so we must derive the expression for the Natario shape function.

$f(rs)$ is the Alcubierre shape function defined as:(see eq 6 pg 4 in [1])

$$f(rs) = \frac{\tanh[@(rs + R)] - \tanh[@(rs - R)]}{2\tanh(@R)} \tag{33}$$

According to Alcubierre we have 3 possible values for $f(rs)$:(see eq 7 pg 4 in [1])

- 1)- inside the warp bubble where the ship resides $f(rs) = 1$
- 2)- outside the warp bubble where Earth resides $f(rs) = 0$
- 3)- in the Alcubierre warped region (warp bubble walls) $1 > f(rs) > 0$

Note that this is the situation seen by two observers C and D placed on Earth watching the ship passing by them with an apparent velocity of $10c$ due to the "fast forwarding film" concept.While Alcubierre worked with the Earth-based coordinates frame,Natario worked with a Ship-based coordinates frame.

But how can we be so sure that Natario worked with a Ship-based coordinates frame??

Recalling the definition of the Natario vector nX from pg 4 in [2] and the Natario shape function from pg 5 in [2].
The Natario Vector $nX = -vs(t)dx = 0$ vanishes inside the warp bubble because inside the warp bubble there are no motion at all because $n(rs) = 0$ while being $nX = -vs(t)dx \neq 0$ not vanishing outside the warp bubble because $n(rs) = \frac{1}{2}$ do not vanish.Then an external observer would see the warp bubble passing by him with a speed defined by the shift vector $nX = -vs(t)$ or $nX = vs(t)$.

$n(rs)$ is the Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $n(rs) = 0$ for small rs(inside the warp bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the Natario warped region(pg 5 in [2])

But two astronauts A and B inside the bubble and stationary with respect to each other when one look to the other both will see themselves at the rest with respect to each other because inside the bubble $nX = -vs(t)dx = 0$ while watching the external objects passing by them with a speed vs because outside the warp bubble $nX = vs(t)$.

The Natario shape function $n(rs)$ for a Ship-based coordinates frame that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble written in function of the Alcubierre shape function $f(rs)$ for an Earth-based coordinates frame is given by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{34}$$

- 1)- inside the warp bubble where the ship resides $n(rs) = 0$
- 2)- outside the warp bubble where Earth(or Proxima Centauri)resides $n(rs) = \frac{1}{2}$
- 3)- in the Natario warped region (warp bubble walls) $\frac{1}{2} > n(rs) > 0$

Rewriting again the equations for the apparent speed vs in function of the real speed v_i in the Natario spacetime we get:

$$vs = v_i \times \gamma = v_i \cosh(\theta) \tag{35}$$

-)apparent speed - generic case:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|]) \tag{36}$$

-)-apparent speed - equatorial plane motion:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2|]) \tag{37}$$

-)real speed - generic case:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|])} \tag{38}$$

-)-real speed - equatorial plane motion:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\ln[|1 - (X^{rs})^2|])} \tag{39}$$

When two observers on-board the ship A and B "looks" to each other both "sees" the $n(rs) = 0$ inside the warp bubble and the terms X^{rs} and X^{θ} reduces to zero..Hence inside the warp bubble the White potential and Lorentz Boost are given by:

$$\theta = \ln[|1|] = 0 \tag{40}$$

$$\gamma = \cosh(\theta) = 1 \tag{41}$$

Inside the bubble $\gamma = 1$.There is no Lorentz Boost inside the bubble and consequently no "fast forwarding" concept regarding both observers A and B because both observers are stationary with respect to each other.The apparent speed vs is equal to the real speed v_i.

Now when both observers A and B inside the bubble sees the regions outside the bubble where Earth (observer C)and Proxima Centauri(observer D) resides both sees a region of spacetime where $n(rs) = \frac{1}{2}$ and hence[6]:

-)-generic case:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^\theta)^2|]) \tag{42}$$

-)-equatorial plane motion:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2|]) \tag{43}$$

Recalling the expressions for X^{rs} and X^θ when $n(rs) = \frac{1}{2}$ we have:

$$X^{rs} = -2v_s n(rs) \cos\theta = -v_s \cos\theta \tag{44}$$

$$X^{rs} = 2v_s n(rs) \cos\theta = v_s \cos\theta \tag{45}$$

$$X^\theta = v_s(2n(rs) + (rs)n'(rs)) \sin\theta = v_s \sin\theta \tag{46}$$

$$X^\theta = -v_s(2n(rs) + (rs)n'(rs)) \sin\theta = v_s \sin\theta \tag{47}$$

$$\theta = \ln[|1 - (X^{rs})^2 - (X^\theta)^2|] = \ln[|1 - (v_s \cos\theta)^2 - (v_s \sin\theta)^2|] = \ln[|1 - vs^2|] \tag{48}$$

In the equatorial plane $\theta = 0$ and $\cos\theta = 1$.Hence we have:

$$\theta = \ln[|1 - (X^{rs})^2|] = \ln[|1 - (v_s \cos\theta)^2|] = \ln[|1 - vs^2|] \tag{49}$$

Note that both results (generic and equatorial) are remarkably similar.

[6]considering the function $n(rs) = \frac{1}{2}$ constant and its derivative equal to zero

$$\theta = \ln[|1 - vs^2|] \tag{50}$$

$$\gamma = \cosh(\theta) \tag{51}$$

Both sees Earth (observer C) moving away from the bubble and Proxima Centauri (observer D) coming towards the bubble in a "fast forwarding" apparent velocity of $vs = v_i \times \gamma$

Although the real speed v_i of the warp bubble is $0,1$c taking 43 years to reach Proxima Centauri both A and B "sees" Proxima Centauri coming closer to them with an apparent velocity of 10c taking only $4,3$ months to complete the journey due to the "fast forwarding film" concept.

A and B "sees" the clocks on Earth and Proxima Centauri "accelerated" due to the "fast-forwarding" completing 43 years in $4,3$ months.A leap into the Future.[7]

Now we must analyze the point of view of 2 observers C and D one located on Earth(C) and another on Proxima Centauri(D) observing the ship passing by them carrying inside the bubble the observers A and B.

[7] we know this is unbelievable.but White mathematics is correct

3 The Natario Warp Drive using Lorentz Boosts according to the Harold White Spacetime Metric potential θ: Boost observed by an Earth-Proxima Centauri Frame Coordinates System

Writing again the equation of the warp drive spacetime according to Natario for the coordinates rs and θ(see Appendix E in [10] for details)

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs}drs + X^{\theta}rsd\theta]dt - drs^2 - rs^2d\theta^2 \tag{52}$$

And also the expressions for X^{rs} and X^{θ}:(see pg 5 in [2],see also Appendix D in [10] for details)

$$X^{rs} = -2v_s o(rs) \cos\theta \tag{53}$$

$$X^{rs} = 2v_s o(rs) \cos\theta \tag{54}$$

$$X^{\theta} = v_s(2o(rs) + (rs)o'(rs)) \sin\theta \tag{55}$$

$$X^{\theta} = -v_s(2o(rs) + (rs)o'(rs)) \sin\theta \tag{56}$$

Note that the Natario warp drive equation for an Earth-Proxima Centauri coordinates frame is exactly the same for a Ship based coordinates frame.The difference lies now in the form of the Natario shape function $o(rs)$ that returns 0 outside the warp bubble and $\frac{1}{2}$ inside the warp bubble.Note that this is exactly the inverse situation when compared to the previous Natario shape function presented in the last section.

The Earth-Proxima Centauri coordinates frame "sees" exactly the opposite of what is being seen by a Ship based coordinates frame.

Observer C(Earth) remains stationary with respect to the observer D(Proxima Centauri) and both sees the warp bubble passing by them with an apparent velocity $vs = 10c$ due to the "fast forwarding" concept.Both sees the clocks in the bubble accelerated and although observers A and B inside the bubble are moving with a real velocity $v_i = 0,1c$ these observers A and B are being seen by observers C and D with the clocks accelerated by the "fast forwarding" making a journey of 43 years in $4,3$ months.

The Natario shape function $o(rs)$ for an Earth-Proxima Centauri coordinates frame that gives $\frac{1}{2}$ inside the bubble and 0 outside the bubble written in function of the Natario shape function $n(rs)$ for Ship-based coordinates frame is given by:

$$o(rs) = \frac{1}{2} - n(rs) \tag{57}$$

- 1)- inside the warp bubble where the ship resides $o(rs) = \frac{1}{2}$
- 2)- outside the warp bubble where Earth(or Proxima Centauri)resides $o(rs) = 0$
- 3)- in the Natario warped region (warp bubble walls) $\frac{1}{2} > o(rs) > 0$

Rewriting again the equations for the apparent speed vs in function of the real speed v_i in the Natario spacetime we get:

$$vs = v_i \times \gamma = v_i \cosh(\theta) \tag{58}$$

-)apparent speed - generic case:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|]) \tag{59}$$

-)-apparent speed - equatorial plane motion:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2|]) \tag{60}$$

-)real speed - generic case:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^{\theta})^2|])} \tag{61}$$

-)-real speed - equatorial plane motion:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\ln[|1 - (X^{rs})^2|])} \tag{62}$$

When two observers one on Earth C and another on Proxima Centauri D ”looks” to each other both ”sees” the $o(rs) = 0$ outside the warp bubble and the terms X^{rs} and X^{θ} reduces to zero..Hence outside the warp bubble the White potential and Lorentz Boost are given by:

$$\theta = \ln[|1|] = 0 \tag{63}$$

$$\gamma = \cosh(\theta) = 1 \tag{64}$$

Outside the bubble $\gamma = 1$.There is no Lorentz Boost outside the bubble as seen by observers C and D and consequently no ”fast forwarding” concept.Both observers are stationary with respect to each other.The apparent speed vs is equal to the real speed v_i.This means to say that if Earth send a probe to Proxima Centauri with a real speed v_i and the probe do not trigger a warp bubble then this probe would reach Proxima Centauri in 43 years.

Now when both observers C and D outside the bubble sees the regions inside the bubble where observers A and B resides both C and D sees a region of spacetime where $o(rs) = \frac{1}{2}$ and hence[8]:

-)-generic case:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2 - (X^\theta)^2|]) \tag{65}$$

-)-equatorial plane motion:

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\ln[|1 - (X^{rs})^2|]) \tag{66}$$

Recalling the expressions for X^{rs} and X^θ when $o(rs) = \frac{1}{2}$ we have:

$$X^{rs} = -2v_s o(rs) \cos\theta = -v_s \cos\theta \tag{67}$$

$$X^{rs} = 2v_s o(rs) \cos\theta = v_s \cos\theta \tag{68}$$

$$X^\theta = v_s(2o(rs) + (rs)o'(rs)) \sin\theta = v_s \sin\theta \tag{69}$$

$$X^\theta = -v_s(2o(rs) + (rs)o'(rs)) \sin\theta = v_s \sin\theta \tag{70}$$

$$\theta = \ln[|1 - (X^{rs})^2 - (X^\theta)^2|] = \ln[|1 - (v_s \cos\theta)^2 - (v_s \sin\theta)^2|] = \ln[|1 - vs^2|] \tag{71}$$

In the equatorial plane $\theta = 0$ and $\cos\theta = 1$.Hence we have:

$$\theta = \ln[|1 - (X^{rs})^2|] = \ln[|1 - (v_s \cos\theta)^2|] = \ln[|1 - vs^2|] \tag{72}$$

Note that both results (generic and equatorial) are remarkably similar.

$$\theta = \ln[|1 - vs^2|] \tag{73}$$

$$\gamma = \cosh(\theta) \tag{74}$$

Both Earth (observer C) and Proxima Centauri (observer D) outside the warp bubble sees the observers A and B inside the warp bubble passing by them in a "fast forwarding" apparent velocity of $vs = v_i \times \gamma$

Although the real speed v_i of the warp bubble is $0,1$c taking 43 years to reach Proxima Centauri both C and D "sees" the warp bubble carrying inside the observers A and B approaching Proxima Centauri with an apparent velocity of 10c taking only $4,3$ months to complete the journey due to the "fast forwarding film" concept.

Earth (observer C) and Proxima Centauri (observer D) sees the observers A and B inside the warp bubble with the clocks "accelerated" due to the "fast-forwarding" completing 43 years in $4,3$ months.A leap into the Future.[9]

[8]considering the function $o(rs) = \frac{1}{2}$ constant and its derivative equal to zero

[9]we know this is unbelievable.but White mathematics is correct

4 Doppler Blueshifts from the Lorentz Boost and "fast forwarding" motion of Proxima Centauri approaching the ship

Consider the negative energy density distribution in the Alcubierre warp drive spacetime(see eq 6 pg 4 in [8],see eq 19 pg 8 in [1])[10]:

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{75}$$

And consider the negative energy density in the Natario warp drive spacetime(see pg 5 in [2])[11]:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{76}$$

In pg 6 in [2] a warp drive with a x-axis only is considered.In this case for the Alcubierre warp drive $[y^2+z^2]=0$ and the negative energy density is zero but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{77}$$

According to White(pg 4 in [8]) the energy density along the x-axis(equatorial plane) in the Alcubierre warp drive is zero. Look to the fig 2 pg 5 in [8].It depicts the negative energy distribution in the Alcubierre warp drive.The negative energy is in a toroidal region above and below the ship and there is nothing in front of the ship to protect it against impacts of hazardous objects as we outlined in our introduction of this work.

On the other hand in the Natario warp drive the negative energy exists in front of the ship to protect it against impacts of hazardous objects because in the equatorial plane of the Natario warp drive the negative energy density is not zero.

This is a major advantage of the Natario geometry when compared to its Alcubierre similar.The negative energy in front can deflect incoming objects protecting the ship from hazardous collisions.

Adapted from the negative energy in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

Now consider the White idea of the "fast forwarding" concept as being seen by the observers A and B inside the ship.They are at the rest with respect to each other and at the rest with respect to the ship itself.If the ship moves with a real velocity $v_i = 0,1c$ generating an apparent velocity $vs = 10c$ due to the "fast forwarding" as seen by Earth or Proxima Centauri then with respect to the observers A and B it is Proxima Centauri that is coming closer to them at $10c$ and Earth is receding away from them also at $10c$.If

[10] $f(r_s)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c = G = 1$
[11] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

Proxima Centaury approaches the observers A and B with the apparent velocity of $10c$ and the journey will be completed in $4,3$ months and not 43 years then its photons are Doppler Blueshifted by a factor proportional to $10c$ and not $0,1c$ and will reach them with the hazardous consequences already stated in our introduction.This is true for the Alcubierre spacetime but not for the Natario one.

Recalling the equations of the Lorentz Boost:

$$\theta = \ln[|1 - vs^2|] \tag{78}$$

$$\gamma = \cosh(\theta) \tag{79}$$

Note that as higher is the apparent speed vs the Lorentz Boost γ grows proportionally.

For a relation between v_i and vs in the case described by White $\gamma = 100$ in order to make a real velocity $v_i = 0,1c$ looks like an apparent velocity of $vs = 10c$.

But in order to reach the star Gliese 581 at 20 light-years away with possible exo-planets in the habitable zone[12] $vs = 10c$ is not enough.It would take two years to complete the journey.If someone wants to reach Gliese 581 in a month or so then an apparent speed of $vs = 500c$ is required and the γ would be immensely higher generating a devastating hazardous Doppler Blueshift for the ship in the Alcubierre case but not in the Natario one.[13]

Computing the relativistic Doppler Blueshift in function of γ is relatively easy:[14]

$$f_o = \gamma \times (1 + vs) f_r \tag{80}$$

In the above equation f_r is the frequency of the real photon emitted by Proxima Centauri and f_o is the observed frequency of the photon measured by A and B. Note that as higher is the γ the observed frequency grows proportionally.

A similar equation can be found in eq 3 pg 2 in [13]. Note that it is mentioned that as higher γ becomes then the COBE photons are Doppler Blueshifted to visible light and the visible light is Blueshifted into hazardous X-rays and they are mentioning "low" γ factors.

Fortunately a ship in the Natario warp drive spacetime do not have to worry about Doppler Blueshifts.

[12]see Wikipedia.The Free Encyclopedia

[13]recall our mention to Chad Clark,Will Hiscock and Shane Larson in our introduction

[14]see Relativistic Doppler Blueshift:Wikipedia:The Free Encyclopedia.Note that the equation presented there is for a photon moving away and the factor $1 - vs$ appears.The equation presented here is for a photon moving closer and the factor must be $1 + vs$

5 Conclusion

In this work we demonstrated that the Harold White idea of the "fast-forwarding" concept is entirely correct and applicable to the Natario warp drive Both Earth and ship makes "leaps" into the Future. This may somewhat sounds to be unbelievable but perhaps White is the correct solution to solve the problem of the interstellar navigation and the Natario geometry looks better when compared to its Alcubierre counterpart .

6 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[15]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[16]

7 Remarks

Some references of this work came from scientific web-servers available to consultation by the general public(eg:arXiv,HAL). We can provide the other references in PDF Acrobat Reader for those interested.

[15] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[16] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

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