AN EXACT VALUE FOR AVOGADRO'S NUMBER, THE FINE STRUCTURE CONSTANT, AND A FUNDAMENTAL DEFINITION OF THE KILOGRAM

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Abstract

I propose an exact value for Avogadro's number of 6.0221396992 $x10^{23}$ that is derived from theoretical considerations and that is independent of experiment and the Le Gran K. Naturally, a fundamental definition of the kilogram follows by dent of this fixed value. As a further consequence of these considerations, I also propose a fixed and exact value for the fine structure constant of $7.29735252341 \times 10^{-3}$. These results are precise, elegant, and unchanging with the passage of time.

I. INTRODUCTION

Avogadro's number N_A is the physical constant that links the macroscopic physical world to the invisible world of atoms. Presently, N_A is formally defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Its current state of the art estimated value is not based on experimental results using carbon-12, however, but on those obtained by x-ray diffraction in crystalline silicon lattices [1] or by the watt-balance method [2]. This definition of N_A and current experiments to estimate it both depend on the present definition of the gram that since 1889 has been defined as one-thousandth of the mass of the Le Gran K, a platinum-iridium cylinder kept in a vault in Sevres France. This creates a problem, however, as the mass of Le Gran K is known to change in time [3] and these changes cannot be exactly quantified because there simply is no perfect reference against which to measure them; Le Gran K is always exactly one kilogram by definition. The implied consequence of this is that the official mass of a single carbon-12 atom is changing with time, simply an unacceptable circumstance. *Le Systéme international d'Unites* (SI) identifies exactly seven basic units and their standards of measurement. Of these seven, the kg is the only unit that is still defined in terms of a physical artifact. For these reasons, a method to eliminate the need for this final SI artifact is highly desirable. Recent proposals to redefine the kilogram experimentally using manmade silicon spheres and the watt balance apparatus suffer from the same problem as Le Gran K; the experiments are inherently inexact, and the results are changing with time, equipment, and laboratory, and require combining the results of two experiments, thus the compounding of experimental errors. Decoupling N_A from Le Gran K and fixing it at an exact value could provide a new and fundamental definition of the kilogram and gram that is numerically fixed. Fixing the value of the kilogram numerical is identical to past decisions to fix the value of the second to exactly 9,192,631,770 vibrations of particular transition between two states of a cesium-133 atom, and that of the meter to exact distance light travels in1/299,792,458 seconds [4].

II. PRESENTATION AND DISCUSSION

A better definition of the kilogram would be to base it on the relationship between the gram and the gram-mole/gram- atom via Avogadro's number:

> *One gram-mole/gram-atom of any entity (element, chemical compound, etc.) is exactly equal to N_A units of those entities, implying that one gram is exactly the mass of N_A units of an entity divided by its (gram-molecular/ gram-atomic) weight.*

Ronald F. Fox and Theodor P. Hill [4][5] have proposed two alternate scenarios that fix possible values for N_A that are exact numbers and are independent of Le Gran K.

One is based on a hypothetical twelve gram, one gram-atom, cube of carbon-12 having exactly n atoms of carbon-12 on each edge, and for which:

$$n^3 = N_A* \quad (1)$$

In an initially publication, Fox and Hill [5] appear to take the cube root of the 2007 NIST recommended value for N_A, that when rounded to the nearest whole number is 84,446,888. In a later publication [4] they reduce this number to 84,446,886, as its cube is evenly divisible by twelve. Upon taking the cube results an integer value for N_A* that factors as 216 x $(14074481)^3$, which upon dividing by twelve implies that:

One gram is the mass of exactly 18 x ($14,074,481^3$) carbon- twelve atoms.

This result obviously is also an exact integer, as it must be to avoid an untenable situation. It is obvious that any integer value for n when cubed will yield an integer result. However, not all will yield a result evenly divisible by twelve, the gram-atomic weight of carbon-12. Their original n-value of 84,446,888 is one such number.

The second scenario results from the practical standpoint that carbon does not conform to an extended simple cubic structure but to that of a face-centered cubic (FCC) structure in three dimensions, the same as diamond and silicon. Thus, in reality, the carbon atoms are arrayed not only along the edges of a simple cube but also at the centers of faces made by a square of four adjacent planer atoms and at certain interior tetrahedral centers of cubes made of eight adjacent corner atoms. The number of atoms in such an actual FCC array with n atoms on each edge can be calculated as $8n^3 – 18n^2 + 15n – 4$ [5]. Thus, if one wished for a definition of N_A* specifically tied to the actual physical FCC lattice structure of carbon-12, one must replace Equation (1) with:

$$8n^3 – 18n^2 + 15n – 4 = N_A* \quad (2)$$

Fox and Hill report choosing n = 42,223,444 because it results in a value for N_A* that lies within the currently accepted range of values for N_A and is easy to remember. Although any integer value for n yields an exact integer result with Equation 2, the one chosen by Fox and Hill has the above described flaw, in that the result is not evenly divisible by twelve, implying that one gram is the mass of exactly a non-exact fractional number of carbon-12 atoms, again an untenable situation. Furthermore, it is easily shown that the only values for n that do result in integers evenly divisible by twelve must satisfy the formula

$$n = 12\mathcal{M} + 8 \tag{3}$$

where $\mathcal{M}$ is a whole number multiplier. However, there are only three multipliers that generate n-values that result in plausible N_A* integers. The three are 3,518,619; 3,518,620; and 3,518,621, resulting in n-values of 42,223,436; 42,223,448; and 42,223,460 respectively, thereby yielding in accordance with Equation (2) integer values for N_A. The results to fifteen significant figures are:

1. $6.02213766678450 \times 10^{23}$
2. $6.02214280130319 \times 10^{23}$
3. $6.02214793582480 \times 10^{23}$

Be assured, however, if carried out to 24 significant figures the results would be integers that divide evenly by twelve.

Even though the FCC model might be closer to physical reality than one based on a simple cube, practically speaking, once a fixed value for N_A is chosen, there no longer would be scientific necessity for an actual cube anyway, just as there has been none since 1983 to construct the perfect meter stick [4]. Therefore, in that sense, the use of a model is unnecessary. I will now propose and demonstrate a theoretical basis for fixing the exact value of N_A, dependent only upon a fixed value for the fine structure constant that results in a fundamental definition of the kilogram.

In a 2009 Physics Essay paper [6], I develop theoretical considerations that lead to two apparently equivalent but independent expressions for the Planck length l_p, as follows:

$$l_p = a_o/(\pi N\sqrt{3})$$

where a_o is the first Bohr radius and N is a large dimensionless number, and also:

$$l_p = a_o\alpha^{10}/(32\,\pi^3\sqrt{2})$$

Upon equating the right hand members of the above equations and solving for N results:

$$N = (4\pi/\alpha^5)^2\,\sqrt{(8/3)} \simeq 6.02213958208 \times 10^{23} \qquad (4)$$

using the 2006 CODATA recommended value for α of $7.2973525376 \times 10^{-3}$.

Within the stated limits of uncertainty, the above is virtually identical with the 2006 CODATA recommended value for N_A. However, result (4) is not evenly divisible by twelve, nearly so, but not quite:

$$6.02213958208 \times 10^{23} \simeq 12 \times 18 \times (14074479.9085)^3$$

Therefore, let:

$$N = \text{exactly } 12 \times 18 \times (14074480)^3 \qquad (5)$$

that upon substitution into Equation (4) and solving for alpha-apparent, yields:

$$\alpha_* = 7.29735252341 \times 10^{-3} \qquad (6)$$

to twelve significant figures, and differs from the above referenced CODATA value for α by only 1.94 ppb. Thus, if it is stipulated that $N = N_A{}^*$, then $N_A{}^*$ becomes fixed and unchanging according to result (5), which obviously yields an exact integer value evenly divisible by twelve. It should be apparent that if the decimal strings for $\boldsymbol{\pi}$ and α_* were carried to infinity, then by definition:

$N_A* = (4\pi/\alpha_*^5)^2\sqrt{(8/3)}$ exactly = $(84446880)^3$exactly =12 x18 x $(14074480)^3$ exactly = $6.0221396992 \times 10^{23}$ exactly. (7)

Even though the Equation (7) relationships are approximate when $\boldsymbol{\pi}$ and α_* are fixed at twelve significant figures, the approximations are so precise that I propose the value for the fine structure constant be fixed and exactly equal to α_*, and by dent of Equation (5) and the gram-atom/ Avogadro's number connection that :

One gram is the mass of exactly $(1/12)(4\pi/\alpha^5)^2\sqrt{(8/3)}$=

18 x$(14074480)^3$ *carbon-12 atoms in their standard state.*

and accordingly that*:*

One kilogram is the mass of exactly of $(1000/12)(4\pi/\alpha^5)^2\sqrt{(8/3)}$ =

18000x$(14074480)^3$*carbon-12 atoms in their standard state.*

Interestingly, this result is nearly identical to that proposed by Fox and Hill based on a simple cube of carbon-12.

Because these findings are nearly identical with those of Fox and Hill, one might conclude that they represent merely a play on the carbon-12 simple cube model that has exactly n carbon atoms on each edge, the only difference being the choice of the next lowest n-value cubed evenly divisible by twelve. There is a noteworthy difference however. Upon the one hand, Fox and Hill appear to have chosen their n-value (84446886) more or less randomly to fit the simple cube model and to result in N_A* closest to the then current literature value for N_A and be evenly divisible by twelve. Upon the other hand, with only a slight tweaking of the fine structure constant, independently derived Equation (4) results in a value for N_A* that is the perfect cube of 84446880 and that is evenly divisible by twelve. Thus, the simple cube model did not preface the result; rather the result appear to preface the model.

III. **CONCLUSIONS**

Theoretical considerations have led to a proposed exact value for the fine structure constant that differs from the 2007 CODATA recommended value by only +1.94 ppb. Adopting this exact value for the fine structure constant would eliminate forever its ever changing value with time due to changing state of the art equipment and experimenter, and also result in the unique number N_A*, that differs from the 2007 recommended CODATA value for Avogadro's number by only a – 0.35 ppm. Adopting N_A* for Avogadro's number would be simple and would have profound experimental and theoretical benefits. Most importantly, it would result in a new definition of the kilogram that is precise, elegant, and unchanging in time, eliminate forever dependence upon the manmade kilogram artifact, and have the advantage of making the definitions of the atomic mass unit and mole explicit, clean, and simple.

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