Zero Division Algebra

An Introduction to Temporal Mechanics

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Introduction to Temporal Mechanics is the result of the merger a merger of Zero Division mathematics principles with Special Relativity. This text shall lay the groundwork for a full Temporal Mechanics course, a union of Zero Division mathematics with General Relativity and Quantum mechanics. Though this text will focus on introductory principles it remains fundamental to Temporal Mechanics. The fundamentals of this course shall describe our space-time as a set of space-like directions. At its heart are concepts which allow us to make sense of and deal with infinities in mathematics. As you read you will be reintroduced to mathematical ideas which have perplexed the human mind for millennia. The text will provide instruction in what is actually implied when we divide by zero and show real solutions to such problems. You shall see how the process of evaluating such an expression implies the existence of additional directional axes within which our universe exists and though present are not normally noticed.

With this text you will learn to tackle mathematical curiosities such as the negative radical and the indeterminate form as you put them to work in solving problems you once thought unsolvable. You will see how a necessary and critical difference between the way we define direction and dimension ultimately resolves what we think of as time to a space-like concept. At the end of this text we'll explore implications of what this means for definition of where an object is located in space time and what happens as its speed approaches that of light.

Please be patient with these new concepts. Work your way through the examples and sample problems and you will see how easily these processes can be applied to attain accurate and useful answers in the real world. When you complete the text you will have a new and powerful mathematical tool at your side. You will never be able to see space or time the same way again.

Robert S. Miller

Chapter 1

Introduction

Subspace—1.a:

This is not the Subspace you've learnt about in Linear Algebra classes. Instead of vector spaces it is literally a layer of space in which we all live in. We see it every day without realizing we are looking right at it; it is hidden from us in a sense. The simplest explanation for subspaces is that they are a space-like representation of time. There exists a subspace axis for every basis vector in a complete basis set. These subspaces collectively define the various places where events, in what is normally understood to be time have, are and will occur. This type of time we shall define below as *dimensional time*. As you will soon learn this idea is markedly different than the idea of *directional time.* Conversion to subspace follows specific rules and reasoning. In order to understand exactly what a subspace is and how to convert to it from a given equation we'll start with an examination of number lines, multiplication and division.

Understanding Subspace—1.b:

What exactly is a number line? It's really nothing more than a pictographic representation of counting. We assign values by combining two or more number lines, and then define operations based upon gain or loss within that system. Zero is a transition point along any number line where values flip from negative to positive. In contrast to the usual holding that 0 is neither positive nor negative Temporal mechanics maintains zero is both positive and negative simultaneously.

Zero can be several things. Yes, it can represent the idea of nothingness or emptiness. However within the number line, its meaning is one of balance, not absence. There is no gain or loss represented as this point has an equal amount of positive and negative numbers extending out to infinity to its left and right. This idea, that a number line axis, represented by a variable, is infinite, is important. We'll return to build more on this idea momentarily.

If you're guessing we are about to delve into the topic of division by zero you're correct. This operation is central to how we define subspace and all other mathematics which follows. Division and multiplication are opposite functions. For any mathematical operation to be determined accurate we must be able to both check that operation by its opposite and be able to use it to accurately portray real world situations. We know that $\frac{10}{5} = 2$ because $5 \cdot 2 = 10$. Just as surely we know that a car traveling at 5mph for 2 hours shall travel a total of 10 miles. The math is verifiable and it can be used for predictive results in the real world. Later I will show this can be done for functions which result in division by zero in the same fashion. For as long of humans have used mathematics problems creep up when a zero exists in the denominator of a fraction. Consider the following two examples.

$$\frac{2}{0} = x \qquad \text{and} \qquad 0x = 2$$

Without the inclusion of subspaces the traditional understanding of both multiplication and division say there are logic issues and the operations cannot be executed. The first expression $\frac{2}{0}$ has been previously labeled as undefined because the output value will be infinity. In the second instance traditional mathematics claims there is no definable value which can satisfy the expression. Any value for *x* will equal *0* and $0 \neq 2$.

This reasoning is normally satisfactory for most operations. However it remains inaccurate because this mathematical reasoning is incomplete, based on an underdeveloped understanding of what multiplication and division by zero imply. 0 is not meaningless. Its presence in any such operation will have a verifiable output value. One of these expressions does have logic issues because its just not possible. But not for $\frac{2}{0} = x$. The output value for this expression is found via separate axes which, though present and implied, aren't written. The solution relies on rewriting the expression to include the existence of these output axes which are, subspaces.

Simple math can be used to rewrite $\frac{2}{0}=x$ as $0x=2$ however these two expressions only look equivalent. In the following chapters we shall find $\frac{2}{0}=0$. Although $0x$ cannot equal 2 it definitely does equal 0 and is in keeping $\frac{2}{0}=0$.

Using the steps for subspace transforms you'll find that $\frac{2}{0}=x$ implies the point $(x|s)=(0|0)$. The term s, a subspace of the x-axis, will be described in detail shortly. The circle-plus operator $\oplus$ will be seen a lot in subspace math. It is used to indicate an expression is simultaneously positive and negative. Whenever any non-zero term is divided (*or multiplied*) by zero there will exist a definable value in a subspace of that system whether or not a real solution can be obtained within the confines of the original expression alone. These subspaces always exist. An explanation of how to arrive at this understanding must begin with previous attempts and failures to define division by zero.

Early Attempts—1.c:

The Brahmasphutasiddhanta of Brahmagupta—1.c.1:

Brahmagupta lived in Ujjain, India from 598 to 668 AD and made notable contributions to mathematics and astronomy. In 628 AD he is reported to have written a text called the Brahmasphutasiddhanta, *The Opening of the Universe* in English, within which is found the earliest known attempt of defining division by zero (Wikipedia: The Brahmasphutasiddhantaof Brahmagupta, *http://en.wikipedia.org/wiki/Brahmasphutasiddhanta*, and *http://www-history.mcs.st-andrews.ac.uk/Biographies/Brahmagupta.html*). Brahmagupta attempts to define division by zero as equaling zero:

$$\frac{a}{0}=0 \text{ where } a \text{ is any constant}$$

As you'll see this is in a sense is correct but only if subspace is introduced to the logic. Otherwise you cannot reach this conclusion. The definition given by Brahmagupta appears to be drawn from the idea of defining any function by its opposite. For example we know that $a \cdot 0 = 0$, and $\frac{0}{a} = 0$ are both true statements. It's easy to see how the assumption could be made but it's still unverifiable without a subspace connection. Consider the function $y = \frac{1}{x}$. The output value approaches infinity as x approaches 0. Infinity cannot be defined in traditional mathematics as within which it represents and impossibly huge value—not zero.

If you divide 1 by a very small number you get a very large output. The reasoning is simple logic. The smaller the denominator, the more times a unit of that value can be added together to get one whole—the numerator. The smaller that denominator gets, the larger the quotient becomes. When *x* equals 0 the reasoning says the output has to reach infinity. Again the answer is clearly not zero within the logic of traditional mathematics.

Something has to be happening at this value that traditional reasoning has failed to explain. If we can divide by other values and obtain real results one must exist for this expression as well, it just requires understanding beyond that of Brahmagupta's time. We shall see later that saying an output is infinity does not actually mean there is no answer. *y* cannot reach the assumed value of infinity. Instead it implies the output on the *y*-axis in obtainable by examining the value reached on its subspace axis. This axis is turned 90 degrees to all other which is why we express it as $y = \infty$ without the subspace's inclusion. The expression at the point of division by zero requires understanding of an additional output axis which though present was never included. By including subspace we shall see where the output value actually lies, removing the infinity and defining the value of *y* itself.

Mahavira and Bhaskara II—1.c.2:

Two others followed Brahmagupta in an attempt to correct his definition of division at zero by asserting their own. In 830 A.D. Mahavira wrote a text titled *Ganita Sara Samgraha* in which he claims *A* *number remains unchanged when divided by zero* (Wikipedia: Division by Zero—*http://en.wikipedia.org/wiki/Division_by_zero*).

$$\frac{a}{0} = a \text{ where } a \text{ is any constant.}$$

Mahavira was undoubtedly brilliant but still made an error. If a number remains unchanged after division, it has either been divided by 1 or not divided at all. With the function $y = \frac{1}{x}$ we clearly see this. One divided by one is one. As *x* becomes larger or smaller than one, no matter how slightly so, *y* will become infinitely small or large respectively. So just avoiding the issue of division at zero and claiming no change takes place won't work.

Bhaskara II, another Indian mathematician who made contributions to calculus, tried his hand at the problem too. He is said to have made the assumption that…*when a finite number is divided by zero, the result is infinity* (http://en.wikipedia.org/wiki/Bhaskara_II*and*http://en.wikipedia.org/wiki/Division_by_zero).

$$\frac{a}{0} = \infty$$

This expression is the foundation of our traditional argument for this operation. It is true, and verifiable in the function $y = \frac{1}{x}$ that dividing by 0 will result in an infinite value for *y* but it doesn't explain what that means. In reality *y* will reach a limiting value. This occurs when the accuracy of the measuring apparatus we use to define how close we are to 0 on the *x* axis can no longer, with certainty, tell the difference between 0 on any other definable non-zero point. When that occurs, though we could mathematically define an ever smaller number for *x* we could not move that distance without actually saying $x = 0$.

I mentioned earlier that saying something is infinite is actually not the same thing as saying you have no answer. Though the presence of infinity is no different than an expression which is divided by 0 it does provide a clue how to proceed. The answer comes from a close examination of our ideas of the space we use to define points, lines, and curves on a graph. The examination of this real space will show solutions are more complex than previously thought and inseparably linked to the presence of these extra, hidden directions. Before making this examination let's look at one method of how to obtain a subspace equation.

All & One—The Indeterminate Form—1.d:

Consider the following Limit:

$$\lim_{x \to 0} y = \frac{1}{x}$$

The process obtaining a Subspace equation requires multiplication on both sides of a real space equation by an expression which can simultaneously relate an infinite quantity to a real number. That expression must equal infinity to represent and replace the value *y* attempts to reach when dividing by zero, and yet still hold the value of 1 to prevent from changing the value of the equation despite altering its form. There is only one figure which satisfies this requirement; the Indeterminate Form $\frac{0}{0}$.

I'm certain that everyone reading this is appalled, their mind recoiling in horror at the absurdity they have just been presented with. Is it really that absurd? Think about it. If accepted that any value divided by the same value equals one then we can define $\frac{0}{0}$. If I made the substitution that $a = 0$, then the indeterminant becomes $\frac{a}{a}$. If one didn't know what *a* was this expression it would likely be defined as equaling 1 so long as $a \neq 0$. Try to keep an open mind about this. If 0 represents a set of nothingness or *emptiness,* then dividing it into the same size whole would be one; the division process leaves the set unchanged. Thus we would define $\frac{+0}{+0} = \frac{-0}{-0} = 1$. Also note this is representative of this value as a constant, not as the result of a function, a distinction of importance that will be explained later.

Though this is true a reason the indeterminate is not traditionally accepted to take this value is because other values apply to the ratio. Every number has an infinite amount numbers to its positive and negative side. 0 however is different in that it has an infinite amount of *only* positive numbers to its right and *only* negative numbers to its left. Zero is usually defined as being neither positive nor negative but this ignores what happens at the origin. Zero is not separate from the number line axis. It is the transition point from positive to negative. It cannot be neither positive nor negative. Instead it is positive and negative at the same time.

When considering 0 itself in basic, traditional mathematics this wouldn't even be noticed. In the expressions $0 \cdot a$ and $\frac{0}{a}$ we will still obtain 0 as a result regardless of whether we consider 0 positive or negative. The indeterminate is different. Zero being simultaneously positive-and-negative means it is possible for either the numerator or the denominator to have the opposite sign of the other. Hence it also equals $\frac{+0}{-0} = \frac{-0}{+0} = -1$.

Additionally we can say this operation asks how many times can we place nothing into a space holding nothing—an infinite number of times. Since zero is positive-and-negative it is possible to define this evaluation of the indeterminate as either $+\infty$ or$-\infty$. However, since infinity is infinite we shall simply label it as the unsigned infinity, ∞ as it includes both of the signed values within it.

None of the three possible evaluations of the indeterminate are any more correct than the other two and the reason for which I suspect this expression is called *The Indeterminate form.* Nonetheless which value applies depends on the situation in which we find this unique figure and how it happens to show up. The only real mathematical object which can equal infinity is a number line. The presence of an infinite will allow the substitution of a new variable representing a subspace number line within the system defined by an equation. We define the indeterminate expression as $\oplus 1 = \frac{0}{0} = \infty$. The infinite will ultimately be replaced by a new variable with its own name. The $\oplus 1$ used in this particular relation is called the Surface Value.

Think about it. If an infinite occurs as a limit for an expression it means two things. Firstly the value that output attempts to reach cannot be achieved. Secondly, the reason it cannot be achieved is it implies the output, without association to a subspace, cannot lie on that axis because it is infinitely far away. What is there in traditional mathematics which is known to equal infinity? The only thing which satisfies this is an entirely new number line which by its very nature contains an infinite set of numbers. It has to be this way. We cannot simply say the infinite resulting from division by 0 applies to the entire number line representing the actual written output term. For $= \frac{1}{0}$, the *y* already represents the entirety of its own number line just by being present. This is the same for *x.* The function tells which specific value of *y* we are defining. Calling the value *y* tries to reach when *x* = 0 infinity is misleading. Infinity is impossible because it is infinitely far away. So the infinite must be something else. That something is a different, third axis.

If you have only the *y* axis as a number line and are told to plot the point *z* = 1 on that line it can't be done. The point *z* = 1 lies on the z-axis. It doesn't exist anywhere on the *y*-axis. From the perspective of the *y*-axis this point is infinitely far away. Though it's not the z-axis, if $y = f(x)$ results in an infinite for the y-value you must look to a paired subspace axis to understand what the infinite means. So when we obtain an infinite output in an expression it's a clue that the output sought lies on an axis which isn't included in the expression. To determine the expression of the subspace axis we have to deal directly with that infinite.

When obtaining a subspace equation, a mathematical procedure called a Transform, the Indeterminate Form does not equal one or the other of the answers discussed. Instead it equals all of them at the same time. Though 1 clearly does not equal infinity this not a logic issue but rather a manifestation of two separate ways of interpreting what this specific division process describes. It's not surprising the term is considered undefined in modern mathematics as it will cause mathematical absurdities if not applied correctly. When used properly, by following a set of rules not unlike those we use to define order of operations, the absurdities will not arise.

The Indeterminate Form is necessary for making a mathematical transformation into the subspace form of an equation because it can relate real number values to infinites. When we convert to subspace we are exchanging a variable of one axis for another. Since an infinite in an equation indicates that a value on the given output axis cannot be reached within the confines of the original expression, we must multiply that side of the equation by a term to cancel the infinite and replace it with a real number. The other side of the equation will be multiplied by the same term but will reinsert an infinite value in the form of a new subspace axis.

So, whether or not an infinite value will be generated in an equation, we can use the infinite solution of the indeterminate form obtain the subspace form of an equation but for a singular exception. The subspace conversion removes the output term replacing it with $\oplus 1$. The opposite side of the equation which contains the input values will always take the new variable. The exception occurs when solving for a subspace against the function side of an equation instead of its output variable. This value is the axis from which rotations are made in either Polar or Spherical coordinates to reach other values. The reason why it is special will be made clear in the trigonometric subspace transformations. When performing a subspace transformation on the function side of an equation a unique Subspace is added and signs of terms will be adjusted but nothing is removed. The reason for this will be covered in greater detail in the section on Subspace Transformations. For now know that it hinges upon this subspace having a direct relation to the concept of the passage of time.

The following statements will collectively provide a definition for division by zero, provide the transformation equations for subspace terms and fully represent all values of the indeterminate form. Some of the statement require explanation to verify their accuracy; this is provided throughout the following sections.

Lemmas 1 through 7—1.d.1:

Lemma 1:

A subspace is a number line axis paired with but orthogonal to other given defined axes, as well as any other subspace axis which may be present, used to plot point positions via an equation. It / They exist(s) simultaneously with other defined axes, implied by, but not necessarily included in the original equation.

Lemma 2:

Real space shall refer to the three space axes, *x, y,* and *z,* used alone or in combination with each other, to define locations within the visually observable universe.

Lemma 3:

Subspaces shall refer to the three subspace axes, *s, u,w*, and *A* used alone or in combination with each other, to define locations within the space of time.

Lemma 4:

Traditionally, any value divided by 0 will result in an infinite. The only object which can be used in mathematical operations and still represent an infinite value is a number line, represented by a variable term, which also represents an axis labeled by that term. The operation which relates a real space axis to a subspace will show the real value of any non-zero number divided by 0 to be 0 via relation of the output axis and its subspace to an infinite implied by the zero division expression. The constant $\frac{0}{0}$ arising in a function must be evaluated depending upon whether its presence represents and infinite which shall resolve to 0, or a constant which shall resolve to 1 wherein it originates.

Lemma 5:

Any term either divided by zero, or multiplied by 0 while set equal to a non-zero value, is an improper expression, which must be rewritten in a proper format. Only then can the subspace form it implies be determined and the real output obtained.

Lemma 6:

Transformation between the given real space equation and the subspace(s) it implies is achieved by multiplying both sides of the equation by $\frac{0}{0}$. The expression is evaluated depending upon whether it lies on the output or function side of the equals sign using the relation that:

Surface Value = Indeterminate = Subspace term

$$\oplus 1 \quad = \quad \frac{0}{0} \quad = \quad \infty$$

Lemma 7:

Information must be conserved in a transformation so as to not destroy the primary input(s) of a function. Values for the indeterminate on each side of the equal sign must support this consideration. When applying the Indeterminate to an output, it is distributed first and then evaluated. When applying the Indeterminate to a function and its inputs, it is evaluated and then distributed.

Chapter 2

Subspace Transformations

The Transformations—2.a:

The idea of time will immediately show up when exploring the several types of subspace transformations. Disturbing, will be the implication that three very different ideas for time exist. Mentioned in the introduction, there is a difference between time as a direction and time as a dimension. Additionally there is a special time variable *T* obtained when we take the subspace of the function side of any equation. Though a simplification, this indicates a flow of time, commonly called the arrow of time. Its quality is seen manifest in progression of directional time.

The rate of change of events, as the flow of time progresses is what most people think of when they consider the idea of time. i.e. How long it takes for something to occur or when something happened. We call this *Directional Time.* It is indicated by *t* and used to indicate length of time as perceived rates of change in traditional three-space. Subspace values are related to this idea but are not simply interchangeable with it. The subspaces themselves show where events occur in what we would consider the past, present and future for all possible outcomes. The direct relation between subspace and *t* is more complex and will be covered in depth during progression through the material. The subspace values we define as *Dimensional Time*. The descriptions for why subspaces must take this role are provided in the next chapter.

Transform 1a—2.a.1:

Consider an expression of the form $y = \frac{a}{x}$

Here *a* equals any non-zero constant, and x is a variable. Two methods exist for transforming an equation like this into its subspace form.

Method 1—Direct Conversion—2.a.1.*i*:

1. Multiply *y* by $\frac{0}{0} = \oplus 1$. Distribute *y* first and then evaluate.

2. Multiply the input / function side by $\frac{0}{0} = \infty = New\ Subspace\ Term$. Evaluate, and solve for this term. You may NOT distribute the new term through the equation before you have solved for it alone, otherwise it would alter form of the function, disrupting the transformation process.

$$\left(\frac{0}{0}\right) y = \left(\frac{a}{x}\right)\left(\frac{0}{0}\right)$$

$$\left(\frac{0}{0}\right) = \left(\frac{a}{x}\right) u$$

$$\oplus 1 = \left(\frac{a}{x}\right) u$$

$$u = \oplus 1 \left(\frac{x}{a}\right)$$

3. The $\oplus$ indicates that the value is a subspace value. It cannot be plotted on the *XY*-plane; only on the *XU*-plane. The $\oplus$ term can be thought of as $(+\,-)$ since the transform is not yet complete. The multiplication by the $+$ sign of the $\frac{x}{a}$ will multiply all of the signs together, causing the value to take only one sign. (*The specifics of this process and other arithmetic with the* $\oplus$ *sign is detailed in Chapter 5-Zero Division Algebra*).

$$u = (+-1)\frac{+1}{+a}$$

$$u = -\frac{x}{a}$$

The *u*-axis is the subspace of the *y*-axis output in the $f(x)$ equation. The same process is used on the *x*-axis as a singular variable to obtain its subspace. Being a free variable the *x* input has no defining equation, so it is simply set equal to itself. It has to be this way for any input parameter.

$x = x$ Mathematical truism

$$\left(\frac{0}{0}\right) x = x \left(\frac{0}{0}\right)$$

$$\oplus 1 = (x)s$$

$$s = -\frac{1}{x}$$

<u>Method 2—Trigonometric Conversion—2.a.1.*ii*:</u>

The second method is more general. It involves trigonometric manipulation which represents how events in the 2D plane are separated from each other within the Hyperplane formed by the addition of the subspace directions.

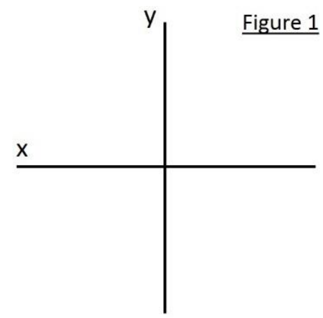


Figure 1

The Subspace axes are there, they're just hidden. If we are interested in just *u* or just *s,* from the point of the Cartesian Plane w*e* are in a sense looking at it head on. This is a way of viewing time as a space-like direction. This is not the rate of change of events, how long it took for something to occur. Instead it defines the place in subspace, where events in space have occurred—past, present and future—the separation of individual events through time in separate portions of continuous subspace.

Part of the Hyperplane is shown here at right in Figure 2. The *u*-axis is orthogonal to *x, y* and *s* but this is still a plane. Events occurring on the plane, defined by $y = f(x)$, occur at a rate of change defined by derivatives of that function. Each event or point though persists in subspace even though we view it as being in our past or perhaps distant future. Should one find a way to move directly through subspace it would be possible to, at a later time *t*, to return to an already passed point, even though the return may exist in the past as defined within subspace.

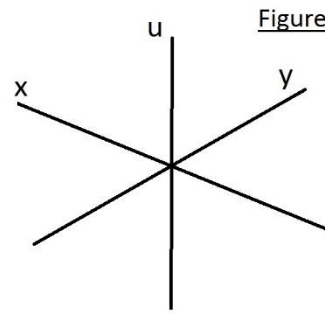


Figure 2

Another way of looking at the hyper plane is that advances through the time direction are rotations of the entire plane through subspace. The picture here below shows how this rotation might be conceived.

a. Trigonometric descriptions of points in the Cartesian plane are governed by polar coordinates

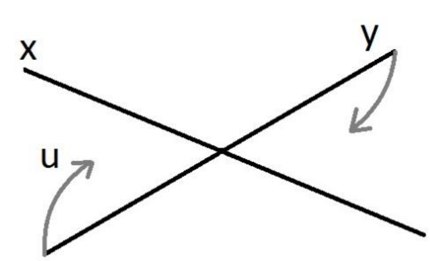
b. These are: $x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$r^2 = x^2 + y^2$

Figure 3



Method 2:

The original equation is in the form of $y = f(x)$. This method directly relates the circular rotation of angle Theta in real space to the value attained on the *u* and *s* axes. Note however that actual subspace angles are, hyperbolic, not circular. More will be covered on this in a later section. From here forward subspace conversions shall be performed by applying two new operators. We define The Surface Value Operator as $\frac{0}{0} = \Xi$. We shall also define the Subspace Operator $\frac{\mathrm{s}}{\mathrm{sn}}$ where *n* indicates a variable in a given expression upon which a subspace transformation is made.

1. Use the equations $x = r\cos\theta$, $= r\sin\theta$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ and $r^2 = x^2 + y^2$ to determine the values for r and θ.

2. With these values known apply the Surface Value and Subspace Operators to both sides of the *x* and *y* equations. The *x* value has an actual equation in this instance and will be take the transform in the same fashion as the *y* equation.

   a. Distribute through the output and evaluate the indeterminate on the left side of the equation with the Surface Value Operator Ξ.

   b. On the right, function side of the equation, evaluate $\frac{s}{s(r\,,\theta)}$, and solve for the subspace term. Remember it cannot be distributed. You must solve for it first to prevent disrupting the transformation process.

   $\Xi x = \frac{s}{s(r\,,\theta)} r\cos\theta$ $\qquad\qquad$ $\Xi y = \frac{s}{s(r\,,\theta)} r\sin\theta$

3. The left side of the equation is replaced by ⊕1 while the right side takes the new subspace term.

   $\oplus 1 = (r\cos\theta)s$ $\qquad\qquad$ $\oplus 1 = (r\sin\theta)u$

4. Finally solve for *s* and *u*

   $s = -\frac{1}{r}\sec\theta$ $\qquad\qquad$ $u = -\frac{1}{r}\csc\theta$

These two methods will yield the same results as method one. Consider the following example. Evaluate the equation $y = \frac{1}{x}$ for the values $x = \{2\,,3\}$. Show the subspace equations and solve using both methods. *Note that decimals in the trigonometric operations have been rounded off. If you perform the operations without rounding the values you will receive the answer shown. Otherwise you will receive an approximation accurate to the decimal value included.*

<u>Method I:</u>

$x = 2$

$y = \frac{1}{x} = \frac{1}{(2)}$

<u>Method II:</u>

$x = 2$

$r = \sqrt{x^2 + y^2} = \sqrt{4.25} \approx 2.062$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(.25) \approx 0.245\, rads$$

<u>For the Y-subspace *u*:</u>

$\Xi y = \frac{\mathrm{s}}{sx}\frac{1}{2}$

$\oplus 1 = \left(\frac{1}{2}\right)u$

$u = -2$

$\Xi y = \frac{\mathrm{s}}{\mathrm{s}(r,\theta)}(2.062)\sin(0.245)$

$\oplus 1 = \left((2.062)\sin(0.245)\right)u$

$u = -\frac{1}{2.062}\csc(0.245) = -2$

<u>For the X-subspace *s*:</u>

$\Xi 2 = \frac{\mathrm{s}}{sx}2$

$s = -\frac{1}{2}$

$\Xi x = \frac{\mathrm{s}}{\mathrm{s}(r,\theta)}(2.062)\cos(0.245)$

$s = -\frac{1}{2.062}\sec(0.245) = -\frac{1}{2}$

If we perform the subspace transforms again we return to the original *x* and *y* values.

<u>By Method I:</u>

$\Xi u = -\frac{\mathrm{s}}{sx}2$

$\oplus 1 = (-2)y$

$y = \frac{1}{2}$

$\Xi s = -\frac{\mathrm{s}}{ns}\frac{1}{2}$

$\oplus 1 = \left(-\frac{1}{2}\right)x$

$x = 2$

The same processes are used to obtain the subspace values when *x* = 3. Repeating them will reverse the operation and again provide the original values.

<u>By Method I:</u>

$u = -3$

<u>Method II:</u>

$u = -\frac{1}{3.018}csc(0.1107)\ rads$

$u = -\frac{9.052\ldots}{3.018\ldots}$

$u = -3$

$s = -\frac{1}{3}$ $\qquad$ $s = -\frac{1}{3.018}\sec(0.1107) = -\frac{1}{3}$

We can see from the process of the conversion that subspace values are the negative reciprocates of their real space counterparts. This is the general meaning of a ⊕ value; the real space component seen is partnered with its negative reciprocate in subspace. A ⊕ number appears positive and negative because the subspace axis is being viewed edge on. We can see the real space value on its axis but not the subspace one without a transform operation. Despite this the influence of both signs are present.

<u>Transform 1b—2.a.2:</u>

When one performs either the Direct or Trigonometric transformation on the function side of an equation, called Inverse Transformation, what is obtained is not a subspace of either the input or the output. To perform a subspace transform on the actual function containing the inputs the application of the Surface Value and Subspace Operators is reversed. However the restriction that input values not be removed is maintained. The removal of any part of the equation would alter its form. Since nothing is removed the result will always be a constant. The value obtained, though not itself a subspace of either input or outs variables, does represent a quality possessed by all of real space and subspace together. Subspaces represent space-like time directions. The constant obtained by performing a subspace conversion on the function portion of an equation generates *T.* This value represents the flow of time. Defined as the Temporal Constant it is its own subspace.

<u>Method 1—Direct Inverse Transformation—2.a.2.*i*:</u>

The Temporal Constant is obtained as if it were a subspace of $f(x)$. in a $y = f(x)$ equation the *y* component now will not be removed. The subspace operator $\frac{\mathrm{s}}{sn}$ acts on the *y* side inserting the temporal constant *T*. The surface value operator Ξ multiplies the function side with ⊕1.

1. Given any expression of the form $y = \frac{a}{x}$ where *a* equals any non-zero constant, and x is a variable, multiply both sides by $\frac{0}{0}$ using the operator Ξ upon $f(x)$ and $\frac{s}{sn}$ on y.

2. This transformation process provides the temporal constant. As this is not a subspace there is no need to remove the output variable. First evaluate Ξ and then distribute. Because nothing has been removed these terms may be multiplied directly (*Had this been used on y it would have been distributed and then evaluated*.) Then operate $\frac{s}{sn}$.

$$\frac{s}{sy} y = \frac{a}{x}\ \Xi$$

$$Ty = \frac{a}{x}(\oplus 1)$$

3. Divide both sides by *y.* Note that this is itself known to be equal to the original expression. Substitute and simplify.

$$T = \frac{-\frac{a}{x}}{y} \qquad T = -\frac{\frac{a}{x}}{\frac{a}{x}} \qquad T = -\frac{a}{x}\left(\frac{x}{a}\right) \qquad T = -1$$

So far so good. Now what about returning to the original form? Nothing was removed from the expression. It just all got wrapped up into that -1. Further, T is not a subspace of any directional axis. In repeating the process the only thing T can be replaced with is T.

$$T = -1 \qquad \Xi T = -\frac{s}{sy}1 \qquad \oplus 1 = -(T)$$

Replaced with itself in the transformation there is nothing to multiply the $\oplus 1$ by except the -1 which is modifying the *T*. Multiply both sides by -1 to make *T* positive.

$$T = 1$$

In this situation *T* really does literally equal $\oplus 1$. Don't fret about the presence of the negative 1. There are two ways of looking at what this means. Temporal Mechanics defines our motion through time literally as motion through a seven-directional, multi-dimensional space. Three of these are the space directions we are familiar with. The distance one travels through the others (*dimensional time*) represents the passage of what we perceive as time (*directional time*), from past to present to future.

$T = -1$ implies the idea of the observer being stationary in time. As choices are made events from the future flow backward toward you before receding away into your past. Just the same, $T = 1$ may be understood that all events and outcomes, for all possible choices are present within subspace and it is you who moves along a given path through that domain as you make your choices, plotting your own path to a unique reality you define by your past-present alignment. Regardless the picture of time this creates, it is one reason we always perceive t as a positive, forward moving value.

Method 2—Trigonometric Inverse Transformation—2.a.2.*ii*:

Using the equations for polar coordinates first determine the values of *r* and θ. Then perform the subspace transforms. Regardless of whether one choses the *x* or *y* component the inputs are *r* and θ. Evaluate the function side with the Ξ operator and the solution side with $\frac{\text{s}}{sn}$ operator. The original output value is not removed.

$$\frac{\text{s}}{\text{sx}} x = r\cos\theta \;\; \Xi \qquad\qquad \frac{\text{s}}{\text{sy}} y = r\sin\theta \;\; \Xi$$

$$Tx = \oplus 1(r\cos\theta) \qquad\qquad Ty = \oplus 1(r\sin\theta)$$

$$Tx = -r\cos\theta \qquad\qquad Ty = -r\sin\theta$$

$$T = -\frac{r\cos\theta}{x} \qquad\qquad T = -\frac{r\sin\theta}{y}$$

$T = -\frac{r\cos\theta}{r\cos\theta}$ $\qquad$ $T = -\frac{r\ \sin\theta}{r\sin\theta}$

$T = -1$ $\qquad$ $T = -1$

Repeating the operation again provides $T = 1$. There are three other general formats for a given equation upon which subspace transformations may be applied. Method 1 or 2 may be used to make the transformation. They are shown here, completed by Method 1. You may try method 2 at your leisure.

Transform 2a—2.a.3:

An expression of the form $y = \frac{a}{x}$ where *a* equals 0, and x is a variable. An interesting event occurs here. When you attempt the transform you end up with another undefined.

$$\Xi y = \frac{\mathrm{s}}{\mathrm{sx}}\left(\frac{0}{x}\right)$$

$$\oplus\ 1 = \left(\frac{0}{x}\right)\infty$$

$$\oplus\ 1 = \left(\frac{0}{x}\right)u$$

This equation is in terms of *u*. So we want to keep *u.*

Multiply by *x* and then divide by zero.

$$-\frac{x}{0} = u$$

$$u = -\infty$$

Look at the first three steps to this equation very carefully:

$$\Xi y = \frac{\mathrm{s}}{\mathrm{sx}}\left(\frac{0}{x}\right) \qquad \oplus\ 1 = \left(\frac{0}{x}\right)\infty \qquad \oplus\ 1 = \left(\frac{0}{x}\right)u$$

Whenever a subspace transformation is conducted you are linking a new variable value with an infinite and the output which was removed from the equation. In this example that new variable is u. If we repeat the subspace transform on $-\frac{x}{0} = u$ we get back the original expression $y = \left(\frac{0}{x}\right)$. So an infinite arising in $y = f(x)$ at a specific value of x may be replaced by the corresponding value on u at the same value of x. Likewise an infinite occurring in $u = f(x)$ may be replaced by the value y equals at the same value of x.

We can evaluate the expression $u = -\infty$ such that $-\infty = y$ without changing the nature of what it implies. In other words, in $u = f(x) = -\frac{x}{0}$, we have $u = y = 0$ for all x. When x = 0 we get $\frac{0}{0}$ in both equations. Because the term comes from an equation instead of being inserted as a constant it must be evaluated within the construct of the function. By direct evaluation within the function this value will take infinity and is then evaluated with its relation to its subspace.

$u = \frac{0}{0} = \infty$ $\qquad \Xi u = \frac{\text{s}}{\text{su}}\infty$ $\qquad \oplus 1 = \infty y$ $\qquad y = \frac{1}{\infty} = 0$

$\Xi y = \frac{\text{s}}{\text{sy}}0$ $\qquad \oplus 1 = 0u$ $\qquad u = \frac{\oplus 1}{0} = \infty$

$$u = \frac{0}{0} = \infty = y = 0$$

.

<u>Transform 2b— 2.a.4:</u>

The reverse of Transform 2a results in the temporal constant just like in transform 1b.

$\frac{\text{s}}{\text{sy}}y = \left(\frac{0}{x}\right)\Xi$ $\qquad T \cdot y = \oplus 1\left(\frac{0}{x}\right)$ $\qquad T = \frac{-\frac{0}{x}}{y}\ T = \frac{-\frac{0}{x}}{\frac{0}{x}}\ T = -\frac{0}{x}\left(\frac{x}{0}\right)$

These last two steps are the same ratio divided by the same ratio and finally $\frac{0}{0}$ respectively. In this instance it takes its $\oplus$ 1 value as a constant because it results from same ratio divided by same ratio—its already 1 .

$$T = -\left(\frac{0}{0}\right) = 1$$

Repeating the transformation process will again solve for $T = -1$ as it is not a subspace.

$T = \oplus 1.$

Transform 3a—2.a.5:

An expression of the form $y = 0x$. The y *value* will not become infinite for any value of x. In fact y will equal 0 for all values of *x*. When you seek the subspace of this form of an equation it will result in division by 0 in the subspace.

$\Xi y = \frac{\mathrm{s}}{\mathrm{sx}} 0x$ $\oplus 1 = (0x)u$ $\oplus 1 = (0)u$ $u = -\frac{1}{0} = -\infty$

Recall that the act of performing a subspace transform links the original output, to the new variable through the infinite. Since the *u* is already present the infinite which remains may be substituted for the *y* value corresponding to the same x value input.

$u = -\infty$ $u = y$ $u = 0$ for all x

The u subspace will have non-zero values for non-zero values of ax where in the same expression a constant $a \neq 0$.

$\Xi y = \frac{\mathrm{s}}{\mathrm{sx}} ax$ $\oplus 1 = (ax)u$ $u = -\frac{1}{ax}$

Transform 3b—2.a.6:

Again, an attempt perform subspace transforms on the function side of the equation results in the temporal constant. Using the operation repeatedly to find the actual value of the final stage yields $T = 1$ and $T = -1$.

$\frac{s}{sy}y = \Xi 0x$ $\quad T \cdot y = \oplus\ 1(0x)$ $\quad T = \oplus\ 1\frac{(0x)}{y}$ $\quad T = \left(\frac{0}{0}\right) = -1$

The repeat transformation yields $T = 1$.

$T = \oplus\ 1$

<u>Transform 4a—2.a.7:</u>

An expression of the form $y = a \cdot x$, where a is any non-zero constant.

$\Xi y = \frac{s}{sx}ax$ $\quad \oplus\ 1 = (ax)u$ $\quad u = -\frac{1}{ax}$

<u>Transform 4b—2.a.8:</u>

The subspace transform on the input results in the temporal constant.

$\frac{s}{sy}y = \Xi ax$ $\quad T \cdot y = \oplus\ 1(ax)$ $\quad T = -\frac{ax}{y}$ $\quad T = -\frac{ax}{ax}$ $\quad T = -1$

Repeating the transformation yields $T = 1$.

Chapter 3

The Seven Directional World

Directional and Dimensional Time—3.a:

Before exploring subspace more deeply, and how the existence of these directions arise its necessary to explore the meaning of time itself within the context of Temporal Mechanics. As mentioned in earlier pages there are really three ideas of time. There is the temporal constant, a time-flow quality denoted as *T*. We indicate directional time by the value *t*. This value is used when we express rates of change with coordinate systems and is synonymous with the idea of the ticks of a clock. Subspaces supply the idea of Dimensional Time. They are related to the idea of time *t* as they represent places where events exist in the space of time.

The temporal constant shows up in the transforms whenever you solve for the subspace of function side of an equation and always equals $\oplus 1$. T and the subspace values are not the rate of change of events, or how quickly something occurred. T merely implies that a progression of time occurs and that this fact has a kind of constancy independent of the perceptions of how much time has passed. It conveys the idea of Past moves to Present, moves to Future which we experience as a series of locations in the space of time; the subspaces where all events continue to exist.

The flow of events as perceived by the human mind always seem to move forward. Consider a time traveler. He leaves home Wednesday, January 1st 2025 in his time machine and arrives on Thursday, January 1st 2015. He may have travelled 10 years into the past but in his own mind, standing in his living room on New Year's Day 2015, he still remembers *yesterday* as December 31st, 2024. For him time still moved forward, he is just in a place he considers to be a part of his past. We'll return to build more on this idea in a later chapter.

Neither expression for $T = 1$ or $T = -1$ is any more correct than the other. Our present is itself a confluence of both perspectives. Every point-present in subspace exists at an intersection of all possible paths from the past which arrived at that configuration, and probable paths away into the future for what may occur. All events and moments in time are themselves a result of the combination of past and future events.

There are as many subspaces in a function as there are degrees of freedom. In a three directional world planes have equations of the form $z = f(x, y)$. The subspace directions for *x, y* and *z* are labeled respectively *s, u* and *w.* The Temporal Mechanical analog of Directional Time is satisfied by another axis; *The Alternate.* Denoted *A* this value is not itself a direction under the Temporal Mechanics definition. The manner in which it is calculated shows that it is itself a dimension, linking directly the concepts of directional and dimensional time. The method of calculating the Alternate will be covered in detail later. For now simply know that it is used as a Temporal Mechanical subspace direction and is as real as the other six listed here.

The subspaces *s, u*, *w* and *A* define a unique location in the space of time where an event exists independent of the flow of time. In other words any event, occurring in the past, present or future will have a location definable in subspace. Those events continue to exist at that definite location in a subspace domain even after our present has moved past them in our perceived flow of time.

Developing Subspace—Hidden Directions and Dimensions—3.b:

Before moving forward let's restate a clear distinction between two terms—direction and dimension. I mentioned earlier this difference existed for time. It's also true for space. Temporal Mechanics defines a direction by a number line axis, while a dimension is defined as multiple directions used in combination with one another to plot point positions. Directions will be denoted by d^{α} where α is the total number of directions in a system. Dimensions will be denoted by D^{β} where β is the total number of space dimensions coexisting within that reference frame. R_m will denote space where *m* is the total number of *perceived* space directions and the minimum number by which dimensionality of the space can be considered. Now let's discuss how time and space fit together and how subspaces arise.

R_0 The Point—3.b.1:

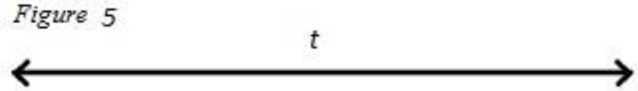
Zero Space means they are no space directions. Figure 1 at right is a singular point; a dot. Though we may draw a point to represent this and even label it, but it cannot convey the reality of the point's existence. This is just a thought experiment. To be able to place a point means we have space to place it in regardless of how small it may be. However, if we imagine this point is infinitely small then it need not have space to exist. It's infinitely small. We can't even assign it the 0 vector. The Zero Vector can be said to point in any or all directions simultaneously with no magnitude. Yet since we have no space, the idea of the point being in some direction is meaningless.

Figure 4

From the perspective of our infinitely small point there is no space but, provided we accept that this point is in-fact *there*, we can measure whether or not it exists. Given it does in fact exist, the only meaningful idea is directional time *t.* As we shall soon see this variable is really a dimension, even though used as a direction and linked directly to the Alternate, a fact for which that axis is named.

Now we may draw the directional time axis, labeled t. It is shown at right in figure 5. We can measure directional time *t* as how long the point exits but nothing more. This also means that this axis only has values that build from 0 toward the positive end of the axis; negative time has no meaning. Now let's add in space.

Figure 5



R_1 The Line—3.b.2:

By expanding a point out to infinity as a one directional axis we arrive at one-space. This is best represented by a line; a one dimensional domain with no up, down, left or right. Only forward and backward exist on this line.

Figure 6

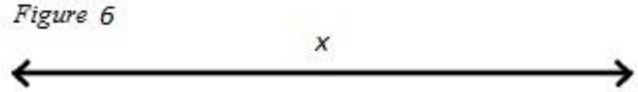
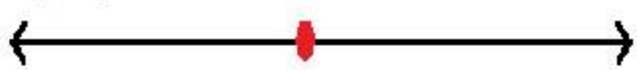


Figure 7



All points are identical and the line infinite, preventing measurement from anything. Without something else for a reference point there is no other way to make measurement except by the concept of directional time, as will be seen shortly, using the subspace *s.* To illustrate consider a single point on the line shown in figure 7. No matter where it's placed no information can be conveyed. There is nothing else within this domain. Sure, the point is there but how do we define its location aside from the length of its existence?

In our 3-directional world we might actually label the line with a numerical scale allowing measurement. But from the perspective of the point, it's not possible. This is 1—space. There is only one space direction. That means there is no other direction we can use to mark the graph indicating the point's position. So from the perspective of anything on the line no numerical scale can be immediately created aside from the idea of directional time. As an example, imagine the dot on the line is the value x = 4. The graph cannot be drawn because there is no Y axis. Because it does not exist the point can be drawn anywhere and the information it conveys is identical to all other points.

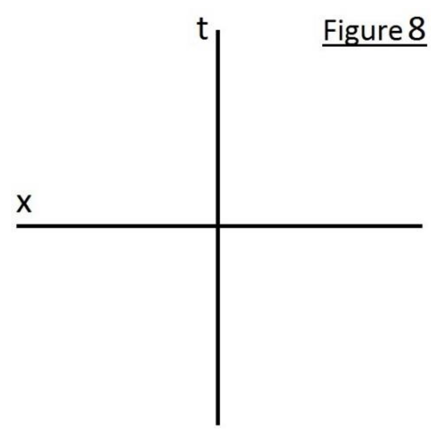


Figure 8

Without at least one other direction present those values and positions are not capable of being understood as part of space because there is nothing to define them by. Yet the idea of time is still present and can be used for measurement. On the line time can be perceived even from the perspective of a singular point directionally as how long it exists even without a known definable location, and dimensionally in the existence of a flow of time, understood in differences in how long it takes for it to move from one position to another. Thus we may partner this one directional axis with a directional time axis, with equations taking the form of $x = f(t)$.

We previously discussed that subspaces are the space of time and a subspace exists for each degree of freedom. When we expand from the point into the one directional line we also expand for it a subspace. Consider the idea of $x = \infty$ and $x = -\infty$. Within our one directional world it would take our point an infinite amount of time to reach this place. It's infinitely far away toward both ends of the axis. Using the logic from the previous chapters the presence of the *x* and *t* variables already imply an infinite number of possible values along their own axis simply by being present. So saying either $x = \infty$ or $x = -\infty$ implies the value lies beyond the range of either axis, on some other axis not included in the expression. This is the subspace labeled *s.*

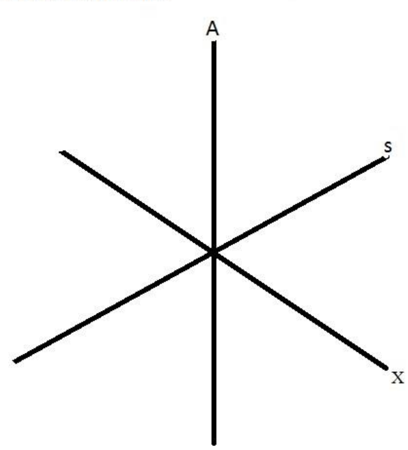
Directional time was already present, linked to the x axis in equations $x = f(t)$. The inclusion of the *s*-subspace now provides the where in the space of time for all possible positions of x. But what about the actual idea of how long it remains in a place or how long it took to get there? This is the idea of directional time, t. This idea is still present but must be linked to the idea of the subspace. Since the subspaces represent *the where* in time an events exists, it must be that the distance across subspace is the value we see as directional time. However, it will be found that *s* generally does not equal *t*. Something is missing from the equation. That something is the subspace equivalent to the idea of directional time, the Alternate.

The Alternate's contribution to the measurement of time *t* is represented as the length traveled across a section of subspace. Yet since it was not obtained from any other axis its value can only be obtained from the distance equation. Distance is given by the square root of the sum of the squares of the change in distance across the subspace axes. We have two for the one directional line: *s* and *A.*

$$dt = \sqrt{ds^2 + dA^2} \qquad \text{where} \qquad dA = \sqrt{dt^2 - ds^2}$$

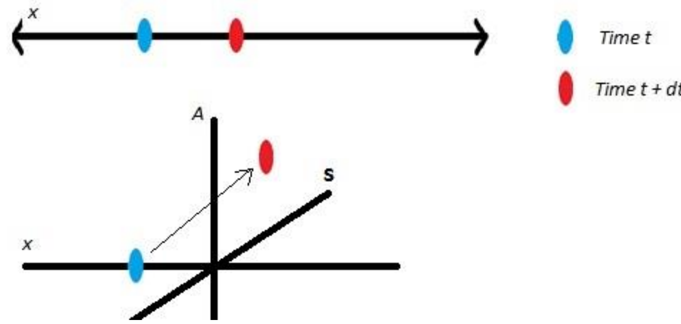
The Alternate is synonymous with directional time. Even if nothing else is changing for the orientation of x and its subspace s, the value of the Alternate will progress as the flow of time. Together these three values create a unique location in space for

Figure 9
1 Direcitonal Line



all possible forms of equation $x = f(t)$ at every point in time. Figure 9 at right shows the 1-directional line paired with its subspace and the Alternate.

Figure 10



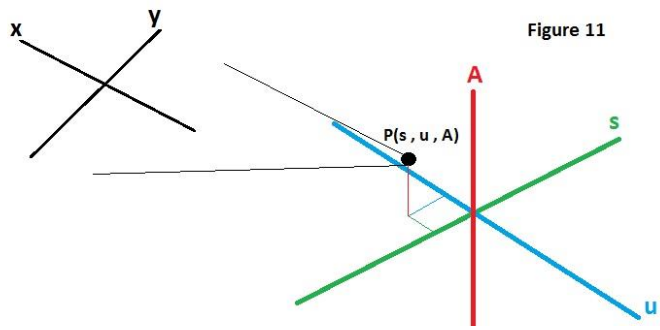
Consider how the addition of another point is expressed in this way. These two points can be the same body at two different times or two separate bodies at the same instance. Though space to the left of the blue point and to the right of the red point are both infinite the space between them is finite. That makes it measurable to us 3-directional humans, and at least perceivable from the 1-directional point of view as a function of time where equations will take the form of $x = f(t)$. The reality though remains that although this equation would generate points $(x\,, t)$ they are actually more accurately described as $(x|s\,, A\,)$. The notation for points as well as how to rewrite the equations to include the missing subspace values will be covered in a later chapter.

If we were to record the positions of any point as it moved back and forth according to the rules of some defining equation we could develop a map of where it was in relation to any and all previous locations based upon the time it took to get to another. The first recorded location of the point is labeled the origin. Positions to the left are negative and to the right are positive. The 1-Space position equation is then a function of time input and a time distance from the origin as an output. A single line will have one perceived space direction (R_1), and thee space directions (d^3: the *x*-axis, its subspace the *s*-axis and the Alternate), and three dimensions (D^3: The XS, XA, and SA Planes). By definition the Alternate is itself a dimension. However its use in this case is an axis so it is not counted as one of the dimensions used by the line.

We can draw the conclusion from the expansion to a line from a point that space is an emergent property of time. It allows any space direction to be definable even alone in a single 1-Space. No form of space can exist without this aspect of time, a fact which complicates issues when considering an R_n where $n > 1$. Note that the Alternate will have a negative root for any instance that $t^2 < s^2$. This is described in detail in chapter 6 in the section on the negative radical.

<u>R_2 The Plane—3.b.3:</u>

Figure 10 at right depicts the standard Cartesian plane with the *x* and *y*-axis. When we expand the line into a plane the newly created y-axis gets the same considerations as did the x-axis when it exited alone. i.e. it has the same properties as the x-axis did in section 3.b.2. Thus the y-axis will have a subspace associated with it, the u-axis, with its value is found via the subspace transformations.



An image of what this looks like cannot be accurately drawn since there are now five axis present (*x, y, s, u and A*) which comprise a hyperplane. Additionally the distance across the temporal hyper plane, perceived as directional time, composed of the subspace axes must include the new axis.

$$dt = \sqrt{dA^2 + ds^2 + du^2} \qquad dA = \sqrt{dt^2 - ds^2 - du^2}$$

The addition of the *y*-axis allows us to define new functions. Equations within the plane are lines of the form of $y = f(x)$ or are parameterized against a common input variable. The Alternate subspace axis is clearly taking on the role of directional time even though it is a dimension by definition. Even if nothing else were changing there would be a rate of change, and forward progression to the Alternate due to its inclusion of t, synonymous with the ticks of a clock in its calculation.

Though it's not possible to draw the 5-direcitonal hyperplane axes it is possible to examine graphs of limited sections of it. The 5-directional hyperplane has 10 dimensions; 10 separate 2-directional hyper subplanes suspended within 3-directional hypervolume subspaces. Each of these may be individually considered. It is also possible to define a Catesian style plane of any two axes and plot their points respective to a third chosen axis as if it were a volume. The possible combinations which may be examined are listed here:

3.b.3.i

The $xy|suA$ Hyper Plane

<u>Graphable Systems:</u>

1. 2-Directional Hyper-Subplanes (*HSp*)—

   Any two of the various subspace axes can be thought of as its own version of the Cartesian Plane existing within a subspace volume definable by the remaining three axes.

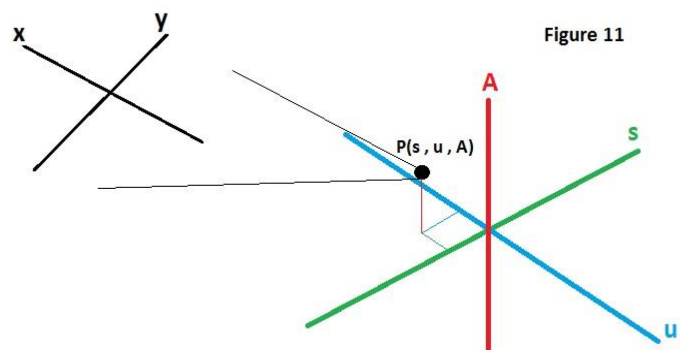


Figure 11

HSp1:	$xy \mid suA$	*This is the standard two directional Cartesian Plane*	
HSp2:	$xA \mid ysu$	HSp3:	$yA \mid xsu$
HSp4:	$xs \mid yuA$	HSp5:	$ys \mid xuA$
HSp6:	$xu \mid ysA$	HSp7:	$yu \mid xsA$
HSp8:	$su \mid xyA$	HSp9:	$sA \mid xyu$
HSp10:	$uA \mid xys$		

No other combinations are possible. Grouping more than two of the directions together (*the left side notation*) as anything but subspace (*the right side notation*) would not describe a hyperplane. Instead it would describe a hypervolume.

2. 3-directional Hyperplane subvolume (HPsV)—

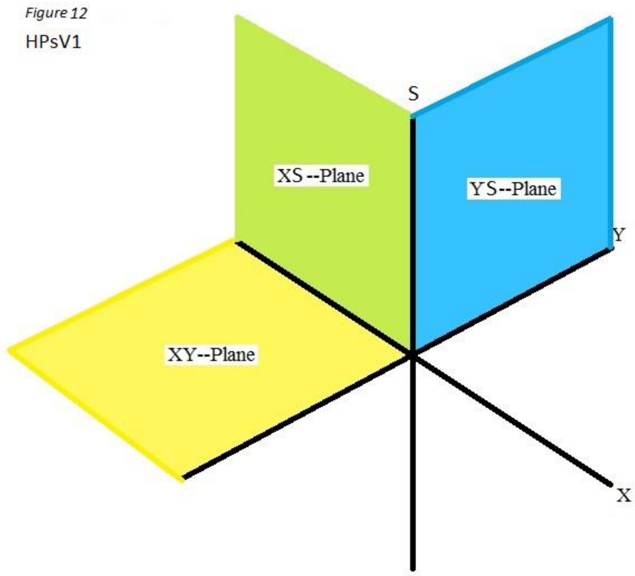
Only ten unique pairings exists for this type of comparison. Any other pairs are simply rearrangements of the orderings listed here bellow and merely correspond to a rotation of the graph.

HPsV1:	xys	HPsV2:	xyu
HPsV3:	xyA	HPsV4:	xsu
HPsV5:	xsA	HPsV6:	xuA
HPsV7:	ysu	HPsV8:	ysA
HPsV9:	yuA	HPsV10:	suA

The graph shown here displays the orientation of HPsV1. Note that each of these pairings is displayed above as the subspace components for each unique Cartesian plane combination.

Figure 12

HPsV1



Each of the Hyper Subplanes can be thought of as consisting of its own two directional version of the Cartesian Plane plotted with a subspace volume.

Each of the five total directional axes are orthogonal to each other. They have to be. If we take the dot product of any two corresponding vectors they will equal zero, making them orthogonal. This is seen most easily with the realspace x and y axes.

$$x \cdot y = \langle x_n , 0\rangle \cdot \langle 0 , y_n\rangle \qquad 0= [(x_n \cdot 0) + (0 \cdot y_n)]$$

In other words both the x and y axis are turned 90° to each other. This can be expanded to include all five axes.

$$x \cdot y = \langle x_n , 0 , 0 , 0 , 0\rangle \cdot \langle 0 \cdot y_n \cdot 0 \cdot 0 \cdot 0\rangle \qquad 0 = [(x_n \cdot 0 \cdot 0 \cdot 0 \cdot 0) + (0 \cdot y_n \cdot 0 \cdot 0 \cdot 0)]$$

We determined the s-axis subspace value earlier $\left(s = -\frac{1}{x}\right)$ from its transformation with a mathematical truism, $x = x$. This may be written more abstractly as $s = -\frac{1}{f(x)} = h(x)$. We can obtain the inverse of this function such that $x = -\frac{1}{f(s)} = j(s)$. Given an equation of the form $y = f(x)$ it's easy to see could just as easily substitute $y = f\left(-\frac{1}{f(s)}\right) = g(s)$. Finally it's also possible to solve for the equation $x = f(y)$ from which it is possible make substitutions into $s = -\frac{1}{f(x)} = h(x)$ and obtain a function $s = r(y)$. Thus x, y and s may all be represented as a function of only one of the remaining two components:

$$x = f(y) \qquad x = j(s)$$

$$y = f(x) \qquad y = g(s)$$

$$s = h(x) \qquad s = r(y)$$

The x, y and s axes are all orthogonal to each other.

The u-axis was similarly obtained with a subspace transformation upon $y = f(x)$ such that $u = -\frac{1}{f(x)}$. Again solving for the inverse of various equations we find $x = -\frac{1}{f(u)}$ and in $y = f(x)$ we substitute to obtain $y = f\left(-\frac{1}{f(u)}\right)$. The inverse of $y = f(x)$ is $x = f(y)$. Substitutions may be made with $u = -\frac{1}{f(x)}$ to determine $u = f(y)$. We also know $x = -\frac{1}{f(s)}$. This allows us to substitute into $x = -f\left(\frac{1}{f(u)}\right)$ and obtain $s = -\frac{1}{\left(f\left(\frac{1}{f(u)}\right)\right)} = f(u)$ and $u = f(s)$. Thus x, y, s and u are all orthogonal to each other.

Because the Alternate is defined as $A = \sqrt{t^2 - s^2 - u^2}$ for the hyperplane, and x, y, s and u can all be expressed in terms each other, it is possible to express the alternate completely in terms of only one variable via substations. Even the t component can be included if the input of the equation is parameterized in terms of t. Thus all five axes of the hyperplane are orthogonal to each other.

$x \cdot y = \langle x_n , 0 , 0 , 0 , 0\rangle \cdot \langle 0 \cdot y_n \cdot 0 \cdot 0 \cdot 0\rangle = 0$

$x \cdot s = \langle x_n , 0 , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , s_n , 0 , 0\rangle = 0$

$x \cdot u = \langle x_n , 0 , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , u_n , 0\rangle = 0$

$x \cdot A = \langle x_n , 0 , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , 0 , A_n\rangle = 0$

$y \cdot s = \langle 0 , y_n , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , s_n , 0 , 0\rangle$=0

$y \cdot u = \langle 0 , y_n , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , u_n , 0\rangle = 0$

$y \cdot A = \langle 0 , y_n , 0 , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , 0 , A_n\rangle = 0$

$s \cdot u = \langle 0 ,0 , s_n , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , u_n , 0\rangle = 0$

$s \cdot A = \langle 0 , 0 , s_n , 0 , 0\rangle \cdot \langle 0 , 0 , 0 , 0 , A_n\rangle = 0$

$u \cdot A = \langle 0 ,0 , 0 , u_n , 0\rangle \cdot \langle 0 , 0 , 0 , 0 , A_n\rangle = 0$

No other parings are shown as they are simply mirrored reversals of orders already shown here. For all the value obtained is 0. All five axes are orthogonal to each other and will form a complete orthonormal basis for the hyperplane.

Subspace directions are present for all equations and expressions. The process above can be adapted for any number of directions and their subspace partners. In every instant all such axes are mutually orthogonal to each other. By acknowledging the inclusion of subspace it is clear that all perceivable space is actually a hyperspace.

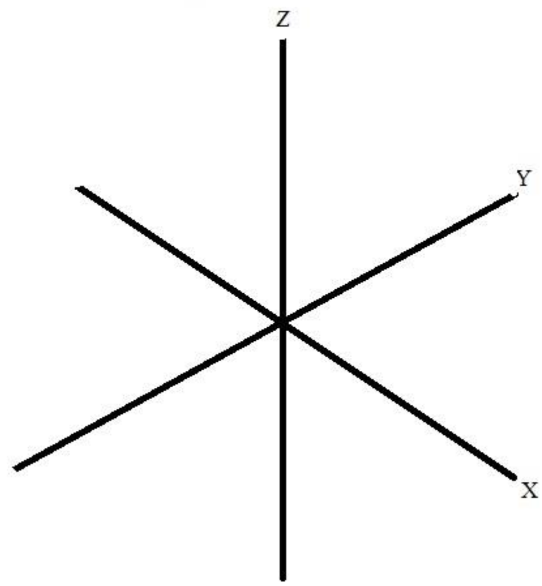
In summation a plane is actually a hyper plane. It has two perceived space directions (R_2), five total space directions (d^5: x-axis, y-axis, s-axis, u-axis and A-axis), and 10 dimensions (D^{10}: XY-plane, XS-plane, YS-plane, XU-plane , YU-plane , XA-plane, YA-plane, SU-plane , SA-plane and the UA-plane).

$\underline{R_3 \text{ The Volume—3.b.4:}}$

Volumes have three perceivable directions and are a direct expansion of a plane orthogonal to its two perceived directions. Figure 13 depicts the standard 3-space volume. Equations of the form $z = f(x, y)$ define planes within a given volume with two inputs, *x* and *y.* The x-axis and y-axis are both free variables. They have their own already discussed subspaces, the s-axis and u-axis. In this instance both are found by transformation on mathematical truisms.

Figure 13

Standard 3-Space Volume



$$\Xi x = \frac{s}{sx} x \qquad \Xi y = \frac{s}{sy} y$$

$$s = -\frac{1}{x} \qquad u = -\frac{1}{y}$$

The *z* output carries its own subspace, the w-axis: $z = f(x\,, y)$ $\quad w = -\frac{1}{f(x\,,y)}$

The Alternate is likewise adjusted: $A = \sqrt{t^2 - s^2 - u^2 - w^2}$

Like the two-directional plane subspaces of a three-directional volume may be found by either direct transformation or trigonometric conversion using Spherical Coordinates. The trigonometric conversions will relate the circular angles of rotation in three space to values in subspace.

A volume is actually a hyper volume. Including the Alternate, there are seven total space directions in a hyper volume. So there are a large number of subvolume spaces. The reality of the whole structure cannot accurately be portrayed all at once. Like the subplanes of the hyper plane, instead we have to examine individual three directional subvolumes in order to obtain accurate graphic representations.

In summary any volume is a hypervolume having three perceived space directions (R_3), seven total space directions all orthogonal to each other (d^7), and thirty-five subvolume dimensions (D^{35}).

xyz **suwA**

syz	xuwA	xsu	yzwA	xuA	yzsw
xsz	yuwA	ysu	xzwA	yuA	xzsw
xys	zuwA	zsu	xywA	zuA	xysw
uyz	xswA	xsw	yzuA	xwA	yzsu
xuz	yswA	ysw	xzuA	ywA	xzsu
xyu	zswA	zsw	xyuA	zwA	xysu
wyz	xsuA	xsA	yzuw	suw	xyzA
xwz	ysuA	ysA	xzuw	suA	xyzw
xyw	zsuA	zsA	xyuw	swA	xyzu
Ayz	xsuw	xuw	yzsA	uwA	xyzs
xAz	ysuw	yuw	xzsA		
xyA	zsuw	zuw	xysA		

Chapter 4

Properties of Subspace

Theorem 1—Number of Subspace directions—4.a:

Let d equal the total number of space directions in any hyperspace. Where the number of perceived space directions is m, the number of subspaces is n and $n = m + 1$. *There will be a total of d = m + n space directions in the hyperspace.*

Subspace Properties—4.b:

The following properties will be true for 2—Space, and 3—Space. No higher examples will be given here due to increasing complexity. These properties can be expanded to any coordinate system to define values previously thought undefinable.

2—Space—4.b.1:

Two space will have three extra directions in subspace. These are *s*, *u* and the Alternate. The subspace ***s*** directly corresponds to the *x*-axis while ***u*** corresponds to the y-axis. The parameters for 2-Space are:

$R_2 \ d^5 \ D^{10}$:

Directional axis: $d_1 = x \quad d_2 = y \quad d_3 = s \quad d_4 = u \quad d_5 = A$

Dimensional Plane: $D^1 = \mathrm{HSp1} = XY \qquad D^2 = \mathrm{HSp2} = XA$

$D^3 = \mathrm{HSp3} = YA \qquad D^4 = \mathrm{HSp4} = \mathrm{XS}$

$D^5 = \mathrm{HSp5} = \mathrm{YS} \qquad D^6 = \mathrm{HSp6} = \mathrm{XU}$

$D^7 = \mathrm{HSp7} = \mathrm{YU} \qquad D^8 = \mathrm{HSp8} = \mathrm{SU}$

$D^9 = \mathrm{HSp9} = \mathrm{SA} \qquad D^{10} = \mathrm{HSp10} = \mathrm{UA}$

Alternate Definition: $A = \sqrt{t^2 - s^2 - u^2}$

As mentioned before, the Cartesian **XY-Plane** exists within a higher Hyperplane. The ***XY*** plane is one of ten subplane dimensions which comprise the 2-directional hyperplane. These subplanes collectively contain every configuration of points and lines possible within the Cartiesion XY-Plane, for all points in time.

Any point definable within any given subplane will have some aspect of it present in each of the other nine. This is because each point has five basis vectors which define its exact location in the space time of the 2-direcitonal plane.

Notation for points in the 2-direcitonal hyperplane take the form of $P_n = (v_1 , v_2 | v_3 , v_4 , v_5)$. The first two components define the actual plane which is directly perceived from the point of view of the observer. The remaining components correspond to the subspace axes. Assuming one is plotting points in the XY Cartesian Plane point notation is:

$$P_n = (x , y | s , u , A)$$

<u>3—Space—4.b.2</u>:

Three space has four subspace directions. This results in 7 total space directions, and 35 subvolume dimensions. We will label the subspace directions ***s*** , ***u*** , ***w*** and ***A.*** The Parameters for 3-Space are:

$R_3 \;\; d^7 \;\; D^{35}$:

Directional axis: $d_1 = x \quad d_2 = y \quad d_3 = z \quad d_4 = s \quad d_5 = u \quad d_6 = w \quad d_7 = A$

Dimensional Subvolumes:

$D^1 = \text{HSv1} = XYZ$	$D^2 = \text{HSv2} = \text{SYZ}$	$D^3 = \text{HSv3} = \text{XSZ}$
$D^4 = \text{HSv4} = \text{XYS}$	$D^5 = \text{HSv5} = \text{UYZ}$	$D^6 = \text{HSv6} = \text{XUZ}$
$D^7 = \text{HSv7} = \text{XYU}$	$D^8 = \text{HSv8} = \text{WYZ}$	$D^9 = \text{HSv9} = \text{XWZ}$
$D^{10} = \text{HSv10} = \text{XYW}$	$D^{11} = \text{HSv11} = \text{AYZ}$	$D^{12} = \text{HSv12} = \text{XAZ}$

$D^{13} = \text{HSv13} = \text{XYA}$	$D^{14} = \text{HSv14} = \text{XSU}$	$D^{15} = \text{HSv15} = \text{YSU}$
$D^{16} = \text{HSv16} = \text{ZSU}$	$D^{17} = \text{HSv17} = \text{XSW}$	$D^{18} = \text{HSv18} = \text{YSW}$
$D^{19} = \text{HSv19} = \text{ZSW}$	$D^{20} = \text{HSv20} = \text{XSA}$	$D^{21} = \text{HSv21} = \text{YSA}$
$D^{22} = \text{HSv22} = \text{ZSA}$	$D^{23} = \text{HSv23} = \text{XUW}$	$D^{24} = \text{HSv24} = \text{YUW}$
$D^{25} = \text{HSv25} = \text{ZUW}$	$D^{26} = \text{HSv26} = \text{XUA}$	$D^{27} = \text{HSv27} = \text{YUA}$
$D^{28} = \text{HSv28} = \text{ZUA}$	$D^{29} = \text{HSv29} = \text{XWA}$	$D^{30} = \text{HSv30} = \text{YWA}$
$D^{31} = \text{HSv31} = \text{ZWA}$	$D^{32} = \text{HSv32} = \text{SUW}$	$D^{33} = \text{HSv33} = \text{SUA}$
$D^{34} = \text{HSv34} = \text{SWA}$	$D^{35} = \text{HSv35} = \text{UWA}$	

Alternate Definition: $A = \sqrt{dt^2 - ds^2 - du^2 - dw^2}$

****Note, assuming change in these variables is measured from the origin, and the origin can be relabeled as any beginning position, this may be simplified to* $A = \sqrt{t^2 - s^2 - u^2 - w^2}$*. This simplification will be use often in coming sections.*

The Dimensions of 3-Space—4.b.2.i:

Three Space, the perceivable universe in which we live, is defined spatially by points (x , y, z). Its subspace directions are ***s*** , ***u***, ***w*** and **A**. The seven space directions combine to form above listed 35 Dimensions in one Hypervolume. Every dimension indicates a unique combination of the seven total directions into a perceivable three-space volume with four subspace, dimensional time directions.

Each point in the XYZ volume itself exists at unique position in a subspace SUW volume, at a given instant in time definable on A. All points, lines, planes and volume configurations possible exist for all points in time within the hypervolume. Their vector basis components for any given point are of the form $P_n = (v_1 , v_2 , v_3 | v_4 , v_5 , v_6 , v_7)$. Assuming one is plotting a position in the XYZ volume points will bear the notation: $P_n = (x , y , z | s , u , w , A)$.

Chapter 5

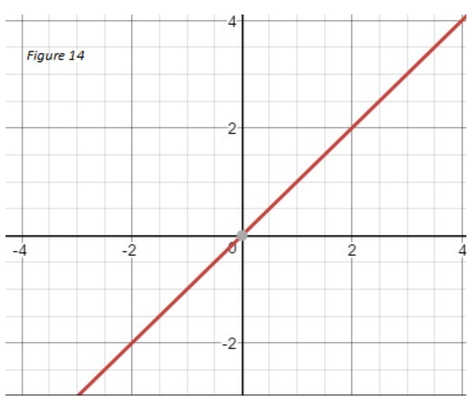
Zero Division Algebra

Earlier it was mentioned in order for mathematical operations to be considered real they had to be able to accurately describe events in the real world. To develop a full understanding of Temporal Mechanics one must return to simpler mathematical subjects and examine how they function with subspaces. This chapter will explore Algebra with subspace values as well as show how accurate solutions are obtained with the math.

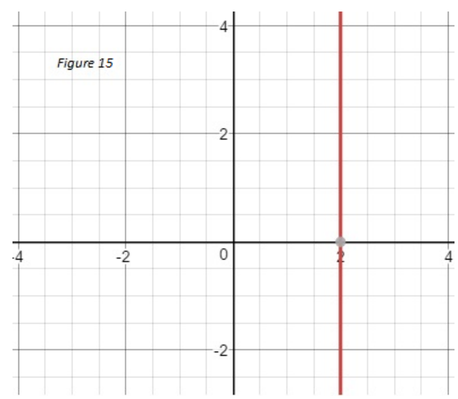
Plotting Subspace Points—5.a.i:

It seems odd that something as simple as plotting points could be a new concept but it's true. Consider Figures 14 and 15 below. The graphs of both equations are fairly straight forward and don't require a lot of visualization.

$y = x$



$x = 2$



However, what do we do with an equation of the form $y = \oplus 2$? This looks different from $y = 2$. It's not $y = -2$. The value $\oplus 2$ indicates simultaneously plus and minus, telling us this is a subspace value. When such a value is part of an equation, absent another subspace value multiplying it, it shall require a series of subspace transforms be used to determine which values are being implied.

The subspace transform conducted on the subspace term will provide its value on the subspace axis.

$$\Xi\, y = \oplus \frac{s}{sn} 2 \qquad \oplus 1 = \oplus 2(u) \qquad u = -\frac{1}{2}$$

The final step in this process involves dividing $\oplus 1$ by $\oplus 2$. Through this is clearly division the $\oplus$ signs can be thought of as multiplying each other, essentially squaring this sign. This process is covered fully in Chapter 6 on the section with the Negative Radical. For now simply know that $\oplus \cdot \oplus = -$, and numbers modified by $\oplus$ are subspace values related to the complex plane.

The expression $y = \oplus 2$ implies $u = -\frac{1}{2}$. This is seen in the subspace transformations, where a subspace is a negative reciprocate of the real space term. Repeating the subspace transformation will provide the real space term corresponding to the subspace.

$$\Xi\, u = -\frac{s}{sn} \frac{1}{2} \qquad \oplus 1 = -\frac{y}{2} \qquad y = 2$$

The final step in this situation is a key element to basic arithmetic within Temporal Mechanics.

5.a.ii—Multiplying and Dividing Subspace Numbers:

It's not possible to directly multiply a subspace number with a real space number unless performed within a subspace transform. The multiplication and division processes are shown below but can only be performed in this fashion if occurring as part of a subspace transform. This usually means doing a subspace transformation on the entire expression. However it is also possible to use a transformation process on just a lone number as well as a special transformation based off of the Alternate equation to find the value of a subspace number within it originating equation.

Consider the expression: $1 \cdot \oplus 1$

In direct multiplication of a subspace term with a real number, something that only occurs during a transform, the $\oplus$ sign may be literally regarded as $(+\,-)$. Thus the subspace term $\oplus 1$ multiplied by 1 will result in -1.

<u>Multiplication of subspace and real space number within a Subspace Transform:</u>

$$1 \cdot \oplus 1 \quad = \quad +1 \cdot (+ - 1) \quad = \quad 1 \cdot (-1) \quad = \quad -1$$

This action applying to subspace transforms was demonstrated in the various transformations and in section on point plotting earlier. If such a value appears it can only be directly applied if it occurs as part of the transformation. Otherwise you must evaluate $\oplus 1$ first to find the value it takes within the equation it originates.

Given $y = 1 \cdot \oplus 1$ let us perform a subspace transformation on the $\oplus 1$ alone. It is a subspace value so it has be reduced to its real space term before anything in the y equation can act upon it. The x-axis subspace was defined by using the subspace operators on the mathematical truism $x = x$. This means we can do the following:

$$\Xi x = \frac{s}{sx} \oplus 1 \quad \rightarrow \quad s = \frac{\oplus 1}{\oplus 1} = -1$$

Since this is a transform on a subspace number alone the use of the *x*-axis here is just a convention. Any dummy notation could be used to keep track of the values for the transformation. We could even have begun the transform on $\oplus 1 = \oplus 1$. The result is the same. AS an example we could just as easily used:

$$\Xi R = \frac{s}{sx} \oplus 1 \quad \rightarrow \quad G = \frac{\oplus 1}{\oplus 1} = -1$$

$$\Xi \oplus 1 = \frac{s}{sn} \oplus 1 \quad \rightarrow \quad \frac{0 \cdot \oplus 1}{0} = \oplus 1 G \quad \rightarrow \quad G = \frac{\oplus 1}{\oplus 1} = -1$$

In either case the return transform from subspace to the real space on the number itself provides the value it takes within the equation it originated, which in this case will be $+1$. For $y = 1 \cdot \oplus 1$, we can rewrite the equation as $y = 1 \cdot \oplus 1 \equiv y = 1 \cdot 1$. It will be this way anytime a subspace number originates within an equation independent of a subspace transform; it will take the positive value of the number implied.

Two unique features of Temporal Mechanics must be understood. First is that subspace numbers are directly related to the Temporal Constant, a mathematical object synonymous with $\frac{0}{0}$.

Multiples of that value are directly representative of values listed on the complex plane asis. The Temporal Constant is also its own subspace. Second is that the Alternate, one of our seven total directions, has no subspace of its own. From this property of the Alternate the second method of finding the value of a subspace number occurring naturally in an equation or expression is derived.

Recall that, for an equation of the form $y = f(x)$, the Alternate is defined by $A = \sqrt{t^2 - s^2 - u^2}$. Whenever we have equations which do not include the value t, or hold a value of $t < (s^2 + u^2)$ we are guaranteed to have a negative radical. i.e. $A = \sqrt{-n}$. In chapter 6 the section on the negative radical explores why this is a ⊕ number. Thus the expression for the Alternate can be used to determine the value a subspace term represents in its originating equation as well as solutions to negative radicals.

Consider the following: $\mathrm{y} = \oplus\, 1$ using $\mathrm{A} = \oplus\, 1$ reduce the subspace term to its actual y axis value.

$\frac{s}{sn}\mathrm{A} = \oplus\, \Xi 1$ The Alternate has no subspace. So a reverse transform must be used in this case and will result in manifestation of the temporal constant. Now we solve for A. Unique to this transform is a need to know which value T takes. Although A has no subspace the value held by T will result in ability to identify the values which correspond to its real space and subspace values if it actually had a subspace. The completion of the first loop is the solution for a negative radical on the A-axis itself. The completion of the second loop gives the value of the ⊕ number on any other axis that it originated.

$\mathrm{TA} = \oplus (\oplus 1)$	The squared $\oplus 1 = -1$ (*more provided on this later*)—Recall in a reverse transform nothing is removed; thus the subspace number survives the process, squaring its sign. T, which is its own subspace and as shown previously already equals $\oplus 1$. Yet we began this example saying that $\mathrm{A} = \oplus 1$. This can be used to solve directly for T and see which of the two values applies to this specific setup with known values for A. We complete solving for A in the next step using $\mathrm{T} = 1$. $\mathrm{TA} = \oplus (\oplus 1) \qquad \mathrm{T} = \frac{-1}{\mathrm{A}} \qquad \mathrm{T} = \frac{-1}{\oplus 1} = 1$
$\mathrm{TA} = -1$ where $\mathrm{T} = 1$	
$\mathrm{A} = \frac{-1}{1} = -1$	Again—A has no subspace. The first rotation of solving for T aligned it to $T = 1$ and A to value -1. Although the attempt to resolve the value appears to have been successful it has provided the opposite sign of the value the subspace term will take in the equation it originates in. A transform moves you away from the space arrangement described by the original equation to an equation describing either a subspace or the Temporal Constant, even if the process acts on a system which has no true subspace. As said above this is the value represented by a $\oplus$ number on the A-axis itself. It is also the value of a negative radical on the A-axis. We must repeat the process solving for A in step identified by $\mathrm{TA} = -1$ using $\mathrm{T} = -1$, the value which corresponds to the reverse transform operation. This will provide the value which corresponds to that on the y-axis and is

identical to that found by way of a subspace transform on a lone subspace number.

To prevent confusion we use the term L to represent actual value the subspace term will take in equation it originates and L_o to represent this opposite—note this is NOT the value it takes in a subspace. Again, it's related to negative radical solutions to the Alternate. The reason is that the equation for the Alternate will result in ⊕ numbers when of $t < (s^2 + u^2)$. However since we are not here solving for the Alternate and instead merely using its relation to provide the value a subspace value when it naturally originates in an equation the process is repeated to find the value on the real space axis. This process is called the Alternate-Temporal Constant conversion (*abbreviate to ATC2*).

For T = 1:	$\frac{s}{sn}A = \oplus\Xi\ 1$	$TL_o = \oplus 1\ (\oplus\ 1)$	$TL_o = -1$	$L_o = -1$
For T = -1:	$\frac{s}{sn}A = \oplus\Xi 1$	$TL = \oplus 1\ (\oplus\ 1)$	$\mathrm{T}L = -1$	$L = 1$

When confronted by a situation of $m(\oplus n)$, where m is a real space number and n is a subspace number the result is identical to $m(n)$ within the equation $\oplus n$ arises. The only time it is negative is when it arises from the A-axis equation when t is less than the sum of the squares of all remaining inputs. A summary lies below.

<u>Positive Multiplication Space:</u>	<u>Negative Multiplication :</u>
$1 \cdot \oplus 1 = 1$	$1 \cdot \oplus 1 = -1$

Positive Division:		Negative Division:	
$\frac{\oplus 1}{1} = 1$	$\frac{1}{\oplus 1} = 1$	$\frac{\oplus 1}{-1} = -1$	$\frac{-1}{\oplus 1} = -1$

Whether by subspace transform or conversion with the Alternate equation we can obtain the value a subspace number takes within the equation it originates. This will usually occur as a result of a negative radical and will be covered later.

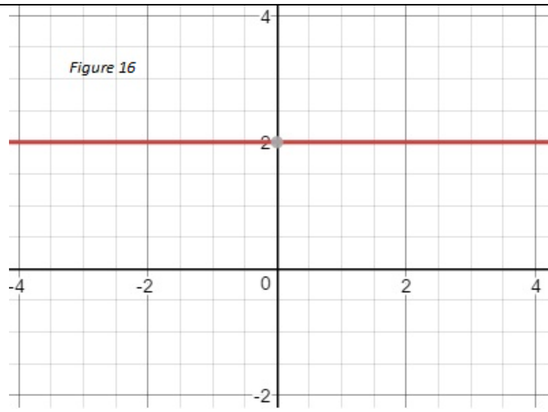
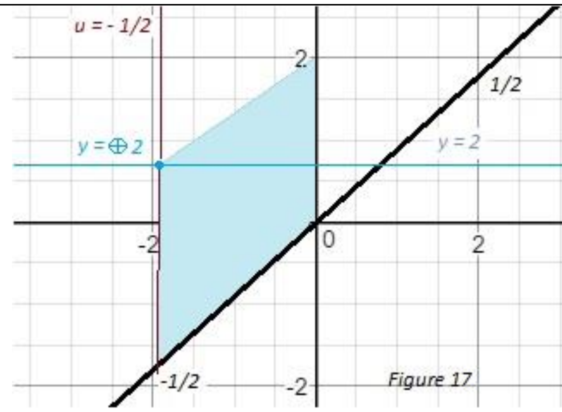
For now let's return to the plotting of points. Consider the following example with $y = \oplus 2$.

A subspace transform implies $y = \oplus 2$ means the positive value on the *y*-axis is $y = 2$ and simultaneously indicates its subspace $u = -\frac{1}{2}$.

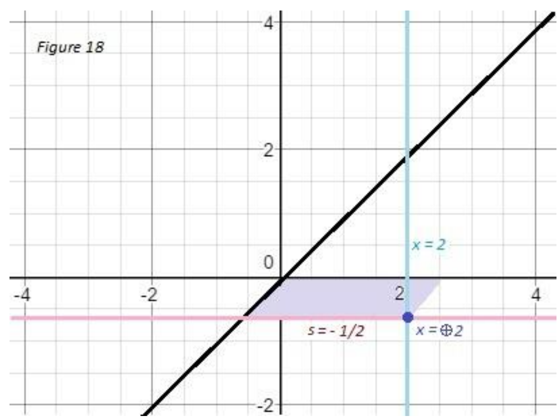
$$\Xi y = \oplus 2 \frac{s}{sy} \qquad \oplus 1 = \oplus 2(u) \qquad u = \oplus\oplus\left(\frac{1}{2}\right) \qquad u = -\frac{1}{2}$$

$$\Xi u = -\frac{1}{2}\frac{s}{su} \qquad \oplus 1 = -\frac{1}{2}(y) \qquad -2 = -y \qquad y = 2$$

The sign $y = \oplus 2$ implied a necessary axis was missing. Consider the plot of the point in figures 16 and 17 below. Figure 16 shows the point we see on the Cartesian plane while figure 17 shows the actual location the point lies in the YU-Plane.

Graph of $y = \oplus 2$	
	

It is possible to graph the same type of point on any axis. Consider the placement of the point $x = \oplus 2$. We know that the subspace of the x-axis is the s-axis.



$\Xi\, x = \oplus \frac{s}{sx} 2$ $\qquad \oplus 1 = \oplus 2s$ $\qquad s = -\frac{1}{2}$

$\Xi\, s = -\frac{s}{ss}\frac{1}{2}$ $\qquad \oplus 1 = -\frac{x}{2}$ $\qquad x = 2$

The full point positions for either of these examples will include the Alternate. As t is not included in the equations its value in the Alternate will be 0, and will result in the root of a negative number. The value of the Alternate axis is the negative of the number represented by the root. It will be covered in detail in the following chapter in the section on the negative radical that the radical of a negative number is a ⊕ number. Using the method of ATC2 transform this will resolve to a negative value on the Alternate.

$$y = \oplus 2 \quad \equiv \quad Point\ P_1: (x\,, y \mid s\,, u\,, A) = \left(0\,, 2 \;\middle|\; 0\,, -\frac{1}{2}\,, -\frac{1}{2}\right)$$

$$x = \oplus 2 \quad \equiv \quad Point\ P_2: (x\,, y \mid s\,, u\,, A) = \left(2\,, 0 \;\middle|\; -\frac{1}{2}\,, 0\,, -\frac{1}{2}\right)$$

Plotting Negative Subspace Points—5.a.iii:

A negative subspace point is represented as a negative real space output. For example $-y = \oplus 2$ and $-x = \oplus 2$. Using the same methods in sections 5.a.i and 5.a.ii we find the points are identical to those when solving $y = \oplus 2$ and $x = \oplus 2$. The Alternate is found by its own equation, $A = \sqrt{t^2 - s^2 - u^2}$ with *t* = 0.

$$-\Xi y = \oplus \frac{s}{sy} 2 \qquad -(\oplus 1) = \oplus 2u \qquad u = \frac{1}{\oplus 2} \qquad u = -\frac{1}{2}$$

$$\Xi \mathrm{u} = -\frac{1}{2}\frac{s}{su} \qquad \oplus 1 = -\frac{y}{2} \qquad y = 2$$

Thus—

$$-y = \oplus 2 \equiv y = 2 \qquad Point\ P_3\colon (x, y \mid s, u, A) = \left(0, 2 \,\middle|\, 0, -\frac{1}{2}, -\frac{1}{2}\right)$$

$$-x = \oplus 2 \equiv x = 2 \qquad \equiv \qquad Point\ P_4\colon (x, y \mid s, u, A) = \left(2, 0 \,\middle|\, -\frac{1}{2}, 0, -\frac{1}{2}\right)$$

Plotting Positive and Negative Points in Real Space and Subspace—5.a.iv:

Theorem 1:

For any point on the Cartesian plane in real space $P_n = (a, b)$, where *a* corresponds to a value on the x-axis *and b* to a value on the y-axis, there shall exist a point in subspace $P_s = (c, d)$ where *c* corresponds to the s-axis definable as $c = -\frac{1}{a}$, and *d* to the u-axis definable by $d = -\frac{1}{b}$. The Alternate, the A-axis is definable as $\sqrt{t^2 - (c^2) - (d^2)}$.

The reasoning for the why the Alternate takes the negative value of a root when taking a negative radical is not easy to grasp. The chapter on the Negative Radical results in a ⊕ number which by ATC2 resolution terminates the negative value of the root. Once again, unlike the x, y, z, s, u, and w the Alternate has no subspace. Neither is it by Temporal Mechanical definition a true direction. Though it *has its own axis and used as a direction it is actually defined as a dimension within Temporal Mechanics. Though Dimensions are composed of real space and subspace axes, Dimensions do not have subspaces. The Alternate Dimension is composed of all subspace directions, relating it thereby to all placement in space directions and a perception of directional time. In the next chapter the section on the negative radical will provide additional details.*

<u>Adding and subtracting subspace values—5.a.v:</u>

Addition and subtraction are not difficult concepts. Expressions such as $y = x + 1$ and $y = x^2 - 3$ are straightforward and require little thought to understand. What about an the expression $y = x + (\oplus 2)$ and $y = x - (\oplus 2)$?

In such an instance the subspace value is not itself set equal to any specific axis like x or y. Instead it's a constant that we need to add or subtract. So a direct subspace conversion on the entire equation won't help determine how its value. Performing a subspace conversion on the full equation looks like $\Xi y = \frac{s}{sx}\left(x + (\oplus 2)\right)$ and will yield $u = -\frac{1}{(x+(\oplus 2))}$. The problem still persists; *what do we make of the* $\oplus 2$*?*

Solving the equation requires either a direction subspace transform on the subspace number itself or conversion using a reverse transform with ATC2 equation, both of which were previously discussed.

Using methods described in section 5.a.ii we find

$$y = x + (\oplus 2) \quad \rightarrow \quad y = x + 2$$

$$y = x - (\oplus 2) \quad \rightarrow \quad y = x - 2$$

Thus any instance of addition or subtraction of a subspace value in an equation may be understood as adding or subtracting the positive value of that number.

<u>Multiplication and Division of Subspace Values—5.a.vi:</u>

Recall from section 5.a.ii that direct multiplication and division of subspace values can only occur as part of a subspace transform. Consider the following equations:

$$y = x(\oplus 2) \qquad\qquad y = \frac{x}{\oplus 2}$$

These operations cannot be completed without the Alternate-Temporal Constant conversion (ATC2) detailed in section 5.a.v. With that process we find the following.

$$y = x(\oplus 2) \quad \equiv \quad y = 2x \qquad\qquad y = \frac{x}{\oplus 2} \quad \equiv \quad y = \frac{x}{2}$$

<u>Squares and Radicals of Subspace Numbers—5.a.vii:</u>

The square of a subspace constant will equal a negative number. For a further description on this process see chapter 6 section on the negative radical.

$$(\oplus\, n)^2 \quad = \quad (\oplus\, n)(\oplus\, n) \quad = \quad -n^2$$

$$(\oplus\, 2)^2 \quad = \quad (\oplus\, 2)(\oplus\, 2) \quad = \quad -4$$

The root of a subspace number is solved using processes detailed in section 5.a.v., using ATC2 with T = -1.

$$\sqrt{\oplus n^2} \quad = \quad \sqrt{n^2} \quad = \quad \pm n$$

$$\sqrt{\oplus 4} \quad = \quad \pm 2$$

Higher Powers and Roots of Subspace Numbers—5.a.viii:

Cubes and higher powers of subspace numbers are more complicated. If two ⊕ numbers are multiplied together direct multiplication IS allowed. Both numbers are subspace terms so there is no need to use a transform on it and it will be a negative, real space number.

What about cubing one? We run into a problem: $\oplus a \cdot \oplus a \cdot \oplus a = -a^2 \cdot \oplus a$. Now the subspace number has to be converted to a real space number to permit the operation. This will result in $-a^2 \cdot \oplus a = -a^2 \cdot a = -a^3$.

What about $\oplus a^4$? If this were a real space number we could easily multiply sususesive copies of the same number to each other, or work them out in groups. For example:

$$1 = \begin{cases} -1^4 \\ 1^4 \\ (-1^2) \cdot (-1^2) \\ (-1^3) \cdot (-1) \\ etc. \end{cases}$$

This won't work with exponents of ⊕ numbers. Although the properties of multiplication say choosing which of these numbers we want to multiply together first, second and so forth will not change the final value of the exponent, this arbitrary choosing does not change the process of raising to a power which actually is:

$$a^n = a \cdot a \cdot a \cdot a \cdot a \ldots..$$

$$a^n = a^2 \cdot a \cdot a \cdot a \cdot a$$

$$a^n = a^3 \cdot a \cdot a \cdot a$$

$$a^n = a^4 \cdot a \cdot a$$

$$\vdots$$

$$a^n$$

The commutative property of multiplication is clearly not valid within the construct of raising $\oplus$ number to a power. You must multiply each successive instance of the value to its last raised total. Nonetheless this remains identical to raising an exponential value. So we for $\oplus a^4$ we get:

$$\begin{aligned} \oplus a^4 &= \oplus a \cdot \oplus a \cdot \oplus a \cdot \oplus a \\ &= -a^2 \cdot a \cdot \{\oplus a \\ &= -a^3 \cdot a \\ &= -a^4 \end{aligned}$$

5.a.ix
For any n

$$(\oplus a)^n \quad = \quad -(a^n)$$

$\oplus a \cdot \oplus a = -a^2$ is permissible because the subspace value is not interacting with other real space values in an equation. Additionally this first multiplication of two $\oplus$ number signs has a unique correlation with the complex plane which will be discussed with the negative radical. Any higher applications must default to (5.a.ix) $(\oplus a)^n \quad = \quad -(a^n)$ because it will result multiplication of real space numbers by subspace numbers.

From this we can define a **modified Commutative Law of Multiplication**—

Given the integers $\oplus a$, $\oplus b$ and $\oplus c$ with $d = |a| \cdot |b| \cdot |c|$

$$-d = (\oplus a \cdot \oplus b) \cdot \oplus c \quad = \quad (\oplus a \cdot \oplus c) \cdot \oplus b \quad = \quad (\oplus b \cdot \oplus c) \cdot \oplus a$$

Polynomials raised to Powers

Consider the equation $y = (x + (\oplus 2))^2$. This should look familiar. Consider the equation $y = x^2 + 4xi - 4$. You've likely guessed they are the same equation in two different forms. This is a peak ahead at the negative radical, but a third form of the equation, and the full evaluation is as follows:

$$y = (x + (\oplus 2))^2 \qquad y = (x + (\oplus 2))(x + (\oplus 2)) \qquad y = x^2 + 4x - 4$$

To get to the third step the inner terms were first multiplied by a real space term, the *x*, which required they be resolved to their real space correspondences and then finally added together. The numeric outer term is the result of a squared ⊕ sign which generates a negative.

Why could we not simply have first resolved the individual $x + (\oplus 2)$ terms before evaluating the square of this polynomial? Each of these $x + (\oplus 2)$ represents an actual number based on a variable unknown. If these terms were instead $x + 2$ nothing needs to be done to resolve the value of any subspace numbers so it wouldn't matter if you simply added x to 2 and then multiplied rather than reverse order. In the case of $y = (x + (\oplus 2))^2$ the attempt to evaluate the interior first will alter the equation.

$$(x + (\oplus 2))^2 \neq (x + 2)^2$$

$$x^2 + 4x - 4 \neq x^2 + 4x + 4$$

$y = (x + (\oplus 2))^2$ tells the mathematician to square a polynomial containing subspace numbers and this must be performed first. It doesn't matter if you are squaring, cubing or distributing. Leave the number in its subspace form and work out any math as you would normally treating the entire term as a single number.

Correct	**Incorrect**
$(x + (\oplus 2))^2 = \mathbf{x^2 + 4x - 4}$	$(x + (\oplus 2))^2 = \mathbf{x^2 + 4x + 4}$

$(x + (\oplus 2))(x - (\oplus 2)) = \boldsymbol{x^2 + 4}$	$(x + (\oplus 2))(x - (\oplus 2)) = \boldsymbol{x^2 - 4}$

In the next example distributing through won't result in any difference. The reason is this is not two or more polynomials multiplying or dividing each other. Instead one polynomial is just being multiplied by whole number.

$$(x + (\oplus 2)) \cdot 3 \quad = \quad \boldsymbol{3x + 6}$$

Here must first resolve the ⊕2 by ATC2 and then distribute the 3. Section 5.a.v. details why $y = x + (\oplus 2)$ is identical to $y = x + 2$.

Roots

Roots aren't difficult to understand within traditional mathematics. Any positive number's root is a positive or negative number which when squared equals that value. An odd numbered root of a negative number is a negative value which when raised to that power equals the original negative number. We also say that any even root of a negative number is a positive number whose value raised to that even power, times i as the root of -1, equals the original negative number.

Where n is any even number, m is any odd number and a is any integer:
$\sqrt[n]{a^n} = a$ $\qquad$ $\sqrt[4]{16} = 2$
$\sqrt[m]{a^m} = a$ $\qquad$ $\sqrt[3]{8} = 2$

$$\sqrt[m]{-a^m} = -a \qquad \sqrt[3]{-8} = -2$$

$$\sqrt[n]{-a^n} = ai$$

First consider the expression $\sqrt[n]{-a^n} = ai$. As mentioned already this will be covered in the section dealing with the negative radical. For now know this expression is resolvable as $\sqrt[n]{-a^n} = \oplus\, a = a$ on any axis except the Alternate and –a on the Alternate axis itself.

Any axis except the Alternate:	$\sqrt{-4} = \oplus\, 2 = 2$
On the Alternate Axis:	$\sqrt{-4} = \oplus\, 2 = -2$

What about roots of an actual ⊕ number? There is no combination available which, when raised to any power, results in a ⊕ number. The operation can be performed by reducing the ⊕ number to the value it holds within its originating equation. Using methods detailed earlier this is the positive value held by the number.

Where n is any integer (*even or odd*)

$$\sqrt[n]{\oplus a^n} \quad = \quad \sqrt[n]{a^n} \quad = \quad \pm a$$

Evaluating Expressions—5.b

The use of the $\frac{0}{0}$ has been discussed when we use this term modify input / output sides of an expression to alternate between real and subspace forms of an equation. We have discussed what to do with ⊕ numbers when they arise in an equation. What however do we do with $\frac{0}{0}$ when it arises in an equation? The answer will depend on whether or not the value is part on equation or brought in as a consonant. Additionally it will follow the pattern an equation is *trending* toward in selecting which value is appropriate for the expression. Consider $= \frac{0}{x}$, $y = \frac{0}{0}$, $y = 1 + \frac{0}{x}$, $y = 1 + \left(\frac{0}{0}\right)$, $y = \left(\frac{2\cdot0}{0}\right)$ and $y = 2\left(\frac{0}{0}\right)$.

These equations have to be handled in a proper order. $\frac{0}{0}$ is used to modify values in a specific way. Its use in subspace transforms uses both ∞ and ⊕1 to generate real values for equations which otherwise were unsolvable. However, when it arises naturally within an equation no conversion is taking place. Instead of using both of these concepts to transform the equation, the value which *fits the equation* must be used.

This is not a new concept in mathematics. Consider a car accelerating down a rod. We start the vehicle at a point labeled as the origin and it accelerate it down the road in the direction we labeled as the positive x-axis. Assuming its position on the road at any point was determined by its speed at a given instant as $x = \sqrt{speed}$. When the car's speed is 100 mph it position is $x = \pm 10$. Effectively there are two answers which fit the math; 10 and -10. However we know the vehicle began at $x = 0$ and accelerated toward the positive side. It was never at -10. This value must be 10.

Unless performing a transform on an equation or expression, $\frac{0}{0}$ must be considered as either ∞ or ⊕1. In a transform it had to represent both but on different sides of the equal sign. On the function side it took ∞ to relate the value that *y* approached. On the solution side it took ⊕1 to relate to the real numbers expressed in function. When arising naturally in an equation you have to decide if the value is trending toward real numbers or infinity.

$y = \frac{0}{x}$

The graph of this equation is a line is identical to the x-axis. Traditionally we are taught to redefine the graph at x = 0 so that y = 0 because otherwise the function will equal $\frac{0}{0}$. Here the value is set equal to a lone variable. For any value of x you get 0 expect at x = 0. In this instance when x = 0 within the confines of our example function $\left(y = \frac{0}{x}\right)$ the expression is clearly trending toward infinity. The expression is not modifying anything else and this value will actually remove the discontinuity in the graph once its resolved. So here $\frac{0}{0} = \infty$. By way of the transforms we relate the infinity to the value the y-axis subspace, the u-axis, approaches when x = 0.

$$y = \frac{0}{0} = \infty \qquad \Xi y = \infty \frac{s}{sn} \qquad u = \frac{\oplus 1}{\infty} = 0$$

$$y = \infty = u = 0$$

y Attempts to reach infinity but this is impossibly far away. Recall that the use of the ∞ in a subspace transform is what relates the real space and subspace axes. This relation is used whenever a value equals infinity to obtain what actual numeric value it approaches. In this instance that is 0.

$$y = \frac{0}{0} = \infty$$

$$y = \infty \qquad u = 0$$

$$y \quad \leftarrow \infty \rightarrow \quad u$$

$$y = 0$$

$y = \frac{0}{0}$

Here the value is not arising as part of a function. Instead *y* has been set directly equal to a constant. Since its not coming from an equation and it being used as a constant the value must be directly resolved as a constant that equals $\oplus 1$. The value is resolved using methods described above to:

$$y = \frac{0}{0} = \oplus 1 \rightarrow \ 1$$

$y = 1 + \frac{0}{x}$ and $y = 1 + \left(\frac{0}{0}\right)$

Here when x = 0 we have two very different things that take place. Both look very similar. The first will have a $\frac{0}{0}$ only when x = 0 as a result of a function. So there is the idea of the value being modified by division and clearly approaches infinity for all values of x. In the second instance it is already a there as a constant indicated by its placement alone within parentheses. As will see it necessary to make this distinction.

In the first instance $\frac{0}{x}$ is 0 for all x with a 1 added to it, except at the value x = 0. As the function is clearly trending toward infinity we use $y = \frac{0}{0} = \infty$. For reasons discussed above this resolved to y = 1 + 0. Adding 1 to the value becomes y = 1 for all values of x. The graph is a horizontal line parallel to the x-axis at the value y = 1.

$y = 1 + \left(\frac{0}{0}\right)$

In this instance the $\frac{0}{0}$ is constant, not a result of a function trending toward infinity. Its presence is trending toward a constant and must take on its real number value via $\oplus 1$.

In $y = 1 + \left(\frac{0}{0}\right)$

$y = 1 + \left(\frac{0}{0}\right) \quad y = 1 + (\oplus 1) \qquad y = 1 + 1$

$y = 2$

$y = \left(\frac{2 \cdot 0}{0}\right)$ and $y = 2\left(\frac{0}{0}\right)$

Here again are two similar looking functions with very different meanings. $y = \left(\frac{2\cdot 0}{0}\right)$ is fairly straight forward. An equation of the form $y = \left(\frac{a\cdot 0}{0}\right)$ will first evaluate the numerator resulting in $y = \left(\frac{0}{0}\right)$. Though this is a function any value of multiplied by 0 will be 0. So even though we have $\frac{2\cdot 0}{0}$ really this is already identical to $\frac{0}{0}$ and is trending toward a constant. So this will resolve to $y = \left(\frac{2\cdot 0}{0}\right) = \frac{0}{0} = \oplus 1 \rightarrow 1.$

In the second instance $y = 2\left(\frac{0}{0}\right)$ is modifying the value of 2 by multiplication. No infinite needs to be removed and there is no need for a subspace transform. Here again $\frac{0}{0}$ is trending toward a constant and must resolve to a real number value as $\oplus 1$. $y = 2\left(\frac{0}{0}\right) = 2(\oplus 1) = 2.$

Had this equation instead been written as $y = 2\left(\frac{0}{x}\right)$ the $\frac{0}{0}$ which arises when x = 0, would be coming from division in a function which is trending toward 0 at all other values of x and toward infinity at x = 0. It would be evaluated to 0 whether or not the 2 was ever distributed. In the given $y = 2\left(\frac{0}{0}\right)$ where the $\frac{0}{0}$ is a constant it have to first be evaluated to the value of the positive 1 it implies before the 2 can be distributed.

These rules for dealing with the natural emergence of $\frac{0}{0}$ in an equation are critical to dealing with polynomials within fractions as will be seen in the next set of examples.

Fractions Containing Polynomials

Consider the equation: $f(x) = \frac{x^2+x-12}{x^2-x-6}$

When evaluating an Algebraic expression we are taught three methods of approach; 1) Direct Substitution, 2) Factoring and 3) The Conjugate Method. A fourth method can be used if these don't seem to work where an insanely close value is plugged in from the positive and negative sides of a specific value. For the example expression above we will find zeros exist when $x = \{-4, -2, 3$. Yet we are also met with trouble if *x* equals either -2 or 3. If you attempt to plug in 3 you arrive at the indeterminate form $\frac{0}{0}$. When we use $x = -2$ we get $-\frac{10}{0}$.

Temporal Mechanics has no issues with either of these values. By using the methods for arriving at subspace equations real solutions are obtained. Let's begin with the traditional route to the solution for this type of problem; Factoring.

$$\frac{x^2+x-12}{x^2-x-6} = \frac{(x+4)(x-3)}{(x+2)(x-3)}$$

1. The first step would be to factor the problem and look to see if anything would cancel out. As it happens we're in luck. The $x - 3$ appears in the numerator and denominator and may be canceled out. This leaves us with:

$$\frac{(x+4)\cancel{(x-3)}}{(x+2)\cancel{(x-3)}} = \frac{(x+4)}{(x+2)}$$

2. Now we can see when we evaluate for the values of $x = \{-4, -2, 3$ we get

$$f(x) = \left\{0, \frac{2}{0}, \frac{7}{5}\right.$$

3. Traditionally, two of these answers are fine but we still have an issue at $x = -2$. The division by 0 will traditionally show there exists a vertical asymptote at the point $x = -2$ in the graph.

By using the subspace equations we can determine what the actual values of this or any other expression is when division by 0 or the Indeterminate arise. We will also see that the different values we get for outputs before and after factoring division by 0 are actually identical despite appearing different. For example before factoring at $x = -2$ we get $-\frac{10}{0}$, afterward we get $\frac{2}{0}$. Using methods already described these both resolve to 0.

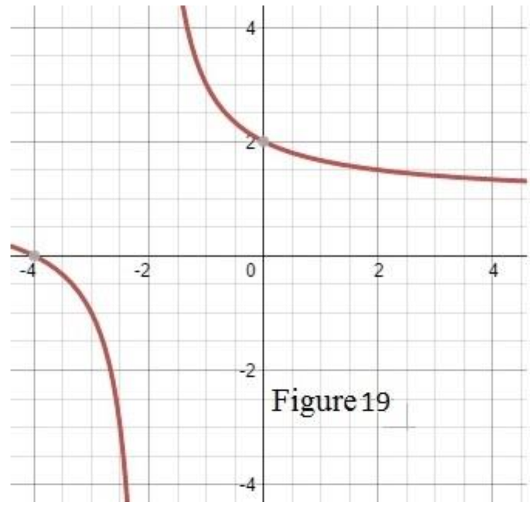


Figure 19

The equation $f(x) = \frac{x^2+x-12}{x^2-x-6}$ will have a corresponding subspace equations. We obtain them by performing their subspace transforms.

$$\Xi y = \frac{s}{sx}\frac{x^2+x-12}{x^2-x-6} \quad \longrightarrow \quad \oplus 1 = \frac{x^2+x-12}{x^2-x-6}\infty \quad \longrightarrow \quad \oplus 1 = \left(\frac{x^2+x-12}{x^2-x-6}\right)u$$

$$u = -\left(\frac{x^2-x-6}{x^2+x-12}\right)$$

$$\Xi x = \frac{s}{sx}x \quad \longrightarrow \quad s = -\frac{1}{x}$$

Recall that the $\oplus$ operator is essentially $(+\,-)$ during a transform and will change the sign of what multiplies or divides it.

$$u = -\left(\frac{x^2 - x - 6}{x^2 + x - 12}\right)$$

Factored u is equal to:

$$u = -\left(\frac{x^2 - x - 6}{x^2 + x - 12}\right) = -\left(\frac{(x+2)(x-3)}{(x+4)(x-3)}\right)$$

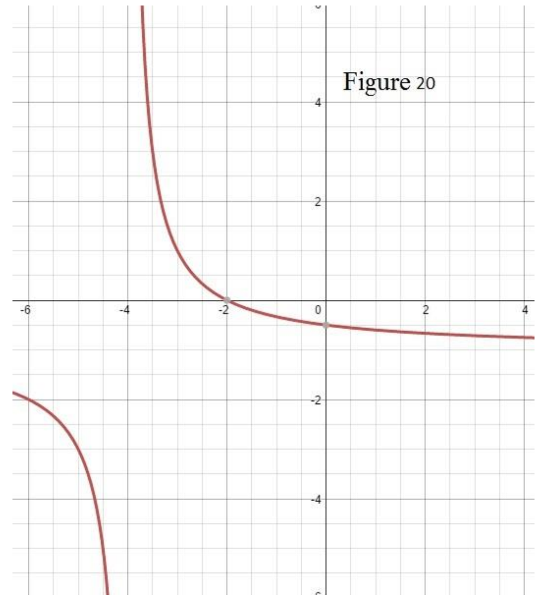


Figure 20 shows a graph of the u equation which is indeed the negative reciprocate of the y equation.

If we directly substitute for the *x* values and use *traditional* means to evaluate solutions:

$x = \{-4\,, -2\,, 3$

we get:

$$u = f(x) = \begin{cases} f(-4) = \frac{14}{0} \\ f(-2) = 0 \\ f(3) = \frac{0}{0} \end{cases}$$

After factoring the $u = f(x) = -\left(\frac{(x+2)}{(x+4)}\right)$ we find the terms provide the outputs:

$$u = f(x) = \begin{cases} f(-4) = \dfrac{2}{0} \\ f(-2) = 0 \\ f(3) = -\dfrac{5}{7} \end{cases}$$

Looking at the $u = f(x)$ graph above we see that there is indeed a zero at *x* = -2. There is also a vertical asymptote at *x* = -4, and the u value equals $-\frac{5}{7}$ when *x* = 3. At first glance there still appears to be logic issues. The before and after factoring values are not identical. As we explore further we will find these values actually are identical and we will be able to augment the graphs to correct them accordingly; $\frac{14}{0} = \frac{2}{0} = 0$.

Let's start with the *x* = 3. At this value we find that $y = \frac{7}{5}$ and $u = -\frac{5}{7}$. Whether we perform the subspace transform on the subspace equations, returning it to the form of the original real-space equation, or perform it on the solution we will arrive at the same answer.

$\Xi u = -\frac{s}{sx}\frac{x^2 - x - 6}{x^2 + x - 12}$	$\Xi \mathrm{u} = -\frac{s}{sx}\frac{5}{7}$
$\oplus 1 = -\frac{x^2 - x - 6}{x^2 + x - 12}\infty$	$\oplus 1 = -\frac{5}{7}\infty$
$\oplus 1 = -\frac{x^2 - x - 6}{x^2 + x - 12}y$	$\oplus 1 = -\frac{5}{7}y$
$\oplus 1 \cdot (x^2 + x - 12) = -(x^2 - x - 6)y$	$-7 = -5y$
$-(x^2 + x - 12) = -(x^2 - x - 6)y$ $y = \frac{x^2 + x - 12}{x^2 - x - 6}$ $y = \frac{(x+4)}{(x+2)}$ $f(3) = \frac{7}{5}$	$y = \frac{7}{5}$

Thus the subspace transform returns us to the same answer we originally obtained in the real-space equation.

All of this is interesting but what about directly evaluating? Can't be done? Don't be so certain. Let's examine the polynomial equation again.

$$y = \frac{x^2 + x - 12}{x^2 - x - 6} \quad \longrightarrow \quad y = \frac{(x+4)(x-3)}{(x+2)(x-3)}$$

We know that when *x* = 3 we get what is traditionally called an undefined value; $\frac{0}{0}$. However because this specific expression is implies subspace values we cannot simply multiply the numerator and denominator by 0. Instead $\frac{0}{0}$ must be evaluated directly. It is already part of the expression and is modifying it through multiplication. In this instance it is equivalent to a subspace constant, $\oplus 1$. It cannot be set to the infinite value—Its clear in the function that $\frac{(x-3)}{(x-3)}$ it trending toward 1. In this instant $\frac{0}{0}$ is being used within the $y = f(x)$ equation as a real value so it must be $\oplus 1$. The ATC2 operation outlined in section 5.a.v. above must be applied to the constant before it can multiply the remainder of the expression. That value will be positive 1.

$$f(3) = \frac{(7)(0)}{(5)(0)} \quad \longrightarrow \quad \frac{(7)}{(5)}\,(\oplus 1) \quad \longrightarrow \quad \frac{7}{5}\,(1)$$

$$f(3) = \frac{7}{5}$$

$\oplus 1$ is a point which combines the position of 1 on a real number line and -1 on either a subspace axis or via the ATC2 transform. As a check see the below transformation using x.

$x = \oplus 1$	Perform a subspace transform to see what the subspace value is.
$\Xi x = \frac{s}{sx} \oplus 1$ $\oplus 1 = (\oplus 1) \cdot s$	When multiplying the $\oplus 1$ its sign will change to negative.
$s = -1$	Now repeat the transformation and return to the original expression to see what the actual *x*

	value is which corresponds to this subspace value, and is simultaneously represented by the expression $$x = \oplus 1$$
$\Xi s = -\frac{s}{sx}1$ $\oplus 1 = -x$ $\frac{\oplus 1}{-1} = x$ $x = 1$	It is this value which is used when the $\oplus 1$ from the indeterminate occurs in the $y = f(x)$ equation.

So it is possible to do direct substitution despite the presence of $\frac{0}{0}$. In this type of polynomial expression we still have to factor out both the numerator and denominator into its individual components to see where its showing up and deal with it accordingly. Let's now consider the other previously discussed solution values. The output at $x = -2$ is interesting. It exists as a zero in the subspace equation graph (*see figure 20*), but is a vertical asymptote in the graph of the real-space equation (*see figure 19*). Recall when we attempted to evaluate $y = f(-2) = \frac{x^2+x-12}{x^2-x-6}$, we obtained $-\frac{10}{0}$ before factoring $\frac{2}{0}$ and after factoring.

Whenever a term is divided by zero it signifies we are dealing with a subspace output to resolve the infinite it would otherwise imply. If you convert from the $y = f(x)$ equation to the $u = f(x)$ subspace equation and evaluate $f(-2)$ you get 0. Though this equation is more complex than those discussed earlier the subspace transform still links the y value to the u value of its subspace by way of an infinite.

$$\Xi y = \frac{s}{sx}\frac{x^2+x-12}{x^2-x-6} \quad \longrightarrow \quad \oplus 1 = \frac{x^2+x-12}{x^2-x-6}\infty \quad \longrightarrow \quad \oplus 1 = \frac{x^2+x-12}{x^2-x-6}u$$

Likewise when converting back to the original real-space equation

$$\Xi u = -\frac{s}{sx}\frac{x^2-x-6}{x^2+x-12} \qquad \oplus 1 = -\frac{x^2-x-6}{x^2+x-12}\infty \qquad \oplus 1 = -\frac{x^2-x-6}{x^2+x-12}y$$

So whenever a non-zero term is divided by zero in the *y*-equation we may substitute it the value reached on u-axis at the same value for x. By the same caveat whenever a non-zero term is divided by zero in the *u*-equation we may substitute it with value reached on the *y*-axis at the same x value. Or more generally whenever a non-zero term is divided by 0, the resulting infinite may be substituted for the variable corresponding to the subspace of that system due to the relation to the infinite. So at $x = -2$, regardless of whether we use $-\frac{10}{0}$ or $\frac{2}{0}$ we would make the substitution that:

$$y = f(-2) = -\frac{10}{0} = \frac{2}{0} = \infty = u \qquad \text{Where } u = f(-2) = 0$$

The same thing is found when we examine the u-equation at the value of$x = -4$.

$$u = f(-4) = \frac{14}{0} = \frac{2}{0} = \infty = y \qquad \text{Where } y = f(-4) = 0$$

Theorem 2—5.b.i:

If $y = f(x) = \frac{h(x)}{g(x)}$ and $g(x) = 0$

$$f(x) = \frac{h(x)}{0} \quad \longrightarrow \quad \Xi f(x) = \frac{\text{s}}{\text{sx}}\frac{h(x)}{0}$$

For $h(x) \neq 0$

$$\oplus 1 = \frac{h(x)}{0}\infty \quad \longrightarrow \quad 0 = h(x)u$$

$f(x) = 0$

$$u = k(x) = \frac{g(x)}{h(x)} = \frac{0}{h(x)} = 0$$

$$\Downarrow$$

$$y = f(x) = \frac{h(x)}{0} = \infty \quad \equiv \quad u = k(x) = 0$$

Seeing this we now have a real value for our out-put values where our graphs above say we should have vertical asymptotes. This means we have to redraw our graphs. In the $y = f(x)$ equation the vertical asymptote occurred at $x = -2$. However we now see that this output value is not infinite but rather $y = 0$; the point $(x\ , y|u) = (-2\ , 0|0)$. Likewise at the value $x = 4$ the $u = f(x)$ equation shows a vertical asymptote but it is not infinite. Instead we have shown the actual output value to be 0, defined as the Point $(x\ , u|y) = (-4\ , 0|0)$.

Figures 10 and 11—5.b.ii:

If we redraw the graphs we get:

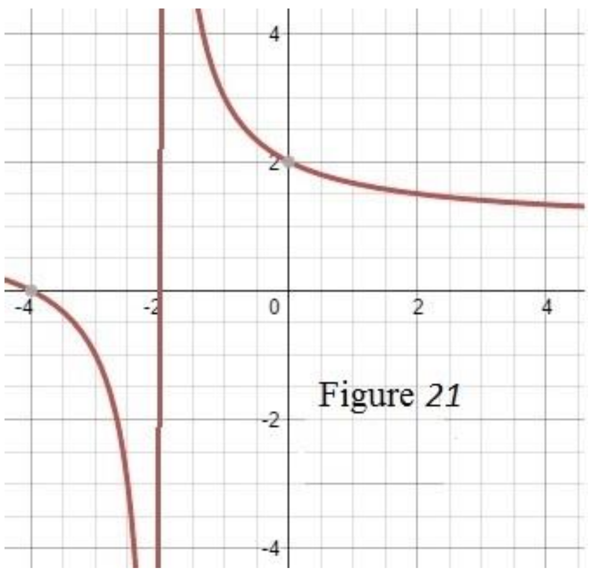


Figure 21

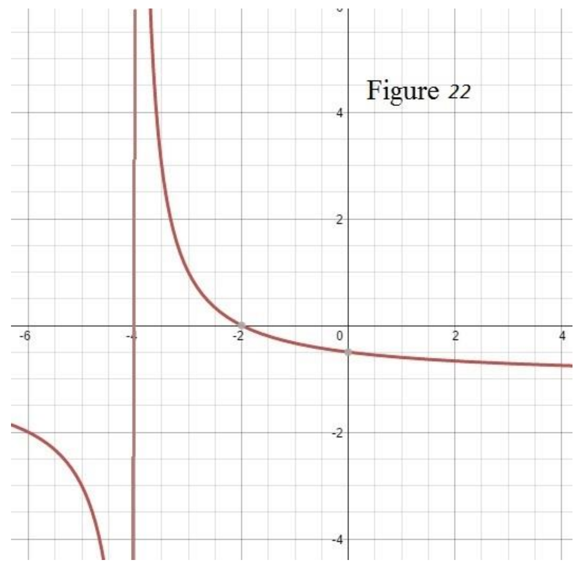


Figure 22

Figures 21 and 22 illustrate this point. This is not an artifact of the math but rather the actual values obtained when $x = \{-2\ , -4$ in both the y and u equation graphs. The asymptotic values have traditionally been accepted to attempt to reach infinity, a value we know to be *impossibly huge* (emphasis added here). This can be understood in two ways. The first is that the actual location at y = infinity, is described in this graph, as a looping back to the point y = 0—it actually is this point. The same argument can be made for the u = f(x) equation.

The second way of understanding this deals with limits of measurement. In Quantum Mechanics, for example, we are limited in how accurate our measurements can be by the Hysenburg Uncertainty Principle. Just as we cannot measure exactly the position of an electron without disturbing greatly its momentum, we can only measure as close to an x value without actually reaching that value as our instruments will allow without error. When we reach this level of error we must disregard any smaller measurement; the next measurement is that value on the *x*-axis we were avoiding.

This means there is a natural limit to how high an asymptotic value can reach. i.e. It cannot reach the impossible value of infinity—it's impossible. Instead it has a real value at that an asymptotic line not previously defined. Regardless of how close one can measure to a given x value, or whether we maintain a continuousness to equations output values which appear to reach infinity, will equal 0 at the input associated with the asymptote.

One will likely require the usage of an equation solver but it is possible to find what *x* equals in the u-equation. With that information, the inverse u-equation $x = f(u)$ can be substituted into $y = f(x)$, obtaining the $y = f(u)$ equation associated with the YU-Subplane. Continuing with the example $f(x) = \frac{x^2+x-12}{x^2-x-6}$ we find that $x = f(u) = \frac{-2(2u-1)}{u+1}$.

Making the substitution we derive the equation for the YU-Plane:

$$y = \frac{\left(\frac{-2(2u-1)}{u+1}\right)^2 + \left(\frac{-2(2u-1)}{u+1}\right) - 12}{\left(\frac{-2(2u-1)}{u+1}\right)^2 - \left(\frac{-2(2u-1)}{u+1}\right) - 6}$$

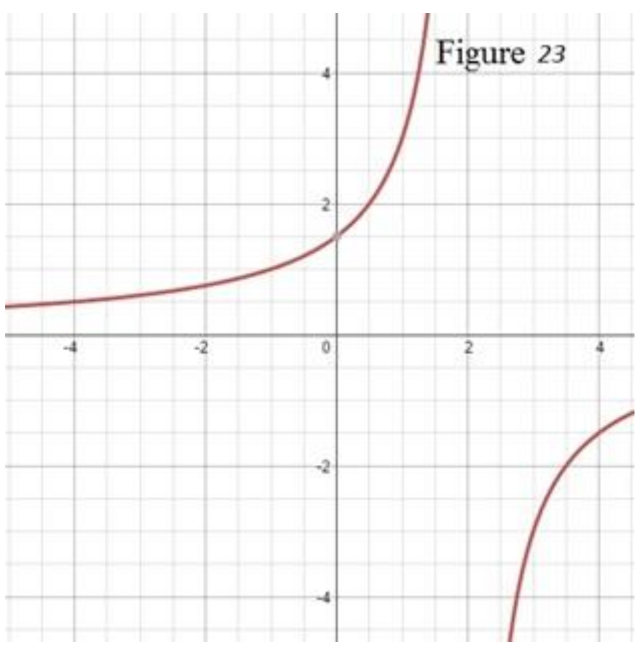


Figure 23

Notice there doesn't appear to be any zeros at first glance. There are however two asymptotes. The u-axis is the horizontal in this graph. There exists a vertical asymptote as a limit when u approaches 2. The other is a Horizontal asymptote which exists as a limit at infinity along the u-axis itself, such that $\lim_{u \to \pm\infty} f(u) = 0$.

Can we perform a subspace transform on this equation? Yes but not with the traditional subspaces. The u and y-axes are each the subspace of the other. Since they are both represented in this equation, related to one another in the YU-plane, no subspace transformation can use either of these relations.

In this instance we can simply use a dummy axis to examine what its subspace equation would look like and use the relationship between these equations to find the value the y = f(u) equation equals at u = 2. Here D-axis represents the dummy axis.

$$\Xi y = \frac{s}{su} \frac{\left(\frac{-2(2u-1)}{u+1}\right)^2 + \left(\frac{-2(2u-1)}{u+1}\right) - 12}{\left(\frac{-2(2u-1)}{u+1}\right)^2 - \left(\frac{-2(2u-1)}{u+1}\right) - 6} \qquad \rightarrow \qquad D = -\frac{\left(\frac{-2(2u-1)}{u+1}\right)^2 - \left(\frac{-2(2u-1)}{u+1}\right) - 6}{\left(\frac{-2(2u-1)}{u+1}\right)^2 + \left(\frac{-2(2u-1)}{u+1}\right) - 12}$$

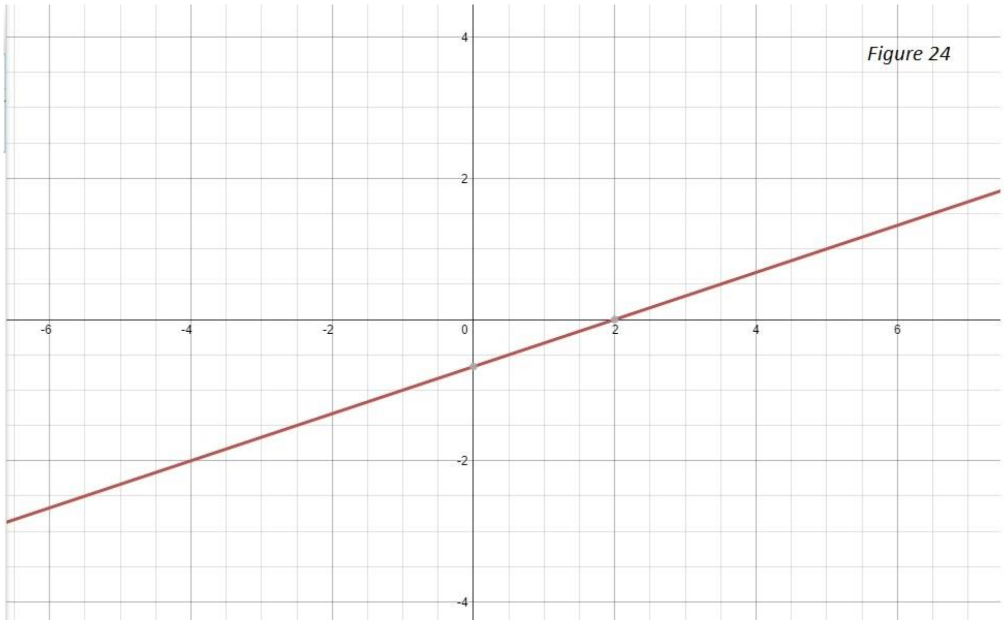
From the $D = f(u)$ equation we see at u = 2, D = 0. This is seen in the graph below in Figure 24, where the D-axis is the vertical axis and the u-axis is horizontal.

Using the relation: $y = f(u) = f(2) = \infty \;\leftrightarrow\; D = f(u) = f(2) = 0.$

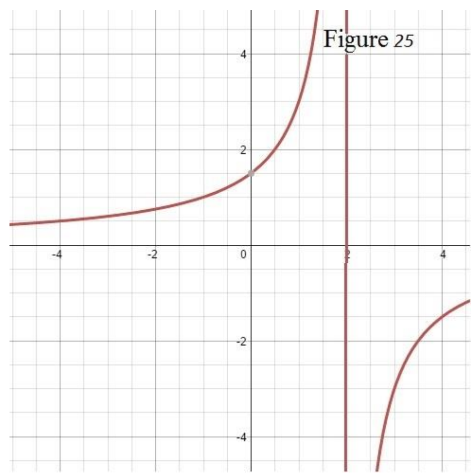
Likewise can use: $\Xi y = \frac{s}{sn}\infty \;\rightarrow\; \oplus 1 = D\infty \;\rightarrow\; D = -\frac{1}{\infty} = 0$

$y = \infty = D = 0$

In either case the value y equals in the y = f(u) equation at f(2) = 0. Again this is not an artifact of the math. Within the confines of the given y = f(u) equation in this example, the top of the infinitely high spire at u = 2 actually is the point (u , y) = (2 , 0).



$$D = -\frac{\left(\frac{-2(2u-1)}{u+1}\right)^2 - \left(\frac{-2(2u-1)}{u+1}\right) - 6}{\left(\frac{-2(2u-1)}{u+1}\right)^2 + \left(\frac{-2(2u-1)}{u+1}\right) - 12}$$



The corrected graph for $y = f(u)$ is shown in Figure 25.

$u \to 2 \quad y = f(u) \to \infty$

$\infty = D \; value \; in \qquad D = f(u)$

$y = f(u) = f(2) = \infty = D = 0$

<u>Recombination—5.c:</u>

Before we move to evaluating the horizontal limits at infinity let's look again at the graph of the hyper-plane. Clearly the s and u equations are not present in the original $y = f(x)$ equation. To include them we need to use substitutions. Consider the inclusion of the u-equation obtained by expressing both x and u in terms of y.

$y = f(x)$ and $y = f(u)$

$$y = \frac{x^2+x-12}{x^2-x-6} \quad \text{and} \quad y = \frac{\left(\frac{-2(2u-1)}{u+1}\right)^2 + \left(\frac{-2(2u-1)}{u+1}\right) - 12}{\left(\frac{-2(2u-1)}{u+1}\right)^2 - \left(\frac{-2(2u-1)}{u+1}\right) - 6}$$

Begin by simply setting them all equal to one another.

$$f(u) = y = f(x) \qquad \longrightarrow \qquad y = f(x) - f(u)$$

This has actually altered the form of the equation. To correct the problem $f(u)$ has to be added back in again. By both adding and subtracting $f(u)$ the equation is unchanged.

$$y = f(x) - f(u) + f(u)$$

The final change is made with the substitution of $f(u) = -\frac{x^2-x-6}{x^2+x-12}$.

$$y = \left(\left(\frac{x^2+x-12}{x^2-x-6}\right) - \frac{\left(\frac{-2(2y-1)}{y+1}\right)^2 + \left(\frac{-2(2y-1)}{y+1}\right) - 12}{\left(\frac{-2(2y-1)}{y+1}\right)^2 - \left(\frac{-2(2y-1)}{y+1}\right) - 6} + \frac{\left(\frac{-2\left(-2\left(\frac{x^2-x-6}{x^2+x-12}\right)-1\right)}{-\left(\frac{x^2-x-6}{x^2+x-12}\right)+1}\right)^2 + \left(\frac{-2\left(-2\left(\frac{x^2-x-6}{x^2+x-12}\right)-1\right)}{-\left(\frac{x^2-x-6}{x^2+x-12}\right)+1}\right) - 12}{\left(\frac{-2\left(-2\left(\frac{x^2-x-6}{x^2+x-12}\right)-1\right)}{-\left(\frac{x^2-x-6}{x^2+x-12}\right)+1}\right)^2 - \left(\frac{-2\left(-2\left(\frac{x^2-x-6}{x^2+x-12}\right)-1\right)}{-\left(\frac{x^2-x-6}{x^2+x-12}\right)+1}\right) - 6} \right)$$

That looks horrendous. Se we'll just simplify it as $y = f(x) - f(u) + f\big(u(x)\big)$. When $f(u) = f\big(u(x)\big)$ the equation reduces to the original form of $y = f(x)$. This same process may be used to include the s-equation by determining $y = f(s) = \left(\frac{\left(-\frac{1}{s}\right)^2 + \left(-\frac{1}{s}\right) - 12}{\left(-\frac{1}{s}\right)^2 - \left(-\frac{1}{s}\right) - 6}\right)$. The function is then added and subtracted to the y equation as was done for u.

$$y = f(x) - f(u) + f\big(u(x)\big) - f(s) + f\big(s(x)\big)$$

$$y = f(x) - f(u) + f\big(u(x)\big) - \left(\frac{\left(-\frac{1}{s}\right)^2 + \left(-\frac{1}{s}\right) - 12}{\left(-\frac{1}{s}\right)^2 - \left(-\frac{1}{s}\right) - 6}\right) + \left(\frac{\left(-\frac{1}{\left(-\frac{1}{x}\right)}\right)^2 + \left(-\frac{1}{\left(-\frac{1}{x}\right)}\right) - 12}{\left(-\frac{1}{\left(-\frac{1}{x}\right)}\right)^2 - \left(-\frac{1}{\left(-\frac{1}{x}\right)}\right) - 6}\right)$$

One axis remains to be added; The Alternate. Given $A = \sqrt{t^2 - s^2 - u^2}$ for the hyperplane its clear it may be redefined $A = \sqrt{t^2 - s^2 - \left(-\frac{1}{y}\right)^2}$. Solving for y provides A subspace transformation provides $y = -\frac{1}{\sqrt{t^2-s^2-A^2}}$. This value must also be added to and then subtracted from the y equation.

$$y = f(x) - f(u) + f\big(u(x)\big) - f(s) + f\big(s(x)\big) + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{\sqrt{t^2 - s^2 - A^2}}$$

The final element must be substituted to remove the subspace values and replace them with real space equivalents.

$$y = f(x) - f(u) + f\big(u(x)\big) - f(s) + f\big(s(x)\big) + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{\sqrt{t^2 - \left(-\frac{1}{x}\right)^2 - \left(\sqrt{t^2 - \left(-\frac{1}{x}\right)^2 - \left(-\frac{1}{y}\right)^2}\right)^2}}$$

It may seem trivial at first glance. The original equation was $y = f(x)$. It was designed to describe the *XY*-Plane, so why do we need to see the hyper-plane? The subspace conversions from the original equation allow us to solve for all values on the Cartesian Plane.

So, why are the subspace values are needed? The reason becomes increasingly clear when we move into the idea of 3-Space, the world in which we exist, where we often concern ourselves with the *what-ifs* of various possible situational outcomes. Assuming $y = f(x)$ describes the path taken by say a particle, we may ask the next logical question. If circumstances had been different would the particle have taken a different path? Undoubtedly it would be at least possible for us to derive equations for motion and position of the particle regardless of the path it took.

For each new equation, describing a different set of events and outcomes there are corresponding subspace equations. In the hyper-plane described above are all of the possible paths this given particle could take, for all points in time, which includes its known path defined by one particular $y = f(x)$ equation, the given example in this reality defined by the original equation— $y = \frac{x^2+x-12}{x^2-x-6}$.

The other paths represent the Alternate realities for this particle's path, as they too develop over time, all separated by distances in space-like directions of time we call subspace.

<u>2d Algebraic Equation Chart—5.e</u>

Type	Equation $y = f(x)$	Subspace Forms $u = g(x)$ $s = h(x)$ $A = r(t, s, u)$	Real Space Relations and Recombination $y = f\left(g^{-1}(u)\right)$ $y = f\left(h^{-1}(s)\right)$ $y = r(t, s, A) = -\frac{1}{\sqrt{t^2 - s^2 - A^2}}$ Let $r^{-1}(t, s, A) = -\frac{1}{\sqrt{t^2-\left(s(x)\right)^2-\left(\sqrt{t^2-\left(s(x)\right)^2-\left(u(x)\right)^2}\right)^2}}$
Linear	$y = x$	$u = -\frac{1}{x}$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \frac{1}{u} + \frac{1}{s} - \frac{2}{\frac{1}{x}} + r(t, s, A) - r^{-1}(t, s, A)$
Linear Plus	$y = x + n$	$u = -\frac{1}{(x+n)}$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \left(\frac{1}{u} - n\right) + \frac{1}{s} - \left(\frac{1}{-\frac{1}{(x+n)}} - n\right) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A) - r^{-1}(t, s, A)$
Linear X Constant	$y = ax$	$u = -\frac{1}{ax}$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \left(\frac{1}{au}\right) + \frac{1}{s} - \left(\frac{1}{-\frac{1}{x}}\right) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A) - r^{-1}(t, s, A)$
Fraction	$y = \frac{1}{x}$	$u = -x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \left(\frac{1}{u}\right) + \frac{1}{s} - \left(-\frac{1}{x}\right) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A) - r^{-1}(t, s, A)$
Quadratic	$y = x^2$	$u = -\frac{1}{x^2} = x^{-2}$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \left(\left(\sqrt{-\frac{1}{u}}\right)^2\right) + \frac{1}{s} - \left(\left(\sqrt{-\frac{1}{x^{-2}}}\right)^2\right) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A) - r^{-1}(t, s, A)$
n^{th} Power	$y = x^n$	$u = -\frac{1}{x^n} = x^{-n}$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \left(\left(\sqrt[n]{-\frac{1}{u}}\right)^n\right) + \frac{1}{s} - \left(\left(\sqrt[n]{-\frac{1}{x^{-n}}}\right)^n\right) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A) - r^{-1}(t, s, A)$

2d Trigonometric Equation Chart—5.f

Sine	$y = \sin x$	$u = -\csc x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \sin(-\text{arccsc}(u)) + \frac{1}{s}$ $- \sin(-\text{arccsc}(-\csc x)) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$
Cosine	$y = \cos x$	$u = -\sec x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \cos(-\text{arcsec}(u)) + \frac{1}{s}$ $- \cos(-\text{arcsec}(-\sec(x))) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$
Tangent	$y = \tan x$	$u = -\cot x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \tan(-\text{arccot}(u)) + \frac{1}{s}$ $- \tan(-\text{arccot}(-\cot(x))) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$
Cosecant	$y = \csc x$	$u = -\sin x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \csc(-\arcsin(u)) + \frac{1}{s}$ $- \csc(-\arcsin(-\sin(x))) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$
Secant	$y = \sec x$	$u = -\cos x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \sec(-\arccos(u)) + \frac{1}{s}$ $- \sec(-\arccos(-\cos(x))) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$
Cotangent	$y = \cot x$	$u = -\tan x$ $s = -\frac{1}{x}$ $A = \sqrt{t^2 - s^2 - u^2}$	$y = f(x) + \cot(-\arctan(u)) + \frac{1}{s}$ $- \cot(-\arctan(-\tan(x))) - \frac{1}{\left(-\frac{1}{x}\right)} + r(t, s, A)$ $- r^{-1}(t, s, A)$

Chapter 6

Trigonometric Properties of Temporal Mechanics

6.a—Review of Trigonometric Properties

The entirety of all Trigonometric identities can be surmised in several formulas—6.a.1:

1. $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

2. $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

3. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{e^{i\theta}-e^{-i\theta}}{i(e^{i\theta}+e^{-i\theta})}$

4. $e^{i\theta} = \cos\theta + i\sin\theta$

5. $e^{-i\theta} = \cos\theta - i\sin\theta$

The mathematical value of $i = \sqrt{-1}$ is a necessary inclusion in trigonometry. Verifiable measurements in real space, which correspond to specific trigonometric identities, are not possible unless it is included as shown in the above expressions. Temporal Mechanics provides a solution to the negative radical.

This chapter will show that $i = \frac{0}{0}$. Whenever i shows up in an equation $\frac{0}{0}$ is already part of the equation. Using methods described above its presence is clearly trending toward a constant as $\oplus$ 1. However in this instance it is also describing the complex plane, wherein it lays on its own axis representing multiples of $\oplus$ numbers, and thereby what can equally be described as multiples of either $\frac{0}{0}$ or T. The temporal constant being its own subspace is seen in the automatic defining of conjugate values for any bi value on the complex plane. As i, despite trending toward a constant we are describing the value of $\frac{0}{0}$ on its very own axis where again all of its possible values apply.

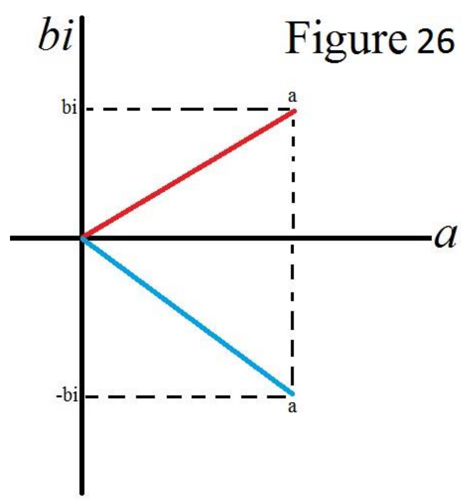


Figure 26

As a constant its value is resolvable where it arises to +1. Trigonometrically this will result in hyperbolic instead of circular relations. $\frac{0}{0}$ being both positive and negative simultaneously already marks both the positive and negative magnitude of the bi representing a multiple of it on its own axis. Because we are uniting this value with a real component a to both the positive and negative components of the number bi, the bi components must both be resolved to their real numbers values before calculating $a \pm bi$.

Consider the complex plane seen here in figure 26. This plane is composed of two axes; one having real number labeled a and one having imaginary numbers bi. Points on the plane can be identified as a complex number of the form $a \pm bi$.

These numbers are such that one point mirrors the other over the a axis. We label numbers in the upper quadrants $z = a + bi$ whilst those in the lower quadrants are $z^* = a - bi$. These are called complex conjugates and have unique feature of squaring. The squared complex number z^2 is always real and positive.

$$z^2 = z^*z = (a - bi)(a + bi) = a^2 + b^2$$

For this to be so, whatever the value of i is, it must equal -1 when squared. We cannot set i equal to either 1 or -1 without changing the outcome of the squaring of these complex numbers.

To see what's happening and to really understand where the -1 comes from let's first consider the nature of the complex plane. From the perspective of Temporal Mechanics, what's complex about the plane, is that the imaginary part exists along a direction that cannot be directly experienced. The bi axis represents what $\frac{0}{0}$ does when used to relate any and all real numbers to their subspaces.

To reiterate the bi axis not only represents multiples of $\frac{0}{0}$, a term which represents infinites and their relation to real values via subspace but also multiples of the Temporal Constant. To see how this works we want to examine only the number i. In place of i we substitute $\frac{0}{0}$. Then set a to 0 and b to 1.

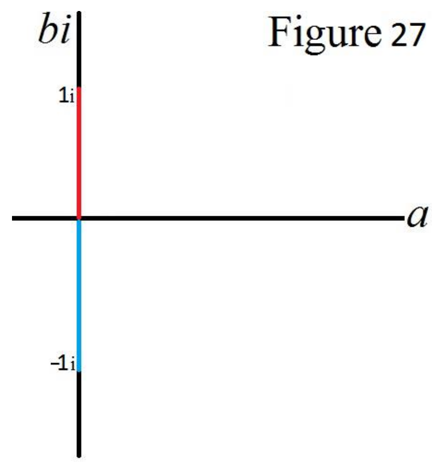
This provides the following complex number pairs:

$$(a+bi) = \left(0 + 1\left(\frac{0}{0}\right)\right)$$

$$(a-bi) = \left(0 - 1\left(\frac{0}{0}\right)\right)$$

Important to note, we are now dealing with the complex axis alone. We have the indeterminate, which is representative of the unitary subspace constant. Yet the complex bi-axis is non-representative of any actual axis, real or subspace. Used to relate one axis to another it is isolated within its own axis; it is its own subspace. Should we perform a subspace transform we find it not only persists in equaling $\oplus\, 1$ but that T must be invoked during the process as nothing can be removed by the transform.

This persistence of $\frac{0}{0} =\oplus\, 1 = T$ is indicative of the situation described by the complex plane when a is 0 and b is 1. Observe figure 27.



From this situation direct multiplication of the value as it lies on the complex plane is identical to multiplying a number constant by $\frac{0}{0}$ as a constant. Further, multiplication of a real space value with the Temporal Constant within the complex plane is identical to its multiplication during and reverse subspace transform upon the function side of an equation; i.e. direct multiplication is permitted. Likewise Repeated attempts to solve for $\oplus\, 1 = T$ by repeated transforms will result in multiplication of the real space constant by positive 1 and again by negative as depicted in figure 27.

Ignoring the presence of the a component we have $z = i =\oplus\, 1$. Using *n* to avoid confusion with the full form of the complex plane equation we resolve the value by first solving to the subspace of the number on the $\frac{0}{0}$ complex axis as: $\Xi n = i = \frac{s}{sn}(\oplus\, 1) \rightarrow \; n^* = -1$. As the $\frac{0}{0}$ complex axis is its own subspace the return value will find n as $\Xi n^* = \frac{s}{sn}(-1) \rightarrow \; n = 1$. From this we find the following:

$$z = (a + bi) = \left(0 + b\left(\frac{0}{0}\right)\right) \rightarrow \quad b(\oplus 1) = b \quad \text{correspondence to real space}$$

$$z^* = (a - bi) = \left(0 - b\left(\frac{0}{0}\right)\right) \rightarrow \quad -b(\oplus 1) = -b \text{ correspondence to subspace}$$

This is representative of the Complex Plane, bi-axis only due to its direct relation to the Temporal Constant because, even though these processes have derived two separate values z and z^*, they ultimately still represent just one simultaneously $\oplus$ value on the complex axis. Whenever $\frac{0}{0}$ arises on real axis component it will take the value of either 0 or +1 depending upon its relation to the equation in which it arises.

Thus any number $\oplus\ b$ represents a position upon the complex bi-axis, where $\oplus\ b$ is composed of simultaneously +b and –b multiples of $\frac{0}{0}$ on the complex axis. These values equal the roots of negative numbers discussed in chapter 5, and can be resolved to the positive sign of the magnitude they represent within the equation they originate by the ATC2 process. On the complex bi-axis the numbers remain $\oplus$ as represented in figure 27.

The most fascinating feature occurs when we attempt to square the complex number. Recall that the square of a complex number is actually the multiplication of Complex Conjugates which together represent singular $\oplus$ on the complex axis.

$$z^2 = z^*z = (a - bi)(a + bi)$$

From the above example we found, where b equals 1:

$$z = (a + bi) = (0 + 1i) \rightarrow \quad 1(\oplus 1) = 1 \qquad \text{Real space correspondence}$$

$$z^* = (a - bi) = (0 - 1i) \rightarrow \quad -1(\oplus 1) = -1 \quad \text{Subspace correspondence}$$

The square then is:

$$z^2 = z^*z = (a - bi)(a + bi) = \left(0 - 1\left(\frac{0}{0}\right)\right)\left(0 + 1\left(\frac{0}{0}\right)\right) = [-1][+1] = -1$$

Theorem 3—6.b.1:

$i = \sqrt{-1}$

Given that $\frac{0}{0} = \frac{\pm 0}{\pm 0} = \oplus 1$

And: $(a + bi) = \left(0 + 1\left(\frac{0}{0}\right)\right) \rightarrow 1(\oplus 1) = 1$ as a real space correspondence

$(a - bi) = \left(0 - 1\left(\frac{0}{0}\right)\right) \rightarrow -1(\oplus 1) = -1$ as a subspace correspondence

Must conclude that:

$$\left(\frac{0}{0}\right)^2 = z^2 = z^*z = (0 - 1i)(0 + 1i)$$

$$= \left(0 - 1\left(\frac{0}{0}\right)\right)\left(0 + 1\left(\frac{0}{0}\right)\right)$$

$$= [-1][+1] = -1$$

Thus: If $\left(\frac{0}{0}\right)^2 = -1$

Then: $\sqrt{-1} = \frac{0}{0} = i$

The final step is representative of the fact that the square root and square of 0 is 0, and via the complex plane is extended to $\frac{0}{0}$ and $\oplus$ numbers. For example if given $y = \sqrt{-4}$ we find $y = \oplus 2$. This value can be represented on the complex plane using multiples of $\frac{0}{0}$, i or T: $\oplus 2 = 2(i) = 2(T)$ and thereby both $bi = 2$ and $-bi = -2$. The positive and negative values are simultaneously represented on the complex axis for any given $\oplus$ number. All values are resolvable by way of transform or ATC2 conversion. Originating on any axis except the Alternate, this is the positive form of that number; the negative form is on the Alternate itself.

It can be seen that the i is a subspace term. This term may be replaced by solving for the possible values of $\frac{0}{0}$.

Trigonometric Identities with $i = \frac{0}{0}$—6.c:

Let's begin by taking another look at the five Trigonometric Identities listed earlier in 6.a.1. Only this time we are going to solve for the various values of $\frac{0}{0}$. When we do this we get four possible outcomes for each equation. Since we are using $\frac{0}{0}$ to replace i we are dealing with the complex plane. This means $\frac{0}{0}$ will not be set to 0 for any instance in which it shows up. Instead it can be kept as is, $\frac{0}{0}$, set equal to +1 or -1 (*the independent values of a* $\oplus$ *number*), or in the event of infinity as a new axis variable which we represent here as κ.

Trigonometric i—ssubstitution: 6.c.1:

$\left(\frac{0}{0}\right)$	+1	-1	κ
$\sin\theta = \frac{e^{\left(\frac{0}{0}\right)\theta} - e^{-\left(\frac{0}{0}\right)\theta}}{2\left(\frac{0}{0}\right)}$ $\cos\theta = \frac{e^{\left(\frac{0}{0}\right)\theta} + e^{-\left(\frac{0}{0}\right)\theta}}{2}$ $\tan\theta = \frac{e^{\left(\frac{0}{0}\right)\theta} - e^{-\left(\frac{0}{0}\right)\theta}}{\left(\frac{0}{0}\right)\left(e^{\left(\frac{0}{0}\right)\theta} + e^{-\left(\frac{0}{0}\right)\theta}\right)}$ $e^{\left(\frac{0}{0}\right)\theta} = \cos\theta + \left(\frac{0}{0}\right)\sin\theta$ $e^{-\left(\frac{0}{0}\right)\theta} = \cos\theta - \left(\frac{0}{0}\right)\sin\theta$	$\sinh\omega = \frac{e^{\omega} - e^{-\omega}}{2}$ $\cosh\omega = \frac{e^{\omega} + e^{-\omega}}{2}$ $\tanh\omega = \frac{e^{\omega} - e^{-\omega}}{(e^{\omega} + e^{-\omega})}$ $e^{\omega} = \cosh\omega + \sinh\omega$ $e^{-\omega} = \cosh\omega - \sinh\omega$	$\sinh\omega = \frac{e^{-\omega} - e^{\omega}}{-2}$ $\cosh\omega = \frac{e^{-\omega} + e^{\omega}}{2}$ $\tanh\omega = \frac{e^{-\omega} - e^{\omega}}{(e^{-\omega} + e^{\omega})}$ $e^{-\omega} = \cosh\omega - \sinh\omega$ $e^{\omega} = \cosh\omega + \sinh\omega$	$\sin\theta = \frac{e^{\kappa\theta} - e^{-\kappa\theta}}{2\kappa}$ $\cos\theta = \frac{e^{\kappa\theta} + e^{-\kappa\theta}}{2}$ $\tan\theta = \frac{e^{\kappa\theta} - e^{-\kappa\theta}}{\kappa(e^{\kappa\theta} + e^{-\kappa\theta})}$ $e^{\kappa\theta} = \cos\theta + \kappa\sin\theta$ $e^{-\kappa\theta} = \cos\theta - \kappa\sin\theta$

The chart listed here provides all possible ways of interpreting the presence of i in an equation. In both the first set shown, where i is kept equal to $\frac{0}{0}$, and the last set where it is set equal to κ, we will obtain the usual Circular Trigonometric Identities. In the second and third set, where substitutions for $\frac{0}{0}$, remove i using +1 or -1, the angle theta changes to omega and we obtain the Hyperbolic Trigonometric Identities. We must change the angle to Omega when this occurs. Theta is not Omega. They are not equivalent and are truly are different angles.

Consider the Pythagorean Identity: $1 = \cos^2\theta + \sin^2\theta$. It is known that this relation holds true. Yet if it were not for the i in the expressions $e^{i\theta} = \cos\theta + i\sin\theta$ and $e^{-i\theta} = \cos\theta - i\sin\theta$, the squaring of the function would be the difference, not the sum of trigonometric squares.

$e^{\theta}e^{-\theta} = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$ vs $e^{i\theta}e^{-i\theta} = (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)$

$$1 = \cos^2\theta - \sin^2\theta \quad \neq \quad 1 = \cos^2\theta + \sin^2\theta$$

Hence the Pythagorean Identity, a relationship known to be accurate, requires the root have i, a figure which Theorem 3 has shown to be $\frac{0}{0}$. If we solve for i using any other value than $\frac{0}{0}$ or κ we will get the hyperbolic relations which serve their own purpose in mathematics, science and physics.

$i = \frac{0}{0}$ The Circular Trigonometric Pythagorean 6.c.2

$e^{i\theta}e^{-i\theta} = (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)$

$e^{\left(\frac{0}{0}\right)\theta}e^{-\left(\frac{0}{0}\right)\theta} = \left(\cos\theta + \left(\frac{0}{0}\right)\sin\theta\right)\left(\cos\theta - \left(\frac{0}{0}\right)\sin\theta\right)$ Squaring of Terms

$e^{(\oplus 1)\theta}e^{-(\oplus 1)\theta} = (\cos\theta + (\oplus 1)\sin\theta)(\cos\theta - (\oplus 1)\sin\theta)$

$e^{(\oplus 1)\theta}e^{-(\oplus 1)\theta} = \cos^2\theta - (\oplus 1)\sin\theta\cos\theta + (\oplus 1)\sin\theta\cos\theta - (\oplus 1)^2\sin^2\theta$

$e^{(1)\theta}e^{-(1)\theta} = \cos^2\theta - (1)\sin\theta\cos\theta + (1)\sin\theta\cos\theta - (-1)\sin^2\theta$

$\oplus 1$ resolved by ATC2 process

$1 = \cos^2\theta + \sin^2\theta$ $(\oplus 1)^2$ resolved by Theorem 3

$$\cos^2\theta + \sin^2\theta \quad = \quad \cos^2\theta - \left(\frac{0}{0}\right)^2 \sin^2\theta$$

Although $\frac{0}{0}$ represents ⊕1 it has been shown this value will take the positive value of one whenever it arises and modifies another term, short of being itself squared, by multiplication. This positive 1 is therefore substitutable into the expression from the get go. This is also true of its complex conjugate, negative 1, which is simultaneously present. Doing this removes the presence of $\left(\frac{0}{0}\right)^2$ in the final term. Without this term we are now using Hyperbolic Trigonometry. As these substitutions represent the values of $\frac{0}{0}$ on the complex bi axis, a line representative of the Temporal Constant, the Hyperbolic Trigonometry is synonymous with the uses of subspace angles.

$$i = \frac{0}{0} = 1 \qquad 6.c.3$$

$$e^{\omega}e^{-\omega} = (\cosh\omega + \sinh\omega)(\cosh\omega - \sinh\omega)$$

$$1 = \cosh^2\omega - \sinh^2\omega$$

Hyperbolic Trigonometric Pythagorean

The same result is found by using -1

$$i = \frac{0}{0} = -1 \qquad 6.c.4$$

$$e^{-\omega}e^{\omega} = (\cosh\omega - \sinh\omega)(\cosh\omega + \sinh\omega)$$

$$1 = \cosh^2\omega - \sinh^2\omega$$

Hyperbolic Trigonometric Pythagorean

The final value for $\frac{0}{0}$ is ∞, a representation of conversion to coordinates that exist on a subspace axis. Although no subspace transform is being attempted, what would happen if we assumed it equaled infinity? We would have to substitute that value with the axis represented by the infinite. We shall use the dummy axis of κ, kappa. The result is identical to keeping $\frac{0}{0} = i$.

$i = \frac{0}{0} = \kappa$	6.c.5
$e^{\kappa\omega} e^{-\kappa\omega} = (\cosh\omega + \kappa \sinh\omega)(\cosh\omega - \kappa\sinh\omega)$	The presence of κ in the exponential is not an issue. Essentially it is an unknown constant multiplying angle omega and may be absorbed into it.
$1 = \cosh^2\omega - \kappa^2\ \sinh^2\omega$	The expression and angle is still hyperbolic so now just solve for κ.
$\kappa = \sqrt{\frac{1-\cosh^2\omega}{\sinh^2\omega}}$	The numerator may be substituted using the Hyperbolic Pythagorean Identity.
$\kappa = \sqrt{\frac{-\sinh^2\omega}{\sinh^2\omega}} = \sqrt{-1} = \frac{0}{0}$	

This last step shows us that when we set $\frac{0}{0}$ equal to infinity, another axis, it's the same things as setting equal to i. The subspace axis it is set to when $\frac{0}{0} = \kappa$ is the complex axis which is its own subspace. We get the same value back out. By substituting this value for κ into the earlier expression we have returned to the Circular Trigonometric Pythagorean and the angle Theta.

$$1 = \cosh^2 \omega - \kappa^2 \sinh^2 \omega \quad \longrightarrow \quad 1 = \cos^2 \theta - \left(\frac{0}{0}\right)^2 \sin^2 \theta$$

$$1 = \cos^2 \theta + \sin^2 \theta$$

Revisiting Trigonometric Subspace Conversions for 2-Space—6.d

Let us consider the equation $y = \frac{1}{x}$. After performing the subspace transform on the equation we get $s = -\frac{1}{x}$ and $u = -x$. We can expand on this to that $y = -\frac{1}{u}$ and $-s$. From these values we solve for the Alternate as $A = \sqrt{t^2 - s^2 - u^2}$. The recombination equation is obtained by combining these in the manner prescribed before.

$$-\frac{1}{u} = -s = y = \frac{1}{x} = -\frac{1}{\sqrt{t^2-s^2-A^2}}$$

These five terms are all equivalent in value.
The subspace values are added in and then subtracted back out.

$$y = \frac{1}{x} + \frac{1}{u} - \frac{1}{u}$$

Begin with u. Substitute the value of u in $-\frac{1}{u}$ for its corresponding x components

$$y = \frac{1}{x} + \frac{1}{u} - \frac{1}{-x} \qquad y = \frac{2}{x} + \frac{1}{u}$$

Next add in and subtract back out the s values. $-s$ is added and subtracted out just like above. The same procedure is repeated for the Alternate followed by appropriate substitutions.

$$y = \frac{2}{x} + \frac{1}{u} + s - s + \frac{1}{\sqrt{t^2-s^2-A^2}} - \frac{1}{\sqrt{t^2-s^2-A^2}}$$

$$y = \frac{2}{x} + \frac{1}{u} + s - \left(-{}^{1}\!/_{x}\right) + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{\sqrt{t^2 - \left(-{}^{1}\!/_{x}\right)^2 - \left(\sqrt{t^2 - \left(-{}^{1}\!/_{x}\right)^2 - (-x)^2}\right)^2}}$$

$$y = \frac{3}{x} + \frac{1}{u} + s + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{\sqrt{t^2 - \left(-{}^{1}\!/_{x}\right)^2 - \left(\sqrt{t^2 - \frac{1}{x^2} - x^2}\right)^2}}$$

$$y = \frac{3}{x} + \frac{1}{u} + s + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{\sqrt{t^2 - \frac{1}{x^2} - t^2 + \frac{1}{x^2} + x^2}}$$

$$y = \frac{3}{x} + \frac{1}{u} + s + \frac{1}{\sqrt{t^2 - s^2 - A^2}} - \frac{1}{x}$$

$$y = \frac{2}{x} + \frac{1}{u} + s + \frac{1}{\sqrt{t^2 - s^2 - A^2}}$$

This is the form of the equation which represents all possible outcomes for all points in time in the form of $y = f(x, s, u, A)$. Note t is not included as it too is included within the definition of the Alternate. Not all of the variables are really free variables in this equation. Although all possible outcomes are simultaneously represented u, s and A are still dependent on values of x and y. When u, s, and A all take the values expressed by their subspace relations the equation will reduce to the original expression.

$$y = \frac{2}{x} + \frac{1}{u} + s + \frac{1}{\sqrt{t^2 - s^2 - A^2}}$$

$$y = \frac{2}{x} + \frac{1}{(-x)} + \left(-\frac{1}{x}\right) + \frac{1}{\sqrt{\left(\sqrt{A^2 + s^2 + u^2}^2\right)^2 - s^2 - A^2}}$$

$$y = \frac{1}{\sqrt{\left(\sqrt{A^2 + s^2 + u^2}\right)^2 - s^2 - A^2}}$$

$$y = \frac{1}{\sqrt{A^2 + s^2 + u^2 - s^2 - A^2}}$$

$y = \frac{1}{\sqrt{u^2}}$ $\quad y = \frac{1}{\sqrt{(-x)^2}}$ $\quad y = \frac{1}{\sqrt{x^2}}$ $\quad y = \frac{1}{x}$

Likewise the Parametric solutions for the curve corresponding to $y = \frac{1}{x}$ through the hyper plane using parameter *p* are $f(p) = \left(p\ , \frac{1}{p}\ , -p\ , -\frac{1}{p}\ , \sqrt{t^2 - p^2 - \frac{1}{p^2}}.\right)$. In an earlier example we used the Polar equations to solve for two separate values of x.

Recall:

Polar Coordinate:

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$r^2 = x^2 + y^2$

In evaluating $y = \frac{1}{x}$ for x = {2 , 3 it was determined

x	y	s	u	A	r	Theta
$r\cos\theta$	$r\sin\theta$	$-\frac{1}{r}\sec\theta$	$-\frac{1}{r}\csc\theta$	$\sqrt{t^2 - s^2 - u^2}$ $for\, t = 0$	$r = \sqrt{x^2 + y^2}$	$\theta = \tan^{-1}\left(\frac{\frac{1}{x}}{x}\right)$
2	$\frac{1}{2}$	$-\frac{1}{2}$	-2	-2.06	2.062	0.245°
3	$\frac{1}{3}$	$-\frac{1}{3}$	-3	-3.018	3.018	0.1107°

The values for s and u were obtained by using a transformation based on the angle Theta. Although this provides the solution for the subspace values Theta is not the actual subspace angle. Given notes 6.c.2 through 6.c.5 we know that angles in subspace are hyperbolic. Since Theta and Omega are not the same angle we still need to solve for Omega.

Using forms of equations which relate real space values to those of subspace it is possible to obtain the value of the subspace angles. The below equations are used to convert to Hyperbolic Polar coordinates and obtain the hyperbolic angle which distends through subspace.

$$Subspace\ Angle = \ln\sqrt{\frac{Subspace\ Axis}{Real\ Space\ Axis}}$$

$$Subspace\ Geometric\ Mean = \sqrt{(Subspace\ Axis)(Real\ Space\ Axis)}$$

$$(Subspace\ Axis) = (S.G.M.)\mathbf{e}^{(\angle_s)} \qquad (Real\ Space\ Axis) = (S.G.M.)\mathbf{e}^{-(\angle_s)}$$

This method of Hyperbolic coordinates is adapted for use in Temporal Mechanics to determine the Hyperbolic angle which exists in subspace. In the image at right the blue lines indicate values marked by values of the subspace hyperbolic axis. Red lines mark the position of hyperbolic arcs at the value of the Subspace Geometric Mean. As it is also possible to express the original x and y equation in terms of hyperbolic coordinates.

Figure 28

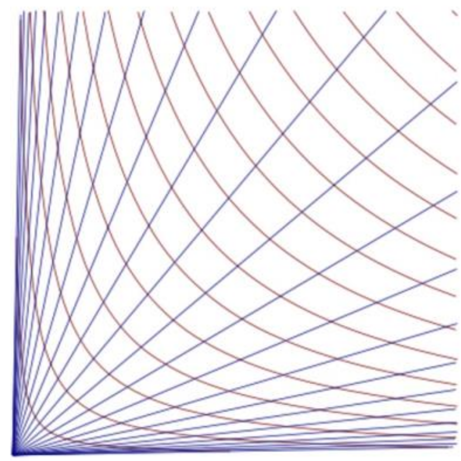


Image obtained from: https://en.wikipedia.org/wiki/Hyperbolic_coordinates

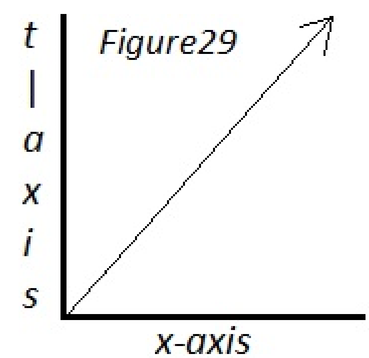
Chapter 7

Special Relative Displacement

Newtonian Reference Frames:

Special Relativistic Displacement (SRD) is a combination of the precepts of Zero Division Algebra (ZDA) and Calculus (ZDC) with Special Relativity (SR). Reference frames of Special Relativity involve coordinates systems of *x* , *y* , *z* and *t*. These reference frames may be moving or stationary. One major difference, in addition to the inclusion of directional time *t* , SRD adds in the subspace axes of *u, s, w* and *A.* These Dimensional Time directions are analogs of directional time in SR and are critical to understanding and removing the complex infinities which arise in application of Relativity.

Like reference frames of SR those of SRD, whether stationary or in motion with a constant velocity, are inertial. The laws of physics remain the same in all reference frames. All events in Special Relativistic Displacement Space (SRDS) are characterized by a coordinate system of *x , y , z , s, u , w, A* points.



Consider Figure 29 at right. An observer in motion relative to only the *x*-axis. Since there is no motion relative to the *y* and *z* axis they may be effectively ignored. The observer makes a diagonal path through such a diagram with respect to the *x*-axis and his perception of the passage of time on the t-axis as he moves from point to point.

Though there is no change in *y* or *z* each point remains a combination of the *x , y , z , s, u , w* and *A* axes. Every point in *x* , *y* , *z* space exists within a higher dimensional s, *u* , *w, A* space. Though its possible to represent these additional directions as if they were free variables through recombination they are in fact never free variables. Their existence is defined through subspace transformation based upon known values of real space equations. If they are not linked directly to real space via an equation of the form $z = f(x , y)$ they are via a parametric linkage.

In our example of the moving observer, moving at a constant velocity along the *x*-axis, we shall find he is in an inertial reference frame. He may consider his position to be stationary though we, seeing him in our own reference frame K consider him to be in motion. Our moving observer we shall place in reference frame K'. To him, he is always at potion $x' = 0$.

In reference frame K we see the moving observer pass us with a position defined by $x = vt$ where v is a constant velocity. This is Distance and equals Rate times Time. We may ask how is it useful to assume there will only be motion along one axis instead of along two or three in real space? The answer lies in the orientation of reference frames. If there is in fact motion in all three real-space directions in one reference frame with respect to some other, then it is possible to orient one's self so that motion appears to occur along one reference frame only. Once this is done motion and coordinate points may be mapped from the system of motion along all three real-space axes into one with motion only along a single chosen axis.

Because all of the coordinate axes are functions based on directional time the dimensional time axes must also be functions based on the parametric variable *t*. Yet the subspace axes *u, s, w* and *A* are space like time directions themselves. This means that the parameter *t* is

$$t = \sqrt{s^2 + u^2 + w^2 + A^2}$$

This definition for *t* is the three-directional equivalent to the version provided for two-directional systems in previous chapters. The value of subspace directions, as dimensional time, define the places in the space of time (*time as space directions*), where all the various possible choices of our moving observer exist—all possible realities at all points in time. To find the values of *s, u, w* and *A* we need only take the subspace conversions of their corresponding real space equations to define the progression of our moving observer within Temporal Mechanics.

Since motion is only along the x-axis the y and z-axis equations will be set to 0

X, Y, Z, S, U, AND W:

$$x = vt \qquad y = 0 \qquad z = 0$$

$$\Xi x = \frac{s}{sn} vt \qquad \Xi y = \frac{s}{sn} 0 \qquad \Xi z = \frac{s}{sn} 0$$

$$s = -\frac{1}{vt} \qquad u = -\frac{1}{0} = \infty = y = 0 \qquad w = -\frac{1}{0} = \infty = z = 0$$

The Alternate:

$$A = \sqrt{t^2 - s^2 - u^2 - w^2}$$

$$A = \sqrt{t^2 - s^2 - 0 - 0}$$

$$A = \sqrt{t^2 - \frac{1}{v^2 t^2}}$$

Directional Time:

t

All seven axes, *x , y , z, s, u, w* and *A*, are all orthogonal to each other and represent where in the space of time, places we consider to be past, present and future, any given set of choices exist.

We now have our subspace and real space equations. Because the Alternate is taking the place of the idea of time for the system of equations we should find that directional time *t* drops out of the equations when substitutions are applied. Since Relativistic four-vectors use directional time this is necessary to maintain correct values for the math. By the same caveat substitutions within the equation for the Alternate do not change its values either.

$$\mathrm{t} = \sqrt{s^2 + A^2} \qquad\qquad A = \sqrt{t^2 - s^2}$$

$$\mathrm{t} = \sqrt{\frac{1}{v^2t^2} + \left(\sqrt{t^2 - s^2}\right)^2} \qquad\qquad A = \sqrt{\left(\sqrt{A^2 + s^2}\right)^2 - \frac{1}{v^2t^2}}$$

$$\mathrm{t} = \sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}} \qquad\qquad A = \sqrt{A^2 + \frac{1}{v^2t^2} - \frac{1}{v^2t^2}}$$

$$\mathrm{t} = t \qquad\qquad A = A$$

By making the exchange we find that the time used by observers to describe the rate of change in three space is the same idea used to measure rate of change in subspace. Just as Special Relativity shows, that though the moving observer and stationary observer will not agree on how much time has passed they both use the same idea of how long a given interval of time measurement it is. Essentially each will still find an hour is sixty minutes long and minute is sixty seconds. These rates of change of time are identical in each of their Reference Frames. Yet when they compare their watches they will find they differ and can only agree on the concept of Proper Time.

The equations for x, y, z, s, u, w and A fully define the path taken by the moving observer, from the perspective of the stationary observer, through space and time. The value $\mathrm{t} = \sqrt{s^2 + A^2}$ is the same as the ticks of the clock the stationary observer uses to define the progress of the moving observer. Those ticks of the clock are identical to a measurement of length across subspace.

We must acknowledge that there are in fact an infinite number of combinations for s, u, w, and A as free variables which would also equal the chosen value of directional time t for each moment of perceived time by an observer in stationary reference frame K. However, although any given value of t could be generated by choosing various inputs for the subspace values only the values defined by the subspace conversions actually apply to the given equation for motion defined in real space. Instead these other infinite variety of combinations are alternate realities, at various locations in subspace, which are identical to the various points along the moving observer's path per unit of time within the reality defined by a different set of parametric points: $x = f(v, t)$ $y = 0$ $z = 0$

Even if the values chosen resulted in different values of *t* than the one specified in the given real space equation, because the exact position of the moving observer is defined by $x = vt$, this would show a position on a line at an alternate point in time defined by *t,* which may or may not correspond to the reality through which the moving observer lies defined by the original given equation of motion. Only the values defined by the subspace conversions actually apply to the given equation for motion defined in real space. If this is not so the subspace coordinates define an equation for which $x \neq vt$, a different path and set of choices taken by a different moving observer whose path in three space is defined by a different equation.

Understanding Subspace Terms:

What does $s = -\frac{1}{vt}$ tell us about the path of the moving observer? It is a single continuous line wherein *t* is permitted to have both positive and negative values. The negative values correspond to points in time occurring before the point at which measurements begin.

Even with constant velocity the longer the amount of time that passes the farther away from us our moving observer shall pass along the path of $x = vt$. We begin at $t = 0$. As t becomes larger it becomes quite difficult of tell where a point is on the s-axis. This is because s approaches 0 as time approaches infinity. It does not indicate that there are no longer any choices for our moving observer. Rather that the longer he maintains his course the greater the number of realities which share that path he has taken along $x = vt$ from *t* = 0 till he alters his path or velocity, a point which we could define as a new *now moment*. i.e. a place which $t = 0$ and a new velocity vector takes over.

Another way of looking at this same situation is that the longer one remains on a given course the less options there are available to make changes which would be noticeable to the outcome one is headed toward. For this reason it should come as a no surprise that the s tapers off so quickly whilst A begins to settle toward a definite value. For very large values of *t,* A and t are indistinguishable because s is so small.

As there are an infinite number of outcomes for all possible situations only those possible choices one can make at the start or very near thereto will make the most dramatic differences. All other outcomes, no matter how similar or radical will lie in a compressed region of either the ever distancing future or past where direct connections to the present become hazy. All possibilities not linked to the curve on which the moving observer travels, lie in the infinite region outside the curve. It always remains possible to redefine the equations used to describe a new now moment within the local space of the moving observer where all of the value regain clear distinction and easy plotting.

Finding Primed Coordinates:

For both the moving observer of reference frame K' and our stationary reference frame K, we shall have $x' = 0$ being equivalent to $x = vt$. By simple algebra we find if $x' = 0$ and $x - vt = 0$ then $x' = x - vt$.

The remaining primed coordinates aren't difficult to obtain either and can be found by way of similar associations.

Primed Real & Subspace Coordinates—Moving Observer:

$$x' = 0 \qquad y' = 0 \qquad z' = 0$$

$$\Xi x' = \frac{s}{sn}0 \qquad \Xi y' = \frac{s}{sn}0 \qquad \Xi z' = \frac{s}{sn}0$$

$$s' = 0 \qquad u' = 0 \qquad w' = 0$$

The Primed Alternate:

The following is obtained based upon the Galilean transformation that $t' = t$.

$$t = t' = \sqrt{A'^2 + s'^2 + u'^2 + w'^2}$$

$$A' = \sqrt{t'^2 - s'^2 - u'^2 - w'^2}$$

$$A' = t' = t$$

Primed—Unprimed Relations:

$$x' = x - vt \qquad y' = y \qquad z' = z$$

$$s' = s + \frac{1}{vt} \qquad u' = u \qquad w' = w$$

$$A' = -A + \sqrt{t^2 - \frac{1}{v^2 t^2}} + t$$

Primed t relation to unprimed t :

$$t = \sqrt{A^2 + s^2} \qquad t' = \sqrt{A'^2 + s'^2}$$

$$t - \sqrt{A^2 + s^2} = 0 \qquad t' - \sqrt{A'^2 + s'^2} = 0$$

$$t' = t - \sqrt{A^2 + s^2} + \sqrt{A'^2 + s'^2}$$

$$t' = t - \sqrt{t^2 - s^2 + s^2} + \sqrt{t'^2 - s'^2 + s'^2}$$

$$t' = t - t + t'$$

$$t' = t - t + t$$

$$t' = t$$

Recall that $s' = 0$ and it had better too. From the perspective of the moving observer, he considers himself stationary. For him his x position is and remains 0 for all points in time. His s-axis positon will also remain constant and unchanging from his point of view (*which for this example is 0*). The moving observer is only moving in the stationary observer's reference frame, with constant velocity. From his own reference frame it is he who sits stationary. Yet time is continuing to tick by on his watch. So his position along the alternate will continue to change even though he considers himself stationary. Within the Galilean transformations $t = t' = A = A'$. It has to be this way. Even though the moving observer is not making decisions in his movement through *x, y, z, s, u, or w* he is still moving through our traditional concept of time from past to present to future.

Exploring Differences in Time:

Thus we have the following sets of Coordinates:

Unprimed	Primed	Combined
$x = vt$	$x' = 0$	$x' = x - vt$
$y = 0$	$y' = 0$	$y' = y$
$z = 0$	$z' = 0$	$z' = z$
$s = -\frac{1}{vt}$	$s' = 0$	$s' = s + \frac{1}{vt}$
$u = 0$	$u' = 0$	$u' = u$
$w = 0$	$w' = 0$	$w' = w$
$A = \sqrt{t^2 - \frac{1}{t^2v^2}}$	$A' = t$	$A' = A - \sqrt{t^2 - \frac{1}{t^2v^2}} + t$
$\mathbf{t} = \sqrt{s^2 + A^2} = t$	$\mathbf{t}' = \sqrt{s'^2 + A'^2} = t'$	t=t'

These values hold true for the Galilean transformations. However, these values are only accurate with velocities which are slow compared to the speed of light. Einstein sought a transformation which would maintain the speed of light at the same rate in all reference frames. These are known as the Lorentz transformations.

$$x' = x\cos\theta + t\sin\theta \qquad t' = -x\sin\theta + t\cos\theta$$

θ is the angle of rotation between the primed and unprimed axes. When squared they satisfy that the distance from the origin to the moving observer is the same in both the moving and stationary reference frames.

In these transformations the velocity of the moving observer is given by $v = \frac{\sinh\omega}{\cosh\omega}$.

We also have the corrections:

$$x' = \frac{x - vt}{\sqrt{1-v^2}} \qquad t' = \frac{t - vx}{\sqrt{1-v^2}}$$

If we plug in the subspace values for the primed and unprimed coordinates we shall find they relations are maintained.

$$\mathbf{t} = \sqrt{s^2 + A^2} = \sqrt{\left(-\frac{1}{vt}\right)^2 + \left(\sqrt{t^2 - \left(\frac{1}{vt}\right)^2}\right)^2} = \sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}} = t$$

$$\mathbf{t}' = \sqrt{{s'}^2 + {A'}^2} = \sqrt{(0)^2 + (t')^2} = t'$$

Plugging these into the Lorentz Transforms:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x' = \frac{x - v(\sqrt{s^2 + A^2})}{\sqrt{1 - v^2/c^2}} \qquad t' = \frac{(\sqrt{s^2 + A^2}) - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x' = \frac{x - v\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}}}{\sqrt{1 - v^2/c^2}} \qquad A' = \frac{\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}} - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

We shall also find the distances between the origin and the moving observer in space-time, called proper time as always defined by:

$$d\tau = \sqrt{dt'^2 - dx'^2 - dy'^2 - dz'^2}$$

$$d\tau = \sqrt{\frac{d}{dt}\left(\frac{\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}} - vx/c^2}{\sqrt{1 - v^2/c^2}}\right)^2 - \frac{d}{dx}\left(\frac{x - v\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}}}{\sqrt{1 - v^2/c^2}}\right)^2}$$

What does this mean for a moving observer accelerating toward the speed of light? Let's consider the Lorentz Transformations with the subspace substitutions

$$x' = \frac{x - v\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}}}{\sqrt{1 - v^2/c^2}} \qquad A' = \frac{\sqrt{\frac{1}{v^2t^2} + t^2 - \frac{1}{v^2t^2}} - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Without even needing to examine the rates of change we can see as one's velocity approaches the speed of light time will slow and stop for that person. For the x' equation if we substitute the value q for the numerator we simply have $x' = \frac{q}{\sqrt{1-v^2/_{c^2}}}$. As the velocity v approaches the value of c experiments have verified that time will slow down. If we set $v = c$ we receive $x' = \frac{q}{\sqrt{1-c^2/_{c^2}}} = \frac{q}{0} = \infty$. This value however is related to the s' as it would be defined by the stationary observer describing the moving observer via a transformation. Here we find:

$$\Xi x' = \frac{s}{sn}\frac{q}{0} \qquad \bigoplus 1 = \frac{qs'}{0} \qquad s' = \frac{0}{q} = 0$$

From the infinite relation of subspace to real space we find when $v = c$

$$x' = \frac{q}{0} = \infty = s' = 0$$

Because this value represents the moving observer's movement through time form the perspective of the stationary observer it's clear that at the speed of light, his movement through time from the perspective of stationary observers has stopped. The same effect is seen in the A' equation. Nonetheless from the perspective of the moving observer he maintains his position and motion are 0, experiencing what feels as a normal flow of time in his own reference frame.

Can we use this to manipulate the idea of position in time? Yes but it requires an understanding of material planned to be released in a later text, Temporal Mechanics 1. This text shall examine Zero Division as it applies to Calculus and then how it applies to both General Relativity and Quantum Mechanics. However, let us consider a hypothesis which will be explored further in later texts.

Study in Electric and Magnetism shows the student that an electrically charged sphere will produce an electric field whose calculation is equivalent to that generated by the same strength point size charge placed at the center of the charged sphere. However, inside the charged sphere there is actually no electric field at all. As the charge is actually on the surface of the sphere the interior fields are cancelled out leaving the interior electrically neutral.

With gravity we have a similar effect. Ignoring that fact you'd be burned up by the sheer intensity of the heat radiating from the core of the earth, if you could get to the center of the planet you would be weightless. The reason is the gravitational fields form all directions are cancelling each other out leaving the space gravitationally neutral.

For the following to work will require the use of technology capable of manipulating the gravitomagnetic force and thereby generating gravitomagnetically induced gravitoelectric fields. Before going further note the gravitomagnetic London Moment has been detected and such technology is, though in its infancy, currently under development.

If a device generates a gravitational field about itself and the device operator in a perfectly spherical pattern the operator of the device will not feel the presence of gravitation. Provided the device is placed adequately far away from objects which would now *fall* toward the operator and his device, or shielded by additional apparatus to gravitationally naturalize the extent of the operator's device before it reaches some containing structure or materials which would fall, the observer need not worry about how strong the field from his device becomes.

If the device then generates a field whose strengths creates an infinitely collapsing well in space time commonly referenced to as a black hole the operator and his device will appear to disappear from space time. The center of the region is infinitely far away from the event horizon. Thereby the observer is infinitely far away from anything which would exist to enter the event horizon.

The specifics of the math will be covered in a later text—however, movement now by the operator and his device within this space is now equivalent to movement not in x, y and z space but through s, u and w space due to the gravitational separation from that original volume via the infinite curvature of the space to the source of the generating device.

Since the actual mass present is not enough to maintain the presence of a gravitational singularity the operator will return to normal x, y, z space upon turning off the device. The Alternate axis represents the idea of an equivalence to *directional time* within subspace. From the equation $A = \sqrt{t^2 - s^2 - u^2 - w^2}$ it is possible to enter a negative value for t^2.

If we assume the operator of the device is at some location we define as the origin such that $0 = \{x, y, z$ subspace conversion relating infinite values will define $0 = \{s, u, w$ as well. Let us assume we are working with units of time in minutes, if we then set $t^2 = -4$, using methods described in earlier chapters for the Alternate we have:

$$A = \sqrt{-4} = -2$$

This idea of a negative square for time implies we squared $\oplus$ number to allow this value. In the sense of the operator and his gravitating device this is realistic as the time he experiences is in a sense a subspace. Within the now gravitationally accessible subspaces of s, u and w he experiences time as a subspace of that in x, y and z space. This value, related to the Alternate is still its own subspace but allows this to be a realistic idea as you'll soon see.

Thus $A = -2$ in our example here represents two minutes into the past of the operator's *now* moment when he activated the device. His moment of activation is at t = 0 at the x, y , z origin which would provide $A = 0$. If the observer to were to move to the point $A = -2$ and turn off his machine he would be two minutes in his past but would still define this new place as his new *now* moment. Thus this point is accessible by setting up the equation as $A = -2 = \sqrt{0 - s^2 - u^2 - w^2}$ and then solving for value of s, u and w which will result in $A = -2$. If for example the operator moves his device from his start point while within the s, u, w space to a point defined by $(s, u, w) = (\pm 1, \pm 1, \pm 1.414)$ we have

$$A = \sqrt{0 - (\pm 1)^2 - (\pm 1)^2 - (\pm 1.414)^2} = \sqrt{-4} = -2$$

This is only one such choice which satisfies this arrangement. The operator will exit at a point that from the perspective of the Alternate which is in his past regardless of the direction he chooses to move while operating his device. He will emerge at a point define by subspace transform as $(x, y, z) = (\pm 1, \pm 1, \pm 0.70721)$. Depending on units chosen for space this location could be quite far from the original location or very near to it. Also note if this point is not defined on a path taken by the observer when he was actually experiencing events marked by time $t = -2$ there is no guarantee he will totally recognize the place he arrives at. Though technically his past if the point is not on the path he took to pass through this place it will describe places in the space of time where different events occurred and may be quite different.

Finally note since the operator never passes beyond his own light cone he is never actually in his own past despite how similar it may seem to him. Instead he is in a place which resembles it and now includes a future version of him. Another way of looking at this is that because our present is a confluence of our past and future the operator's arrival with his device in this place he considers his past is enough of an alteration in and of itself to vastly alter not only the future which results form actions he makes after his arrival but also the past before he arrived to include his sudden existence.

This material as well as that for travel to what an observer would consider the future will be covered in depth in Temporal Mechanics 1, which at the time of this text is still under development. This is only provided to show the implications and use of zero division math for plotting such passages. One chapter remains which covers equations of multiple inputs followed by several example problems.

Chapter 8

Multi-Variable Subspaces

Earlier was discussed the development of space and time from a point. From this discussion came the idea of subspaces, space-like time directions. It was also stated that these directions had a relationship with directional time. Chapter 7 explored the inclusion of subspaces into Special Relativity. We found that $t = \sqrt{A^2 + s^2 + u^2 + w^2}$. The subspace direction s, u, w and A are the components which collectively define t, a Time Dimension, and the Alternate A is simply a rearrangement of these values; $A = \sqrt{t^2 - s^2 - u^2 - w^2}$. These subspace directions map the places where all possible events, past, present and future occur for—all possible realities.

There are as many subspaces in a system of equations as there are degrees of freedom within a perceivable dimension plus one which is representative of time. A dimension is the union of two or more directions used to define placement of points. In three directions it is the x, y, z dimension we exist within. All movements through time are the length across the reaming 4-directional subspace.

We can extend this idea relatively easily into multi-variable functions, those which represent $z = f(x\ , y)$ or parametrically as $x\ , y\ , z\} = f(p)$ where *p* is the parameter. (*t* should be avoided as a parameter since it is the length across subspace—only use *t* if the equation includes time.)

The end of this section will provide example problems. Solutions are provided for all of them. Work through them to build confidence in usage of the materials. First consider what it means to solve for subspaces of a multiple variable function.

$$z = 2x^3y + \frac{x^2}{3y^2}$$

There isn't much different to tackling a multi-variable problem. In this example there are two separate inputs; *x* and *y.* Because these are inputs we know the mathematical truism holds that $x = x$ and $y = y$. From these two relations we may begin subspace transforms to obtain the corresponding subspace equations.

$$\Xi x = \frac{s}{sx}x \qquad\qquad \Xi y = \frac{s}{sx}y$$

$\oplus 1 = xs$ $\qquad\qquad$ $\oplus 1 = yu$

$s = -\frac{1}{x}$ $\qquad\qquad$ $u = -\frac{1}{y}$

What remains to find is the z-subspace *w* and the Alternate. The first of these is found by subspace transform on the *z* equation.

$$\Xi z = \frac{s}{s(x,y)}\left(2x^3y + \frac{x^2}{3y^2}\right)$$

$$w = -\frac{1}{2x^3y + \frac{x^2}{3y^2}}$$

Last is the Alternate definable as $A = \sqrt{t^2 - s^2 - u^2 - w^2}$.

$$A = \sqrt{t^2 - \left(-\frac{1}{x}\right)^2 - \left(-\frac{1}{y}\right)^2 - \left(-\frac{1}{2x^3y+\frac{x^2}{3y^2}}\right)^2}$$

Where, if we say $n = -\left(-\frac{1}{x}\right)^2 - \left(-\frac{1}{y}\right)^2 - \left(-\frac{1}{2x^3y+\frac{x^2}{3y^2}}\right)^2$ then we have$A = \sqrt{n}$. By rules established earlier we find if n is positive $A = \sqrt{n}$. If n is negative $A = -(\sqrt{n})$.

The final values for all seven forms are points of the form $P_n = (x\,, y\,, z|s\,, u\,, w\,, A)$ definable by

$x = x$ $\qquad$ $y = y$ $\qquad$ $z = 2x^3y + \frac{x^2}{3y^2}$

$s = -\frac{1}{x}$ $\qquad$ $u = -\frac{1}{y}$ $\qquad$ $w = -\frac{1}{2x^3y+\frac{x^2}{3y^2}}$

$$A = \sqrt{t^2 - \left(-\frac{1}{x}\right)^2 - \left(-\frac{1}{y}\right)^2 - \left(-\frac{1}{2x^3y + \frac{x^2}{3y^2}}\right)^2}$$

An attempt at recombination will likely require the aid of an advanced computer program and will not be attempted here. However the values represented by these several equations can also be arrived at by the trigonometric transformations.

$$x = \rho \sin\varphi \cos\theta \qquad s = -\frac{1}{\rho}\csc\varphi \sec\theta$$

$$y = \rho \sin\varphi \sin\theta \qquad u = -\frac{1}{\rho}\csc\varphi \csc\theta$$

$$z = \rho \cos\varphi \qquad w = -\frac{1}{\rho}\sec\varphi$$

$$A = \sqrt{t^2 - (s)^2 - (u)^2 - (w)^2}$$

$$\rho^2 = x^2 + y^2 + z^2$$

Example Problems:

A number of problems are listed below for examination. Attempt them on your own or directly examine the provided solution. The first several are provided with more detail than later examples to remind the reader of how to complete these zero-division algebra problems. The later examples bare only the solutions. If need more information on how to complete problems review the sections dealing with the appropriate applications of the math.

Problem 1:

Given $y = \frac{1}{x}$ plot point $(x\,, y) = (0\,, y)$

Solution:

1)	Work out the equations for the y-subspace.	$\Xi y = \frac{s}{sx}\frac{1}{x} \quad \rightarrow \quad u = -x$
2)	Substitution for x = 0 yields:	$y = \infty \qquad u = 0$
3)	Use the Infinite Transform Relation to show that when y reaches infinity it takes the value of its subspace at the same input value.	$y = \infty \rightarrow u = 0$

4)	Solve by entering y = 0 at x = 0.	In $y = \frac{1}{x}$ for x = 0, y = 0.

Problem 2:

Given $u = x$ plot the point $(x\,, y|u) = (0\,, y|u)$.

$$\Xi u = \frac{s}{sx} x \qquad \oplus 1 = xy \qquad y = -\frac{1}{x} \qquad y = -\frac{1}{0} = \infty$$

$$y = \infty = u = 0$$

Problem 3:

Given $y = \frac{0}{x}$ plot point $(x\,, y) = (0\,, y)$

$\Xi y = \frac{s}{sx}\frac{0}{x} \qquad \oplus 1 = \frac{0}{x} u \qquad \mathrm{u} = \frac{x}{0} = \infty$ for all x not 0 $\quad y = \frac{0}{x} = 0$

$$\mathrm{u} = \frac{x}{0} = \infty = y = 0$$

In this instance the *y*-function is trending toward zero for all x. When x = 0 the function will take its value as an infinite leaving $y = \infty$. Given the above relation

$\mathrm{u} = \frac{x}{0} = \infty = y = 0$ this infinite is resolved to y = 0.

$$y = \frac{0}{0} = \infty = 0$$

Problem 4:

Given $y = 2x + \frac{0}{x}$ plot point $(x\,, y) = (0\,, y)$

There are two parts to consider in the equation. $2x$ is easy enough. When x = 0 this component will equal 0 as well. Using methods already detailed the $\frac{0}{x}$ element is clearly trending toward 0 for all values of x and will be resolved to 0 when x = 0.

$$y = 2(0) + \frac{0}{(0)} = 0$$

Problem 5:

Given $y = 2x + \left(\frac{0}{0}\right)$ plot point $(x\,, y) = (0\,, y)$

This time the $\frac{0}{0}$ element is added in as a constant, ⊕1. This value will resolve to 1.

$y = 2(0) + \left(\frac{0}{0}\right) = 1$

Problem 6:

Given $y = 2x + \frac{0}{x} + \left(\frac{0}{0}\right)$ plot point $(x\,, y) = (0\,, y)$

compare to: $y = 2x + \frac{1}{x} + \left(\frac{0}{0}\right)$

Compare to: $y = 2x + \frac{1}{x} + \left(\frac{x}{x}\right)$

Compare to: $y = \frac{0}{0}$ vs. $y = \frac{0}{x}$ and $y = \frac{x}{0}$

When x = 0 in $= 2x + \frac{0}{x} + \left(\frac{0}{0}\right)$ the $2x$ will equal 0 and so will the $\frac{0}{x}$ element. The $\frac{0}{x}$ is trending toward 0 for all values of x. When x = 0 although this becomes $\frac{0}{0}$ the pattern taken by the equation tells us it will take the value of infinity and resolve to 0 using methods described earlier. The final $\frac{0}{0}$ element is already in place as a constant and will resolve to 1.

$$y = 2(0) + \frac{0}{(0)} + \left(\frac{0}{0}\right) = 1$$

In $y = 2x + \frac{1}{x} + \left(\frac{0}{0}\right)$ the only difference is with the $\frac{1}{x}$ element. As x approaches 0 this value will trend toward infinity. The infinite will resolve to 0 using methods described earlier. Again the final $\frac{0}{0}$ element is already in place as a constant and will resolve to 1.

$$y = 2(0) + \frac{1}{(0)} + \left(\frac{0}{0}\right) = 1$$

In $y = 2x + \frac{1}{x} + \left(\frac{x}{x}\right)$ the new addition of $\frac{x}{x}$ is already a one-to-one ratio….i.e. 1. When x = 0 this term becomes $\frac{0}{0}$ and is trending toward a constant. The term will result to 1.

$$y = 2(0) + \frac{1}{(0)} + \left(\frac{(0)}{(0)}\right) = 1$$

Using methods already described we know that $y = \frac{0}{x}$ trends toward 0 and $y = \frac{x}{0}$ trends toward infinity for all values of x. When x equals 0 both values use this relation with infinity to resolve to 0 due to how the arise. For $y = \frac{0}{0}$ the term is instead trending toward a constant since it did not originate form a function. Here it resolves to 1.

Problem 7:

Given $y = 2\left(\frac{0}{x}\right)$ plot point $(x\,, y) = (0\,, y)$

Regardless if one distributes the 2 first or divide 0 by x the $\frac{0}{x}$ term is trending toward 0 for all x. When x = 0 the value will take infinity and resolve to 0.

$$y = 2\left(\frac{0}{(0)}\right) = 0$$

Problem 8:

Given $y = 2\left(\frac{0}{0}\right)$ plot point $(x\,, y) = (0\,, y)$

The $\frac{0}{0}$ is entered as a constant and will tend toward a constant value resolving to 1.

$$y = 2\left(\frac{0}{0}\right) = 1$$

Problem 9:

Given $y = \frac{x^2-2x-8}{x^2-9x-20} = \frac{(x+2)(x-4)}{(x-5)(x-4)}$ plot point $(x\,, y) = (0\,, y)$

The difficulty arises with the $\frac{(x-4)}{(x-4)}$ element. When x = 0 this term will become $\frac{0}{0}$.

However the expression $\frac{(x-4)}{(x-4)}$ is already in a one-to-one ratio and is clearly trending

toward a constant for all value of x. The $\frac{0}{0}$ arising here at x = 0 will resolve to 1.

$y = \frac{((0)+2)((0)-4)}{((0)-5)((0)-4)} = -\frac{2}{5}$.

Incidentally when x = 5 the expression becomes $\frac{7}{0}$. This value will take infinity which will

resolve to 0 by methods already discussed.

Problem 10:

Given $y = 2\left(\frac{x^2-2x-8}{x^2-9x-20}\right) - 1$ plot point $(x\,,y) = (0\,,y)$

Look at the values for the solutions in problem 9 above. When x = 0 the $\frac{x^2-2x-8}{x^2-9x-20} = -\frac{2}{5}$.

The final value then for this example is $y = 2\left(-\frac{2}{5}\right) - 1 \;\rightarrow\; -\frac{9}{5}$.

Problem 11:

Given $y = \frac{x^2-2\left(\frac{x-4}{x-4}\right)-8}{2}$ plot point $(x\,,y) = (0\,,y)$

$$y = \frac{(0)^2-2\left(\frac{(0)-4}{(0)-4}\right)-8}{2} = -5$$

Problem 12:

Given $y = \frac{x-4}{x-4}$ plot point $(x\,,y) = (0\,,y)$

$$y = \frac{(0)-4}{(0)-4} = 1$$

Problem 13:

Given $y = \frac{x}{x}$ plot point $(x\,,y) = (0\,,y)$

$$y = \frac{(0)}{(0)} = 1$$

Problem 14:

Given $y = \sqrt{-4}$ plot point $(-4, y)$

$y = \sqrt{-4} = 2$ ATC2 resolution for non-Alternate Axis.

Problem 15:

Given $A = \sqrt{-4}$ plot point $(-4, A)$

$A = \sqrt{-4} = -2$ ATC2 resolution for value on the Alternate

Problem 16:

Given $y = \sqrt{a}$ for $a = \{1, -1, \frac{0}{0}, \frac{x}{0}, \frac{0}{x}$

$$y = \sqrt{1} = \pm 1$$

$$y = \sqrt{-1} = 1$$

$$y = \sqrt{\frac{0}{0}} = \pm 1$$

$$y = \sqrt{\frac{x}{0}} = 0$$

$$y = \sqrt{\frac{0}{x}} = 0$$

Problem 17:

Given $A = \sqrt{a}$ for $a = \{1, -1, \frac{0}{0}, \frac{x}{0}, \frac{0}{x}$

$$A = \sqrt{1} = \pm 1$$

$$A = \sqrt{-1} = -1$$

$$A = \sqrt{\frac{0}{0}} = \pm 1$$

$$A = \sqrt{\frac{x}{0}} = 0$$

$$A = \sqrt{\frac{0}{x}} = 0$$

Problem 18:

Given $y = \sqrt[3]{a}$ for $a = \left\{1\,, -1\,, \frac{0}{0}\,,\ \ \frac{x}{0}\,, \frac{0}{x}\right.$

$$y = \sqrt[3]{1} = \pm 1$$

$$y = \sqrt[3]{-1} = -1$$

$$y = \sqrt[3]{\frac{0}{0}} = \pm 1$$

$$y = \sqrt[3]{\frac{x}{0}} = 0$$

$$y = \sqrt[3]{\frac{0}{x}} = 0$$

Problem 19:

Given $A = \sqrt[3]{a}$ for $a = \left\{1\,, -1\,, \frac{0}{0}\,, \frac{x}{0}\,, \frac{0}{x}\right.$

$$A = \sqrt[3]{1} = \pm 1$$

$$A = \sqrt[3]{-1} = -1$$

$$A = \sqrt[3]{\frac{0}{0}} = \pm 1$$

$$A = \sqrt[3]{\frac{x}{0}} = 0$$

$$A = \sqrt[3]{\frac{0}{x}} = 0$$

Problem 20:

Given $y = \frac{x^2-2}{\left(\frac{x-1}{x-1}\right)}$ plot point $(x\,, y) = (1\,, y)$ $\qquad y = \frac{(1)^2-2}{\left(\frac{(1)-1}{(1)-1}\right)} = -1$