A list of known root prime-generating quadratic polynomials producing more than 23 distinct primes in a row

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. A simple list of known such polynomials, indexed by the value of discriminants, containing no analysis but the introduction of the "root prime generating polynomial" notion.

I listed below the polynomials (after the value of discriminant). In the brackets we have the polynomials that generate same primes but in reverse order (any prime-producing polynomial has such a reversal). The list contains 42 polynomials (84 with their reversals). I didn't consider redundant primes to not complicate the list furthermore. I discovered myself all the polynomials with the font italic (32(64)). I know the other ones from the articles available on Internet like *Prime-Generating Polynomial* from Wolfram Math World or sites like Rivera's *The Prime Puzzles & Problems Connection*.

Note: because of the special nature of the number 1, I considered the polynomials that generate that number too as prime-generating polynomials, but, for the purists, I indexed with specification "d.p." distinct primes and "d.p.1." distinct primes plus number 1 (in absolute value).

Note: a "**root prime-generating polynomial**" I consider to be the prime-generating polynomial that has two properties:

(1) for n = -1 gives a non-prime term (for instance, 8n^2 + 88n + 43 is not a root prime-generating polynomial because for n = -1 we have the prime term (in absolute value) -37 and for n = n - 39 we have the "complete" root prime-generating polynomial: 8n^2 – 488n + 7243);

(2) there is no other prime-generating polynomial with the same value of discriminant that generates the same amount of primes in a row, having coefficients of smaller values.

Note: I submitted few of these polynomials to OEIS.

Discriminant equal to -222643:
   35 d.p.: 43n^2 – 537n + 2971 (43n^2 – 2387n + 34421).
Discriminant equal to – 23472:
   26 d.p.: 36n^2 – 408n + 1319 (36n^2 – 1392n + 13619).
Discriminant equal to – 13203:
   28 d.p.: 81n^2 – 1323n + 5443 (81n^2 – 3051n + 28771);
   25 d.p.: 9n^2 – 219n + 1699 (9n^2 – 213n + 1627).
Discriminant equal to – 10432:
   23 d.p.: 64n^2 – 1192n + 5591 (64n^2 - 1624n + 10343).
Discriminant equal to – 8523:
   23 d.p.: 27n^2 – 489n + 2293 (27n^2 - 699n + 4603).
Discriminant equal to – 7987:
   23 d.p.: 49n^2 – 469n + 1163 (49n^2 – 1687n + 14561).
Discriminant equal to – 4075:
   32 d.p.: 25n^2 – 365n + 1373 (25n^2 – 1185n + 14083).
Discriminant equal to – 2608:
   31 d.p.: 16n^2 – 292n + 1373 (16n^2 – 668n + 7013);
   30 d.p.: 16n^2 – 300n + 1447 (16n^2 – 628n + 6203).
Discriminant equal to – 1467:
   40 d.p.: 9n^2 – 231n + 1523 (9n^2 – 471n + 6203).
Discriminant equal to – 708:
   29 d.p.: 6n^2 + 6n + 31 (6n^2 – 342n + 4903).
Discriminant equal to – 652:
   40 d.p.: 4n^2 – 154n + 1523 (4n^2 – 158n + 1601).
Discriminant equal to – 232:
   29 d.p.: 2n^2 + 29 (2n^2 – 112n + 1597).
Discriminant equal to – 163:
   40 d.p.: n^2 + n + 41 (n^2 – 79n + 1601).
Discriminant equal to 293:
   24 d.p.1.: n^2 + n - 73 (n^2 – 47n + 479).
Discriminant equal to 437:
   28 d.p.1.: n^2 + n - 109 (n^2 + 55n + 647).
Discriminant equal to 677:
   25 d.p.1.: 13n^2 – 313n + 1871 (13n^2 – 311n + 1847);
   23 d.p.: n^2 + 3n - 167 (n^2 – 49n + 431).
Discriminant equal to 1077:
   24 d.p.1.: 3n^2 + 3n - 89 (3n^2 – 141n + 1567).
Discriminant equal to 1172:
   29 d.p.1.: 4n^2 – 90n + 433 (4n^2 – 142n + 1187).
Discriminant equal to 1253:
   27 d.p.1.: 7n^2 + 7n - 43 (7n^2 – 371n + 4871).
Discriminant equal to 1592:
   28 d.p.1.: 2n^2 – 199 (2n^2 + 108n + 1259).
Discriminant equal to 6368:
   31 d.p.: 8n^2 + 8n - 197 (8n^2 – 488n + 7243).
Discriminant equal to 19808:
   23 d.p.: 104n^2 - 2200n + 11587 (104n^2 - 2376n + 13523).
Discriminant equal to 25472:
   35 d.p.: 4n^2 + 12n - 1583 (4n^2 – 284n + 3449);
   31 d.p.: 32n^2 - 944n + 6763 (32n^2 - 976n + 7243);

29 d.p.: 16n^2 - 408n + 2203 (16n^2 - 488n + 3323).
Discriminant equal to 57312:
35 d.p.: 72n^2 - 1416n + 6763 (72n^2 - 1752n + 10459).
Discriminant equal to 64917:
35 d.p.1.: 27n^2 - 741n + 4483 (27n^2 - 1095n + 10501);
33 d.p.: 81n^2 – 2247n + 15383 (81n^2 - 2937n + 26423);
32 d.p.: 27n^2 - 753n + 4649 (27n^2 - 921n + 7253);
24 d.p.: 9n^2 + 9n - 1801 (9n^2 - 423n + 3167).
Discriminant equal to 78008:
28 d.p.: 98n^2 - 2128n + 11353 (98n^2 - 3164n + 25339).
Discriminant equal to 101888:
31 d.p.: 4n^2 - 428n + 5081 (4n^2 + 188n - 4159);
24 d.p.1.: 128n^2 - 1216n + 2689 (128n^2 – 4672n + 42433);
Discriminant equal to 159200:
27 d.p.: 100n^2 - 2820n + 19483 (4n^2 - 2380n + 13763).
Discriminant equal to 259668:
45 d.p.: 36n^2 – 810n + 2753 (36n^2 – 2358n + 36809);
24 d.p.: 108n^2 - 2130n + 9901 (108n^2 - 2838n + 18043).
Discriminant equal to 979373:
43 d.p.: 47n^2 – 1701n + 10181 (47n^2 – 2247n + 21647).
Discriminant equal to 1038672:
29 d.p.: 144n^2 – 2196n + 6569 (144n^2 – 5868n + 57977).
Discriminant equal to 1398053:
43 d.p.: 103n^2 – 4707n + 50383 (103n^2 – 3945n + 34381).

I also submit the following problem: find a value of discriminant, beside the ones from the following list: -222643, -4075, -2608, -1467, -652, -163, 6368, 25472, 57312, 64917, 101888, 259668, 979373, 1398053, for which a quadratic polynomial having this discriminant generates 30 or more distinct primes in a row.

I list below the polynomials that I know that generates 30 or more distinct primes in a row (in the brackets are the reverse polynomials, that generates same primes in reverse order):
43n^2 – 537n + 2971 (43n^2 – 2387n + 34421);
9n^2 – 231n + 1523 (9n^2 – 471n + 6203);
4n^2 – 154n + 1523 (4n^2 – 158n + 1601);
n^2 + n + 41 (n^2 – 79n + 1601);
8n^2 + 8n – 197 (8n^2 – 488n + 7243);
36n^2 – 810n + 2753 (36n^2 – 2358n + 36809);
47n^2 – 1701n + 10181 (47n^2 – 2247n + 21647);
103n^2 – 4707n + 50383 (103n^2 – 3945n + 34381).

I list below the polynomials that I discovered myself that generates 30 or more distinct primes in a row (few of them are posted on OEIS):

25n^2 – 365n + 1373 (25n^2 – 1185n + 14083);
16n^2 – 292n + 1373 (16n^2 – 668n + 7013);
16n^2 – 300n + 1447 (16n^2 – 628n + 6203);
4n^2 + 12n - 1583 (4n^2 - 284n + 3449);
32n^2 - 944n + 6763 (32n^2 - 976n + 7243);
72n^2 - 1416n + 6763 (72n^2 - 1752n + 10459);
81n^2 – 2247n + 15383 (81n^2 - 2937n + 26423);
27n^2 - 753n + 4649 (27n^2 - 921n + 7253);
4n^2 - 428n + 5081 (4n^2 + 188n - 4159).