The quantum gravity lagrangian

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Abstract

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A previous paper (http://vixra.org/abs/1111.0111) makes some calculations from a quantum gravity theory and sketches a framework for further predictions. This paper defends in detail the lagrangian for quantum gravity, based on the theory in our earlier paper, by examining the simple physical dynamics behind general relativity and gauge theory.

General relativity predictions from Newtonian gravity lagrangian, with a relativistic metric

In 1915, Einstein and Hilbert derived the field equation of general relativity from a very simple lagrangian. The classical "proper path" of a particle in a gravitational field is the minimization of action:

$$S = \int L dt = \int \mathcal{L} d^4 x = \int R(-g)^{1/2} c^4/(16\pi G) d^4 x$$

where the Lagrangian energy $L = E_{\text{kinetic}} - E_{\text{potential}}$, and energy density is $\mathcal{L} = L/\text{volume}$), which gives Einstein's field equation of general relativity when action is minimized, i.e. when $dS = 0$, found by "varying" the action S using the Euler-Lagrange law. To Weyl and his followers today, the "Holy Grail" of quantum gravity research remains the task of obtaining a theory which at low energy has the Lagrangian gravitational field energy density component, $\mathcal{L} = R(-g)^{1/2} c^4/(16\pi G)$, so that it yields produces Einstein's field equation as a "weak field" limit or approximation.

But the Einstein-Hilbert Lagrangian density $R(-g)^{1/2} c^4/(16\pi G)$ is a relativistic, spacetime curvature-based generalization of classical Newtonian gravity, since Ricci's scalar, in the isotropic radial symmetry of gravitational fields produced by a fundamental particle, is: $R = g^{\mu\nu} R_{\mu\nu} = R_{00} = \nabla^2 k = 4\pi G\rho$. Since the Newtonian gravitational potential energy is proportional to mass, $L_{\text{field}} = - E_{\text{potential}} = -\int_r^\infty (-GMm/x^2) dx = GMm/r$, it follows that Poisson's equation is the weak field, non-relativistic Newtonian limit for the Ricci scalar, $R = 4\pi G\rho$, so the non-relativistic Lagrangian energy density is:

$$\mathcal{L} = \rho = (\nabla^2 k)/(4\pi G) = R/(4\pi G),$$

which upon multiplication by $(-g)^{1/2} c^4/4$ gives general relativity's Einstein-Hilbert relativistic lagrangian. This $c^4/4$ factor is the conversion from time to space units for 4-dimensional spacetime ($d^4 t$ is changed into $d^4 x$ by employing multiplication factor $d^4 x / d^4 t = c^4$), and the fact that the fraction of the gravitational field energy density acting in one dimension (say the x direction) out of four dimensions is only 1/4 of the total isotropic energy density. The relativistic multiplication factor $(-g)^{1/2}$ occurs because g is the determinant $g = |g_{\mu\nu}|$ of metric tensor $g_{\mu\nu}$ and Ricci's scalar, $R = g^{\mu\nu} R_{\mu\nu}$, is clearly a function of the metric.

For isotropic radial symmetry ("spherical symmetry") coordinates ($r = y = z = x$), the Schwarzschild metric implies $(-g)^{1/2} = [1 + 2GM/(r c^2)]^{1/2}$, the gravitational field equivalent of the inertial mass Lorentzian spacetime contraction, $(1 - v^2/c^2)^{1/2}$. In other words, the Schwarzschild metric is the Lorentzian metric converted from velocity v to gravitational field effect $2GM/r$ by Einstein's equivalence principle between inertial and gravitational mass, which states that *the effects of a given inertial acceleration are indistinguishable from those of a corresponding gravitational acceleration*, or equivalently (if the accelerations on both sides of this equivalence may be integrated to yield velocities), that the properties of any body having velocity v are equivalent to those of a similar body which has acquired an identical velocity as a result of a gravitational, e.g., $v = (2GM/r)^{1/2}$ (the escape velocity of any body from mass M, which is equivalent to the velocity acquired in falling to the same mass). For weak gravitational fields, $(-g)^{1/2} \approx 1$, thus the non-relativistic approximation: $R(-g)^{1/2}/(16\pi G) \approx R/(16\pi G)$. So general relativity by writing Newton's law as $\mathcal{L}_{\text{Newtonian}} = R/(16\pi G)$ and then including a relativistic metric determinant $(-g)^{1/2} = [1 + 2GM/(r c^2)]^{1/2}$, i.e. the empirical Lorentz contraction factor with Einstein's equivalence principle to convert from inertial accelerations and velocities to corresponding gravitational accelerations and velocities.

However, while $\mathcal{L} = R(-g)^{1/2}c^4/(16\pi G)$ appears mathematically incontrovertible for the Lagrangian energy density of quantum gravity, it isn't logical or correct physically, simply because this Lagrangian is *contrived to model only the classical path of least action (i.e. the real or onshell path in a path integral), unlike quantum field theory Lagrangians, which are generally applicable for all paths.*

Hence, we point out that the Einstein-Hilbert free-field classical lagrangian of general relativity $\mathcal{L} = R(-g)^{1/2}c^4/(16\pi G)$ is "Newtonian-Poisson gravity, $Rc^4/(16\pi G)$, multiplied by a relativistic spacetime contraction correction factor, $(-g)^{1/2}$, which the contraction due to motion in Lorentzian/FitzGerald/special relativity, $[1 + 2GM/(r c^2)]^{1/2}$, with Einstein's equivalence principle of inertial and gravitational motion, $v = (2GM/r)^{1/2}$." However, the fact remains that the Einstein-Hilbert lagrangian $\mathcal{L} = R(-g)^{1/2}c^4/(16\pi G)$ cannot itself be taken as a "rigorous piece of mathematics" when it merely gives the correct path of *least action*. In quantum field theory, we need a lagrangian that holds good for *all paths,* not merely the path of *least action*. The way that Einstein and Hilbert "derived" $\mathcal{L} = R(-g)^{1/2}c^4/(16\pi G)$ was contrived as the simplest free-field lagrangian that would - when "varied" with respect to action - yield the correct path of least action. In other words, while we accept $\mathcal{L} = R(-g)^{1/2}c^4/(16\pi G)$ to *include* correctly the classical path of least action, that *doesn't mean that we accept it to also include off-shell (non-least action) paths that must be included in a quantum gravity theory, and which in the low energy limit are merely unobservational due to multipath interference in the path integral.*

Our approach to quantum gravity consistency with "general relativity" in http://vixra.org/abs/1111.0111 is to point out that general relativity was derived by Hilbert and Einstein in the first place as a rank-2 spacetime curvature mathematical modelling exercise, as opposed to the rank-1 curving field lines of Maxwell's equations for electromagnetism. If only Maxwell's laws of electromagnetism were reformulated as spacetime curvature, we'd have a rank-2 version of electromagnetism in which forced are propagated by spin-1 field quanta, thus breaking the popular but mathematically false dogma that the rank of the field equation model is equal to the spin of the field quanta (as spin-1 for rank-1 Maxwell equations, and spin-2 for rank-2 general relativity).

There are numerous deficiencies mathematically and physically in the dogma that gravitons are spin-2, some of which are discussed in http://vixra.org/abs/1111.0111. First of all, it's reductionist. If only two masses existed in this universe, and they were found to attract, then yes, that observation could prove the need for a spin-2 graviton. But this is not the case, since the Casimir effect shows how metal places can be "attracted" (pushed together) by spin-1 electromagnetic field quanta in the surrounding vacuum (virtual photons with wavelengths longer than the separation distance between the metal plates don't contribute, so the force of repulsion between the plates is less than the force pressing them together). This Casimir effect relies on offshell field quanta, so the old arguments against LeSage onshell gas pressures in a vacuum are negated. No offshell field quanta cause drag or heating effects on planets (the traditional anti-mechanism no-go theorem).

So it is quite possible for spin-1 gravitons to exist, causing repulsion of masses (e.g. our correct quantitative prediction in 1996 of the positive sign amount of "dark" graviton energy causing the cosmological acceleration first observed by Perlmutter in 1998). The only reason why apparent "attraction" ("gravity") arises is that the immense surrounding masses in the universe are practically isotropic and are unable to avoid exchanging gravitons with all other masses. Local masses are proved in http://vixra.org/abs/1111.0111 to precisely have the right gravitational interaction cross-section to produce the cosmological acceleration. The real reason why spin-2 gravitons have become dogma, aside from the simplistic reductionist idea of ignoring graviton exchange with distant masses (Occam Razor's, gone too far) is Edward Witten's M-theory, which relies on a spin-2 graviton to defend 11 dimensional supergravity bulk and its 10 dimensional superstring brane as being the only intelligent theory of quantum gravity. In Witten's marketing process (*Physics Today,* April 1996), all the caveates that should be included about the assumptions behind the Pauli-Fierz spin-2 gravity wave are eliminated, to leave the impression that the mere existence of observed gravity is itself proof that gravitons have spin-2. (Similarly, Ptolemy dismissed Aristarchus's solar system with the allegation that anybody can see that the sun orbits the earth. This kind of political claim to "close down the debate" before the facts are checked, is very convenient for censors and peer-reviewers!)

What we point out in http://vixra.org/abs/1111.0111 is that the correct theory of quantum graviton is in effect what is now taken to be QED, namely an Abelian U(1) theory. This quantum gravity U(1) gauge theory literally replaces the electromagnetic (hypercharge, prior to mixing) U(1) in the Standard Model, and electromagnetism is transferred to SU(2) massless gauge bosons, replacing Maxwell's equations by the SU(2) charge-carrying Yang-Mills equations, which merely reduce to what appear to be Maxwell equations when the field quanta are massless. This is because the two charged SU(2) massless field quanta of electromagnetism have infinite magnetic self-inductance which prevents a net flow of charge. Their magnetic fields (self inductance) are only cancelled when there is an equilibrium in the exchange of charged massless field quanta between similar charges. Thus, the positive electric field around a proton is simply composed of virtual photons with a positive charge sign. The dynamics of this are inherent in the properties of the Heaviside energy current, which is precisely the charged electromagnetic field quanta in electricity. The two "extra polarizations" which the electromagnetic gauge boson has in order to allow both attraction and repulsion are simply the two electric charge signs.

The charge transfer term of the Yang-Mills equations is constrained to a value of zero, because two protons can only exchange positively charged massless gauge bosons for "electromagnetic" repulsion when the rates of transfer of charge from one to the other and vice-versa are identical. This means that there is an equilibrium in the transfer of positive charge between the two protons, so they both appear to have a static, constant amount of positive charge. Hence, the Yang-Mills charge transfer term is effectively suppressed, reducing the Yang-Mills equations for SU(2) into Maxwell equations, and this explains why historically Maxwell's equations look good superficially (like Ptolemy's superficially good-looking dogma that the sun orbits the earth daily), but it obfuscates electroweak theory.

Schwinger argued correctly but disingeniously in 1949 (see *Physical Review,* v76, p790) that electric charge conservation forces the photon to be massless. We reverse this: massive electrically neutral photons do exist but have 91 GeV so Maxwell never discovered them or had any need to include them in his equations! The photon is not forced to be massless. The Z boson (which is the field quanta of neutral currents) is a photon which has acquired mass and thus has "weak isospin charge." The only reason why these massive Z photons were not known to Schwinger in 1949 is that their energy is 91 GeV, far beyond 1949 physics. In other words, what happens in the history of physics is that incomplete observations are "explained" in a contrived way, and these explanations become hardened dogma. So theories are developed in an ad hoc way to incorporate previous dogma, so that they will overcome peer-review and get published, without questioning dogma.

Gauge field theory's local and global symmetries of physics are, by Noether's theorem, dualities to energy and charge conservation mechanisms, respectively. This is a physical process. In 1929, Weyl differentiated the complex exponential solution to the Schroedinger equation, $\psi_S = \psi_0 e^{iS/\hbar}$. Because the right-hand side, $\psi_0 e^{iS/\hbar}$, is a product, the product rule of differentiation is used, $d(uv) = (u\ dv) + (v\ du)$, thus, remembering $de^{f(x)}/dx = f'(x)e^{f(x)}$, we obtain the following derivative of $\psi_S = \psi_0 e^{iS/\hbar}$. Dirac's bra-ket notation was introduced a decade later, in 1939, and isn't of interest to us since we're only interested in obtaining real numbers, i.e. "resultant vectors" which as arrows always lie parallel to the real plane on a complex/Argand diagram; for all real world tested probability and cross-section predictions from the path integral of 2nd quantization we can thus replace $e^{iS/\hbar}$ with real scalar cos $(S/\hbar)$.

$$\begin{aligned} d_\mu \psi_S &= d_\mu(\psi_0 e^{iS/\hbar}) \\ &= e^{iS/\hbar}\, d_\mu\psi_0 + \psi_0(d_\mu\, e^{iS/\hbar}) \\ &= e^{iS/\hbar}\, d_\mu\psi_0 + \psi_0(ie^{iS/\hbar}\, d_\mu S/\hbar) = e^{iS/\hbar}\, [d_\mu \psi_0 + (i/\hbar)(d_\mu S)\psi_0] \end{aligned}$$

The term $(i/\hbar)(d_\mu S)\psi_0$ here introduces a dependence on the derivative of the action that prevents local phase invariance, which would require $d_\mu \psi_S = e^{iS/\hbar}\, d_\mu\psi_0$ instead of the actual result $d_\mu \psi_S = e^{iS/\hbar}\, d_\mu\psi_0 + \psi_0(d_\mu\, e^{iS/\hbar})$. This is because a field must supply energy to a particle in order to make its wavefunction amplitude change by the amount $\psi_S = \psi_0 e^{iS/\hbar}$, so this work takes energy away from the Dirac's field lagrangian, replacing $\mathcal{L}_{\text{Dirac}} = \bar{\psi}\,(i\gamma^\mu\, d_\mu - m)\psi$ by $\mathcal{L}_{\text{Dirac}} = \bar{\psi}[i\gamma^\mu (d_\mu - iqA_\mu) - m]\psi$, which replaces Dirac's non-covariant derivative d_μ with the covariant derivative $d_\mu - iqA_\mu$, and this must be accompanied by the change in the potential of the field vector, $(A_\mu)_S = (A_\mu)_0 + d_\mu S/(q\hbar)$ where q is the charge (or coupling). E.g., if a magnet causes an electron's spin direction to flip, or a compass needle to rotate, then some energy of the magnetic field is used, so the magnetic field is affected. This is an undeniable, provable consequence of energy conservation or, viewed (even more simply), work-energy mechanics. Work must be paid for. Newton's classical theory *simply makes no correction for the Earth's gravitational field energy when some of it is used up when accelerating an apple's fall to the ground* (which converts gravitational potential energy of the Earth's field - the apple's gravitational field is relatively trivial - into sound waves and mechanical action, bruising the apple). Weyl's utilization of Noether's theorem to relate symmetry to conservation of field energy, correcting the field potential for the energy that is used in doing work by changing a wavefunction amplitude, is thus applicable to gravitational as well as electromagnetic fields. Hence, there is no physical inconsistency.

We conclude that the mathematics of general relativity and quantum field gauge (interaction) theory are fully consistent with the simple mechanism in http://vixra.org/abs/1111.0111; the the quantum gravity lagrangian is similar to U(1) QED. The hardened fanatics of Ptolemy's epicycles developed trignometry, but that didn't prove their sloppy muddled-up physics.