The new system that is concerned about Rindler theory

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ABSTRACT

In the general relativity theory, discover the new system that is concerned about Rindler coordinate theory. In this time, $a_0 = \frac{m_0 c^3}{h}$.The new system uses the tetrad on the new method and it discovers the new inverse-coordinate transformation of the new system.

PACS Number:**04,04.90.+e**

Key words:The general relativity theory,

The new system,

The Rindler's theory

The coordinate transformation,

The tetrad

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I.Introduction

This theory is that it discovers new system that is concerned about Rindler theory.

Think the motion that use following the formula.

$$x \approx \frac{1}{2}a_0 t^2 = \frac{1}{2}(\frac{m_0}{h}c^3)t^2 \quad , a_0 = \frac{m_0 c^3}{h} \qquad (0)$$

It treats the motion by Eq(0).

Finding the new coordinate theory, use following the formula.

$$x = \frac{c^2}{a_0}(\cosh(\frac{a_0 \tau}{c}) - 1) = \frac{h}{m_0 c}(\cosh(\frac{m_0 c^2 \tau}{h}) - 1),$$

$$t = \frac{c}{a_0}\sinh(\frac{a_0 \tau}{c}) = \frac{h}{m_0 c^2}\sinh(\frac{m_0 c^2 \tau}{h}) \qquad (1)$$

x and t is the coordinate and the time in the inertial system, τ is invariable time, c is light speed in the inertial system in the free space-time, m_0 is the particle's stationary mass. h is the plank constant.

$$dt = \cosh(\frac{m_0 c^2}{h}\tau)d\tau,$$

$$dx = c\sinh(\frac{m_0 c^2}{h}\tau)d\tau,$$

$$dy = dy' = 0, \ dz = dz' = 0$$

$$V = \frac{dx}{dt} = c\tanh(\frac{m_0 c^2}{h}\tau) \qquad (2)$$

II. Additional chapter-I

The tetrad $e_a{}^{\mu}$ is the unit vector that is each other orthographic and it use the following formula.

$$e_a{}^{\mu} e_b{}^{\upsilon} g_{\mu\upsilon} = \eta_{ab} \qquad (3)$$

$e^a{}_{\mu}$ is

$$e^a{}_{\mu} = \eta^{ab} g_{\mu\upsilon} e_b{}^{\upsilon} \qquad (4)$$

and it is $e_a{}^{\mu}$'s inverse-matrix. And it is

$$e^a{}_{\mu} e_b{}^{\mu} = \delta^a{}_b \ , e^a{}_{\mu} e_a{}^{\upsilon} = \delta_{\mu}{}^{\upsilon}$$

$$e^a{}_{\mu} e^b{}_{\upsilon} \eta_{ab} = g_{\mu\upsilon} \qquad (5)$$

The $e^{\alpha}{}_{\mu}(\tau)$ is the tetrad that if $\xi^1 = \xi^2 = \xi^3 = 0, d\xi^1 = d\xi^2 = d\xi^3 = 0$ .In this time, in Eq(5) it does $g_{\mu\upsilon} = \eta_{\mu\upsilon}$.

Therefore, Eq(5) is

$$\eta_{\alpha\beta} e^{\alpha}{}_{0}(\tau) e^{\beta}{}_{0}(\tau) = \eta_{00} = -1$$

$$d\tau^2 = -\frac{1}{c^2}\eta_{\alpha\beta} dx^{\alpha} dx^{\beta}$$

$$\rightarrow \quad -1 = \eta_{\alpha\beta}(\frac{1}{c}\frac{dx^{\alpha}}{d\tau})(\frac{1}{c}\frac{dx^{\beta}}{d\tau}) = \eta_{\alpha\beta} e^{\alpha}{}_{0}(\tau) e^{\beta}{}_{0}(\tau) \qquad (6)$$

According to Eq(2),Eq(6)

$$e^{\alpha}{}_{0}(\tau) = \frac{1}{c}\frac{dx^{\alpha}}{d\tau} = (\cosh(\frac{m_0 c^2}{h}\tau), \sinh(\frac{m_0 c^2}{h}\tau), 0, 0) \qquad (7)$$

About y -axis's and z -axis's orientation

$e^{\alpha}{}_{2}(\tau) = (0,0,1,0)$, $e^{\alpha}{}_{3}(\tau) = (0,0,0,1)$

And the other unit vector $e^{\alpha}{}_{1}(\tau)$ has to satisfy the tetrad condition, Eq (5)

$$e^{\alpha}{}_{1}(\tau) = (\sinh(\frac{m_0 c^2}{h}\tau), \cosh(\frac{m_0 c^2}{h}\tau), 0, 0) \qquad (8)$$

III. Additional chapter-II

According to the tetrad $e^{a}{}_{\mu}$,in the flat Minkowski space, the inertial coordinate system $S(t, x, y, z)$ transform the new system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$. Therefore,

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^{\mu}}\frac{\partial x^b}{\partial \xi^{\upsilon}} d\xi^{\mu} d\xi^{\upsilon}$$

$$= -\frac{1}{c^2}\eta_{ab} e^{a}{}_{\mu} e^{b}{}_{\upsilon} d\xi^{\mu} d\xi^{\upsilon} = -\frac{1}{c^2} g_{\mu\upsilon} d\xi^{\mu} d\xi^{\upsilon}$$

$$e^{a}{}_{\mu} = \frac{\partial x^a}{\partial \xi^{\mu}} \qquad (9)$$

Therefore, for saving the new system in the mathematical way, the $e^{\alpha}{}_{\mu}(\xi^0)$ is used by Eq (7),Eq(8) that used ξ^0 instead of τ .

The unit vector $e^{\alpha}{}_{1}(\xi^0)$ is

$$e^{\alpha}{}_{1}(\xi^0) = \frac{\partial x^{\alpha}}{\partial \xi^1} = (\sinh(\frac{m_0 c^2}{h}\xi^0), \cosh(\frac{m_0 c^2}{h}\xi^0), 0, 0) \qquad (10)$$

$$\frac{\partial e^{\alpha}{}_{1}(\xi^0)}{c\partial \xi^0} = \frac{\partial^2 x^{\alpha}}{\partial \xi^1 c\partial \xi^0} = \frac{\partial e^{\alpha}{}_{0}(\xi^0)}{\partial \xi^1} \qquad (11)$$

Therefore, the vector $e^{\alpha}{}_{0}(\xi^0)$ is

$$e^{\alpha}{}_{0}(\xi^{0}) = \frac{\partial x^{\alpha}}{c\partial \xi^{0}}$$

$$= ((1+\frac{m_0 c}{h}\xi^1)\cosh(\frac{m_0 c^2}{h}\xi^0), (1+\frac{m_0 c}{h}\xi^1)\sinh(\frac{m_0 c^2}{h}\xi^0), 0, 0) \quad (12)$$

About y -axis's and z -axis's orientation, the unit vector $e^{\alpha}{}_{2}(\xi^{0})$ and $e^{\alpha}{}_{3}(\xi^{0})$ is

$$e^{\alpha}{}_{2}(\xi^{0}) = \frac{\partial x^{\alpha}}{\partial \xi^{2}} = (0,0,1,0) \quad , \quad e^{\alpha}{}_{3}(\xi^{0}) = \frac{\partial x^{\alpha}}{\partial \xi^{3}} = (0,0,0,1)$$

The differential coordinate transformation is

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = \frac{\partial x^{\alpha}}{c\partial \xi^{0}} cd\xi^{0} + \frac{\partial x^{\alpha}}{\partial \xi^{1}} d\xi^{1} + \frac{\partial x^{\alpha}}{\partial \xi^{2}} d\xi^{2} + \frac{\partial x^{\alpha}}{\partial \xi^{3}} d\xi^{3}$$

$$= e^{\alpha}{}_{0}(\xi^{0}) cd\xi^{0} + e^{\alpha}{}_{1}(\xi^{0}) d\xi^{1} + e^{\alpha}{}_{2}(\xi^{0}) d\xi^{2} + e^{\alpha}{}_{3}(\xi^{0}) d\xi^{3}$$

$$cdt = (1+\frac{m_0 c}{h}\xi^1)\cosh(\frac{m_0 c^2 \xi^0}{h}) cd\xi^0 + \sinh(\frac{m_0 c^2 \xi^0}{h}) d\xi^1 \quad (13)$$

$$dx = (1+\frac{m_0 c}{h}\xi^1)\sinh(\frac{m_0 c^2 \xi^0}{h}) cd\xi^0 + \cosh(\frac{m_0 c^2 \xi^0}{h}) d\xi^1 \quad (14)$$

$$dy = d\xi^2, dz = d\xi^3 \quad (15)$$

Therefore if Eq(13), Eq(14) and Eq(15) integrate, finally the new system's coordinate transformation is found.

$$ct = (\frac{h}{m_0 c} + \xi^1)\sinh(\frac{m_0 c^2 \xi^0}{h}) \quad (16)$$

$$x = (\frac{h}{m_0 c} + \xi^1)\cosh(\frac{m_0 c^2 \xi^0}{h}) - \frac{h}{m_0 c} \quad (17)$$

$$y = \xi^2, z = \xi^3$$

Therefore, the inverse-coordinate transformation of the new system is

$$\frac{ct}{(x+\frac{h}{m_0 c})} = \tanh(\frac{m_0 c^2 \xi^0}{h})$$

$$\xi^0 = \frac{h}{m_0 c^2}\tanh^{-1}[\frac{ct}{(x+\frac{h}{m_0 c})}] \quad (18)$$

$$(x+\frac{h}{m_0 c})^2 - c^2 t^2 = (\frac{h}{m_0 c} + \xi^1)^2$$

$$\xi^1 = \sqrt{(x+\frac{h}{m_0 c})^2 - c^2 t^2} - \frac{h}{m_0 c} \quad (19)$$

$$\xi^2 = y, \xi^3 = z$$

Therefore, the invariable time $d\tau$ of the new system is by Eq(13),Eq(14),Eq(15)

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= (1 + \frac{m_0 c}{h}\xi^1)^2 (d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] \quad (20)$$

Hence, Riemann curvature tensor $R^{\lambda}{}_{\mu\upsilon\rho}(x), R^{\delta}{}_{\alpha\beta\gamma}(\xi)$ is

$$g_{00} = -(1 + \frac{m_0 c}{h}\xi^1)^2, g_{11} = g_{22} = g_{33} = 1,$$

$$g^{00} = -1/(1 + \frac{m_0 c}{h}\xi^1)^2, g^{11} = g^{22} = g^{33} = 1,$$

$$\Gamma^1{}_{00} = \frac{1}{2}g^{11}(-\frac{\partial g_{00}}{\partial \xi^1}) = \frac{1}{2}\cdot 2(1 + \frac{m_0 c}{h}\xi^1)\cdot\frac{m_0 c}{h} = (1 + \frac{m_0 c}{h}\xi^1)\frac{m_0 c}{h}$$

$$\Gamma^0{}_{10} = \Gamma^0{}_{01} = \frac{1}{2}g^{00}(\frac{\partial g_{00}}{\partial \xi^1}) = \frac{1}{2}\cdot -1/(1 + \frac{m_0 c}{h}\xi^1)^2 \cdot -2(1 + \frac{m_0 c}{h}\xi^1)\frac{m_0 c}{h} = \frac{1}{(1 + \frac{m_0 c}{h}\xi^1)}\frac{m_0 c}{h}$$

$$R^{\delta}{}_{\alpha\beta\gamma}(\xi) = \frac{\partial \Gamma^{\delta}{}_{\alpha\beta}}{\partial \xi^{\gamma}} - \frac{\partial \Gamma^{\delta}{}_{\alpha\gamma}}{\partial \xi^{\beta}} + \Gamma^{\sigma}{}_{\alpha\beta}\Gamma^{\delta}{}_{\sigma\gamma} - \Gamma^{\sigma}{}_{\alpha\gamma}\Gamma^{\delta}{}_{\sigma\beta}$$

$$R^1{}_{001}(\xi) = -R^1{}_{010}(\xi) = \frac{\partial \Gamma^1{}_{00}}{\partial \xi^1} - \Gamma^0{}_{01}\Gamma^1{}_{00} = \frac{m_0{}^2 c^2}{h^2} - \frac{m_0{}^2 c^2}{h^2} = 0, \text{ otherwise } R^{\delta}{}_{\alpha\beta\gamma}(\xi) = 0$$

$$0 = R^{\lambda}{}_{\mu\upsilon\rho}(ct, x, y, z) = \frac{\partial x^{\lambda}}{\partial \xi^{\delta}}\frac{\partial \xi^{\alpha}}{\partial x^{\mu}}\frac{\partial \xi^{\beta}}{\partial x^{\upsilon}}\frac{\partial \xi^{\gamma}}{\partial x^{\rho}}R^{\delta}{}_{\alpha\beta\gamma}(c\xi^0, \xi^1, \xi^2, \xi^3),$$

$$0 = R^{\delta}{}_{\alpha\beta\gamma}(c\xi^0, \xi^1, \xi^2, \xi^3) \quad (21)$$

Therefore, the new system is in the flat Minkowski space.

About x -axis's light speed,

$$dy = d\xi^2 = dz = d\xi^3 = 0, y = \xi^2 = z = \xi^3 = 0$$

$$cdt = dx \;,\; ct = x \;,$$

$$cd\xi^0 = \frac{d\xi^1}{(1 + \frac{m_0 c}{h}\xi^1)},$$

$$c\xi^0 = \frac{h}{m_0 c}\ln|1 + \frac{m_0 c}{h}\xi^1|$$

$$\rightarrow (1 + \frac{m_0 c}{h}\xi^1) = e^{\frac{m_0 c^2}{h}\xi^0} \rightarrow (\frac{h}{m_0 c} + \xi^1) = \frac{h}{m_0 c}e^{\frac{m_0 c^2}{h}\xi^0} \quad (22)$$

In this time, if use the new system's coordinate transformation, Eq(16),Eq(17)

$$ct=(\frac{h}{m_0c}+\xi^1)\sinh(\frac{m_0c^2\xi^0}{h})$$

$$=\frac{h}{m_0c}e^{\frac{m_0c^2}{h}\xi^0}(\frac{e^{\frac{m_0c^2}{h}\xi^0}-e^{-\frac{m_0c^2}{h}\xi^0}}{2})$$

$$=\frac{h}{m_0c}(\frac{e^{2\frac{m_0c^2}{h}\xi^0}-1}{2})$$

$$=x=(\frac{h}{m_0c}+\xi^1)\cosh(\frac{m_0c^2\xi^0}{h})-\frac{h}{m_0c}$$

$$=\frac{h}{m_0c}e^{\frac{m_0c^2}{h}\xi^0}(\frac{e^{\frac{m_0c^2}{h}\xi^0}+e^{-\frac{m_0c^2}{h}\xi^0}}{2})-\frac{h}{m_0c}$$

$$=\frac{h}{m_0c}(\frac{e^{2\frac{m_0c^2}{h}\xi^0}-1}{2}) \qquad (23)$$

IV. Conclusion

It found the new system that used the tetrad on the new method.

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