

Thermodynamics of the Primary Eigen Gas and the Postulates of Quantum Mechanics

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Abstract

The author shows that that for each quantum mechanical property of a micro-system there is a corresponding thermodynamic one in the primary eigen gas approach. He further shows that the basic postulates of quantum mechanics have equivalents in the primary eigen gas approach provided time is accorded directional symmetry. The interference pattern obtained in Young's double slit experiment is explained in terms of the primary eigen gas approach using the directional symmetry of time.

PACs numbers: 03.65-w, 3.65 Ta, 03.75 -b, 05.40 -a, 05.70 Ce

Key Words : Primary eigen Gas, Stochastic quantum mechanics, Action- entropy equivalence, Quantization of space and time .

1 Introduction

It was earlier shown that electron can be represented by a confined helical wave (CH wave) formed by the confinement of a plane polarized electromagnetic wave after it acquires half spin. The properties of a particle like mass, electric charge and spin were found to emerge from the CH wave [1],[2][3],[4]. These intrinsic properties of electron are found to be expressed by the space-dependent component of the CH wave which gets compacted into the internal coordinates while the time dependent component becomes the plane wave which represents the particle in the laboratory coordinate system. Extending this idea further it was proposed that any particle can be represented by a CH wave where the confinement takes place on a composite wave which has oscillations not only in the electromagnetic field, but also in other appropriate fields. It was further shown that the states occupied successively in time by its interactions with the vacuum fluctuations may be taken to form a gas called the primary eigen gas. The only difference between the primary eigen gas and the real gas is that while in the real gas the microstates are occupied simultaneously, in the case of the primary eigen gas the micro-states are occupied successively in time. But for this difference, the statistical mechanics of the primary eigen gas is equivalent to that of the real gas [5]. It was observed that the primary eigen gas picture of a particle and the wave picture of a particle are equivalent ones and this equivalence is a direct result of a new symmetry called the Wick symmetry. In fact, it was proposed that quantum mechanics could be understood in terms of the statistical mechanics of the primary eigen gas where time has not lost its directional symmetry [5].

We saw earlier that the probability function representing the state of the primary gas with energy E_i is given by

$$W_i = g(E_i) e^{h^{-1}A} \quad (1)$$

This function can be obtained by applying Wick's operator on the plane wave representing the quantum mechanical state with energy E_i [6]

$$\text{ie; } \bar{R}\phi_i = \bar{R}[B(E_i) e^{ih^{-1}A}] = g(E_i) e^{h^{-1}A} \quad (2)$$

Note that Wick's operator replaces $2\pi iN$ by N wherever N occurs in the function. We saw that this equivalence emerges from the Wick symmetry. In other words, the plane

wave which is the eigen state of a particle in space-time representation becomes the probability function of the primary eigen gas representation.

In this paper we shall examine the compatibility of the basic postulates of quantum mechanics to the statistical mechanics of the primary eigen gas. Besides, using the eigen gas approach we shall try to explain the interference pattern obtained in the double slit experiment using an electron beam.

2 Quantum Mechanics as Reversible-Time Thermodynamics

When we represent a particle in terms of a plane wave, we never specify the number of single waves constituting the wave train or the length of the wave train which constitutes the plane wave. It is left undefined in the usual treatment. This is because, in quantum mechanics, the eigen state is supposed to be the most basic state. But in the primary eigen gas approach we are attributing an inner structure to the eigen state. In order to strike equivalence between the two approaches, we assume that there are N micro-states that constitute a primary eigen gas just as there are N wavelets that constitute a plane wave state. We do not know whether N is a universal constant or an arbitrary number which gets factored out in any observation. In fact, in the interaction between two particles, N can be conveniently taken as a constant. The justification for such an assumption is that the variation can be accounted by n which represents the number of the primary eigen gas states involved in any interaction. One need not take both N and n as variables [5].

The Copenhagen interpretation (CI) treats eigen state as the most basic element of the quantum reality. According to CI any observation on a micro-system can take us only up to the level of an eigen state. Besides, any physical system can be interpreted only in terms of the observable reality [7]. Such a notion did not permit any further analysis of the eigen state. But in terms of the eigen gas approach, we observe that an eigen state can be taken as equivalent to a eigen gas which is a primary gas. In our approach we do not take the primary eigen gas as the most basic state. We assume that the primary eigen gas state is constituted by a group of plane wavelets occupied successively. This may appear rather contrived as the wavelet is below the level of observation. But such a structure is proposed here as it simplifies the picture and we are able to explain the laws of quantum mechanics in terms of thermodynamics of the primary eigen gas. Note that this approach does not accept the view that there is an objective reality to the micro-system. It goes by the approach based on Copenhagen interpretation that the micro-system exists in a state where all possibilities exist in an unmanifested manner and the system crystallizes into reality only in an observation.

We shall now look for the primary eigen gas equivalents of other quantum mechanical properties. We know from the Wick symmetry that the negative action is equivalent to entropy [5].

$$\text{ie; } \quad -A/h \quad = \quad S^\# / K \quad (3)$$

This may be expressed in terms of the energy-momentum of the particle as

$$(Et - px)/h \quad = \quad E_o t_o / \hbar \quad = \quad S^\# / K \quad (4)$$

But we know that $t_o = nNT_{eo} = nNh/K\theta_o$, where θ_o is the temperature of the vacuum fluctuations background which interacts with the particle [5]. Therefore, we have

$$S^\# \quad = \quad nNE_o / K\theta_o \quad (5)$$

Note that the rest energy of the particle E_o behaves like the intrinsic heat content.

We shall now try to list out the reversible time equivalent of other quantum mechanical entities below.

<u>Imaginary time</u>	<u>Reversible time</u>
i) Wave Function	State function of the primary eigen gas
ii) Negative Action	Entropy
iii) Total energy	Internal energy
iv) quantum of time	Inverse of temperature
v) particle velocity	Drift velocity
vi) Langrangean	Negative Intrinsic heat content
vii) Uncertainty principle.	Equation for fluctuations
viii) Zero point energy	Thermal energy of vacuum fluctuations

3 The Basis for the use of Boltzmann's Micro-canonical Distribution

In the approach followed above, one question that may be raised is regarding the use of the Boltzmann's distribution to represent the primary eigen gas state. The justification for this is quite straight forward. First of all we should remember that we are dealing with one path of progression at a time. The system in the primary eigen gas representation occupies only a particular micro-state (wavelet of a plane wave) at a time. Such a wavelet state is occupied out of very large number possible wavelet states available for occupation. In fact there is no limitation to the number of wavelet states that could be occupied here. In that sense, the situation is equivalent to a large number of boxes available for occupation with a limited number of balls. This is the situation where Boltzmann's distribution holds good. If the number of boxes and the number of balls were of the same order, then we would have been forced to apply the Fermi-Dirac distribution.

The concept of the ensemble will have to be given a fresh look in the case of the primary eigen gas. In the case of the canonical ensemble of a real gas, we deal with energy and volume along with their conjugate variables, temperature and pressure. However, in the case of the primary eigen gas we take the translational momentum in place of volume, and velocity in place of pressure. We know [5] that the internal heat of the primary eigen gas is given by

$$Q = Nq = NE - Npv . \quad (6)$$

Therefore, the entropy of the system will be given by

$$S = Q/\theta = N(E/\theta - vp/\theta) \quad (7)$$

Earlier we saw that such a primary gas may be taken to be equivalent to a real gas [5]. One important difference between volume in the case of a real gas and momentum in the case of a primary gas is that volume cannot be transformed out by suitably selecting a frame of reference while momentum can be. This means that we can always deal with the micro-canonical ensemble by introducing suitable transformation and study the system more conveniently. Afterwards, we may transform it back by shifting the frame of reference suitably. This is made possible by the fact that the entropy of the system remains invariant in such a transformation. In short, we do not have to deal with the canonical ensemble to work out the thermodynamics of the system. Micro-canonical ensemble will do.

4 The Primary Gas and the Postulates of Quantum Mechanics

In the earlier papers we saw that a wave function representing a micro-system will become a probability density function on being operated by Wick's operator $\bar{\mathbf{R}}$. We also saw that such an operation does not alter the physical situation. Only the imaginary time picture gets replaced by the reversible (real) time picture which is same as stating that the wave nature of the micro-system gets replaced by the primary

eigen gas picture. This means that the dynamics of the system which is determined by the wave nature of the micro-system will have to be replaced by laws of statistical mechanics. This in turn means that the basic postulates of quantum mechanics could be understood in the light of the laws of statistical mechanics. We shall confine ourselves to the three basic postulates of quantum mechanics for study [8]. These three postulates capture the essence of the operator form of quantum mechanics. There are further postulates, but these three are the most important ones. They firmly establish the existence of the wave function and its status as a complete description of the state of a quantum particle. The replacement of the values of the observable quantities (classical mechanics) with their corresponding operators (quantum mechanics) provides a recipe for using the wave functions and operators to calculate the values of the observables. Let us take the first postulate of quantum mechanics to start with [8].

Postulate I: *The state of a quantum mechanical system is completely described by the wave function ψ_k .*

Here the subscript k serves as a short hand for the set of one or more quantum numbers on which the wave function depends. Let us take the plane wave representation of a quantum mechanical system. Note that a plane wave is the eigen function of a momentum state (taking energy as the fourth component of the momentum) in the coordinate representation. This means that the system may be occupying any of the plane wave states denoted by k . If the wave function ψ_k is represented as a vector in a configuration space (Hilbert space), then ψ_k will be orthogonal to ψ_m unless $k = m$. In this way, the state of a system can be represented as a vector in an n -dimensional configuration space. This assumes that the system may occupy all possible states simultaneously in a virtual way and for such a possibility to be acceptable, time has to be treated as imaginary. It should be kept in mind that the orthogonality of the wave functions becomes important only at the instant of observation as two eigen states cannot be realized simultaneously. The higher the magnitude of the vector ψ_k larger will be the probability of catching the system in that state.

In the reversible time approach the state function of the primary eigen gas state which is a probability density function plays the same role as the plane wave in the imaginary time. However, since we are treating as real time it may appear that the concept of simultaneous occupation of various primary gas states may not be possible. But again we have to remind ourselves that while time in the primary eigen gas approach may be treated as real, it is also treated as reversible. Therefore, by the process of reverse jump in time it is possible for the system to occupy all states at the same instant [7]. In other words, there is no logical inconsistency in assuming that all primary eigen gas states exist simultaneously. Therefore just as in the case of the plane wave representation, in the primary eigen gas approach also it is possible to represent the state of a particle as a linear combination of the primary eigen gas states.

The next question that we have to confront is that when we take the linear combination of the primary eigen gas states, whether they will also undergo interference just as in the case of the plane wave states. One may think that as the primary eigen gas state does not exhibit undulatory behavior it is impossible to account for the interference phenomenon in this approach. But then we should not forget that primary eigen gas approach and the plane wave approach are two ways of viewing the same phenomenon. The phenomenon itself remains the same in both approaches. Here we have to recall that in the case of the primary eigen gas approach, the quantum of time is one period of oscillation of the plane wave. This means that the periodic nature of the wave gets pushed into the internal structure of the quantum and gets compacted into the internal coordinates. But when two micro-states belonging two different primary eigen gases occupy the same spatial region, then of course these internal coordinates come into play and the interference occurs as usual.

This means that there is no difference in the net result whether we go by the plane wave approach or by the primary eigen gas approach.

The concept of the orthogonality of the eigen states emerges from the fact that the probability for observing the system in the k^{th} state at an instant is given by $\Psi_k^* \Psi_k$. Here all the paths of evolution of states occur simultaneously and therefore when we take the integral

$$\int \psi_m^* \psi_n d\tau = \delta_{mn} \quad (8)$$

In the eigen gas approach also the equivalent relation given below will hold good:

$$\int W_m^c W_n d\hat{\tau} = \delta_{mn} \quad (9)$$

here $\hat{\tau}$ denotes the reversible (real) time. This is taken care of by the concept of forward and reverse jump in time. The resultant picture that emerges is no different from the approach based on the plane wave.

One important issue that arises out of this reversible (real) time approach is that the wave equation which applies to the wave nature of the particle will no more be valid in the primary eigen gas approach. Let us take the Schrodinger's equation first and try to understand the implication of replacing imaginary time with real time. We also know that the Schrodinger equation for a free particle on carrying out Wick's operation will transform into the Diffusion equation [9]. We shall take complex conjugate of the Schrodinger equation and apply Wick's operator to obtain

$$\bar{R}\left\{ \frac{\hbar^2}{2m} \nabla^2 \psi^* - i\hbar \frac{\partial \psi^*}{\partial t} \right\} = 0$$

$$\text{ie; } \frac{\partial W^c}{\partial \hat{t}} = \frac{\hbar}{2m} \hat{\nabla}^2 W^c \quad (10)$$

Note that since $t = nNT_e$, where n and N are integers while T_e denotes the period of the plane wave representing the particle. \hat{t} will be obtained by carrying out Wick's operation on t when $2\pi iN$ will be transformed to N . This means that $\hat{t} = 2\pi it$. Similarly $\hat{x} = 2\pi it$. This is the heat equation or diffusion equation where $(\hbar/2m)$ represents the diffusivity. W^c which stands the probability density can also represent the particle density which in turn can denote the energy density.

In this connection we should recall that [5] the rest energy of a particle can be treated as its internal heat. This means that the Schrodinger equation and the diffusion equation are two ways looking at the same reality. Needless to say, we may view a potential as one which introduces a gradient in the vacuum fluctuations field that directs the diffusion process. In this connection it is worthwhile to recall an alternate approach to quantum mechanics called stochastic quantum mechanics has been proposed by Nelson, Yasue et al [10][11][12][13]. This approach has met with partial success. But this approach has not been able to obtain profound insights like the action-entropy equivalence and the Wick symmetry. The main reason for this shortcoming may be that the stochastic approach shied away from proposing a basic structure to the particle and was basing its bet on the concept of point particle. The striking similarity between the plane wave and the probability density function was never probed in depth. The approach seems have focused too much on the similarity between the Schrodinger equation and the diffusion equation. But it is quite obvious that the primary gas approach may be developed further using the methods of

stochastic quantum mechanics for understanding the behavior of the micro-systems in greater depth.

Let us now take the Dirac equation now.

$$(\gamma_\mu \partial / \partial x_\mu + mc/\hbar) \psi = 0, \quad (11)$$

where $\gamma_k = \begin{bmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{bmatrix}$, $k = 1, 2, 3$; $\gamma_4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Here σ_k represent Pauli's spin matrices. On operating with Wick's operator \check{R} , the Dirac equation transforms to give

$$(\gamma_\mu \partial / \partial \hat{x}_\mu + mc/\hbar) W = 0, \quad (12)$$

Note that $\hat{x}_\mu = nNv_\mu T_e$, where $T_e = h/K\theta$. We observe that the wave equation given in (11) exhibits undulatory behavior while that given in (12) does not have such a property since W is just a probability function. Therefore, prima-facie it may appear that these two equations represent different realities. But we should keep in mind that the undulatory behavior holds good only within one wave length. If we treat one wave length as a single unit, then we will not observe any undulatory behavior. Instead, we would observe that the system progresses in a uniform manner. We shall examine this issue in more detail in the next section.

We know that we can extract the Schrodinger equation from the Dirac equation in the non-relativistic region if we ignore spin [9]. In view of this it is reasonable to assume that the Dirac equation in real time also represents some sort of a diffusion equation. But we are not familiar with its classical analogue.

Postulate II: *Observable quantities are represented by mathematical operators. These operators are chosen to be consistent with the position-momentum commutation relations.*

To clarify the picture, let us take the plane wave representation of the eigen state ψ_n given by

$$\psi_n = B e^{i\hbar^{-1} p_n x} \quad (13)$$

The momentum operator $\hat{p} = -i\hbar \partial / \partial x$ so that

$$-i\hbar \partial \psi_n / \partial x = p_n \psi_n \quad (14)$$

In other words, the momentum operator is defined in such a way that ψ_n becomes the eigen function and p_n its eigen value. When we go over to the primary eigen gas picture after introducing Wick's operation, we observe that p_n will be the momentum of a primary eigen gas. The probability density function for the primary eigen gas in forward time representing a particle travelling along the x-axis will be [6] given by

$$W_n = B e^N e^{-\hbar^{-1}(Et - p_n x)} \quad (15)$$

The corresponding momentum operator will be " $\hbar \partial / \partial \hat{x}$ " and we obtain the similar eigen value equation

$$\hbar \partial W_n / \partial \hat{x} = p_n W_n \quad (16)$$

Here also the average value of momentum would be obtained by averaging the values of \mathbf{p}_n . It is obvious that in the operators in the primary eigen gas approach can be obtained by applying Wick's operation on the corresponding operators in quantum mechanics. It can be easily seen that Wick's operation transforms only x and t leaving the form of the other operators unchanged.

Let us now examine how the commutation relation appears in the primary eigen gas approach. We know that the momentum operator in quantum mechanics satisfies the relation

$$(\mathbf{x}\mathbf{p} - \mathbf{p}\mathbf{x}) = i\hbar \quad (17)$$

In the primary eigen gas approach by replacing x by \hat{x} we obtain

$$(\hat{\mathbf{x}}\mathbf{p} - \mathbf{p}\hat{\mathbf{x}})W_n = -\hbar \quad (18)$$

Postulate III : *The mean value of an observable is equal to the expectation value of its corresponding operator.*

For a specific wave function ψ_n , the expectation value of the operator A is defined by the expression

$$\langle A \rangle = \frac{\int \psi_n^* A \psi_n d\tau}{\int \psi_n^* \psi_n d\tau} \quad (19)$$

where τ denotes the generalized volume element. In quantum mechanics this expression is introduced rather arbitrarily. It emerges from the fact that ψ_n is the eigen function of the operator \hat{A} and a_n is the corresponding eigen value. Therefore, we have

$$A\psi_n = a_n\psi_n \quad (20)$$

Multiplying both sides by ψ_n^* and integrating over all spatial elements yields

$$\int \psi_n^* A \psi_n d\tau = a_n \int \psi_n^* \psi_n d\tau \quad (21)$$

This is possible because a_n is a mere number. If ψ_n is normalized, we obtain

$$\langle A \rangle = a_n \quad (22)$$

Since the wave function can be a complex function, the operator too may be a complex function. However, if postulate III is to make sense, the eigen value of an operator representing an observable must be a real quantity because this is something that can be measured in an experiment. Operators whose eigen values are real are called Hermitian operators. As a result of this property of the operators any two eigen functions are orthogonal. That is, taking the eigen functions as normalized, we have

$$\int \psi_n^* \psi_m d\tau = \delta_{nm} \quad (23)$$

Despite ψ being a complex function while probability is a real number, the above equations holds good because the phase part of ψ gets removed as we are multiplying it with its complex conjugate which leaves us with the square of the amplitude. But then we know from (13) and (15) that

$$\psi_n^* \psi_n = W_n^c W_n = B^2$$

This will directly lead us from (19) to

$$\langle A \rangle = \frac{\int W_n^c A W_n d\tau}{\int W_n^c W_n d\tau} \quad (24)$$

Strictly speaking (24) represents the classical definition of the probability. The probability postulate of quantum mechanics appears more contrived and adhoc. Note that in the case of a real gas existing in progressive time, the average value for momentum will be given by the relation

$$\langle A \rangle = \frac{\int A W_n d\tau}{\int W_n d\tau} \quad (25)$$

5 More on Wick Symmetry

We saw that the Wick's operation transforms the plane wave into the state function of the primary eigen gas. In this operation, $2\pi iN$ gets converted to N . Let us examine what is happening here. We know that the basic difference between the primary eigen gas approach and the plane wave approach is that the former treats each single wavelet as the basic unit or quantum. This does not mean that this approach completely ignores imaginary time aspects and the accompanying phase change that takes place within a single wavelet. It is just compacted into its internal coordinates. To put it differently, the primary eigen gas approach accounts for the phase of the wavelet as the intrinsic property of the micro-state. In the plane wave approach there is no quantization and therefore the imaginary nature of space and time gets expressed in the external coordinates themselves.

Another important point that emerges from the Wick symmetry is the relationship between the probability for the forward evolution and the reverse evolution in time. We know from the earlier discussion that the state function $W_k(x,t)$ represents the sum total of all probability for forward jumps in time to the k^{th} state at the space-time point (x,t) from the past states. Likewise, $W_k^c(x,t)$ represents the probability for the reverse jumps to the past from the k^{th} state at the same space-time point. We know that when time has not lost its directional symmetry, the probability for the forward jump should be equal to that for the reverse jump. In other words $W_k = W_k^c$. Let us see if such a relationship actually emerges from the primary eigen gas picture. Let us start with the primary eigen gas state representing the particle travelling along the x-axis given by

$$W_k = g_k e^{-(Et-px)/h} \quad (26)$$

To make matters simple, we shall transform this primary eigen gas to a frame of reference with which it is at rest given by [6]

$$W_k(o) = B_k e^{nN} e^{-nNE_o/K\theta_o} \quad (27)$$

Since the system is in equilibrium with the vacuum fluctuations, we may take $E_o = K\theta_o$. Therefore, (27) can be written as

$$W_k(o) = B_k e^{nN} e^{-nN} = B_k \quad (28)$$

It can be easily shown that the corresponding expression for $W_k^c(o)$ would be given by

$$W_k^c(o) = B_k e^{-nN} e^{nN} = B_k \quad (29)$$

This confirms our assumption.

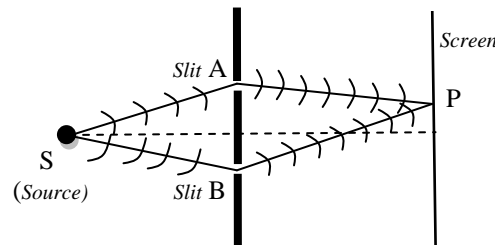
6 Collapse of the Primary Gas in an Observation

Till now we have not explained what happens to the primary eigen gas in an observation. We had assumed that a primary eigen gas state is defined by N micro-states (plane wavelets) occupied successively. This means that a particle state is spread over a spatial length of $N\lambda$, where N is assumed to be a large number and therefore the particle state is spread over a very large region. But in an experimental set up, we select a small region to observe the particle and this small region would contain paths of various primary gas states. Therefore, when we observe a particle, the corresponding gas state need not belong to one single primary gas state. The N states constituting the eigen state may belong to various primary eigen gas states which have their paths passing through the region in question.

In fact in the region of localization, the interference among the micro-states belonging to various individual paths will create a sharp maximum. One should keep in mind that the interference phenomenon takes place in the reversible time also; the only difference is that the phase of the micro-states is defined in their internal coordinates. But that is only an accounting jugglery and does not affect the actual situation. The important point is that N states which constitute the observed gas that represent the particle is no more a primary eigen gas state as the microstates are not occupied successively in time. Such an observed gas state could be called the collapsed primary eigen gas state. But then based on the principle of statistical equivalence, we can always find a primary eigen gas state matching with the collapsed primary eigen gas state. Therefore, it will be proper to say that on observation, the system crystallizes to one of the primary eigen gas states.

7 Explaining Interference pattern of the Double-slit Experiment

Let us now examine the interference pattern obtained in the double slit experiment using an electron beam. We know the interpretation of this phenomenon based on the wave nature of the electron [14]. We shall briefly describe it here (fig.2). The quantum mechanical interpretation is that the waves representing the particles spread around as a wave front from the source. The secondary waves emerging from slit A and slit B arrive at P on the screen where they undergo interference. If both



On the basis of the wave picture, electron from the source S goes through slits A and B and the secondary waves from the slits arrive at the screen and the interference between the two waves fronts creates the fringes at P.

Figure 2

waves reach P in phase the intensity will be maximum and if they reach with a phase difference of π , then, the intensity will be minimum. This explains the formation of the interference pattern on the screen. Note that this interference would be formed even if only one electron is emitted by the source at a time. This would mean that a single electron initially disembodyes into a wave front and passes through both slits simultaneously and arrives at P on the screen when suddenly it throws away its wave-disguise and appears as a particle. This interpretation may appear strange, but that is

what quantum mechanics has to offer. The issue of the wave particle duality is still a topic which is not properly understood in quantum mechanics.

In the primary eigen gas approach, the electron starting from the source jumps forward in time to slit A and from there it jumps forward to P along all possible paths and then jump back in time to reach A and back again to S. In a similar manner, we may imagine the same process taking place through slit B also. In fact, these forward and the reverse jumps in time creates a wave front emanating from S. This will also explain the secondary waves emanating from the two slits. In fact this interpretation is in tune with the Maxwell's equation which has solutions with waves moving forward in time and backward in time. We now obtain the same result as obtained in the wave representation of the particle. The interference pattern is created due to the fact that the quantum has an internal structure of a wavelet and at P the wavelets emerging from slit A and slit B arrive simultaneously and is caught in an observation where their internal phases come into play and creates the interference pattern.

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