Wick Rotation as a New Symmetry

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Abstract

The author discusses the similarity between the expression for the state function of the primary eigen gas representing a particle and that of the wave function. It is observed that the only difference between these two expressions is that in the former time appears as a real function while in the latter it appears as an imaginary function. He shows that the primary eigen gas approach which treats time as real and the quantum mechanical approach which treats time as imaginary are two ways of representing the same reality and points to a new symmetry called the Wick symmetry. He shows that the probability postulate of quantum mechanics can be understood in a very simple and natural manner based on the primary eigen gas representation of the particle. It is shown that the zero point energy of the quantum mechanics is nothing but the energy of the thermal bath formed by the vacuum fluctuations. The author shows that the quantum mechanics is nothing but the thermodynamics of the primary eigen gas where time has not lost its directional symmetry.

1 Introduction

It was earlier proposed that a particle can be represented by a confined helical wave and on imparting a translational velocity, its space dependent part transforms into the amplitude wave that gets compacted in to the internal coordinates. Such a compacted wave determines the intrinsic properties of the particle like its mass, spin and charge while the time dependent part of the confined helical wave can be identified with the plane wave [1],[2][3],[4]**.** It was shown that the plane wave which is the eigen state of the particle in the space-time representation may be treated as an primary eigen gas having N microstates or wavelets [5]. We know that the probability for the primary eigen gas to occupy a state with intensive energy E_o' in the rest frame of reference, is given by [5]

$$W_o' \quad = \quad g_o' \, e^{-E_o t_o / h} \quad = \quad g_o' e^{-nNE_o / K\theta_o} \quad , \tag{1}$$

where E_o is the rest energy of the particle, N the number of microstates in the primary eigen gas state, n the number of primary gas states and θ_o its temperature. For such a gas the energy states lie in a very narrow region $(E_o-\Delta E) < E'_o < (E_o+\Delta E)$ with the action function possessing a minimum. In other words, action can be taken as a constant in the narrow region and we may express it using the average values of energy and momentum of the particle as

$$W(E_o) \quad = \quad g(E_o) \, e^{-h^{-1} E_o t_o} \quad , \tag{2}$$

In the plane wave representation of a particle, the state with energy E_i (we shall use the energy state instead of the momentum state for convenience) can be expressed by

$$\phi(E_i) \quad = \quad B(E_i) \, e^{-i\hbar^{-1}(Et - \bar{p}z)} \tag{3}$$

In the rest frame ϕ can be expressed as

$$\phi(E_o) \quad = \quad B(E_o) \, e^{-i\hbar^{-1} E_o t_o} \tag{4}$$

The above equation has the same form as (2) except for the absence of the factor '$2\pi i$' in the exponential term. This is quite startling. What does this similarity of the plane wave with the

expression for the primary eigen gas mean? Let us go back to the basis of deriving the equation for the plane wave and compare it with the derivation of the state function for the primary eigen gas in order to explain the similarity between the two functions.

We saw that when we examine the concept of a particle from two different approaches, we obtain the same function but with one basic difference. In the case of the primary eigen gas approach, time appears as real while in the wave function approach, it appears as imaginary. Prima facie one may feel that since both functions represent the same system, they should be equivalent. However we observe that there are major differences between the two functions. The function $\phi(E_o)$ in (4) is a periodic function representing a wave and its value oscillates as time progresses while $W(E_o)$ in (2) is a probability function and the exponential factor on the right hand side keeps on decreasing as time progresses. Thus prima facie these two functions seem to represent two different realities which are fundamentally irreconcilable. But we shall shortly show that these differences can be reconciled.

When we talk about the imaginary time, the idea that immediately comes to one's mind is Minkowski's four-dimensional formalism of the relativistic mechanics where time is treated as the fourth coordinate which is imaginary [6]. Here time is taken as an imaginary quantity represented by $\tau = ict$. But the problem with this approach is that it treats energy as imaginary while momentum is taken as real with the result that the action function which has the dimension of the product of energy and time remains real. That is to say, the Boltzmann factor, "$\exp(-E_o t_o/h)$" in (2) will continue to remain real since both time and energy are treated as imaginary. Therefore, treating time as an imaginary fourth-coordinate does not help in resolving the problem.

2 More About Real Time and Imaginary Time

On detailed scrutiny of equations (2) and (4) we observe that the two approaches differ in one fundamental aspect. In the case of the plane wave approach, each of the energy momentum states is assumed to exist at the same instant. In other words, the wave function represents a system where the simultaneous occupation of all energy-momentum states is possible. However, the system evolves without collapsing into any one state. On the other hand, in the primary eigen gas approach the situation is quite different. Here we had assumed that at every instant the system occupies a particular energy-momentum state. This is quite different from the approach based on the wave nature of the particle. In the primary eigen gas approach, since we took it for granted that the system can occupy only a single energy-momentum state in one instant, it is equivalent to assuming that the system crystallizes into reality at every instant and therefore the primary gas we have been dealing with is a real gas. The path which has the maximum probability is the path along which system will progress for all practical purposes. The other paths which the system could occupy remain just as the probable ones. Let us examine the issue taking a specific example.

Let us take the case of a microsystem progressing from point A to point B (figure.1). In the wave picture, the particle will evolve along different paths. We know that action will have

(a)

(b)

Imaginary time picture   *Real time picture*
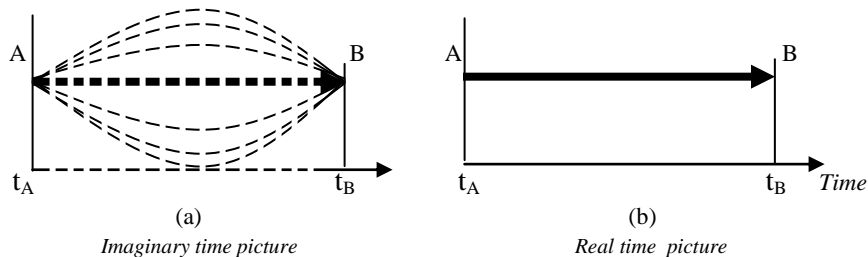
(a) shows the evolution of the system in the imaginary time. Here all the paths exist simultaneously. (b) shows the corresponding picture in the real time. Here only one path out of many possible ones gets occupied.

Figure.1

A minimum along the classical path (shown in thick dotted lines)and it will vary only by second order along nearby paths. But, for paths which are slightly more removed from the classical path, the action changes substantially even for a slight change in the path. Therefore, except along the paths close to the classical paths, the amplitude of all other waves become zero due to the destructive interference. But close to the classical path, the waves interfere constructively making the amplitude of the waves maximum. This does not mean that the system collapses into a particular energy-momentum eigen state.

In the primary eigen gas approach, the situation is different. Here we assumed that at every instant the system occupies a particular energy-momentum state which means that the system crystallizes into reality at every instant. In other words, the primary eigen gas is a real gas. Note that the probability along the classical path will have a sharp peak along the bold line (figure.1-b) because the degeneracy $g_{(E')}$ increases with extensive energy of the primary eigen gas while the Boltzmann's factor decreases with it and the sharp peak is obtained along the classical path. The probability for the occupation of the other paths will be very small. This means that when we take the linear sum of the primary eigen gas states that arrive at B, we end up with the classical path which is represented by the thick line in figure 1(b). The particle will progress along a single path that has maximum probability. Thus the difference between the two approaches can be traced to the state of crystallization of reality. This leads us to conclude that the imaginary time applies to the situation when we treat the system as occupying a large number of states simultaneously while the real time can be associated with the situation where we treat the system as occupying only one state at a time. This means that the wave function is nothing but the state function of the primary gas where real time is replaced by the imaginary time. This gives us a completely new insight into the nature of wave itself. We shall later see that there is a further twist to this argument.

In the wave representation, all states that can be occupied are occupied simultaneously. This raises a very serious problem. How can a particle occupy so many paths at the same time? Quantum mechanics tries to resolve the issue by assuming that the particle gets disembodied into a wave front and occupy all possible paths. But at the point of observation, the particle is assumed to discard the disguise of the disembodied wave front and appears as a localized entity. There is no explanation how the particle takes up the disguise of a wave when not observed off stage and how it takes on the particulate nature when on stage. But this is the best quantum mechanics can offer. But a deeper scrutiny in the light of the discussion in the previous paragraphs allows us to interpret the situation differently. Since the same particle occupies a large number of paths at the same time, we have to conclude that the states occupied cannot be real. The most logical choice is to treat these states along the paths as imaginary. We shall shortly show that this is equivalent to treating time as imaginary.

In the primary eigen gas picture the particle has the choice to occupy all paths, as shown in figure 1(b). But only one particular path can be occupied. All the others remain as potential paths. In short, the fundamental difference between the real time and the imaginary time is this. In the case of the real time, when the system can evolve along a large number of paths, it chooses one particular path. All other paths remain as potential ones which are not occupied. In the case of the imaginary time, the system evolves along all possible paths simultaneously. The more probable a path is, more often that path is occupied compared to the others. It may seem that the primary eigen gas picture is unworkable as it is impossible to observe each microstate at every instant of the evolution of the system so that it remains always crystallized in reality. In actual situation, the observation may be done at very long intervals only, and in between the system may be free to evolve without the intervention by an external observer. In that sense, the imaginary time picture which treats the system of a particle as a wave appears to be better suited for the job. However, the equivalence of the imaginary time picture and the real time picture in terms of the equations (4) and (2) appears to be too good to be discarded outright.

One may feel that the interference phenomenon is essentially a direct outcome of the wave nature which manifests only in the imaginary time. If so it may become impossible to explain the interference phenomenon in the primary eigen gas approach and this will come in

the way of striking the equivalence between the two approaches. Actually it is possible to explain the interference phenomenon in the primary eigen gas picture also. Here we should keep in mind that the primary eigen gas picture treats the plane wavelet as the basic entity or quantum. Therefore time gets quantized in terms of one period of the wavelet T_o and the progression in time is accounted by "NT_o", where N is the number of wavelets. This means, in this approach the sinusoidal structure of the wave gets pushed into the internal space of the wavelet. So what we have done here is just a jugglery. Now although we are treating the wavelet as the basic unit in the primary eigen gas approach, we have to treat the phase of the wavelet as its intrinsic property. Ultimately, the difference between the imaginary time approach and the real time approach is that imaginary time treats the wavelet as a wave while the real time treats it as a quantum with phase appearing as a property defined in its internal space. The problem of the interference phenomenon can be tackled in the primary gas picture with this idea of the phase defined in the internal space. We shall discuss the issue of the double slit interference in detail in a separate paper.

3 Feynman's Reverse Time Travel and the Primary Eigen Gas Approach

Actually there is a simple way to redeem the primary eigen gas approach and put it on the same footing as the wave picture. The explanation for this has been around us thanks to the genius of Feynman. It is based on the picture of a particle jumping into future and jumping back in time [7]. Feynman found that a positron can be taken as an electron traveling backward in time. Let us take the world lines for the electron-positron annihilation that results in the creation of photons as shown in figure.2. Feynman showed that taking a positron as an electron
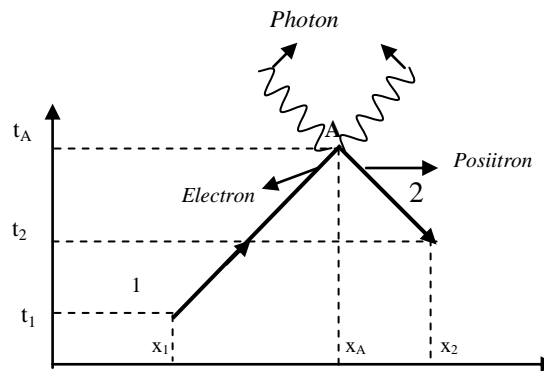
The solid line is the world line of the electron as it goes from (x_1,t_1) to (x_A,t_A). at $A(x_A,t_A)$ it collides with the positron coming from (x_2,t_2) resulting in the annihilation of both, and emission of two photons. This also can be seen as the electron from 1 going backward in time after interacting with photons at A

Figure.2

traveling backward in time simplifies the picture substantially. He developed this idea further and came up with his path integral formalism which revolutionized the approach to quantum electrodynamics.

We shall now use this basic concept to explain the numerous paths followed by the system of a particle in the primary eigen gas picture in its evolution from t_A to t_B (figure 1). We know that in the plane wave representation, the evolution of the system takes place simultaneously along all possible paths. But in the primary eigen gas picture the system is more localized and can be treated as jumping forward from A to B along a particular path. The particle may, then, move back from B to A in a reverse time-travel. The particle may take another path similarly and then travel back in time again. In this manner, we can imagine that the particle exhausts all possible paths. More probable the path is, more number of times it may be traversed compared to others. Note that in this picture we are able to treat evolution along each path just as in the case of real time. The system is allowed to occupy only one state at a time. It has to jump back to the starting instant (in the reverse real time) to initiate a new

path of evolution. Therefore, when we deal with each path, we are able to treat it just like we treat it in real time. We may actually call the time in which the primary eigen gas is defined as the reversible real time or just the "reversible time" because the system is able to travel back in time here. We should keep in mind that this is not the real time which we experience in our daily life because there we cannot go back in time. We shall examine the difference between these different concepts of time later in a separate section.

A detailed study will reveal that even the wave picture also has to take into account the reverse time travel by the particle. This is represented by Ψ^*. But here, the particle traveling in the reverse direction (the anti-particle) is assumed to exist as a shadow of the real particle everywhere [4]. Therefore, the need to treat the particle traveling in the reverse direction in isolation never arises. However when the probability of observing the particle is to be calculated, then Ψ^* becomes an essential part of the computation. Therefore, we may say that the wave picture and the primary gas picture are two ways of looking at the same process. In the wave picture, the system of the particle is assumed to evolve simultaneously along all probable paths while the corresponding reverse motion in time is accounted in terms of the anti-particle that exist as a shadow particle. On the other hand, in the primary gas approach, the particle is assumed to evolve along one path at a time and then travel backward to the starting point in time and then again travel forward along another path and back and so on. This means that the physical content of both approaches are the same. The difference will be only in the interpretation of the process involved. Before we make the equivalence between the two approaches to be total there are many other issues which will have to be resolved. For example, the property of the interference cannot be explained without invoking the wave nature which is an imaginary time property. We shall explain these aspects as we go on.

In the primary eigen gas approach, the microstate is assumed to be real as these states are assumed to be occupied successively. This makes time real. On the other hand, in the plane wave representation, the microstates (wavelets) denoted by N are occupied simultaneously and the only way this can be accounted will be by assuming that these states are imaginary ones. We shall now modify the definition of the extensive time given in the earlier paper [5] using the relation

$$ t_e \quad = \quad nNT_e \quad \rightarrow \quad n(iN)T_e \quad = \quad it_e \tag{5} $$

From the above equation it is clear that when we take N to be imaginary, it means that we are treating extensive time t_e to be imaginary. This is the basis behind the quantum superposition. This shows that the quantum superposition which emerges from the wave nature is a direct consequence of the imaginary nature of time.

In the light of this interpretation of imaginary time, we may have a better understanding of the confined helical (CH) wave structure of the particle. We saw in the earlier paper [2] that a moving particle is represented by a CH wave which may be aligned along any direction, and by the process of quantum superposition it is possible to assume that the confinement could take place in all possible directions at the same time. In this manner, with the concept of imaginary time we are able to retrieve the spatial symmetry of the particle in the rest frame of reference.

We now notice that along with time, spatial coordinate also has to be treated as imaginary. This is because the spatial coordinate is defined by the relation [5]

$$ x_e \quad = \quad nNvT_e \quad \rightarrow \quad n(iN)vT_e \quad \rightarrow \quad ix_e \tag{6} $$

Compare this treatment with the four-dimensional (Euclidian) formalism of the special theory of relativity where only the time coordinate is taken as imaginary while the space coordinate is treated as real. In the present approach since N is imaginary all extensive quantities become imaginary. Needless to say, action and Langrangean also will have to be treated as imaginary.

4 The Wick Symmetry

We know that the concept of Wick rotation where real time is converted into imaginary time was introduced as an adhoc procedure to facilitate easy manipulation of certain functions in quantum field theory [8]. It is not clear why such an adhoc procedure works, but all the same it became an established procedure. We shall now show that this is actually a new symmetry and points to the equivalence of the wave and the primary gas representations of a particle. Let us now examine the issue of the action-entropy equivalence proposed in the earlier paper [5]. Actually, when we study the issue in depth, we observe that there is one major difference between action and entropy which is not discussed in that paper. The difference is that while action is defined in the imaginary time, entropy is defined in the real time. This calls for further clarification. Let us now introduce an operator called Wick's operator denoted by $\breve{R}$ which operates on a function $f(u, 2\pi i N)$ such that

$$\breve{R} f(u, 2\pi i N) \quad = \quad f(u, N). \tag{7}$$

This means that operator $\breve{R}$ replaces $2\pi i N$ by N wherever it appears. Since $t = nNT$, we have

$$\breve{R} f(u, 2\pi i t) \quad = \quad f(u, t). \tag{8}$$

Note that in the quantum field theory, the Wick rotation operates only on t and converts the function "$\exp(-i\hbar^{-1}E_o t_o)$" to "$\exp(-\hbar^{-1}E_o t_o)$" and leaves the space coordinates unchanged. On the other hand, the Wick operator defined here operates on $2\pi i N$, the number of microstates with the result that it rotates time and spatial coordinates at the same time. This difference does not affect the computational aspects as one can always transform the system to the proper frame of reference where the action function is free of the spatial coordinates. This may provide the theoretical justification why Wick rotation works. Note that we may define a reverse Wick operator such that

$$\breve{R}' f(u, t) \quad = \quad f(u, 2\pi i t) \quad \text{and} \quad \breve{R}'\breve{R} \quad = \quad 1$$

Let us write now the amplitude of the plane wave in a more general form without altering its basic properties as

$$B \quad = \quad B e^{2\pi i n N} \tag{9}$$

Note that the exponential factor in this equation will always be unity as N and n can take only integral values. Therefore, the introduction of the exponential factor in the right hand side of (9) does not alter B in any way. Let us now apply the Wick's operator in (4) to obtain

$$\breve{R}\phi_i \quad = \quad \breve{R}\left(B_i e^{-i\hbar^{-1}E_o t_o}\right) \quad = \quad \breve{R}\left(B_i e^{2\pi i n N} e^{-i\hbar^{-1}E_o t_o}\right)$$

$$= \quad B_i e^{nN} e^{-i\hbar^{-1}E_o t_o} \quad = \quad B_i e^{nN} e^{-nNE_o/K\theta_o} \tag{10}$$

Here while the first exponential function increases with nN, the second one decrease with nN with the result that we obtain a sharp maximum for the probability function. Thus, for large values of nN, all possible primary eigen gas states crowd around the average value of the internal energy. Since we know [5] that $t_o = nNT_o = nNh/K\theta_o$, taking $B_i e^{nN} = g_i$, (10) may be expressed as

$$\breve{R}\phi_i \quad = \quad g_i e^{-nNE_o/K\theta_o} \quad = \quad W_i \tag{11}$$

This shows that the plane wave state on undergoing Wick rotation becomes the state function of the primary eigen gas. When viewed from a moving frame of reference, (11) will become

$$W_i \quad = \quad g_i e^{-nN(E-\text{þ}v)/K\theta} \tag{12}$$

[6]

We have to now understand what the implication is of Wick's operator $\check{R}$ on $\phi_i{}^*$. Since the operation of $\check{R}$ on ϕ_i yields the probability function for the forward evolution, on the same logic, operation of $\check{R}$ on $\phi_i{}^*$ should yield the probability function for the reverse evolution (reverse jump) in time.

$$\check{R}\phi_i^* \quad = \quad \check{R}\left(B_i * e^{-2\pi i n N} \; e^{i\hbar^{-1}E_o t_o}\right) \quad = \quad B_i \; e^{-nN} \; e^{E_o t_o / h}$$

$$= \quad g_i^c \; e^{nNE_o/K\theta} \quad = \quad g_i^c e^{nNE_o/K\theta_o} \quad = \quad W_i^c. \tag{13}$$

Here $B_i \exp(-nN) = g_i^{\,c}$, is the degeneracy of the state and $W_i^{\,c}$ represents the probability density function when nN takes negative values. Note that we have assumed B_i is a real function so that $B_i^* = B_i$. On first scrutiny, the function on the right hand side of (13) appears unsuitable to represent a probability function as the Boltzmann's factor "$\exp[nNE_o/K\theta_o]$" is a divergent function and will keep on increasing as nN increases. Note that this function represents the reverse time evolution of the system. But here we should remember that correspondingly the exponential factor constituting the degeneracy denoted by "$B_i e^{-nN}$" keeps on decreasing. Now the net result is similar to what we obtained when time evolved in the positive direction. There the Boltzmann's function given by "$e^{-nNE_o/K\theta_o}$" was decreasing with nN while the degeneracy factor "$B_i e^{nN}$" was increasing correspondingly. In the case of the reverse time evolution, only the roles of the Boltzmann's factor and the degeneracy factor are interchanged. The net result remains unchanged. In fact, the situation described here is similar to a system having negative temperature.

The concept of the negative temperature is quite well understood in thermodynamics [9]. The best example of the negative temperature is obtained when the external field acting on a paramagnetic material is reversed. In this situation, the system exists in a state of maximum energy. Suppose initially all the molecules in the paramagnetic material were aligned in the direction of the external magnetic field. Now on reversal of the magnetic field, the state of the maximum energy of the material possesses only one way of arranging the molecules among themselves, treating them as indistinguishable. But for slightly lower energy, the number of ways of the arrangement of the molecules increases. In other words, the degeneracy increases as energy decreases. In the case of an ideal gas, we know that

$$d\xi \quad = \quad \theta \, dS + P \, dV \qquad or \qquad \partial \xi / \partial S \quad = \quad \theta \tag{14}$$

But in the case of a system under negative temperature

$$\partial \xi / \partial S \quad = \quad -\theta \tag{15}$$

We know that the entropy S depends on the level of randomness of the system. In the case of the system under negative temperature, the degree of randomness decreases as temperature increases. When we take n to be negative as is the case in (13), we end up getting a system having negative temperature. Therefore, the function on the right hand side of (13) is a well defined function and is well suited as a probability density function.

Now we can afford a guess that the quantum mechanical systems can be understood in terms of the thermodynamics of the primary eigen gas. This means that when we replace $2\pi i N$ in the quantum theory by N, then for each law in quantum mechanics we may have an equivalent law in thermodynamics so that the physical reality remains unchanged. In other words, for every law which determines the dynamics of a particle in the imaginary time, there should be an equivalent law in the real time. We shall first of all take the case of action in the imaginary time. Action is the most important function because we can derive the entire dynamical equations of classical and relativistic mechanics using the least action principle. We are already familiar with the equivalence of action to entropy from the earlier paper [5]. We see that the least action principle can be directly related to the second law of thermodynamics which is based on the maximization of entropy. In other words, Fermat's principle of least

action is nothing but a restatement of the second law of thermodynamics. We shall establish this equivalence of quantum mechanics to thermodynamics of the primary eigen gas in more detail in the following sections. Before that, we should have a clear concept of the various aspects of time as we know it.

Here we have to keep in mind that the localization of a particle which is easily understood in the wave picture in terms of the superposition of a group of waves is not easily explained in the primary eigen gas approach. This is because the interference is a purely imaginary time phenomenon and cannot be explained in the real time of the primary Eigen gas. Therefore, it amy seem that the primary Eigen gas has no option but to be spread in space over N number of successive plane wavelets. We shall resolve this issue in a separate paper when the probability postulate will be interpreted in terms of the primary eigen gas.

5 The Imaginary Time, Reversible time and the Progressive Time

Before we proceed further with our investigation it is necessary to have a clear understanding of what is meant by the imaginary time, reversible time and the progressive time that we experience by our senses. We already saw that when a particle occupies microstates simultaneously, these states have to be treated as imaginary. This results in time and space acquiring imaginary nature. In such an approach, a particle will be represented by a plane wave. When we transformed the wave function of a particle by the Wick's operator, $\breve{R}$ we obtained a probability function. In this picture, various paths of progression of a particle are occupied successively one at a time. But, since the system is able to jump back in time in such an approach, ultimately all paths get occupied simultaneously. In other words, the end results of using the wave picture and the primary eigen gas picture are the same. But since only one path is occupied in the primary eigen gas approach in one channel of progression, notionally it can be treated as a real time process. The probability function gives only the density of occupation of the states. Since the particle is able to jump forward and backward in time with equal felicity, primary eigen gas picture may also be called the reversible time picture.

In our everyday experience, a reverse jump in time is not possible. For example, when we throw a dice, we obtain only one outcome at a time. Other outcomes will occur when the experiment is repeated. In other words, the other outcomes cannot occur simultaneously. They can occur successively. Therefore, the reversible time obtained by the operation of $\breve{R}$ on the imaginary time is not the time with which we are familiar in our experiences with our senses. It pertains to a different category altogether. Actually reversible time is so named because the system can occupy all possible paths successively only if it can jump back in time and again start the forward jump again. This is the only way the system can occupy all possible paths. We know that such a picture has already been introduced by Feynman who created a whole new formalism based on it. This is a very important property which will help us to differentiate the reversible time from the progressive time.

One may feel that the interference phenomenon is essentially a direct outcome of the wave nature which manifests only in the imaginary time. If so it may become impossible to explain the interference phenomenon in the primary eigen gas approach and this will come in the way of striking the equivalence between the two approaches. Actually it is possible to explain the interference phenomenon in the primary gas or reversible real time picture. Here we should keep in mind that the primary gas picture treats the plane wavelet as the basic entity or the quantum. Therefore time gets quantized in terms of one period of the wavelet T and the progression in time is accounted by "NT", where N is the number of wavelets. This means, in this approach the sinusoidal structure of the wave gets pushed into the internal space of the wavelet. So what we have done here is just a jugglery. Now although we are treating the wavelet as the basic unit in the primary gas approach, we have to attribute phase as an intrinsic property of the wavelet. Ultimately, the difference between the imaginary time approach and the reversible time approach is that imaginary time treats the wavelet as a wave while the reversible time treats it as a quantum with phase appearing as a property defined in its internal space. The problem of the interference phenomenon could be tackled in the primary gas

picture with this idea of the phase defined in the internal space. We shall discuss the issue of the double slit interference in detail in a separate paper.

Time as we experience in our daily life is the progressive time. Time progression and the increase in entropy are two sides of the same coin. For example, if we play backwards a very short video recording of the collisions of the billiard balls on a billiards table during a short time interval, then their motion will appear as natural. This is because the laws of mechanics are invariant to time reversal. Actually, this invariance is an idealization. The progressive nature of time will leave its imprint to some extent, even if the time interval involved is not too small. For example, in the experiment with the billiard balls, if we play back the video recordings for slightly longer duration, we will observe that the balls which were standing stationary start moving by themselves and gain momentum. Balls may even pop up from the pockets and come into play on the board. This may appear quite absurd and against the laws of mechanics. This is the reason why we have to segregate time into two categories. The time in which the mechanical laws operate and the time which we perceive in our day to day experience. The time which we experience in our day today experience may be called the progressive time. In this time, entropy keeps on increasing. In fact, we may use the increase in entropy as a marker to measure the progression of time.

We should keep in mind that all those states which remain as probable for occupation in the progressive time actually get occupied in the reversible time and the imaginary time. The probability function only gives the frequency of occupation, or to be more specific, the density of occupation of a certain states. Here we may compare the picture that emerges from the imaginary time with that of the reversible time. We saw that in the imaginary time, a particle has to be described in terms of a group of waves progressing in time along all paths. Or in other words, we have to use the wave picture to describe the particle. On the other hand, when we use the reversible time, we describe the particle in terms of the primary eigen gas which occupy states successively by traveling forward and backward in time along all possible pathways. In the wave picture, there is no need to imagine the forward and backward jumps. The square of the amplitude of the wave represents its energy density and therefore it is identified with the particle density or the probability density for observing the particle. In the case of the reversible time and the progressive time which are both defined in the real time, the common feature is that the primary gas states are occupied successively. This means that only one state is occupied at one instant. It is a different issue that in the case of the reversible time, by the process of reverse jump in time the system is able to occupy all possible states at the same instant.

In the progressive time, time gains the directional property and because of this property it is no more possible to jump back in time. In that sense, the progressive time picture is identical with the primary eigen gas picture provided the reverse jumps in time are disallowed. Note that in the imaginary time picture, the particle disembodies into a large number of waves and they get back the particulate nature only at the instant of observation. This sudden change has been one of the most discussed topics in quantum mechanics and known as the collapse of the wave function. It is also the least understood. One advantage of the reversible time picture over the imaginary time picture is that we do not have to disembody the particle and therefore, there is no sudden change to the structure of the particle at the time of observation. But then we have to pay a price by way of allowing the reversible jumps in time. Actually, the concept of the reverse jump exists in the wave picture also. In fact, when we take the complex conjugate of the wave function, Ψ^*, we are dealing with a wave which is travelling backwards in time. But in the wave picture, Ψ^* represent the anti-particle which is assumed to exist as a shadow of the real particle. This shows that the Wick rotation has not created anything new. It only changed the method of accounting the states. In the wave picture, the forward and the reverse waves are taken together, the reverse wave being treated as a shadow of the forward wave. Therefore, when the system evolves, it evolves along all possible paths simultaneously. In the primary gas approach, the forward jumps and the reverse jumps are segregated and accounted separately. Of course, there is one more difference in that time is quantized in the reversible time picture with the quantum of time being period of the plane wave.

The problem we face in taking the increase in the entropy as a marker for the progressive time is that we do not have a process which can be taken as a norm for the purpose on hand. If we take the case of a billiard ball rolling over the surface and coming to a standstill, then we know that the kinetic energy of the ball is converted into heat energy due to friction. If we denote the increase in the heat energy by dq and the temperature of the board as θ, then the increase in entropy in the slowing down of the billiards ball will be dq/θ. But this creation of heat depends on so many factors that it becomes virtually impossible to use it as a norm to measure the progression of time. But we know that the rest mass can be taken as a measure of the internal heat of the particle [5]. Therefore, it is logical to conclude that the progressive time should be in some way related to the most basic process which results in the creation of the rest mass of a particle. We shall study the relationship of the rest mass with the progressive time in a separate paper and show how gravitation and the progressive time and the expansion of the universe are all the outcome of the same basic process which causes the rest mass of the particles to increase.

The Wick symmetry shows that it is possible to study the evolution of a micro-system either in the imaginary time in which case quantum mechanical laws will apply or in the reversible time in which case the laws of thermodynamics will apply. This means that apart from the canonical approach and the path integral approach we have a third option in the primary eigen gas approach to study the quantum behavior. It is well known that the path integral formalism is eminently suited to solve the problems of quantum electrodynamics while the canonical approach has its own high points. In a similar manner, the primary eigen gas approach may be ideally suited to resolve some of the conceptual problems of quantum mechanics and quantum field theory. We shall examine these aspects as we go on.

Here it is interesting to note that Feynman had used the basic idea of replacing 't' by '-iℏ/Kθ' to interpret the path integral formalism on the basis of statistical mechanics [10]. He obtained the value of the density matrix ρ which is the sum of all contributions from each motion, considering all possible paths, or motions, by which the system could travel between the initial configuration in time to t = ℏ/Kθ as

$$\rho \quad = \quad \int_0^{\hbar/K\theta} \left\{ \exp\left[-\hbar^{-1} \int \left[\tfrac{1}{2} m x^2(u) + V(x) \right] du \right] \right\} Dx(u) \qquad (16)$$

Here u is a parameter having the dimension of time and *Dx(u)* relates to the individual path taken. He comments about the above result in following words. " This is very amusing result, because it gives the complete statistical behavior of a quantum-mechanical system as a path integral without the appearance of the ubiquitous *i* so characteristic of quantum mechanics. This path integral (in reversible time) is much easier to work with and visualize than the complex integrals which we have studied previously. Here it is easy to see why some paths contribute very little to the integral; for there are paths for which the exponential is very large and thus the integrand is negligibly small. Further more, it is not necessary to think about whether or not nearby paths cancel each other's contributions, since in the present case all contributions add together with some being large and others small."

We can interpret the mathematics behind the equation given in (16) as follows. The number of paths that can be taken by a system *Dx(u)* to move from a point A to another point B in a certain time interval will increase as the energy of the system increases. Note that the particle may take all circuitous paths and still make it to point B from A in the given time if it travels faster in the intervening period. But these paths would belong to states of higher energy and momentum. But for the higher energy states, the exponential term within the bracket on the right hand side in (16) will have a lower value. Thus we observe that while *Dx(u)* increases with energy, the exponential term would decrease with the increase in energy. In fact, the integral will have a maximum along the classical path. We know this is exactly what happens in the primary eigen gas approach provided we take *Dx(u)* as representing the degeneracy g.

Actually, the path obtained in the primary Eigen gas (reversible real time) approach may not be as sharp as the one obtained by the imaginary time approach. To that extent the equivalence between the two approaches may be inexact. In the case of the imaginary time, the

amplitudes for all other paths get destroyed completely leaving us with only the classical path. In the case of the reversible time approach, there is no destruction of the degeneracy of paths. This introduces differences between the two approaches. A deeper analysis of the problem shows that this difference can be reconciled if we keep in our mind that in the reversible time a plane wavelet is taken as the basic unit and therefore, the phase of the wave gets compacted into the internal coordinates. Therefore, when two micro-states plane wavelets) of the primary Eigen gas occupy the same region in space, the interference phenomena would kick in and the result will be exactly the same as what we obtain the case of the imaginary time approach.

The imaginary time approach does not bring out in a simple manner the exponential decrease in the amplitude of the paths as they move away from the classical one. Such an exponential decrease in the degeneracy in the states representing the paths as they move away from the classical path is very clear in the primary Eigen gas approach. This shows that the imaginary time approach and the reversible time approach are essentially two self-consistent ways of looking at the same basic reality.

## 6 The Probability Postulate and the Wick Symmetry

We know that quantum theory deals with probability of observing a particle in given state and does not deal with the path of progression of the particle. On the other hand, Feynman's path integral formalism (see Annexure) is based on the idea that the particle evolves along various paths. Of course, instead of the particle, it is the probability amplitude which is assumed to evolve along various paths. This is similar to the primary eigen gas approach where each path of evolution is taken in isolation. The only difference is that path integral formalism treats time as imaginary. This is to say that it accounts for the phase of the wavelet in the laboratory coordinate system itself without compacting it into its internal space. But for this, the two approaches are similar, making it possible to use the tools of path integral formalism in the other one. We shall now re-examine Feynman's approach to the probability postulate in the light of the forward and reverse evolution in time. Keeping this purpose in view we shall take up the special case of the evolution of a system from the initially observed eigen state (x_a, t_a) to the final state (x_b, t_b) where $t_b > t_a$. Note that the eigen state represented by a plane wave in the wave picture will become the primary eigen gas state in the reversible real time approach. Since the wave function $\Psi(x_b, t_b)$ by definition represents the total amplitude arriving at the point (x_b, t_b), in the case on hand it will represent only the amplitude arriving at (x_b, t_b) from just (x_a, t_a). In other words,

$$\psi(x_b, t_b) \quad = \quad K(x_b, t_b; x_a, t_a) \quad . \tag{17}$$

where $K(x_b, t_b; x_a, t_a)$ denotes the probability arriving at the space-time point (x_b, t_b) from (x_a, t_a). Similarly, taking the complex conjugate, we obtain

$$\psi^*(x_b, t_b) \quad = \quad K^*(x_b, t_b; x_a, t_a) \tag{18}$$

Here Ψ^* represents the probability amplitude for the system to devolve from the state (x_b, t_b) to the earlier state (x_a, t_a). Thus the function $\Psi^*\Psi$ represents the probability amplitude for
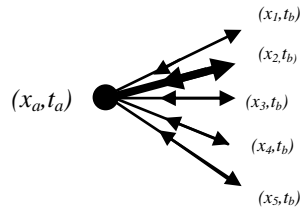
The figure shows the evolution of the system from a single initial state at instant t_a to a large number of states at a subsequent instant t_b and back.

Figure.3

evolving from the state (x_a,t_a) to the state (x_b,t_b) and then back to the state (x_a,t_a). Note that we have taken only (x_a,t_a) as fixed while (x_b,t_b) stands for a large number of states as shown in figure.3. Therefore, when the system evolves from (x_a,t_a) to the states denoted by (x_b,t_b), it takes place along a number of paths. We may say that in any observation at the instant t_b, the system may be caught in any one of the (say) five paths as shown in figure 3. Since the system will be in any one of the five paths at the instant t_b, we may take the probability for the system to be in the first path to be

$$P(x_1,t_b) \quad = \quad \psi*(x_1,t_b)\,\psi(x_1,t_b) \tag{19}$$

provided we take Ψ as normalized such that $\Sigma\ \Psi*(x_b,t_b)\ \Psi(x_b,t_b) = 1$. Here we have taken the initial state (x_a,t_a) to be the eigen state which is given. Actually, the entire chain of reasoning will hold good even if the initial state (x_a,t_a) is not a single eigen state but a large number of eigen states. This means that the system is not observed in any particular eigen state at the instant t_A but retains its potential to occupy a large number of eigen states.

The arguments put forward in the case of the path integral formalism can be extended to the case of the eigen gas approach without any problem. We have to use the probability function $W(x_b,t_b)$ instead of $\Psi(x_b,t_b)$. In that case (17) and (18) will become

$$W(x_b,t_b) \quad = \quad \hat{K}(x_b,t_b;x_a,t_a)\ . \tag{20}$$

$$W^c(x_b,t_b) \quad = \quad \hat{K}^c(x_b,t_b;x_a,t_a) \tag{21}$$

where $\hat{K}$ is the real time equivalent of K and $\hat{K}^c$ that of $K*$. The path of evolution will be exactly as shown in figure 3 except that these paths will represent the frequency of occupation rather than the probability amplitude. Ultimately both imply the same. The probability for the system to occupy the first path in figure 3 will be given by

$$P(x_1,t_b) \quad = \quad \hat{K}^c(x_1,t_b)\ \hat{K}(x_1,t_b) \tag{22}$$

The evolution of the quantum mechanical system into future and back may appear quite absurd, but then we should keep in mind that in the microcosm, time has not lost its directional symmetry unlike in the case the macroscopic world. As a result, the system may evolve into future and back to the present and again into future and so on. In other words, in the imaginary time or reversible time, a particle can be in many places simultaneously. However, when the system comes into contact by way of an observation with the macroscopic world which exists in the progressive time, it will be caught in one of the possible locations. This is because in the progressive time, a particle can be only in one place at an instant. One may say that the observation is a process by which a micro-system shifts from the imaginary time to the progressive time. Note that if the micro-system were to evolve in the progressive time, then it has to be under continuous observation. Therefore, the probability to occupy the state (x_2,t_b) will be given by the probability to jump from (x_a,t_a) to (x_2,t_b) only. There is no possibility of a reverse jump here. On the other hand, in the imaginary time, we have to take both forward and the reverse jumps to arrive at the probability to occupy the state (x_2,t_b). Thus the basis of the probability postulate can be traced to the fact that a micro-system exists in a time with its reversibility or directional symmetry intact.

We shall now try to find out what is the quantity which is conserved in the Wick symmetry. We know that if the Wick symmetry is a universal symmetry, then there should be an invariant property of the system associated with this symmetry. It appears that the probability density is the quantity which remains invariant in the Wick rotation. Let us apply Wick's operator on the wave function to yield

$$\breve{R}P_i \quad = \quad \breve{R}[\psi_i^*\psi_i] \quad = \quad W_i^c W_i \quad = \quad P_i \tag{23}$$

But from (13) we know that $W_i^c = B_i \, e^{-nN} e^{nN\bar{s}/K}$ while $W_i = B_i \, e^{nN} e^{-nN\bar{s}/K}$. Note that W_i denotes the probability of the primary eigen gas which progresses in time while W_i^c denote the probability for regression in time. From the above analysis it is obvious that the probability density P_i is conserved in the Wick rotation provided B_i is taken as real. In other words, we may take the conserved quantity connected with the Wick symmetry to be the probability density.

7 Zero Point energy as the thermal energy of vacuum fluctuations

We shall now investigate the concept of the zero point energy of quantum mechanics and look out for the equivalent law in the reversible time. According to Quantum Mechanics, vacuum is taken as zero energy state or the ground state. This does not disallow the existence of fluctuations in the energy and momentum of the vacuum state. We know that the CH wave is created by its interactions with the vacuum fluctuations [4]. Note that this CH wave is in thermal equilibrium with the vacuum fluctuations. In other words, the vacuum fluctuations act like a thermal bath and the energy of the particle is equal to the energy of the vacuum fluctuations confining it. Here we should keep in mind that a CH wave has two degrees of freedom, one is due to the oscillations in one plane and the other is on account of spin **[2].** Therefore, for one degree of freedom, we have

$$E \quad = \quad \tfrac{1}{2} h \nu \tag{24}$$

And if we take the both degrees of freedom, we obtain

$$E \quad = \quad h \nu \tag{25}$$

This is the energy of the vacuum fluctuations which confines the helical wave resulting in the formation of the particle.

We should keep in mind that if the vacuum fluctuations interacting with the CH wave did not have the same energy, then it would have lost out its energy to the vacuum and faded away. But since the CH wave is in equilibrium with the vacuum fluctuations, we should assume that the rest energy of the particle and the energy of the vacuum fluctuations are perfectly matched. But we have $\nu_o = 1/T_{eo}$, where T_{eo} is the period of oscillations of the CH wave. Therefore, we have

$$E_o \quad = \quad h/T_{eo} \tag{26}$$

But T_{eo}, which is the intrinsic time of the particle in its rest frame of reference is related to the temperature of the primary eigen gas by the relation $T_{eo} = h/K\theta_o$ [5]. Therefore, we have

$$E_o \quad = \quad K\theta_o \tag{27}$$

This result makes sense because the CH wave has got two degrees of freedom and we know that the thermal energy for unit degree of freedom is given by $\tfrac{1}{2}K\theta_o$. In other words the zero point energy is nothing but the energy of the vacuum fluctuations acting as a thermal bath.

In the light of equation (27) let us examine the action function of a free particle expressed in terms of the extensive time of the particle given by

$$-(Et_e - \text{þ}x_e) \quad = \quad -E_o t_o \tag{28}$$

Let us replace the extensive space and time coordinates in the above equation by the intrinsic coordinates (given by $t_e = nNT_e$ and $x_e = nNX_e$) to obtain

$$-(ET_e - \text{þ}X_e) \quad = \quad -E_o T_{eo} \tag{29}$$

But using (26), the above equation can be written as

$$(ET_e - \text{þ}X_e) \quad = \quad E_o h / E_o \quad = \quad h \tag{30}$$

This shows that whatever be the nature of particle, the intrinsic action is a universal constant denoted by h. Therefore

$$A/h \quad = \quad -E_o t_o / h \quad = \quad -nN\,E_o\,T_{eo}/h$$

$$= \quad -nN \tag{31}$$

But since $A/h \quad = \quad -S^{\#}/K$, we have $S^{\#}/K \quad = \quad nN$. $\tag{32}$

8 Conclusion

In view of what is discussed above it is quite obvious that quantum mechanics is nothing but the thermodynamics of the primary eigen gas that exists in the reversible time. In the next paper, we shall examine each of the postulates of quantum mechanics and show how it conforms to the primary eigen gas approach. We shall also show how the localization of the particle and the results of Young's double slit experiment can be explained in the primary eigen gas approach. We already saw here how the probability postulate has its origin in the reversibility of time. The idea that an elementary particle is in thermodynamic equilibrium with the vacuum fluctuations provides us with a simple picture of the inner structure of the elementary particle and in a sense takes us to the concept that all elementary particles as some sort of vortices created by electromagnetic and other composite waves. Descartes was not too much of the mark! But we are still far from understanding why only certain rest energies are allowed for the particles. Could it be understood in terms of the relative strengths of the basic fields?

When we examine Feynman's path integral formalism, it is quite surprising to note that how close he came to the real time approach. He even arrived at the similarity between the imaginary time approach and the real time approach as discussed in section 4. He had already introduced the concept of the forward jump in time and the reverse jump in time. The only reason why he couldn't arrive at the Wick symmetry may be traced to the fact that he couldn't think up the concept of the primary eigen gas. His mind was too busy solving the problems of quantum electrodynamics using the newly discovered path integral formalism.

It is interesting to examine the emergence of the space-time from the interactions of the particles with the vacuum fluctuations. Remember that these interactions are undergone by all particles and the universal nature of these interactions is the basis behind the perception of a universal background of time and space. By the same reasoning, we may attribute space-time for other short range interactions. But the only problem is that such a space-time would not form a continuum. We will have to treat them as space-time which gets compacted and warped into itself. It is worthwhile to note that string theory uses the concept of 10 dimensional space where the higher dimensions get warped and compacted into infinitesimally small regions [12].

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Annexure

Feynman's Path Integral Formalism

Before attempting to understand the basis of the probability postulate, it is necessary to go through Feynman's path integral formalism briefly. The path integral formalism is a completely new way of looking at the quantum phenomena. In Feynman's approach, a particle evolves from one point in space-time to another along various paths and each path may be associated with certain probability amplitude [11]. According to Feynman if $\varphi[x(t)]$ is the probability amplitude for a particle to reach a point (x_b,t_b) from point (x_a,t_a) along a particular path defined by x(t), then

$$K(b,a) \quad = \quad \sum \phi[x(t)] \tag{I}$$

where $K_{(b,a)}$ is the probability amplitude for the particle to move from (x_a,t_a) to (x_b,t_b). The summation means that all paths from (x_a,t_a) to (x_b,t_b) contributes to the probability amplitude. On the other hand the probability for the same movement is given by

$$P(b,a) \quad = \quad \left|K(b.a)\right|^2 \tag{II}$$

The interesting aspect of these paths is that the adjacent ones vary substantially in phase because a change as low as h can alter the phase by 2π. So ultimately all amplitudes cancel each other out except those, which lie very close to the path having least action. Here, a small change in the path creates changes in the phase only in the second order. So the classical path gets maximum amplitude and becomes the observable path. Therefore, in the classical limit

$$K(b.a) \quad = \quad "Smooth\ function". \, e^{-i\hbar^{-1}A_{cl}} \tag{III}$$

Based on this simple line of reasoning, Feynman shows that

$$K(b.a) \quad = \quad \int K(b,c)\, K(c,a)\, dx_c \tag{IV}$$

where (x_c,t_c) is a point lying between (x_a,t_a) and (x_b,t_b). Equation (IV) represents a basic property of the probability amplitude. A wave function by definition represents the total amplitude arriving at the point (x,t). It does not specify the previous state from which it is coming. In other words, a wave function does not give any information about the past history of the particle or system. If a system is in the initial state with coordinates (x_a,t_a) which is not fixed, the wave function for the state (x_b,t_b) will be given by

$$\psi(x_b,t_b) \quad = \quad \int K(x_b,t_b;x_a,t_a)\,\psi(x_a,t_a)\,dx_a \qquad\qquad \text{(V)}$$

In other words, the wave function $\psi(x_b,t_b)$ which is the total amplitude to arrive at (x_b,t_b) is the sum of or integral over all amplitudes from all possible points (x_a,t_a) which are prior to (x_b,t_b). Note that $t_a < t_b$. If $t_a > t_b$, then it is tantamount to assuming that future events are affecting the present which is not acceptable.

Now we shall try to express the amplitude $K(x_b,t_b;\,x_a,t_a)$ in terms of the state functions at b and a. We know from equation (V) that $\psi^*(x_b,t_b)$ can be expressed as

$$\psi^*(x_b,t_b) \quad = \quad \int K^*(x_b,t_b;x'_a,t_a)\,\psi^*(x'_a,t_a)\,dx'_a \qquad\qquad \text{(VI)}$$

It is implicit here that the integral sign represents integration over the region from $-\infty$ to $+\infty$. Now post multiplying both sides of (VI) by $\psi(x_b,t_b)\,dx_b$ and integrating, we have

$$\psi^*(x_b,t_b)\psi(x_b,t_b)dx_b \quad = \quad \iiint K^*(x_b,t_b;x'_a,t_a)\,K(x_b,t_b;x_a,t_a)\psi^*(x'_a,t_a)\psi(x_a,t_a)\,dx_a\,dx'_a\,dx_b$$

$$= \quad \psi^*(x_a,t_a)\,\psi(x_a,t_a)\,dx_a \qquad\qquad \text{(VII)}$$

This represents the conservation of the probability. For this relation to hold good for arbitrary values of Ψ, we must have

$$\int K^*(x_b,t_b;x'_a,t_a)\,K(x_b,t_b;x_a,t_a)\,dx'_a dx_b \quad = \quad \delta(x'_a - x_a) \qquad\qquad \text{(VIII)}$$

That is, in order to interpret ψ as probability amplitude, the kernel must satisfy equation (VIII). To know what $K^*(x_b,t_b;\,x_a',t_a)$ stands for let us multiply (VIII) by $K(x_a,t_a;\,x_c,t_c)$ and integrate over x_a, to obtain

$$\int K^*(x_b,t_b;x'_a,t_a)\,K(x_a,t_a;x_c,t_c)\,K(x_b,t_b;x_a,t_a)\,dx_a\,dx'_a dx_b \quad = \quad \int \delta(x'_a - x_a)\,K(x_a,t_a;x_c,t_c)\,dx_a$$

$$K^*(x_b,t_b;x'_a,t_a)K(x_b,t_b;x_c,t_c)\,dx_b \quad = \quad K(x'_a,t_a;x_c,t_c)$$

Note that the underlying assumption here is that $t_b > t_a > t_c$. Replacing (x'_a,t_a) by (x_a,t_a), we obtain

$$K^*(x_b,t_b;x_a,t_a)K(x_b,t_b;x_c,t_c)\,dx_b \quad = \quad K(x_a,t_a;x_c,t_c) \qquad\qquad \text{(IX)}$$

Keeping in mind that the above relation holds good only if $t_b > t_a > t_c$, let us compare it with the following equation based on the property given by (IV)

$$\int K(x_a,t_a;x_b,t_b)\,K(x_b,t_b;x_c,t_c)\,dx_b \quad = \quad K(x_a,t_a;x_c,t_c) \qquad\qquad \text{(X)}$$

where $t_a > t_b > t_c$. The process behind the second relation can be explained as follows. Starting from at t_c, $K(x_b,t_b;\,x_c,t_c)$ gives us the amplitude at a later time t_b. If we still want to go to a later time t_a, we may do so using the kernel $K(x_a,t_a;x_b,t_b)$. On the other hand, if having the amplitude at t_b, we want to work back to find it at an earlier time $t_a < t_b$, we may do this using the function $K^*(x_b,t_b;\,x_a,t_a)$. In other words, $K^*(x_b,t_b;\,x_a,t_a)$ is undoing the work of $K(x_a,t_a;\,x_b,t_b)$. Another way of looking at this is that while $K(x_a,t_a;\,x_b,t_b)$ stands for movement of amplitude in the direction of the time evolution which is from t_b to t_a where $t_a > t_b$, $K^*(x_b,t_b;\,x_a,t_a)$ represents movement of amplitude going back in time from t_b to t_a where $t_b > t_a$. This means that $\psi^*(x_b,t_b)$ given by

[16]

$$\psi^*(x_b, t_b) \quad = \quad \int K^*(x_b, t_b; x_a, t_a)\, \psi^*(x_a, t_a)\, dx_a \tag{XI}$$

represents the total amplitude going back from the states (x_b, t_b) to (x_a, t_a). To put things in proper perspective, while $\psi(x_b, t_b)$ represents the probability to arrive at the state (x_b, t_b) from all past states, $\psi^*(x_b, t_b)$ represents the probability to go back from (x_b, t_b) to all past states. Thus we may state that while $\Psi(x_b, t_b)$ represents the probability amplitude for evolving from the past states to the state (x_b, t_b), $\Psi^*(x_b, t_b)$ represents the probability amplitude to go back in time to the past states from the state (x_b, t_b). We shall see that this interpretation is ideally suited for the proposed idea of the Wick symmetry.

If ϕ_n represents a set of eigen functions which are orthogonal to each other, and E_n the set of eigen values of energy, then we know that the state function can be expanded as

$$\psi(x, t) \quad = \quad \sum_{\alpha=1}^{\infty} c_n\, e^{-i\hbar^{-1} E_n t}\, \phi_n(x) \tag{XII}$$

If we now take two points in space time (y, t_1) and (x, t_2) where $t_2 > t_1$, then it can be easily shown that

$$K(x, t_2; y, t_1) \quad = \quad \sum_{\alpha=1}^{\infty} \phi_n(x)\, \phi_n^*(y)\, e^{-i\hbar^{-1} E_n (t_2 - t_1)} \tag{XIII}$$

$\phi_n(y)$ represents the nth eigen states of the wave function at the point (y, t_1). It is possible to express the eigen functions as

$$\phi_n(x) \quad = \quad e^{i\hbar^{-1} \mathrm{p}_n (x-y)}\, \phi_n(y) \tag{XIV}$$

With this expression for $\phi_n(x)$, we may write (XIII) as

$$K(x, t_2; y, t_1) \quad = \quad \sum_{n=1}^{n=\infty} \phi_n(y)\, \phi_n^*(y)\, e^{-i\hbar^{-1}[E_n(t_2 - t) - \mathrm{p}_n(x-y)]} \tag{XV}$$

for $t_2 > t_1$. Let us suppose that the states denoted by E_n and p_n are lying very close so that we may take the distribution as a continuous one. Let us now denote by x and y the four vector space-time coordinates and by p the four vector energy momentum coordinates. Now for notational convenience we may switch over to the natural units where $\hbar = 1$ and $c = 1$. In that case (XV) becomes

$$K(x, y) \quad = \quad \left(\tfrac{1}{2\pi}\right)^4 \int |\phi_n(y)|^2\, e^{-i\mathrm{p}.(\mathbf{x}-\mathbf{y})} d^4\mathrm{p} \tag{XVI}$$

The constant $(1/2\pi)^4$ has been introduced for normalizing purpose. It is quite obvious that in a field free situation $K(x, y)$ should be equal to $\delta(x-y)$ because that is the only way that $\psi(x)$ will retain the same form as $\psi(y)$ as the system moves from y to x. This is possible only if $|\varphi(y)|$ does not dependent on p.

In quantum field theory $K(x, y)$ is called the propagator and is denoted by G $_{(x\text{-}x')}$. It is worthwhile to note that the propagator depends only on the interval in the coordinates, which is $(x\text{-}x')$ and the notation brings this out explicitly. In the general case, equation (XVI) may be written as

$$G(x - x') \quad = \quad \left(\tfrac{1}{2\pi}\right)^4 \int |G_n(\mathrm{p})|^2\, e^{-i\mathrm{p}.(\mathbf{x}-\mathbf{x'})} d^4\mathrm{p} \tag{XVII}$$

Note that $G(\mathrm{p})$ will change according to the field acting on the system. The wave function is nothing but the sum total of all such eigen states weighted with their amplitudes. We see from (XVII) that the weightage factor $G(\mathrm{p})$ remains unity if the field acting is zero. But when a field

acts on the system, in the journey from x to x′, the weightage factor for each eigen state of þ undergoes a change due to the action of $G(þ)$ and thus the wave function gets altered.

In Feynman's approach $\Psi(x_b,t_b)$ represents the sum of the probability amplitude reaching the state (x_b,t_b) from the past, while $\psi^*(x_b,t_b)$ represents the sum of all amplitudes starting from the state (x_b,t_b) and going back to the past. This sort of an interpretation is in-built in the Feynman's picture. Such an interpretation means that evolution of a system backward in time is equally possible and therefore one may treat forward and backward evolution in time symmetrically. But in real world, time progresses only in one direction. This may be the reason why Feynman never attempted to seek an alternate interpretation for the probability postulate based on his path integral formalism.