Sub-quantum Thermodynamics
and the Creation of Time and Space

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Abstract

The author introduces the concept of the primary eigen gas which is an abstract gas where the microstates are occupied successively in time unlike in the case of a real gas where the microstates are occupied simultaneously. He shows that the energy-momentum eigen state of a particle represented by a plane wave can be treated as a primary eigen gas which makes it possible to understand the dynamics of a particle in terms of the thermodynamics of such a gas. In this approach, time and space turn out to be the intrinsic properties of the primary eigen gas representing a particle and the quantum nature of time and space emerges from it in a natural manner. It is shown that the action (with a negative sign) of a particle can be identified with the entropy of the primary eigen gas and the principle of least action is nothing but the second law of thermodynamics. Besides, it is shown that the uncertainty relations of quantum mechanics can be derived directly from the equation for the fluctuations.

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1 Introduction

We saw that an electron can be represented by a confined helical wave which is formed by the confinement of a plane polarized electromagnetic wave that acquires half spin. It is observed that such a confined helical wave not only acquires rest mass and the electric charge but also the half spin of a particle like electron. Such a confined helical wave representation of a particle is seen to be compatible with the Dirac equation and it directly leads to the Pauli exclusion principle [1],[2],[3]. It was suggested that the confinement of the helical wave may be effected by its interactions with the virtual helical waves created in the vacuum fluctuations [4]. The approach outlined above is based on the confinement of the helical wave created out of the electromagnetic wave and therefore, the particles created can only be leptons. However, we shall assume that such a confined helical wave representation will also hold good for particles like quarks. The only difference will be that instead of the electromagnetic wave, it may be some other composite wave having oscillations not only in the electromagnetic field but also in other appropriate fields, whose confinement creates these particles. But then whatever be the nature of the interactions, it will be reflected only in the amplitude wave which gets compacted into the internal coordinates. The time-dependent part of the confined helical wave created out of the composite wave will have the form of the plane wave. In other words, whatever be the internal structure of the particle, in the laboratory frame of reference they will always be represented by a plane wave [1]. In the proposed approach we are not interested in the internal structure of the particles, but only in their plane wave representation. We shall now onwards use the term confined helical wave (CH wave) in a more general sense to represent such confined composite waves.

It is now proposed that the states occupied by the CH wave in its interactions with the vacuum fluctuations can be treated as a primary eigen gas. The basic idea is to treat a particle as a thermodynamic system and to understand the dynamics of the particle in terms of the thermodynamic analogues. We know that a gas is one which is made up of millions of molecules moving randomly in all directions within a container having a definite volume. Statistically speaking, a gas is a system containing trillions of molecular states that exists at an instant with each molecule having a specific value of energy and momentum. We might as well imagine an abstract gas which is formed by trillions of molecular states occupied by a single molecule in its successive interactions with the walls of the container. We know that statistically speaking these two gases are equivalent. The only difference is that in the first case, all the states are occupied simultaneously, while in the second case they are occupied successively. However, this will not be important if we are dealing with a system in equilibrium which by definition is time invariant. We know that the thermodynamics of a real gas can be understood in terms of the statistical properties of the molecular states forming it. Based on the statistical equivalence of the two gases, it appears reasonable to extend the laws of thermodynamics to the abstract gas also. We shall call this abstract gas "the primary gas" and study the thermodynamics of this gas in more detail. Later we shall show that the constituent of the primary gas need not be a molecule, it can as well be a CH wave. We shall now onwards use the term "primary eigen gas" only if it is constituted by CH waves.

The idea of a gas formed by a single molecule was first introduced by Leff a few decades back. He called it 'model gas' and it was constructed by the states occupied by a single particle with certain mass and kinetic energy confined in a one-dimensional box [5]. His approach has been based on the kinetic theory and assumes the classical action function for the particle. In the entire analysis, it is presumed that the gas as a whole is stationary with regard to the observer. However, the approach we propose here involves the introduction of the additional thermodynamic coordinates like the translational velocity of the gas and the corresponding translational momentum.

2 The Principle of the Statistical Equivalence

Let us take a real gas with N molecules contained in a vessel with conducting walls kept in a thermal bath. For the sake of generality, we shall use the term "microstates" to denote the state occupied by the molecules. We assume here that the molecular density of the gas is quite low and therefore for all practical purpose the molecules do not interact with each other; they interact only with the walls of the container. In short, we treat the gas as an ideal gas. Let this gas be in thermal equilibrium with the macro-setting. We shall also assume that the ensemble of this gas contains R copies, where R can be a very large number.

We may now imagine another identical container with only one molecule kept in it. It is possible to think of a very large number of states occupied by this single molecule in its successive interactions with the container walls. Note that the average energy of the molecule will be determined by the temperature of the container walls as the system is in thermal equilibrium. If we take L number of such micro-states and divide it into 'R' groups, each group being comprised of N successive micro-states, where R and N are very large then, our assumption is that for each group having a certain profile of occupation of micro-states we can identify with an ensemble copy of the real gas having the same profile of occupation of the micro-states. Such a group of states occupied by a single molecule in successive interactions can be treated as an abstract gas. Here the sequence of the occupation of micro-states is not relevant, only the frequency profile of the occupation of the micro-states is important. In other words, if there are two abstract gases whose micro-states do not tally sequentially although the frequency of occupation of the micro-states is the same, then the two abstract gas states may be treated as identical. In simple terms, what we are doing is analogous to comparing the outcomes of one die thrown thousand times with those of thousand dice thrown simultaneously. Obviously both results will be statistically equivalent and the sequence of the outcomes in the case of a single die is unimportant in this comparison. Let us call this "the principle of the statistical equivalence".

We know that the thermodynamic relations of a real gas can be derived purely on the basis of the probability of occupation of different gas states using the concepts of canonical ensemble and grand canonical ensemble. For example, for a given equilibrium macroscopic condition determined by, say, constant temperature, pressure and chemical potential, the probability of occupation will have a sharp maximum like a delta function for states with a particular value of energy, volume and number of molecules. The thermodynamic variables like energy, volume, number of molecules etc. of a real gas can be identified with such average values. The thermodynamics of the gas can be understood based on the ensemble approach by suitably defining entropy as a statistical quantity [6]. The statistical equivalence of the ensemble of the primary gas with that of a real gas may lead us to the conclusion that thermodynamics of a real gas and that of a primary gas are equivalent.

However, we should keep in mind that on certain aspects there are differences between a real gas and a primary gas. We shall examine these aspects in detail shortly. With this purpose in view, we shall construct a primary gas with its micro-states occupied by a single particle and derive the thermodynamic relations. Subsequently, we shall introduce translational velocity and momentum as additional thermodynamic variables and generalize the thermodynamic relations.

3 Thermodynamics of the Primary Gas

We assume here that the primary gas is formed by a single particle. Here the particle can be a single molecule or even an electron. The spread of the states occupied by the particle in time contributes to certain interesting properties. To avoid confusion regarding the identification of the particle state and that of the primary gas, we shall now onwards call the particle state as the micro-state of the primary gas. It should be noted that the number of micro-states in the primary gas is not constant, but increases as the time interval within which it is defined increases.

Let us now take a real gas whose number of molecules and volume are fixed. If we treat the system as a quantum mechanical one with quantized values for energy, then the notation for the energy of the gas will be ξ_k where 'k' denotes a complete set of quantum numbers. We now propose to derive the thermodynamic relations of the primary gas based on its equivalence to the real gas [7].

Let us consider L number of successive micro-states of the primary gas which are subdivided into R groups, each group being formed by N successive micro-states such that

$$N R = L . \tag{1}$$

It should be noted that R represents the ensemble of the primary gas. L and R can be made as large as required. Let us denote by n_k the number of the ensemble copies for which the energy is ξ_k. Then, the ratio, n_k/R will give the probability for the gas to occupy the states with the specific values ξ_k. The logical foundations of the approach are based on the following. Let us consider a giant abstract gas formed by combining all of the R copies of the ensemble. Then the states with a given value ξ_k will play the same role as played by a box in the Boltzmann's method [8] and the number of copies n_k, the role of the number of balls in the boxes.

The number of ways in which the copies of the ensemble can be distributed among the boxes is equal to

$$M = R! / \Pi_k \, n_k! . \tag{2}$$

The copies of the ensemble will be assumed to be distinguishable as the primary gas state is a collection of micro-states, not an isolated micro-state. We shall later show that even if the states are taken as distinguishable, there will not be any change in the final result. We may assume that there exists a Gibbs micro-canonical distribution for the giant gas containing R copies. Therefore, the number of copies R and the total energy of all the copies $\hat{\xi}$ will be assumed to be fixed. These constraints may be expressed by the equations

$$\sum_k n_k = R \quad and \quad \sum_k \xi_k n_k = \widehat{\xi} . \tag{3}$$

The approach we propose to follow is based on the assumption that the most probable state of the gas can be identified with that particular gas state which recurs most often in the ensemble. For this purpose, we should find out the maximum of M in (2). Keeping this in view, let us define a function σ given by

$$\sigma \quad = \quad K \ln M \, , \tag{4}$$

where K is the Boltzmann's constant. We know that σ has a maximum when M has a maximum. So our effort boils down to finding the maximum of σ. On carrying out some simplifications (Annexure I) we obtain that

$$S \quad = \quad \xi/\theta - F/\theta \ . \tag{5}$$

where S is the entropy of the primary gas while ξ, θ and F are respectively its average energy, temperature and free energy. Further, we obtain the probability for the primary gas to occupy the state with energy ξ′ as (Annexure I)

$$W(\xi') \quad = \quad g(\xi')\, e^{-(\xi'-F)/K\theta}, \tag{6}$$

Reverting back to the quantum number k, the above equation may be written as

$$W_k \quad = \quad g_k \ e^{-(\xi_{\mathbf{k}}-F)/K\theta} \ . \tag{7}$$

Here we should keep in mind that the free energy function, *F* is defined for a specific value of volume and number of micro-states. This is implicit in the steps we followed in the derivation. Taking summation over the entire range of values of k in (7), we obtain

$$e^{-F/K\theta} \quad = \quad Y \quad = \quad \sum g_k \ e^{-\xi_{\mathbf{k}}/K\theta} \ . \tag{8}$$

Note that the function $e^{-F/K\theta}$ in (7) plays the role of the normalizing factor.

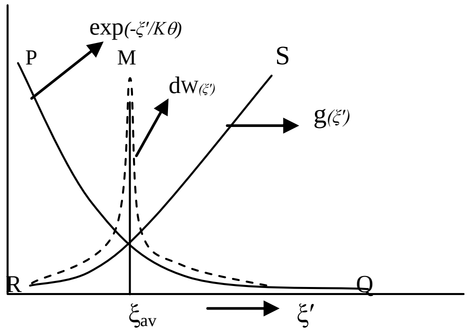
4 Eigen State as a Primary Eigen Gas

In the earlier papers we discussed about the structure of an elementary particle [1],[2],[3],[4]. There we studied the case of the confined helical wave (CH wave). It was observed that the space-dependent part forming the amplitude of the CH wave is defined in the internal coordinates. On the other hand the time-dependent part of the CH wave defined in the laboratory coordinates appears as a plane wave. We also observed that the CH wave and the plane wave are always in phase. This means that by the time the confined wave travels one wave length between the mirrors, the plane wave would have traveled one wave length in the laboratory coordinates. We may use the term wavelet to identify a single plane wave. We shall now introduce the concept of wavelet which represents a single wave and identify it with the microstate of the primary gas. This would mean that the plane wave representation of a particle in quantum mechanics has to be treated as a wave train having finite number of wavelets in it. Note that in the conventional treatment the number of wavelets constituting the plane wave representing a free particle is left undefined. It is pertinent to note that a photon is usually represented by wave train having a certain spatial spread.

We shall now assume that a large number of wavelets, say N wavelets, occupied successively form the plane wave. Here N is left undefined. It can be taken as a large number. The veracity of this assumption can be confirmed only if it leads to a new self consistent picture where N comes out as a universal constant which gets factored out in any observation.

By the same logic, we may assume that the wave train of a photon contains N wavelets. In quantum mechanics the plane wave plays the role of the eigen state in coordinate representation. One should keep in mind that observation is a macroscopic process and it involves a large number of the microstates. We are assuming that the number of microstates involved is fixed and this group of micro-states taken together is treated as a primary gas. Since a CH wave is formed by its interactions with the vacuum fluctuations, the energy and the momentum states occupied by it will be randomly distributed with a sharp maximum close to the average value. Here we treat a plane wave state which is an eigen state as a primary gas state on this basis. Now onwards we shall use the term primary eigen gas exclusively for such a gas.

The properties of a primary eigen gas are similar to that of a real gas with which we are familiar. Let us take the state function represented by (6). One interesting property of $W(\xi')$ is that for given large values of N, as ξ' increases it shows a sharp maximum for the value $\xi' = \xi_{av}$, where ξ_{av} may be taken as the average value of the energy of the primary eigen gas. The reason for the sharp maximum can be found in the fact that as ξ' increases, the exponential term in (6) decreases. However, the degeneracy of the state represented by $g(\xi')$ increases exponentially with the increase in ξ'. As a result, the product of the two functions exhibits a sharp maximum at the average value (figure.1). The sharpness of $W(\xi')$ increases as the number of microstates increases. When the number of microstates is in the range of 10^{21}, the function becomes so sharp that it can be taken as delta function. In such a situation practically all states of the gas will crowd around the average value. This is the reason why it is possible to identify the thermodynamic values of energy, volume etc with the average values. On the basis of the probabilistic considerations, it is obvious that the same situation should hold good in the case of the primary eigen gas representing a particle. Note that $g(\xi')$ represents the degeneracy of the state or the number of ways the microstates can be occupied for a given value of the extensive energy ξ'.



PQ denotes the Boltzmann factor which decreases as ξ' increases while RS represents degeneracy, g(ξ') which increases exponentially as ξ' increases. The resultant curve RMQ has a sharp peak at M similar to a delta function.

Figure.1

In the case of the primary eigen gas, the role of the molecular state will be taken up by the CH wave which we know [2][3][4] can be represented by the plane wave. To be precise, a single CH wave that can be taken as a micro-state will be represented by a plane wavelet. In other words, a plane wave constituted by a train of large number of waves will be equivalent to the primary eigen gas state. We use the term “primary eigen gas” keeping in mind that the plane wave represents an eigen state of the particle in the coordinate representation. On this basis, the confinement will be effected not by the walls of the vessel containing the gas, but by the two abstract mirrors confining the CH waves. An important property of the primary eigen gas so formed is that its energy (or momentum as they are linearly related) determines the distance between the mirrors. Note that in the rest frame, the distance between the mirrors is

$\frac{1}{2}\lambda_o$ while the energy of the CH wave is given by hc/λ_o. Therefore, once we fix the energy, the volume of the primary eigen gas gets fixed automatically. In other words, for the primary eigen gas volume is not an independent variable. Note that this holds good even when the system undergoes translational motion.

Let us now take up the case of the number of microstates contained in a primary eigen gas. Needless to say, for a given duration, the number of microstates, N of the primary eigen gas is not an independent variable. This is because the number of interactions undergone by the helical wave with the mirrors in one second is equal to its frequency which in turn is proportional to its energy. But we know that the time taken by the plane wavelet to complete one oscillation is exactly equal to the time taken by the CH wave to complete one to and fro motion. In other words, the number of plane wavelets formed in the laboratory coordinates will be exactly equal to the number of CH wave states. This shows that the number of CH waves formed in a given duration is not an independent variable and is completely defined by the energy of the CH wave states.

Here we should note that while the period of the CH wave and the plane wavelet are the same, their wave lengths are different. Note that the CH wave has a spread of $\frac{1}{2}\lambda_o/\gamma$, where λ_o is the wave length in its rest frame of reference whereas the wave length of the plane wavelet is much larger [1][2] and is given by

$$\lambda = h/\text{þ} = h/(\gamma E_o \mathrm{v}/c^2) = (\mathrm{c}/\mathrm{v})\lambda_o/\gamma \; . \tag{9}$$

Needless to say, in the rest frame of reference λ will become infinity. Since the corresponding phase velocity given by c^2/v will also become infinity the frequency of the plane wavelet will continue to remain finite given by c/λ_o.

In the micro-canonical treatment followed for the primary eigen gas, it is implied that the volume and the number of states are fixed. However, we now have a situation where the spatial spread which is equivalent to volume is no more an independent variable. It is just half of the wave length which is inversely proportional to the energy of the plane wave. Likewise, it is obvious that the number of wavelets constituting the primary eigen gas state also is completely determined by the frequency or the energy of the plane wave. In other words, the primary eigen gas gets fully defined when we take only the energy of the micro-states into consideration. In other words, in the case of the primary eigen gas the concept of the grand canonical ensemble will not be required as the micro-canonical ensemble will fully define the system.

Here we should keep in mind that we are dealing with the primary eigen gas in its rest frame of reference and therefore it does not possesses translational velocity and translational momentum as its thermodynamic coordinates. We shall introduce them shortly. Since we are dealing with the CH wave remaining in equilibrium with the macro-setting (vacuum), we shall not consider the case where it undergoes expansion in its wave length and does work. Therefore, we may treat F to be a constant. We may now introduce a modified entropy S′ where S′ = (S + F/θ). Accordingly we may express (5) as

$$S' = \xi/\theta = NE/\theta \; ,$$

Dropping the prime from S′, we may re-express the above equation as

$$S = \xi/\theta = NE/\theta \tag{10}$$

where E denotes the average energy of the microstate. Therefore (6) can be expressed as

$$W(\xi') = g'(\xi')\, e^{-\xi'/K\theta}. \tag{11}$$

Note that here $g'(\xi')$ actually stands for $g(\xi')e^{F/K\theta}$

5 Primary Eigen gas in motion at relativistic velocities

If we study a thermodynamic system like a gas having translational velocity in the relativistic range [9], then it is observed that its heat content or enthalpy given by (ξ+PV), will transform exactly the way the rest mass of a particle will transform. We may extent this property to the primary eigen gas also without any logical inconsistency (see annexure). However, in the case of the primary eigen gas, pressure and volume are not independent variables and therefore will not figure in the expression for enthalpy. In its place we may introduce variables representing its translational velocity and momentum. If we denote by G, the translational momentum of the gas as a whole, then we have

$$\xi = \gamma\,\xi_o \quad ; \quad \mathbf{G} = (\xi/c^2)\,\mathbf{v} \quad , \tag{12}$$

where $\gamma = (1-v^2/c^2)^{-\frac{1}{2}}$ and **v** is the translational velocity. It can be easily shown that the above relations will hold good even if the direction of the translational motion does not coincide with that of the confinement. It is obvious that ξ and **G** are related to each other exactly the way the energy and momentum of a particle are related to each other. In fact combining two equations in (12), we obtain

$$\xi^2 = \mathbf{G}^2 c^2 + \xi_o^2 \quad . \tag{13}$$

This is the relativistic energy and translational momentum relation of the primary eigen gas. If we take **v** to be along the z-axis, then by taking the differentials in (12), one can readily show that

$$d\xi = \mathrm{v}_z\, dG_z \quad . \tag{14}$$

But from (10) we may write

$$\xi = NE \quad and \quad G_z = N\text{þ}_z \quad . \tag{15}$$

E may be called the intensive internal energy of the primary eigen gas and þ its intensive translational momentum. Dropping the suffix from $G_{z,}$, þ_z and v_z while assuming they are directed along the z-axis, (11) can be written using (15) as

$$\text{þ} = \mathrm{Ev/c^2} \quad . \tag{16}$$

It is worthwhile to recall here that while the confinement of the helical wave takes place in the internal coordinates, the eigen state which represents the particle is defined in the laboratory coordinates. Therefore, in the laboratory coordinate system the microstates constituting the eigen state will be represented by the plane wavelets. As already discussed, the plane wavelet will be taken as the most basic entity for the primary eigen gas just as the molecular state is taken as the most basic entity forming a real gas state. Although a wavelet has the inner structure of a CH wave, this aspect plays no role in the nature of the eigen state. Only the plane wavelets, which can be treated as the projections of the CH wave onto the laboratory coordinates, are of importance. The need to get into the CH wave structure will arise only to explain the fermionic nature of the particle. For all other purposes, the plane wavelet structure will be adequate.

It should be noted that the introduction of the translational momentum, G is tantamount to the introduction of a new thermodynamic variable. Therefore, apart from (3), and (4) the following additional condition will have to be introduced where $\hat{G}$ is taken as fixed because the total number of copies in the ensemble is fixed. We shall for the sake simplicity take the direction of the translational motion to be strictly along the z-axis so that

$$\sum_k n_k G_k \quad = \quad \widehat{G} \quad \cdot \tag{17}$$

Using (17), equation (3) can be generalized to give

$$\phi \quad = \quad \sigma - \beta\overline{\xi} + \rho\,\widehat{G} + (\delta+1)\,R \quad , \tag{18}$$

where ρ is a constant which has to be determined. $\rho\widehat{G}$ is naturally a scalar. Now following the steps taken earlier (Annexure I), we can write

$$\begin{aligned} S \quad &= \quad -K\sum_k (n_k/R)\{-\beta\xi_k/K + \rho G_k - \ln Y_1(\beta,\rho)\}\ \cdot \\ &= \quad \xi/\theta - \rho G - \varphi/\theta\ , \end{aligned} \tag{19}$$

Here ξ and G represent the average values of the internal energy and the translational momentum while φ denotes the thermodynamic potential given by

$$Y_1(\beta,\rho) \quad = \quad e^{-\varphi/\theta} \quad = \quad \sum g_k\ e^{-(\xi_{\mathbf{k}}/K\theta - \rho G_{\mathbf{k}}/K)} \tag{20}$$

Note that if the energy states are lying close together, we may take them as continuous. Therefore, we may replace the summation by integration and also replace ξ_k and G_k by ξ' and G' respectively. We may now express the probability function corresponding to (6) as

$$W(\xi',G') \quad = \quad Y_1^{-1}\ g(\xi',G')\ e^{-(\xi'/K\theta - \rho G'/K)}, \tag{21}$$

We know that in any reversible change we may take S as an invariant. Therefore taking differentials in (19) , keeping in mind that φ/θ is a constant, we obtain

$$d\xi/\theta \quad = \quad \rho\, dG\ , \tag{22}$$

Since dξ/dG = v, the translational velocity, we have ρ = v/θ. Accordingly we may re-express (19) as

$$S \quad = \quad \xi/\theta - \mathrm{v}G/\theta - \varphi/\theta \tag{23}$$

We may introduce a modified entropy function S′ given by

$$S_1 \quad = \quad S + \phi/\theta \quad = \quad \xi/\theta - \mathrm{v}G/\theta \tag{24}$$

Note that S_1 also will remain an invariant in any reversible change as we know from (20) that φ/θ is a constant. Keeping in mind that S_1 is the modified entropy function, we may drop the suffix from S_1 and express (24) as

$$S \quad = \quad \xi/\theta - \mathrm{v}G/\theta \tag{25}$$

If the velocity and the translational momentum are along any general direction, then we have

$$S \quad = \quad \xi/\theta - \mathbf{v}\cdot\mathbf{G}/\theta \tag{26}$$

We may now express (21) as

$$W(\xi') \quad = \quad g(\xi')\ e^{-(\xi' - \mathbf{v}\cdot\mathbf{G}')/K\theta}\ . \tag{27}$$

Here $g(\xi')$ stands for $e^{\varphi/K\theta}g(\xi')$. Further,we should keep in mind that the translational momentum $\mathbf{G'}$ is taken along one direction only and this means ξ' completely determines the value of $\mathbf{G'}$.

It can be seen (see Annexure) that in a relativistic transformation, an ideal gas for all practical purpose will behave like a particle having rest energy. Needless to say, this applies to the case of the primary eigen gas also. Since the number of microstates is a relativistic invariant, we may define E = ξ/N and þ = G/N respectively as the energy and the translational momentum of a single microstate. For the primary eigen gas, an infinitesimal increase in the energy of the translational motion will be given by $\mathbf{v}\cdot d\mathbf{G}$. In the case of particle mechanics, the equivalent function is $\mathbf{v}\cdot d\mathbf{þ}$.

$$\mathbf{v}\cdot d\mathbf{þ} \quad = \quad (d\mathbf{r}/dt)\cdot d\text{þ} \quad = \quad d\mathbf{r}\cdot(d\mathbf{þ}/dt) \quad = \quad \mathbf{F}\cdot d\mathbf{r} \ .$$

Now combining (26) and (27) we may write

$$W(\xi') \quad = \quad g(\xi')\, e^{-[\xi' - \mathbf{v}\cdot\mathbf{G}']/K\theta\}} \ . \tag{28}$$

(15) can now be written as

$$\mathbf{G} \quad = \quad N\mathbf{þ} \quad \text{and} \quad \mathbf{þ} \quad = \quad E\mathbf{v}/c^2 \ . \tag{29}$$

Equation (26) can be expressed in a more general way as

$$S \quad = \quad Ns \quad = \quad N\left(\frac{E}{\theta} - \frac{\mathbf{v}.\mathbf{þ}}{\theta}\right) \tag{30}$$

where *s* may be called the intensive entropy

6 Introducing the Abstract Giant Primary Gas

We shall now try to express (28) in terms of the intrinsic quantities like E′ and **þ′** instead of ξ′ and **G′**. We know that the probability density function *W(ξ′,G′)* has a sharp peak like a delta function for value ξ′ = ξ and **G′** = **G**, where ξ and **G** are the average values for the ensemble as a whole. Therefore, for all practical purpose the entire ensemble of the primary gas may be assumed to occupy states having average values ξ and **G**. We shall now introduce an abstract gas formed by the entire ensemble of the gas which has Nn_e microstates. For this abstract gas, the average intrinsic energy and intrinsic momentum of the microstate will be E and **þ**. Note that this abstract gas having Nn_e microstates possesses the internal energy $\widehat{\xi}$ and the translational momentum Ĝ. We shall ignore the problem regarding the indistinguishability of the microstates for the present. Now following the Boltzmann's box method, It can be easily shown that the probability for occupying the microstate with energy E′ is given by

$$W(E') \quad = \quad g(E')\, e^{-n_e N(E' - \mathbf{v}\cdot\mathbf{þ}')/K\theta} \tag{31}$$

But we know that the number of microstates have a sharp maximum for energy and translational momentum which are close to the average values. Note that here the situation is similar to what is given in figure 1. In fact, equation (22) may be written as

$$(E' - E)/\theta \quad = \quad \mathbf{v}\,(\text{þ}' - \text{þ})/\theta \ ,$$

The above equation can be modified using (30) to obtain

$$E'/\theta - \mathbf{v}\cdot\mathbf{þ}'/\theta \quad = \quad E/\theta - \mathbf{v}\cdot\mathbf{þ}/\theta \quad = \quad s\,. \tag{32}$$

(31) can now be expressed using (32) as

$$W(E') \qquad = \qquad g(E')\, e^{-n_e N(E\,-\,\mathbf{v}\cdot\mathbf{þ})/K\theta} \tag{33}$$

In the rest frame of reference, the energy of a copy of the primary eigen gas will be ξ_o and (33) may be expressed as

$$W(E'_o) \qquad = \qquad g(E'_o)\, e^{-n_e N\, E_o/K\theta_o} \tag{34}$$

Since the energy of the individual microstates also would have a very sharp maximum for $E'_o = E_o$, we may replace E'_o by E_o for all practical purpose. Here $g(E'_o)$ which represents the degeneracy, increases as E'_o increases while the exponential factor decreases correspondingly with the result that $W(E'_o)$ gets a sharp maximum like a delta function at $E'_o = E_o$, where E_o is taken as the average value. Therefore, the probability to occupy any microstate with energy E'_o can be expressed as

$$W(E^{'}_o) \qquad = \qquad g(E_o)\, e^{-n_e N\, E_o/K\theta_o} \tag{35}$$

Using Sterling’s formula it can be easily shown that for very large values of the microstates, the degeneracy

$$g(E^{'}_o) \quad \rightarrow \quad e^{n_e N} \tag{36}$$

Here it is worthwhile to note that E_o is the average value of energy of the microstate. Therefore it can be equated to $K\theta_o$. In the light of this, substituting for $g(E^{'}_o)$ from (36) into (35) yields

$$W(E_o) \qquad = \qquad e^{n_e N}\; e^{-n_e N} \;=\; 1$$

This result is quite consistent with the observations. Note that on a macroscopic system like an ideal gas, the probability to occupy any state other than that having the average values is negligibly small and can be taken as zero for all practical purpose. It should be kept in mind that when the system is given a translational velocity, E_o will transform into (E-þv) while θ_o will transform into θ.

7 Quantizing Space and Time

We saw from the previous chapter that the intensive energy and the intensive momentum of a primary eigen gas are related just the way the energy and the momentum of a particle are related. That is $\mathbf{þ} = E\mathbf{v}/c^2$. Further we have $E = \gamma\, E_o$ where E_o is the value of E in the rest frame of reference. This leads us to the relativistic relation

$$E^2 \qquad = \qquad þ^2 c^2 + E_o^2 \quad . \tag{37}$$

Let us now introduce two new thermodynamic quantities T_e and $\mathbf{R_e}$ defined by

$$T_e \qquad = \qquad h/K\theta \qquad and \qquad \mathbf{R_e} \qquad = \qquad \mathbf{v}h/K\theta\,. \tag{38}$$

The factor h/K has been introduced into the equation so that T_e will have the dimension of time. We saw from the previous section while deriving (35) from (36) that $E_o = K\theta_o$. From a moving frame of reference the corresponding equation will be $(E-\mathbf{þ.v}) = E/\gamma^2 = K\theta$. Therefore, we have

$$T_e = h\gamma^2/E = \gamma^2 T \quad . \tag{39}$$

Here T denotes the period of the plane wavelet or the microstate. Let us now take (30) and substitute for 1/θ and **v**/θ from (38) and (39) after multiply it with n_e. If we now define a new function $S^{\#} = n_e NS$ representing the extensive entropy, then we obtain

$$S^{\#} = (K/h)[T_e n_e N E - n_e N\mathbf{R}_e \cdot \mathbf{p}] \ . \tag{40}$$

Here $n_e NT_e$ and $n_e N\mathbf{R_e}$ have very interesting properties. $n_e NT_e$ denotes the spread of the eigen state in time while $n_e N\mathbf{R_e}$ represents the spatial distance covered during that period. Equation (40) pertains to n_e eigen states. We may re-express the equation as

$$S^{\#} = (K/h)(Et_e - \mathbf{p} \cdot \mathbf{r_e}) \tag{41}$$

where $t_e = n_e NT_e \quad and \quad \mathbf{r_e} = n_e N\mathbf{R_e} \ .$ (42)

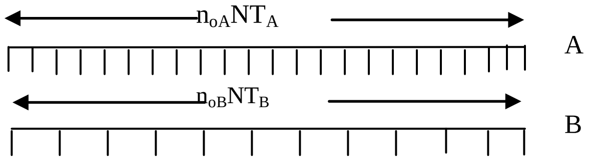
Here as $n_e N$ increases, $\mathbf{t_e}$ also increases. In that sense $\mathbf{t_e}$ has the property of the progressive time. Likewise, $\mathbf{r_e}$ has the property of spatial coordinate.

The question which comes to our mind here is whether the time and the space coordinates denoted by $\mathbf{t_e}$ and $\mathbf{r_e}$ are specific to the primary eigen gas under study or can it be taken to represent the laboratory (external) coordinate system. The equations in (38) which define T_e and $\mathbf{R_e}$ are specific to a primary eigen gas. But we have to consider the fact that if we take certain time interval, then the number of eigen states, n_e contained in it will be inversely proportional to the energy of the eigen gas. In (39) we have equated the intrinsic time of a primary eigen gas, T_e inversely proportional to the rest mass. It is obvious that the period of the plane wavelet representing one particle will be different from that of another if they differ in their rest masses.

Let us now take the case of two primary eigen gases A and B having rest energy E_{oA} and E_{oB} respectively, with $E_{oA} > E_{oB}$. Then, if T_{oA} and T_{oB} are respectively the intrinsic quantum of time (the period of the plane wave) of A and B, it is obvious that

$$T_{oB} > T_{oA} \quad as \quad T_{oA} = h/E_{oA}, \quad T_{oB} = h/E_{oB} \quad . \tag{43}$$

Note that although T_{oB} is greater than T_{oA}, the number of eigen states that exist in a time interval in the case of B is proportionately less than that of A. If we denote by n_{oA} and n_{oB} the number of eigen states contained in a certain time interval by the primary eigen gases A and B respectively, then presuming that n_{oA} and n_{oB} are very large numbers, we have



The interval between two consecutive vertical lines can be taken as the spread of the eigen gas state in time and it is quite clear from the figure that $n_{oA}T_A = n_{oB}T_B$.

Figure.2

$$n_{oA}N\,T_{oA} = n_{oB}N\,T_{oB} = n_o N\,T_o \ . \tag{44}$$

Here n_o and T_o pertain to some eigen gas which can be taken as a norm. Note that T_o can be taken as small as required so that for all practical purpose, n_oNT_o can be taken as a continuous function and therefore it can be used to represent the external time coordinate. Substituting for n_oNT_o by t_o , (44) can be written as

$$n_{oA}N\,T_{oA} \quad = \quad n_{oB}N\,T_{oB} \quad = \quad t_o \quad .$$

If these particles are observed from a moving frame of reference (moving with a velocity –v), then the above equation transforms to

$$n_A N\,T_A \quad = \quad n_B N\,T_B \quad = \quad t \qquad (45)$$

It can be easily seen that this property given by (45) can explain the perception of time as a universal background. Note that if **r** is the spatial coordinate of the primary eigen gas taken as the norm, then **r** = **v**t.

We know that the extensive internal energy of n eigen states, each having N microstates, can be represented by $\xi^{\#}$ where

$$\xi^{\#} \quad = \quad n\xi \quad = \quad nNE \quad . \qquad (46)$$

If we now take two primary eigen gases, A and B interacting with each other for a duration t, then their corresponding extensive energy can be taken as $\xi_A^{\#}$ and $\xi_B^{\#}$. Since the eigen states of A and B involved in the interactions are n_A and n_B respectively, the corresponding number of microstates involved in the interactions will be Nn_A and Nn_B respectively. This means that, instead of taking the extensive internal energy of A and B, we may as well use the average energy of the microstates of A and B denoted by E_A = $(\xi_A^{\#}/Nn_A)$ and E_B = $(\xi_B^{\#}/Nn_B)$ respectively.

Using (45), we may now express the small variations in t and **r** as

$$\Delta t \quad = \quad \Delta n\,N\,T \quad and \quad \Delta\mathbf{r} \quad = \quad \Delta nN\mathbf{R} \; . \qquad (47)$$

where T and **R** pertains to the primary eigen gas used as the norm in the laboratory coordinate system. We saw from the preceding discussion that we may take T and **R** as small as required so that t and **r** may be taken as continuous variables. Therefore, if Δn << n, then Δt can be replaced by dt and Δ**r** by d**r**. Let us now take (45) and replacing the suffix A by e, we obtain

$$t_e \quad = \quad n_eNT_e \quad = \quad nNT \quad = \quad t \qquad (48)$$

$$\mathbf{r_e} \quad = \quad n_eN\mathbf{v}T_e \quad = \quad nN\mathbf{v}T \quad = \quad \mathbf{r} \quad . \qquad (49)$$

Now substituting in (41) for $t_\mathbf{e}$ and $\mathbf{r_e}$ from (48) and (49) we have

$$S^{\#} \quad = \quad (K/h)[Et - \textbf{þ}\cdot\mathbf{r}] \; . \qquad (50)$$

We may also re-write (33) using (38), (48) and (49) as

$$W(E') \quad = \quad \mathrm{g}(\mathrm{E}')\;\; \mathrm{e}^{-(\mathbb{E}\,t - \textbf{þ}\cdot\mathbf{r})/h} \, . \qquad (51)$$

Substituting for $S^{\#}$ from (50) into (51), we have

$$W(E') \quad = \quad \mathrm{g}(\mathrm{E}')\;\; \mathrm{e}^{-S^{\#}/K} \, . \qquad (52)$$

Now we have to define the infinitesimal variations in entropy, energy and momentum. Here we should keep in mind that the eigen state is the most basic level for observation. Therefore, any variation should be measured from one eigen state to another. Let us take n number of eigen states which are occupied successively. Let $S^{\#}$ be the extensive entropy of the group of eigen states. Then we have

$$S^{\#} \quad = \quad nNs \quad = \quad nS \quad and \qquad \Delta S^{\#} \quad = \quad \Delta nS \; . \tag{53}$$

This means that as the laboratory time progresses through Δt, the extensive entropy increases by Δn S. The variation defined by (53) is the extensive variation where only n varies. In a similar manner we may define differentials of ξ and G also.

8 Action Entropy Equivalence of the of the Primary Eigen gas

We saw from the earlier discussion that a particle can be represented by a primary eigen gas. We shall now examine if the mechanics of a particle can be related to the thermodynamics of the primary eigen gas. We saw that in a relativistic transformation, the intensive energy E and corresponding intensive translational momentum þ of a primary eigen gas changes exactly like the energy and linear momentum of a real particle. Let us now take the action function of a free particle given by

$$A \quad = \quad -(Et - \mathbf{þ}\cdot\mathbf{r}) \quad = \quad -\mathrm{E_o} t_o \, . \tag{54}$$

Here E_o is the energy of the particle and t_o its time coordinates in the rest frame of reference. It should be kept in mind that action is a relativistic invariant. Comparing (50) with (54) we obtain

$$S^{\#}/K \quad = \quad -A/h \quad or \quad dS^{\#}/K \quad = \quad -dA/h \; . \tag{55}$$

The invariance of entropy under a relativistic transformation holds good for both the extensive and the intensive aspects. The extensive aspect remains invariant because nN is an invariant in a relativistic transformation.

This equivalence of two of the relativistic invariants, action and entropy brought out by (55), is an exciting result. Being a thermodynamic system, entropy of the primary eigen gas has a maximum in the equilibrium state. We know that entropy remains a constant in a relativistic transformation. From (55), we observe that when $S^{\#}$ is maximum, action A is a minimum. This means that the least action principle of the particle-mechanics is nothing but a restatement of the second law of thermodynamics applied to the primary eigen gas. Let us now try to find out the thermodynamic equivalent of the Langrangean of the primary eigen gas. We know that if we introduce to a system a small number of eigen states having the same average characteristics, then it will result in the increase in its heat content by a small value $dQ^{\#}$. This will result in a change in the extensive aspect of entropy given by

$$dS^{\#} \quad = \quad dQ^{\#}/\theta \quad = \quad (K/h)T_e dn\, N\, q \quad = \quad (K/h)\, q\, dt \quad , \tag{56}$$

where q is the internal heat expressed in terms of a single microstate or one may call it the intensive internal heat. However, for a mechanical system

$$dA \quad = \quad L\, dt \quad , \tag{57}$$

where L is the Langrangean.Comparing (57) with (56) and (55) yields

$$q \quad = \quad -L \quad = \quad mc^2\sqrt{(1 - \mathrm{v}^2/c^2)} \; . \tag{58}$$

When v = 0, q = mc^2. Therefore, one may state that the rest mass of a particle is a measure of its internal heat. Note that the heat which we perceive in our daily experience is due to the random motion at the molecular level. However, the heat referred to here is created by random motion at the innermost or most or sub-quantum level. Therefore it may be more appropriate to call it the **sub-quantum heat**.

In the light of the above discussion, we may examine the equation for the relativistic transformation of energy given by

$$E \;=\; E_o \Big/ \sqrt{(1-\mathrm{v}^2/c^2)} \;=\; E_o\sqrt{(1-\mathrm{v}^2/c^2)} \;+ (E_o\mathrm{v}^2/c^2)\Big/\sqrt{(1-\mathrm{v}^2/c^2)}$$

$$=\; mc^2\sqrt{(1-\mathrm{v}^2/c^2)} \;+\; \text{þ}\mathrm{v}\,. \tag{59}$$

Using (58), the first term on the right hand side of the above equation can be identified with the heat content while the second term can be identified with the free energy given by þv. Note that in the case of an ordinary gas, where the number of molecules is fixed, the free energy is given by PV, where P and V are pressure and volume respectively. But in the case of a primary gas, the equivalent function is þv. Therefore, (59) can be expressed as

$$E \;=\; q + \text{þ}\mathrm{v}\;\;. \tag{60}$$

This represents the first law of thermodynamics. Actually the first law should be expressed using the variations as

$$dE \;=\; \Delta q + \Delta w\,. \tag{61}$$

Here *w* denotes the work done on the system which is equal to þv. Note that Δq and Δw are improper differentials. This is because a thermodynamic system can undergo variations in *q* and *w* along various paths defined by values of pressure, volume and temperature occupied by the system. Note that when a system moves from state $A(P_1,V_1,\theta_1)$ to state $B(P_2,V_2,\theta_2)$, then there are infinite path ways available for this transition. Therefore, when Δq or Δw approaches zero, the paths followed by them are not uniquely defined. Therefore we cannot replace Δq and Δw by d*q* and d*w*. This also means that we cannot arrive at (60) starting from (61). Here it should be noted that the linear combination of the two variations equals to the variation in the internal energy which is seen to be a proper differential and is therefore denoted by dE. In the case of the primary eigen gas, the situation is different. This is so because we know that both *q* and *w* are functions of v and their variations are uniquely defined. Therefore we may replace Δq and Δw by d*q* and dw respectively. This explains why (60) holds good for the primary eigen gas.

Let us now substitute for *q* in (60) from (59) and take the differential to obtain

$$dE \;=\; d(mc^2/\gamma) + d(\text{þ}\mathrm{v}) \;=\; -\text{þ}\,d\mathrm{v} + \text{þ}\,d\mathrm{v} + \mathrm{v}\,d\text{þ} \;=\; \mathrm{v}\,d\text{þ}\,. \tag{62}$$

Note that the variation in the internal heat is exactly compensated by one of the two terms representing the variation in the free energy leaving only vdþ as the resultant variation. This represents the variation in the kinetic energy (recall that d[$\tfrac{1}{2}mv^2$] = v dþ). This is the reason why we could ignore the primary gas structure of a particle and deal with it as a point particle. The idea that the mass of a particle represents its sub-quantum heat opens the window to a new breath-taking view of the internal structure of matter. In fact, this allows us to study the quantum world using the well understood principles of thermodynamics.

9 Statistical basis of the uncertainty principle

We shall now show that the Heisenberg's uncertainty principle can be derived from this idea of the interactions with vacuum fluctuations. Let us take n number of primary eigen gas states having energy ξ_1 to ξ_n. We know that if n is quite large, then the states occupied by the particle would crowd around the average value ξ. Now applying the principle of fluctuations on these n eigen states , we obtain

$$\Delta\xi/\xi \quad = \quad 1/\sqrt{n} \quad . \tag{63}$$

This is the equation for fluctuations [5]. If we take the case of the primary eigen gas with N microstates, then, we know that $\xi = Nh\nu = Nh/T_e$ where T_e is the period of the CH wave. Therefore, we have

ie;
$$\Delta\xi\, T_e \quad = \quad Nh\gamma^2/\sqrt{n} \quad . \tag{64}$$

Let us now define the variation in the intrinsic energy as $\Delta E = \Delta\xi/N$. Now multiplying both sides of (64) by n, we obtain

ie;
$$\Delta E\, n T_e \quad = \quad \gamma^2 \sqrt{n}\; h \quad .$$

If we now take n to be a very small number denoted by Δn, then the above equation could expressed as

$$\Delta E\, \Delta n T_e \quad = \quad \gamma^2 \sqrt{\Delta n}\; h \tag{65}$$

If T is the intrinsic quantum of time of the laboratory coordinate system, then from (48) we can always relate it to the intensive time of a particle by the relation

$$T_e \quad = \quad mNT \quad , \tag{66}$$

where m is an integer. Here T is the intrinsic time of the primary eigen gas which is taken as the norm for the external or laboratory coordinate system. NT will give us the time spread of the primary eigen gas state which is taken as the norm. Note that T can be made as small as required. We may now express (65) as

$$\Delta E\, \Delta n\, mNT \quad = \quad \gamma^2 \sqrt{\Delta n}\; h \quad .$$

Taking $m\,\Delta n = \Delta n'$, the above equation may be written as

$$\Delta E\, \Delta n' N\, T \quad = \quad \gamma^2 \sqrt{\Delta n}\; h \quad .$$

We know from (47) that "Δn'NT" can be taken as the durative time in the external coordinates and could be denoted by Δt. This gives us the relation

$$\Delta E\, \Delta t \quad = \quad \gamma^2 \sqrt{\Delta n}\; h \qquad or \qquad \Delta E\, \Delta t \quad \geq \quad h \quad . \tag{67}$$

In a similar fashion, taking the case of the momentum and the spatial coordinates, we may derive the relation

$$\Delta \text{þ}\, \Delta \mathrm{x} \quad \geq \quad \tfrac{1}{2} h \quad . \tag{68}$$

The reason why $\tfrac{1}{2}$h appears on the right hand side is due to the fact that the CH wave is half wave, not a full wave.

10 Understanding the Nature of the Virtual Interactions

We know that in quantum mechanics the uncertainty principle given in (67) and (68) plays a very important role. If the period Δt is long, then the fluctuation in energy becomes almost zero. We know that any fluctuation in energy which does not comply with the uncertainty relation given in (67) and (68) will remain a virtual one. In other words, if the vacuum fluctuations comply with the inequality ΔE Δt < h, then such fluctuations will be virtual by nature.

Let us examine this issue in the light of the equation for fluctuations given in (63). Here we should keep in mind that when we fix the time duration Δt, it is equivalent to fixing the value of Δn. Therefore, when we take ΔE Δt > h, we are taking Δn > 1. Similarly, when we take ΔE Δt = h, we are basically taking Δn = 1. By the same reasoning, when we take ΔE Δt < 1, we are assuming Δn < 1. Now we should try to understand what is the implication when we take Δn < 1. We know that a primary eigen gas state (equivalent to the eigen state of quantum mechanics) is formed by N microstates. We can assume that the number of microstates, N_i keeps on increasing from 1, 2, 3 etc. till it reaches the value N when the eigen state is formed and the value of n becomes 1. Therefore, it is obvious that n < 1 pertains to the situation where N_i < N or when the primary eigen gas state is not yet fully formed. Let us now examine the fluctuations that could take place in the energy of the microstates. We know from the equation for fluctuations that

$$\Delta\xi_i \quad = \quad \xi_i \big/ \sqrt{N_i}\ , \qquad or \qquad \Delta E_i \quad = \quad E_i \big/ \sqrt{N_i}\ . \tag{69}$$

,

Here ξ_i represents the extensive energy of the system having N_i microstates while $E_i = \xi_i/N_i$ and $\Delta E_i = \Delta\xi_i/N_i$. It is quite obvious from (69) that smaller the time duration (note that N_i is directly proportional to the time interval) larger will be the fluctuations in the energy. This means that the so called virtual interactions undergone by a particle are caused by the usual statistical fluctuations. If we take the case of space and momentum, the requirement for the fluctuations to remain virtual will be ΔþΔx < $\frac{1}{2}$h.

One of the paradoxes of quantum mechanics has been the role played by the virtual interactions. While the interactions are termed virtual, their effect on the particles is real and observable like in the case of Lamb shift and Casimir effect. The interpretation of the uncertainty principle given above makes it clear that the so called virtual interactions with the vacuum are as real as any other interactions. The only reason why they appear virtual is that only a primary eigen gas state formed by a group of N wavelets could only be an observable state. To understand the situation better let us examine an analogous case of a real gas. Let us assume that no sophisticated instruments are available to observe the molecules of a real gas directly, the instruments available can observe only macroscopic systems. In that case, as far as the observer is concerned, only the gas contained in a vessel will be a real entity while the molecules that form the gas will be virtual! Here, the virtual nature of the molecules is a constraint imposed by technology. However in the case of sub-quantum states, there seems to be a universal limit for observation. It seems that N microstates are required to form an observable state. But we do not know the exact value of N as it gets factored out in any calculation. We can only presume that it is not a small number.

11 Conclusion

Since the vacuum fluctuations play the role of the sub-quantum thermal bath, it is obvious that the internal energy of the particle will be equal to the energy of the vacuum fluctuations. Here we should keep in mind that the energy for one degree of freedom will be given by $\frac{1}{2}K\theta_o$. Since the CH wave could be attributed two degrees of freedom, one due to oscillations in one plane and the other due to rotation, the total energy involved would be $K\theta_o$. This would apply even if we have to consider some other composite wave instead of the electromagnetic wave for confinement to represent particles like quarks. Therefore, the energy of the particle in the rest frame of reference would be given by $E_o = K\theta_o$.

The concept that an eigen state is represented by N wavelets means that a primary eigen gas will be spread out in space over N wave lengths of the plane wave where N is not a small number. Therefore, it appears that a particle cannot be localized under the primary gas approach. It will be shown in separate papers that this problem of the localization of the particle can be resolved by going deeper into the nature of time. As such we know from the equation given in (51) that the state function of the primary eigen gas has the same form as that of a wave function except for the absence of the imaginary number "$2\pi i$" in the exponential term as a factor. In other words, we may say that in the wave representation of a particle, time is treated as an imaginary quantity while in the primary gas approach it is treated as a real quantity. This analysis appears to lead us to a new understanding of the nature of time and also to the very basis of quantum mechanics. However, before making such a claim, it is necessary to establish the thermodynamic basis of the basic postulates of quantum mechanics. This will be done in separate papers shortly.

A detailed analysis of the approach shows that in the case of wave picture, time and space coordinates r and t increase parametrically with the increase in N and n while in the primary gas picture the corresponding increase is in the energy and momentum values. In other words, while the plane wave is the space-time representation, the primary eigen gas is the energy-momentum representation of a particle. Note that energy-momentum space may be termed as the reciprocal space of the four dimensional space-time.

Here it is interesting to note that de Broglie, towards the later part of his life, had come around to the view that a particle may be attributed a hidden thermodynamic system and the mass of a particle may be taken as a measure of the hidden heat of the system while action may represent its entropy [10]. However, his idea did not find acceptability due to certain serious shortcomings. For example de Broglie assumed that the particle may be treated like a real gas. However, the action function of a free particle at rest given by "$-E_o t_o$" increases as time progresses while the entropy of a real gas at rest is given by ξ_o/θ_o which does not have such a time dependence. This forced him to restrict the action function to one period of the phase wave artificially before equating it to the entropy. Besides, it was not clear as to how vacuum which acts like a sub-quantum thermal bath can sustain two particles with different rest masses continuously in the same region. We should keep in mind that he had not given up the concept of the point particle while attempting this intuitive jump. Therefore, although the concept of the action-entropy equivalence proposed by him was a laudable attempt, he could not put forward a consistent theory to back it up. That may explain why his attempt did not get the attention it actually deserved. All said and done, one has to salute this genius who was in the forefront in the quantum revolution during 1920s and fifty years hence could still foresee the next revolution in physics which we may be called the "sub-quantum thermodynamics".

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ANNEXURE -A

Let us consider L number of successive microstates of the primary gas which are subdivided into R groups, each group being formed by N successive microstates such that

$$N\,R \quad = \quad L \quad . \qquad \text{(A I)}$$

Here *R* represents the ensemble of the primary gas. *L* and *R* can be made as large as required. If we denote by n_k the number of the ensemble copies for which the energy is ξ_k , then, the ratio, n_k/R will give the probability for the gas to occupy the states with the specific values ξ_k. If we now consider a giant abstract gas formed by combining all of the *R* copies of the ensemble, then the states with a given value ξ_k will play the same role as played by a box in the Boltzmann's method [1] and the number of copies n_k, the role of the number of balls in the boxes.

The number of ways in which the copies of the ensemble can be distributed among the boxes is equal to

$$M \quad = \quad R!/\Pi_k\, n_k! \quad . \qquad \text{(A II)}$$

We may assume that there exists a Gibbs micro-canonical distribution for the giant gas containing R copies. Therefore, the number of copies R and the total energy of all the copies $\hat{\xi}$ will be assumed to be fixed. These constraints may be expressed by the equations

$$\sum_k n_k \quad = \quad R \quad and \quad \sum_k \xi_k n_k \quad = \quad \widehat{\xi} \quad . \qquad \text{(A III)}$$

We know that for the most probable state of the gas, M will have a maximum. Keeping this in view, let us define a function σ given by

$$\sigma \quad = \quad K \ln M \ , \qquad \text{(A IV)}$$

where K is the Boltzmann's constant. We know that σ has a maximum when M has a maximum. Taking the conditions imposed by equations in (A III) and using the Langrangean method of multipliers along with Sterling formula, we look for the maximum of σ in the expression

$$\phi \quad = \quad \sigma - \beta\widehat{\xi} + (\delta + 1)R \ , \qquad \text{(A V)}$$

where $\sigma \quad = \quad K \ln M \quad = \quad K\,R \ln R - K \sum n_k \ln n_k .$

Differentiating the n_k's as independent quantities and equating the first derivative of ϕ to zero, we obtain

$$n_k \quad = \quad e^{(\delta - \beta\xi_k)/K} \quad . \qquad \text{(A VI)}$$

The Langrangean multiplier δ is obtained by substituting for n_k from (A VI) into (A III) and is given by

$$e^{\delta/K} = R/\left(\sum e^{-\beta\xi_k/K}\right) \tag{A VII}$$

and $$n_k/R = [Y(\beta)]^{-1}\, e^{-\beta\xi_k/K}, \quad where \quad Y(\beta) = \sum e^{-\beta\xi_k/K}. \tag{A VIII}$$

Since the number of the ensemble copies and as a consequence n_k can be made as large as desired, the application of the Stirling's formula is quite justified. The ratio $n_k/R = W_k$ is the probability that the system will be in the state with energy ξ_k. If the energy level ξ_k is degenerate by a factor g_k, then (A VIII) may be modified to give

$$W_k = n_k/R = [Y(\beta)]^{-1} g_k\ e^{-\beta\xi_k/K}, \tag{A IX}$$

where $$Y(\beta) = \sum_k g_k\ e^{-\beta\xi_k/K}\ . \tag{A X}$$

Let us now examine the thermodynamic significance of β. Using (A IV), let us introduce a function S given by

$$S = R^{-1}K\ln M = \sigma/R\ \cdot \tag{A XI}$$

S could be identified with entropy. The basis for identifying σ/R with entropy is firstly the additivity of the quantities S and σ. The second reason is that S and σ both reach their maximum values in the most probable state or the equilibrium state. The need to divide by R in (A XI) arises due to the fact that we want to determine the entropy of a real system which is represented by just one copy of the ensemble. Substituting the value of σ and using Sterling's formula to simplify, we get

$$S = -K\sum_k (n_k/R)\ln(n_k/R)\ \cdot \tag{A XII}$$

This equation determines the entropy of an arbitrary state of a system, whether in equilibrium or otherwise. Substituting for n_k/R from (A IX) into (A XII), we obtain

$$S = -K\sum_k (n_k/R)\{-\beta\xi_k/K - \ln Y(\beta)\}\ \cdot \tag{A XIII}$$

Note that (n_k/R) denotes the probability to occupy the kth energy state. Therefore, the term $\Sigma\ (n_k/R)\ \xi_k$ will give the average value of ξ_k and we obtain

$$S = \beta\bar{\xi} + K\ln Y(\beta)\ , \tag{A XIV}$$

where $\bar{\xi} = \hat{\xi}/R$ is the mean value of the energy of the system. It should be noted that the mean value $\bar{\xi}$ as per the statistical approach represents its value in the thermodynamic sense.

Let us now onwards, for the sake of notational convenience, replace $\bar{\xi}$ by ξ keeping in mind that it represents the average value, while ξ' will be used for the individual value in place of ξ_k. Accordingly (A IX) may be written as

$$W(\xi') = Y^{-1}\ g(\xi')\, e^{-\beta\xi'/K}, \tag{A XV}$$

where $$Y = \sum g(\xi')\, e^{-\beta\xi'/K}. \tag{A XVI}$$

The mean energy and the entropy of the system are given by

$$\xi \quad = \quad -K\frac{\partial(\ln Y)}{\partial\beta} \quad and \quad S \quad = \quad K\ln Y + \beta\xi \cdot \qquad \text{(A XVII)}$$

By comparing (A XVII) with a similar function obtained for an ideal gas, β can be easily identified with the inverse of temperature θ and "-K ln$Y$" with the ratio of the free energy F to the temperature of the system. These relations may be expressed as

$$\beta \;=\; 1/\theta \qquad and \qquad K\ln Y \;=\; -F/\theta \;=\; S-\xi/\theta \,. \qquad \text{(A XVIII)}$$

The entropy may now be expressed in the familiar form

$$S \quad = \quad \xi/\theta - F/\theta \;. \qquad \text{(A XIX)}$$

We may now re-express (XV) as

$$W(\xi') \quad = \quad g(\xi')\, e^{-(\xi'-F)/K\theta}, \qquad \text{(A XX)}$$

ANNEXURE B

Relativistic Thermodynamics of an Ideal Gas

We know from the relativistic thermodynamics that the pressure acting on an ideal gas is a relativistic invariant just as its entropy is. If we denote pressure of the ideal gas by P, its internal energy by ξ and its translational velocity by v, then we have that [9]

$$\xi + PV \quad = \quad \gamma(\xi_o + PV_o)\cdot \qquad \text{(B I)}$$

where $\gamma = (1\text{-}v^2/c^2)^{-½}$. The corresponding relation for the translational momentum, **G** of the relativistic ideal gas as a whole is given by

$$\mathbf{G} \quad = \quad \gamma(\xi_o + PV_o)\,\mathbf{v}/c^2 \cdot \qquad \text{(B II)}$$

As pressure is a relativistic invariant, we may treat the relativistic transformation as an isobaric process. We know that in such a process, the enthalpy, denoted by H, is well defined. Therefore, if the translational velocity varies infinitesimally, the variation in the heat content

$$dQ \;=\; d\xi + PdV \;=\; d(\xi + PV) \;=\; dH \cdot \qquad \text{(B III)}$$

where

$$H \quad = \quad (\xi + PV)\cdot \qquad \text{(B IIIA)}$$

Enthalpy represents the heat content of the gas. We now see that in a relativistic transformation, the enthalpy of the gas behaves exactly like the rest mass of a particle. Moreover, the translational momentum **G** also behaves like the momentum of a particle in a relativistic transformation. Combining (B I) and (B II), we obtain

$$H^2 - \mathbf{G}^2c^2 \quad = \quad H_o^2 \cdot \qquad \text{(B IV)}$$

This is the equivalent of the relativistic energy-momentum relation for a particle. These similarities reinforce our conviction that a particle can be treated as a gas.

But when we extend these results, we have to keep in mind that the primary eigen gas does not have an independent dimension in pressure and volume. Therefore, (B IIIA) will have to be modified to give

$$H = \xi . \qquad \text{(B V)}$$

This means that the internal energy of a primary eigen gas plays the role of the heat content. Accordingly, corresponding to (B I) and (B II) we have

$$\xi = \gamma \xi_o \quad \textit{and} \quad \mathbf{G} = \gamma \xi \, \mathbf{v}/c^2 . \qquad \text{(B VI)}$$

Since the number of microstates is an invariant in a relativistic transformation, we may re-express the two equations in (B VI) in terms of the intrinsic values of energy and momentum as

$$E = \gamma E_o \quad \textit{and} \quad \text{þ} = \gamma \, \mathrm{Ev}/\mathrm{c}^2 . \qquad \text{(B VII)}$$

where E = ξ/N and **þ** = **G**/N , N being the number of molecules in the gas.

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