Electromagnetic waves in an expanding 5D universe

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Electromagnetism is analyzed in a 5D expanding universe. Compared to the usual 4D description of electrodynamics it can be viewed as adding effective charge and current densities to the universe that are static in time. These lead to effective polarization and magnetization of the vacuum, which is most significant at high redshift. Electromagnetic waves propagate but group and phase velocities are dispersive. This introduces a new energy scale to the cosmos. And as a result electromagnetic waves propagate with superluminal speeds but no energy is transmitted faster than c.

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INTRODUCTION

In the standard particle model classification in terms of field strengths has led to four types of interactions: the strong, the electromagnetic, the weak and the gravitational. Since the strengths of the interactions are energy dependent this classification is not so clear cut. At high energy the strong interaction becomes weaker and is nearly equal to the electromagnetic interaction. Then there are the expected yet undetected Higgs interactions which are at about the same strength as the weak interaction. The latter involves a spin 0 boson (of undetermined mass) and a W or Z particle. Also there are interactions between a photon and a Higgs particle, which may involve a W or Z particle. Electromagnetic forces involving only photons may not be so useful to advance our understanding of the physics. Would it be possible to unify the Higgs and the electromagnetic interactions at high energy? Until now there has been no framework suitable to do it. Carmeli suggests that a suitable framework may exist within his cosmology (see sections 2.11 and 2.12 of Carmeli [1]).

Carmeli's cosmology, also referred to as Cosmological Relativity, was initially a space-velocity theory of the expanding universe, in 4D. It is a description of the universe at a particular fixed epoch of cosmic time t and involves a new dimension v, the velocity of the expansion of the fabric of space itself, in which the galaxies are sitting. The new dimension requires a new time constant the Hubble-Carmeli time, τ, of order of the age of the Universe. Carmeli then incorporates time to make it a full 5D theory. A full introduction to and an explanation of the theory may be found in *Cosmological Relativity* [1] or in *Relativity: Modern Large-Scale Spacetime Structure of the Cosmos*[2].

Under the framework of cosmological special relativity Carmeli extended Maxwell's equations into five dimensions using the following skew-symmetric tensor $f_{\mu\nu}$ in units where $c = \tau = 1$,

$$f_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z & \mathcal{H} \\ E_x & 0 & B_z & -B_y & W_x \\ E_y & -B_z & 0 & B_x & W_y \\ E_z & B_y & -B_x & 0 & W_z \\ -\mathcal{H} & -W_x & -W_y & -W_z & 0 \end{pmatrix}, \tag{1}$$

where μ, ν = 0,1,2,3,4; $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields. For μ, ν = 0,1,2,3, this is exactly the usual representation for the electromagnetic field. Here the field ($\mathcal{H}$, $\mathbf{W}$) describes a new interaction.

The interpretation that Carmeli applied to the scalar $\mathcal{H}$ and the vector $\mathbf{W}$ in Eq. (1) is that they are the Higgs massless particle and the W or the Z particle. In this way the Higgs interaction is unified with the electromagnetic interaction within the framework of cosmology, where the Higgs interaction is, in some way, associated with the expansion of the Universe.

The potential A_μ is introduced in five dimensions as it is in 4D electrodynamics as

$$f_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu}, \tag{2}$$

with $A_\mu = (A_0, A_m, A_4) = (\phi, -\mathbf{A}, \tilde{\phi})$ $(m = 1, 2, 3)$, where ϕ and $\mathbf{A}$ are the usual scalar and vector electromagnetic potentials, and A_4 is an additional potential related to the expansion of the Universe.

WAVE EQUATION IN 5D

The wave equation, in vacuum, for A_μ in 5D is given by

$$\partial_\nu \partial^\nu A_\mu = 0, \tag{3a}$$

which is explicitly,

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^{\mathbf{2}} + \frac{1}{\tau^2} \frac{\partial^2}{\partial v^2} \right) A_\mu = 0, \tag{3b}$$

where the Lorentz condition

$$\partial_\alpha A^\alpha = 0 \tag{3c}$$

is also required.

Using Cartesian space coordinates (x, y, r) and equating $z = v/c$ (valid for small redshift) to change to redshift space Eq. (3b) becomes

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial r^2} + \frac{1}{c^2\tau^2}\frac{\partial^2}{\partial z^2}\right) A_\mu = 0. \tag{3d}$$

We shall look for a solution for the plane wave in the form

$$A^\mu = \varepsilon^\mu sin(\omega t - kr + \gamma z), \tag{4}$$

where ω, k and γ are constants and ε^μ is a five-vector with constant components. Substituting Eq. (4) into Eq. (3d) yields

$$k^2 = \frac{\omega^2}{c^2} + \frac{\gamma^2}{c^2\tau^2}. \tag{5a}$$

Hence the wave number (propagation constant along the r-direction) is

$$k = \frac{\omega}{c}\sqrt{1 + \frac{\gamma^2}{\omega^2\tau^2}}, \tag{5b}$$

where the positive sign of the square root has been used. This modification to the usual wave number in 4D electrodynamics, in Eq. (5b), is frequency dependent and also depends on the value of the Hubble parameter; remember $H_0 \approx \tau^{-1}$.

Therefore the phase velocity of the wave is

$$v_p = \frac{\omega}{k} = c\left(1 + \frac{\gamma^2}{\omega^2\tau^2}\right)^{-1/2}, \tag{6a}$$

and its group velocity is

$$v_g = \frac{\partial\omega}{\partial k} = c\left(1 + \frac{\gamma^2}{\omega^2\tau^2}\right)^{1/2}. \tag{6b}$$

This means that the wave is dispersive with a dependence on γ and the Hubble parameter. However γ could be real or imaginary. Assuming the imaginary case $\gamma = i\kappa$ where κ is real Eq. (5b) becomes

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\kappa^2}{\omega^2\tau^2}}. \tag{7}$$

Provided $\kappa \ll \omega\tau$ the wave will still propagate. But note that

$$v_p v_g = c^2, \tag{8}$$

and independent of redshift and energy scale.

The relevant speed for the transmission of photons is the group velocity. In the case where γ is real Eq. (6b) yields superluminal speeds. But it follows from Eq. (8) that energy is not transmitted faster than light. When $\tau \to \infty$ the situation becomes the usual non-expanding universe and the group and phase velocities for the wave in Eqs (6) is c.

Now the Lorentz condition Eq. (3c) requires that

$$\frac{\omega}{c}\varepsilon^0 - k\varepsilon^3 + \frac{\gamma}{c\tau}\varepsilon^4 = 0, \tag{9}$$

which means

$$\varepsilon^0 = \varepsilon^3\sqrt{1 + \frac{\gamma^2}{\omega^2\tau^2}} - \varepsilon^4\frac{\gamma}{\omega\tau}. \tag{10}$$

This means we have only 4 independent components of ε^μ; ε^0 can be seen as a linear combination of ε^3 and ε^4.

For $\gamma \ll \omega\tau$, to first order in $\gamma/\omega\tau$, Eq. (10) becomes

$$\varepsilon^0 \approx \varepsilon^3 - \varepsilon^4\frac{\gamma}{\omega\tau}. \tag{11}$$

And when the propagation constant $\gamma \to 0$ then $\varepsilon^0 \to \varepsilon^3$, and we recover the usual result of 4D electrodynamics. If the 4 independent components of ε^μ are $\varepsilon^0 = (1,0,0,1,0)$, $\varepsilon^1 = (0,1,0,0,0)$, $\varepsilon^2 = (0,0,1,0,0)$, $\varepsilon^4 = (0,0,0,0,1)$, then from Eq. (11) the dependent component is $\varepsilon^3 = (1,0,0,1,\gamma/\omega\tau)$. Because the first 4 components of ε^μ are the same as in the usual 4D electrodynamics the same result is obtained for the alternating electric and magnetic fields, which are perpendicular to each other and orthogonal to the propagation direction. The particular solution for the plane wave

$$A^3 = A\varepsilon^3 sin(\omega t - kr + \gamma z), \tag{12}$$

carries no global energy.

The propagating part of the electromagnetic waves A^μ can be written as

$$A^\mu = A\varepsilon^\mu e^{i(\omega t - kr)}e^{i\gamma z}, \tag{13}$$

where the redshift dependent part in 4D can also be interpreted as an additional phase factor. Locally $z \ll 1$ and the phase factor $\exp(i\gamma z) \approx 1$. Only at cosmological distances, at very high redshift ($z \gg 1$) would it contribute significantly to the wave. This is representative of the early universe and much higher energy scales.

MAXWELL'S EQUATIONS IN 5D

Using Eq. (2), the fields maybe determined as

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}, \tag{14a}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{14b}$$

$$\mathbf{W} = -\nabla\tilde{\phi} - \frac{\partial\mathbf{A}}{\partial v}, \tag{14c}$$

$$\mathcal{H} = \frac{\partial \phi}{\partial v} - \frac{\partial \tilde{\phi}}{\partial t}, \tag{14d}$$

in units of $c = \tau = 1$.

Carmeli determines the generalized Maxwell's equations, in vacuum but with source terms. Here we are interested in source free equations but for a general dielectric medium. In terms of the electric displacement field $\mathbf{D}$ and the magnetic field intensity $\mathbf{H}$ we get

$$\nabla \cdot \mathbf{D} + \frac{\partial \mathcal{H}}{\partial v} = 0, \tag{15a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{15b}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{15c}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \frac{\partial \mathbf{W}}{\partial v}, \tag{15d}$$

$$\nabla \cdot \mathbf{W} = \frac{\partial \mathcal{H}}{\partial t}, \tag{15e}$$

$$\nabla \times \mathbf{W} = \frac{\partial \mathbf{B}}{\partial v}. \tag{15f}$$

The fields $\mathbf{D}$ and $\mathbf{H}$ are defined by,

$$\mathbf{D} = \tilde{\epsilon} \cdot \mathbf{E}, \tag{15g}$$

$$\mathbf{H} = \tilde{\mu}^{-1} \cdot \mathbf{B}, \tag{15h}$$

where $\tilde{\epsilon}$ and $\tilde{\mu}$ are permittivity and permeability tensors for the medium. In the following we will assume the medium to be isotropic and homogeneous with scalar permittivity ϵ and permeability μ.

Now if we take the curl of Eq. (15b) we get

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}), \tag{16a}$$

$$= -\mu \left(\frac{\partial^2}{\partial t^2} \mathbf{D} + \frac{\partial^2}{\partial t \partial v} \mathbf{W} \right). \tag{16b}$$

But using the identity

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

and Eq. (15a), Eq. (16b) becomes

$$-\nabla^2 \mathbf{E} = -\mu \left(\frac{\partial^2}{\partial t^2} \mathbf{D} + \frac{\partial^2}{\partial t \partial v} \mathbf{W} \right) - \frac{\partial}{\partial v} \nabla \mathcal{H}, \tag{17a}$$

Rearranging Eq. (17a) we get

$$\nabla^2 \mathbf{D} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{D} = \frac{1}{\tau} \frac{\partial}{\partial v} \left(\frac{1}{c} \frac{\partial}{\partial t} \mathbf{W} + \nabla \mathcal{H} \right), \tag{17b}$$

where the constants have been put in and c is the speed of propagation in the medium.

The left hand side of Eq. (17b) is the usual form of the wave equation for the electric displacement field when the right hand side is zero. That is so where the field $(\mathcal{H}, \mathbf{W})$ does not exist as in the usual 4D representation of electrodynamics. Also the right hand side of Eq. (17b) is zero where $\mathbf{W}$ is time or velocity independent and where $\mathcal{H}$ is space or velocity independent. Similarly it can be shown that for the magnetic field

$$\nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{B} = \frac{1}{\tau} \frac{\partial}{\partial v} (\nabla \times \mathbf{W}). \tag{17c}$$

Each term on the right hand sides of Eqs (17b) and (17c), in general, has a dependence on the velocity of the expansion of the Universe. If that dependence is zero then we have usual result in electrodynamics. Otherwise these terms act as source terms for the propagation of electromagnetic wave through space.

Now if we compare Eqs (15a) and (15d) to the usual Maxwell equations with source terms in 4D we can get effective sources $(\rho_{eff}, \tilde{j}_{eff})$ resulting from the expansion of the cosmos.

$$4\pi \rho_{eff} = -\frac{1}{\tau} \frac{\partial \mathcal{H}}{\partial v}, \tag{18a}$$

$$4\pi \tilde{j}_{eff} = \frac{1}{\tau} \frac{\partial \mathbf{W}}{\partial v}. \tag{18b}$$

If we then substitute these into Eqs (17b) and (17c) we respectively get

$$\nabla^2 \mathbf{D} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{D} = -4\pi \nabla \rho_{eff}, \tag{19a}$$

with the condition that $\partial \mathbf{W} / \partial t = 0$ and

$$\nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{B} = 4\pi \nabla \times \tilde{j}_{eff}. \tag{19b}$$

In order to derive Eq. (19a), which is dependent only on the gradient of the effective source charge density the $\mathbf{W}$ field has to be time independent or static. That is consistent with Eq. (14c). It should also be noted that Eq. (19b) only depends on the curl of the effective current density, and also is static. The sources however are velocity or redshift dependent. Of course a true vacuum is gradient and curl free, which means in a static universe Eqs (19) reduce to the usual 4D vacuum electrodynamics. The new fields introduced in 5D add effective source terms to the 4D electrodynamic equations resulting from the expansion of the vacuum in an expanding universe. So solutions to Eqs (19a) and (19b) are the sum of the usual vacuum solution in 4D electrodynamics and a particular solution of these respective equations, which only comes from the new dimension.

If we further assume the medium not only is linear and isotropic but the fields are weak then the electric field can be expressed in terms of polarization $\mathbf{P}$,

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}, \tag{20a}$$

and the magnetic field in terms of magnetization $\mathbf{M}$,

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}. \tag{20b}$$

Substituting Eq. (20a) into Eq. (19a) we get

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} + 4\pi\left(\nabla^2\mathbf{P} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}\right) = -4\pi\nabla\rho_{eff}, \tag{21a}$$

where the first two terms are zero for the propagation of the $\mathbf{E}$-field in vacuum. This means it can be written

$$\nabla^2\mathbf{P} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{P} = -\nabla\rho_{eff}. \tag{21b}$$

This means the polarization field travels as a damped wave with damping dependent only on redshift and distance. Similarly, using Eq. (20b) in Eq. (19b) we get

$$\nabla^2\mathbf{M} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{M} = \nabla\times\tilde{j}_{eff}, \tag{22a}$$

where it is also assumed the $\mathbf{H}$-field propagates through the vacuum. The magnetization field also is a damped wave with damping dependent on redshift and distance.

If we further assume that $\mathbf{P}$ is time independent we have

$$\nabla^2\mathbf{P} = -\nabla\rho_{eff}, \tag{21c}$$

which may be integrated for the effective polarization of the vacuum. Also

$$\nabla^2\mathbf{M} = \nabla\times\tilde{j}_{eff}, \tag{22b}$$

where we have assumed time independence on $\mathbf{M}$ also. This gives us the effective magnetization of the vacuum.

CONCLUSION

Electrodynamics have been analyzed in a 5D universe starting with the suggested fields of Carmeli. The approach is quite general and need not necessarily depend on the exact details of those fields. The results follow if one assumes the extra dimension is the velocity of the expanding universe, after Carmeli. Electromagnetic waves are constructed in 5D but when compared with 4D electrodynamics the addition of the new dimension can be seen to add a dispersive component to the wave propagation constant. This may appear as a new energy scale for high redshift source photons and is consistent with observation of differences in the arrival times of high energy gamma rays from cosmological sources. However, though the theory indicates superluminal speeds for the wave group velocity its phase velocity remains subluminal by the same factor and hence energy is not passed faster than c. At very high redshift where the effects of the extra dimension are significant we find the vacuum to be both polarized and magnetized with density and currents related to the field $(\mathcal{H}, \mathbf{W})$.

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[1] M. Carmeli, *Cosmological Relativity* (World Scientific, Singapore, 2006)
[2] M. Carmeli, Editor, *Relativity: Modern Large-Scale Spacetime Structure of the Cosmos* (World Scientific, Singapore, 2008)