

# Angular momentum of light with plane phase front

Radi I. Khrapko\*

Moscow Aviation Institute, 125993, Moscow, Russia

We consider two different types of angular momentum of electromagnetic radiation. 1) Moment of linear momentum, which we consider as orbital angular momentum. 2) Spin, which is not a moment of momentum; its origin is a circular polarization. We show that a circularly polarized light beam with plane phase front carries angular momentum of both types, spin and orbital angular momentum, contrary to the standard electrodynamics. Because of the conservation laws of momentum and total angular momentum, spin and moment of momentum have concrete values. These two types of angular momentum are spatially separated. Flux of spin and flux of moment of momentum act on an absorber independently. An experiment is described, which can verify this supposition.

*Key words:* Classical spin; optical torque; optical experiment

## 1. Introduction. Spin of light

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2] that circularly polarized light carries angular momentum. So, a torque acts on a body, which absorbs at least a part of the light or/and changes its polarization state. Poynting wrote,

“If we put  $w$  for the energy in unit volume and  $\mu_z$  for the torque per unit area, we have  $\mu_z = w\lambda / 2\pi = p\lambda / 2\pi$ , where  $p$  is the pressure exerted”.

In other words, according to Poynting, the ratio between the densities is

$$\frac{j_z}{w} = \frac{\mu_z}{S} = \frac{1}{\omega}, \quad (1.1)$$

where  $j_z$  and  $S$  are the angular momentum volume density and the energy flux density, i.e. the Poynting vector, respectively.

Now R. Feynman has explained the torque acting on an absorbing body [3]:

“The electric vector  $\mathbf{E}$  goes in a circle – as drawn in Fig. 17-5(a) [our Fig. 1]. Now suppose that such a light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics. We’ll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b) [our Fig. 1]. The electron is displaced at some displacement  $\mathbf{r}$  from its equilibrium position at the origin and goes around with some phase lag with respect to the vector  $\mathbf{E}$ . As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now let’s look at the work being done on this electron. The rate that energy is being put into this electron is  $v$ , its velocity, times the component of  $\mathbf{E}$  parallel to the velocity:

$$dW / dt = eE_t v. \quad (1.2)$$

But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is  $\tau = eE_t r$  which must be equal to the rate of change of angular momentum  $dJ_z / dt$ :

$$dJ_z / dt = eE_t r \quad (1.3)$$

Remembering that  $v = \omega r$ , we have that

$$dJ_z / dW = 1 / \omega. \quad (1.4)$$

Therefore, if we integrate the total angular momentum which is absorbed, it is proportional to the total energy – the constant of proportionality being  $1 / \omega$ .”

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\* email: [khrapko\\_ri@hotmail.com](mailto:khrapko_ri@hotmail.com), <http://khrapkori.wmsite.ru>

Feynman's reasoning holds for any atom. So, it entails that the angular momentum volume density  $j_z$  is proportional to the energy volume density  $w$ , and the torque per unit area  $\mu_z$  is proportional to the Poynting vector  $S$ , as in (1.1). Feynman explains that light which is right circularly polarized carries an angular momentum and energy in proportion to  $1/\omega$  because photons carry spin angular momentum  $\hbar$  and energy  $\hbar\omega$ . So, the angular momentum density  $j_z$ , which is proportional to the energy density, is the spin density  $s_z$ ,  $s_z = j_z$ .

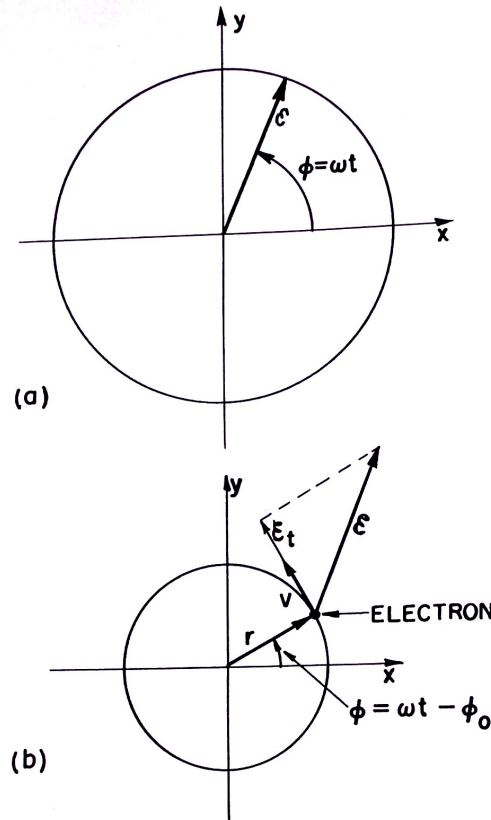


Fig. 17-5. (a) The electric field  $\mathbf{E}$  in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.

Fig. 1. The torque acting on an absorbing body

The Beth experiment [4] and many experiments on micro particles with tweezers confirm a presence of an angular momentum in a circularly polarized light. Beth used a transparent birefringent plate as the body. His reasoning leads to the proportionality between the densities as well:

“The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant  $\mathbf{K}$  is a tensor. Consequently the electric intensity  $\mathbf{E}$  is, in general, not parallel to the electric polarization  $\mathbf{P}$  or to the electric displacement  $\mathbf{D} = \mathbf{K}\mathbf{E} = \mathbf{E} + 4\pi\mathbf{P}$  in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is  $\tau/V = \mathbf{P} \times \mathbf{E}$ ”.

Also N. Carrara [5] wrote:

“If a circularly polarized wave is absorbed by a screen or is transformed into a linearly polarized wave, the angular momentum vanishes. Therefore the screen must be subjected to a torque per unit surface equal to the variation of the angular momentum per unit time. The intensity of this torque is  $\pm S/\omega$ .”

We noted [6] that the spin torque density produces a specific mechanical stress in the absorbing screen, and this effect may be tested experimentally [7].

## 2. Calculation of the light spin

Unfortunately, now there is no conventional expression for the spin torque density  $\mu_z$  in terms of electromagnetic fields. But an expression for the volume density of spin,  $s_z = j_z$ , is in use. This expression is

$$s_z = \varepsilon_0 \mathbf{E} \times \mathbf{A}, \quad (2.1)$$

where  $\mathbf{A}$  is the vector potential. Jackson: “The term  $\varepsilon_0 \int d^3x (\mathbf{E} \times \mathbf{A})$  is sometimes identified with the ‘spin’ of the photon” [8]. Ohanian: “The term  $\varepsilon_0 \int \mathbf{E} \times \mathbf{A} d^3x$  represents the spin” [9]. The expression  $\mathbf{E} \times \mathbf{A}$  is used by Friese et al. [10]. If  $\mathbf{E} = \mathbf{E}_0 \exp[i(kz - \omega t)]$ , then  $\mathbf{A} = -i\mathbf{E}/\omega$  because  $\mathbf{A} = -\int \mathbf{E} dt$ . The authors [10] wrote: “The angular momentum of a **plane electromagnetic wave** of angular frequency  $\omega$  can be found from the electric field  $\mathbf{E}$  and its complex conjugate  $\mathbf{E}^*$  by integrating over all spatial elements  $d^3r$  giving  $\mathbf{J} = (\varepsilon_0 / (2i\omega)) \int d^3r \mathbf{E}^* \times \mathbf{E}$ ”. Crichton & Marston: “The spin angular momentum density,  $s_i = E_j^* (-i\varepsilon_{ijk}) E_k / (8\pi\omega)$ , is appropriately named in that there is **no moment arm**” [11].

The expression (2.1) is successfully used for simple plane waves. But it appears that the expression is wrong for a somewhat complicated wave. Really, consider a standing wave

$$E^x = \sin z \sin t, \quad E^y = -\sin z \cos t, \quad B^x = -\cos z \sin t, \quad B^y = \cos z \cos t, \quad (2.2)$$

$$A^x = \sin z \cos t, \quad A^y = \sin z \sin t. \quad (2.3)$$

According to (2.1), the spin density has a laminated structure in this wave,  $s_z = 2E^{lx} A^{yl} = \sin^2 z$ , and  $s_z = 0$  there where  $\mathbf{E} = 0$ . It is strange.

However, no authors point out that the expression  $\varepsilon_0 \mathbf{E} \times \mathbf{A}$  is a time component of the canonical spin tensor [12 (4-150)]

$$Y_c^{\lambda\mu\nu} = -2A^{[\lambda} \delta_{\alpha}^{\mu]} \frac{\partial \mathcal{L}}{\partial (\partial_\nu A_\alpha)} = -2A^{[\lambda} F^{\mu]\nu}, \quad (2.4)$$

$$Y_c^{ijt} = \mathbf{E} \times \mathbf{A}. \quad (2.5)$$

Here  $\mathcal{L} = -F_{\mu\nu} F^{\mu\nu} / 4$  is the canonical Lagrangian,  $F^{\mu\nu} = F_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta}$  and  $F_{\alpha\beta} = 2\partial_{[\alpha} A_{\beta]}$  is the field strength tensor. The canonical spin tensor is not adequate. The sense of a spin tensor  $Y^{\lambda\mu\nu}$  is as follows. The component  $Y^{ijt}$  is a volume density of spin. This means that  $ds^{ij} = Y^{ijt} dV$  is the spin of electromagnetic field inside the spatial element  $dV$ . The component  $Y^{ijk}$  is a flux density of spin flowing in the direction of the  $x^k$  axis. For example,  $ds_z / dt = ds^{xy} / dt = d\tau^{xy} = Y^{xyz} da_z$  is the  $z$ -component of spin flux passing through the surface element  $da_z$  per unit time, i.e. the torque acting on the element.

In the plane wave

$$E^x = \cos(z-t), \quad E^y = -\sin(z-t), \quad B^x = \sin(z-t), \quad B^y = \cos(z-t), \quad (2.6)$$

$$A^x = \sin(z-t), \quad A^y = \cos(z-t), \quad (2.7)$$

the expression (2.4) promises flux densities of spin flowing in the direction of the  $x$  and  $y$  axes

$$Y_c^{zxy} = A^x B_x = \sin^2(z-t), \quad Y_c^{yzx} = A^y B_y = \cos^2(z-t). \quad (2.8)$$

This is absurdity, of course. As a result, we state that the being used expression (2.1) for spin density is somewhat unsatisfactory, and there is no expression for torque density. Nevertheless, the total spin of a wave in a volume is usually calculated by the integral

$$\text{spin} = \epsilon_0 \int \mathbf{E} \times \mathbf{A} dV \quad (2.9)$$

(we cannot use here the character  $S$ , which is the Poynting vector).

### 3. Angular momentum of another nature

Light spin density, which is proportional to energy density, and which sometimes can be calculated by the formulae (2.1), (2.9), has a distinguishing feature: spin angular momentum has no momentum arm, as [11] noted. However, angular momentum of another nature exists at the lateral surface of a circularly polarized wave, i.e. at the lateral surface of a circularly polarized beam. The point is that there are longitudinal components of electromagnetic fields near the lateral surface of a wave because the field lines are closed loops [9]. It entails a rotary mass-energy flow and, correspondingly, an angular momentum, which is determined by the momentum arm  $\mathbf{r}$ .

$$\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV. \quad (3.1)$$

We name this angular momentum the *orbital* angular momentum and, correspondingly, the density is named the orbital angular momentum density  $\mathbf{l} = (\mathbf{r} \times \mathbf{S}) / c^2$

Heitler: “Consider a cylindrical wave with its axis in the  $z$ -direction and traveling in this direction. At the wall of the cylinder,  $r = R$ , say, we let the amplitude drop to zero. It can be shown that the wall of such a wave packet gives a finite contribution to  $\mathbf{L}$ ” [13]. Note,  $\mathbf{S}$  in (3.1) is the orbital component of the Poynting vector, not the Poynting vector  $\mathbf{S}$  itself as in (1.1). So, the density of angular momentum (3.1) is not connected with the density of spin  $j_z = s_z$  (1.1), (2.1), which is proportional to the mass-energy density  $w$  and the Poynting vector  $S$ .

Many authors, for example [8], consider the cylindrical beam in the form

$$\mathbf{E} = \exp(ikz - i\omega t) [\mathbf{x} + iy + \frac{\mathbf{z}}{k} (i\partial_x - \partial_y)] E_0(x, y), \quad \mathbf{B} = -i\mathbf{E} / c, \quad (3.2)$$

or in the cylindrical coordinates  $r, \varphi, z$  with the metric  $dl^2 = dr^2 + r^2 d\varphi^2 + dz^2$ , in the form

$$\mathbf{E} = \exp(ikz - i\omega t + i\varphi) (\mathbf{r} + ir\bar{\varphi} + \frac{\mathbf{z}}{k} i\partial_r) E_0(r), \quad \mathbf{B} = -i\mathbf{E} / c, \quad (3.3)$$

The orbital angular momentum density was found to be [14,15]

$$l_z = -\epsilon_0 r \partial_r |E_0(r)|^2 / 2\omega. \quad (3.4)$$

Simmonds and Guttman: “From (3.2) the electric and magnetic fields can have a nonzero  $z$ -component only within the skin region of this wave. Having  $z$ -components within this region implies the possibility of a nonzero  $z$ -component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only in which the  $z$ -component of angular momentum does not vanish” [16, p. 226].

It is important to accentuate the orbital angular momentum (3.1) exists though the beam (3.2), (3.3) has no azimuthal phase structure. The beam has plane phase front in spite of the exponential factor  $\exp(i\varphi)$  in (3.3). We do not consider here Laguerre-Gaussian beams of type  $\text{LG}_p^l$ . An azimuthal phase dependence appears because of the exponential factor  $\exp(i\varphi + il\varphi)$ .

Energy volume density in the beam (3.2), (3.3) is

$$w = \epsilon_0 E_0^2. \quad (3.5)$$

Therefore the ratio between the densities,

$$\frac{l_z}{w} = -\frac{r \partial_r |E_0(r)|^2}{2\omega |E_0(r)|^2}, \quad (3.6)$$

has a sharp maximum near the beam boundary, in contrast to (1.1).

#### 4. The Humblet equality is broken

Consider a section of the circularly polarized beam (3.2). It is interesting to compare the total spin (2.9), which is distributed uniformly in the section, and the total angular momentum (3.1), which is located near the beam surface. Humblet [17] and many others [6,18, ...] transformed (3.1) and showed that the quantities equal to each other,

$$\epsilon_0 \int \mathbf{E} \times \mathbf{A} dV = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV. \quad (4.1)$$

Nieminen et al.: “For any physically realisable beam both expressions for the angular momentum density yield the same total angular momentum and torque exerted on an object” [19].

On this ground an inference was made that spin (2.9) and angular momentum (3.1) are the same *matter* in spite of the fact that they are spatially separated. Ohanian: “This angular momentum (3.1) is the spin of the wave” [8].

Jackson [8] and Becker [20] agree that the Humblet equation (4.1) identifies angular momentum (3.1) with spin (2.9). To confirm this identification, they try to generalize the equation (4.1) to a free electromagnetic radiation produced by a source localized in a finite region of space. They apply the Humblet transformation with the integration by parts to the radiation and obtain the equality (4.1).

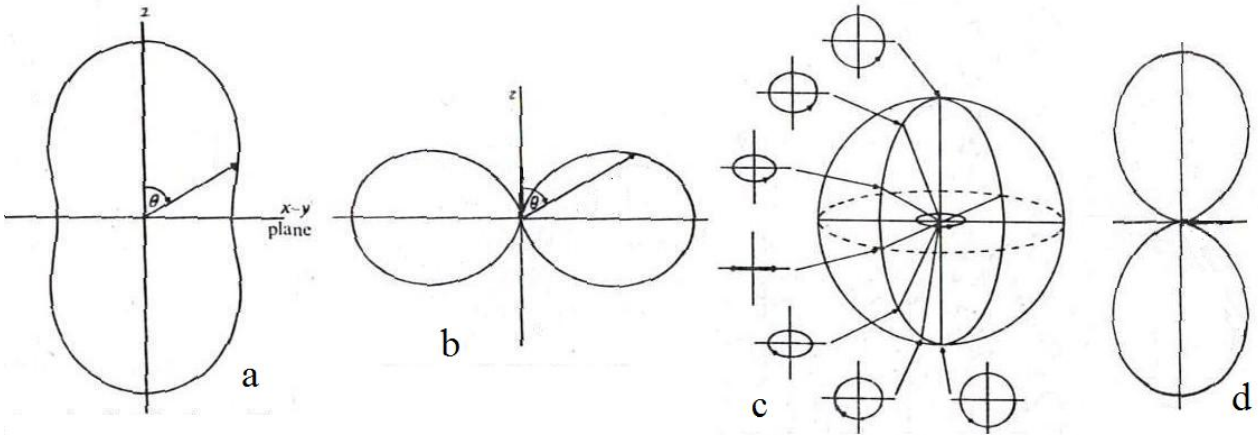
**But they are mistaken!** The integration by parts cannot be used when radiating into space. A straight calculation presented in [21] for the radiation of a rotating dipole gives

$$2\epsilon_0 \int \mathbf{E} \times \mathbf{A} dV = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV. \quad (4.2)$$

Somewhat such result must be expected because when radiating into space photons are variously directed, and their spins are not parallel to each other as in a beam. As a result, equality (4.2) proves the moment of momentum  $\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV$  is not the spin  $\epsilon_0 \int \mathbf{E} \times \mathbf{A} dV$  !

**Our conclusion is: orbital angular momentum (3.1) is not spin!**

The spatial separation of quantities  $\epsilon_0 \mathbf{E} \times \mathbf{A}$  and  $(\mathbf{r} \times \mathbf{S})/c^2$  is obvious for a light beam. The separation for the radiation of a rotating dipole is depicted in Fig. 2. In this case moment of momentum,  $\frac{1}{c^2} (\mathbf{r} \times \mathbf{S})$ , is radiated mainly near the plane of rotating of the dipole (Fig. 2b), while spin,  $\epsilon_0 \mathbf{E} \times \mathbf{A}$ , exists near the axis of rotation (Fig. 2d), where the radiation is circularly or elliptically polarized [22].



**Fig. 2** [23]. (a) Angular distribution of the rotating dipole radiation,  $S_r \propto (\cos^2 \theta + 1)$ . (b) Angular distribution of z-component of the moment of momentum flux,  $dL_z / dtd\Omega \propto \sin^2 \theta$ . (c) Polarization of the electric field seen by looking from different directions at the rotating dipole. (d) Angular distribution of z-component of the spin flux,  $d(\text{spin})_z / dtd\Omega \propto \cos^2 \theta$ .

There is another important circumstance, which prevent the interpretation of the integral  $\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV$  as spin of a *radiation*. Vectors  $\mathbf{E}$  and  $\mathbf{B}$  of a radiation are perpendicular to the direction of the propagation i.e.  $(\mathbf{E} \times \mathbf{B}) \times \mathbf{k} = \mathbf{S} \times \mathbf{k} = 0$ , where  $\mathbf{k}$  is the wave vector. So

$(\mathbf{r} \times \mathbf{S}) \cdot \mathbf{k} = 0$  for any radiation. Therefore the moment of momentum  $\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV$ , must be calculated by the use of the non-radiative field, which is proportional to  $1/r^2$  in the case of a radiation into space. This indicate non-radiative nature of the moment of momentum

$\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV$  while spin is an attribute of a radiation and must be calculated by the use of fields, which is proportional to  $1/r$  only. Heitler, when defending the spin nature of the moment of momentum, refers to a subtle interference effect on this subject [13]. But this explanation seems to be not convincing.

Besides, as we emphasized [6], the possibility to calculate the moment of momentum by integrating over volume (4.2) means nothing. Really, consider an analogous integral  $\int \mathbf{r} \times \mathbf{j} dV$  over the surface of a long solenoid where  $\mathbf{j}$  is an electric current density of the solenoid. We have

$$\int \mathbf{r} \times \mathbf{j} dV = \int \mathbf{r} \times (\nabla \times \mathbf{H}) dV$$

$$= \int (r^i \partial_k H_i - r^i \partial_i H_k) dV = \int [\partial_k (r^i H_i) - H_k - \partial_i (r^i H_k) + \partial_i r^i H_k] dV = \int 2\mathbf{H} dV \quad (4.3)$$

This equality between the moment of an electric current and the integral of  $\mathbf{H}$  over the solenoid volume does not prove that  $\int \mathbf{r} \times \mathbf{j} dV$  and  $\int 2\mathbf{H} dV$  are the same *matter*. So, the hope that “performing an integration by parts **moves** the nonzero values of the density of  $\mathbf{J}$  from the border to the bulk of the beam” [26] is futile.

On our opinion, it is necessary to concede that  $\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{S}) dV$  represents a moment of momentum, which has an orbital nature and does not represent spin of an electromagnetic radiation.

### 5. Absorption of the circularly polarized beam

Nieminen et al. claim that both expressions for the angular momentum density,  $\varepsilon_0 \mathbf{E} \times \mathbf{A}$  and  $\frac{1}{c^2} (\mathbf{r} \times \mathbf{S})$ , yield the same total angular momentum and torque exerted on an object [19]. But the question arises: *where* is the torque exerted?

Suppose that the cylindrical beam of radius  $R$  is absorbed by an absorber, which is divided concentrically at radius  $r_1$  into an inner part where  $r < r_1 < R$  and outer corresponding part ( $r > r_1$ ) such that the skin of the beam is absorbed by the outer part (Fig. 3a). Will the inner part perceive a torque (and rotate)? This question is of critical importance.

Really, if the inner part does not perceive a torque, spin angular momentum of a photon disappears or is absorbed on peripheries of the absorber while energy of the photon is absorbed on the inner region. If the inner part does perceive a torque, this cannot be explained in terms of the Maxwell stress tensor of electromagnetic field because this tensor provides no tangential forces in the inner part [16]. Also note there is no angular momentum flux in the radial direction.

On spring 1999 the distribution of angular momentum across a circularly polarized beam was discussed at V.L. Ginzburg Moscow Seminar, and the problem was formulated in terms of an experiment [24].

Answering the question [24], Allen et al. [25] represent our beam as the superposition of two parts,

$$E_0(r) = E_{in}(r) + E_{out}(r), \quad (5.1)$$

such that the radius of the inner part is  $r_1 < R$ , and the outer part looks like a thick-wall tube located approx between  $r_1$  and  $R$ . The authors conclude the inner part of the absorber does perceive a torque by action of angular momentum (3.1), in accordance with (3.4), because  $\partial_r E_{in}^2$  is not zero. So, as we can understand, Allen et al. conclude that the ratio  $l_z / w$  is constant in the beam's interior and has no maximum in the skin region, contrary to (3.6).

We criticized this conclusion in [6]. It seems that we must take into account the both components,  $E_{in}$  and  $E_{out}$ , of the superposition and the interference between them. Then we obtain zero for the angular momentum density (3.4) because

$$l_z = -\epsilon_0 r \partial_r |E_0(r)|^2 / 2\omega = 0. \quad (5.2)$$

at any point of inner or outer part of the body except the skin region; the zero will hold when the (constant) sum  $E_0(r)$  from (5.1) is substituted into (5.2). So, taking into account the angular momentum (3.1) only, we have to conclude the torque acts on a periphery of the absorber only.

## 6. Experiment

Here we describe an experiment, which will show *where* the torque is exerted in reality. A half-wave plate is used instead of the absorber. The plate reverses the handedness of the circular polarization, so that the plate experiences a torque. If the plate rotates in its own plane, work will be done. This (positive or negative) amount of work must reappear as an alteration in the energy of the photons, i.e., in the frequency of the light, which will result in moving interference fringes in any suitable interference experiment [27].

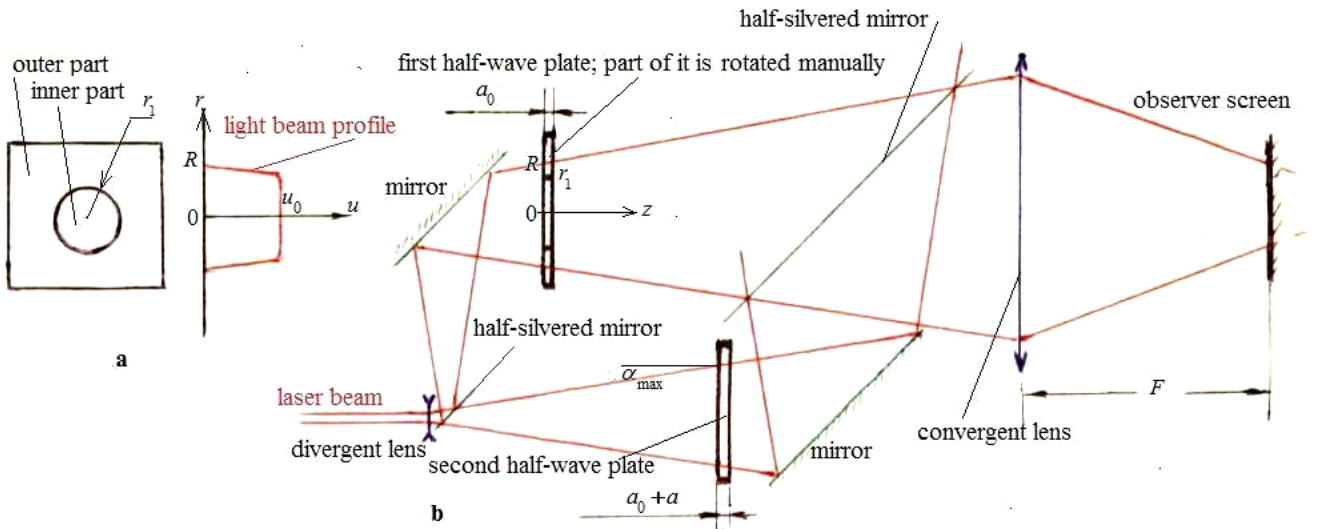


Fig. 3a Half-wave plate, parts of which are rotated manually. Fig. 3b Circularly polarized beam is divided into two beams which go through half-wave plates and then interfere at the screen

Let  $\mu$  denotes an angular momentum flux density into the half-wave plate, i.e., density of torque acting on points of the plate and  $S$  denotes the normal component of Poynting vector. Because of the reverse of the handedness we have

$$\frac{\mu}{S} = 2 \frac{j_z}{w}. \quad (6.1)$$

Thus a rotation of the half-wave plate with the angular frequency  $\Omega$  yields the alteration in the energy of the photons

$$\Delta S = \mu \Omega = 2 \frac{j_z}{w} \Omega S, \quad (6.2)$$

and the light angular frequency alteration

$$\Delta\omega = 2 \frac{j_z}{w} \Omega \omega. \quad (6.3)$$

Corresponding phase shift in time  $t$  is  $\varphi = \Delta\omega t$ . The phase shift per revolution ( $\Omega t = 2\pi$ ) is

$$\Phi = 4\pi \frac{j_z}{w} \omega, \quad (6.4)$$

and the fringes shift per revolution is

$$N = 2 \frac{j_z}{w} \omega. \quad (6.5)$$

To answer the question [24], we propose to place two half-wave plates in the paths of the beams in a two-beams interferometer, but one of the plates must be divided into an inner disc and a closely fitting outer part so that the both parts can be rotated just as in the Righi experiment [27], but independently from each other (see Figure 3b). The half-wave plates differ in thickness by a small value  $a$ . Because of the difference, interference rings occur at the observer screen where the beams are superimposed.

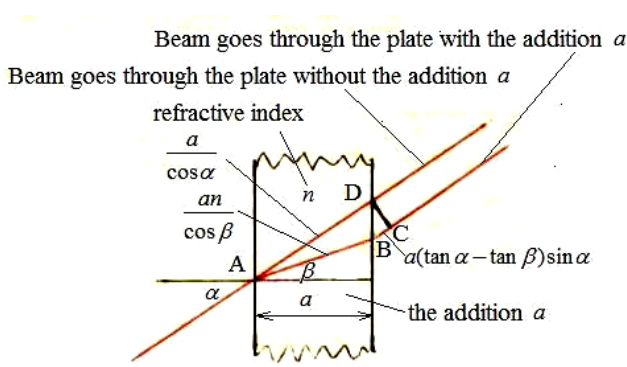


Fig. 4. Calculation of the path difference ABC – AD

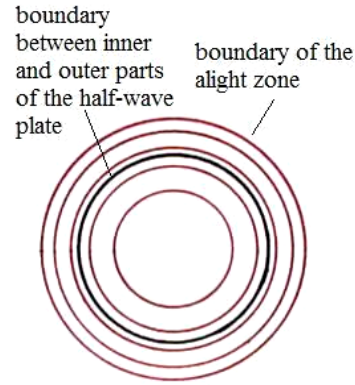


Fig. 5. Interference rings

A calculation of the path difference is presented in Figure 4. If the angles of incidence of the beams are  $\alpha$ , path ABC equals  $an / \cos \beta + a(\tan \alpha - \tan \beta) \sin \alpha$ , and corresponding path AD through air equals  $a / \cos \alpha$ . The condition of constructive interference is  $an / \cos \beta + a(\tan \alpha - \tan \beta) \sin \alpha - a / \cos \alpha = k\lambda$ , i.e.

$$n \cos \beta - \cos \alpha = k\lambda / a, \quad k = 0, 1, 2, \dots \quad (6.6)$$

If  $\sin \alpha = \alpha$ , equation (6.6) gives

$$n - 1 + \alpha^2(n - 1) / n = k\lambda / a. \quad (6.7)$$

Omitting constant term  $n - 1$  we obtain the angular size of a ring number  $k$

$$\alpha_k = \sqrt{\frac{n\lambda k}{(n - 1)a}}. \quad (6.8)$$

Let  $\lambda = 630 \text{ nm}$  and a quartz half-wave plate be in use,  $n = 1.55$ ,  $\Delta n = n_o - n_e = 0.009$ . Then the minimal thickness of a half-wave plate is  $l_{1/2} = \lambda / 2\Delta n = 35 \mu\text{m}$ . If we put

$a = 9l_{1/2} = 315 \mu\text{m}$ ,  $\alpha_k = 0.0751\sqrt{k}$ , and  $k_{\text{max}} \leq 177\alpha_{\text{max}}^2$ . According to Figure 1,  $\alpha_{\text{max}} \approx 10^\circ = 0.175$ . So,  $k_{\text{max}} = 5$ . These five rings are depicted in Figure 5.

If spin (2.9) exists, the fringes shift (6.5) must be equal to 2 when the inner part is rotated. If angular momentum (3.1) exists, the enormous fringes shift must be at the edge of the alight zone when the outer part is rotated.

Now a description of the experiment is published [28].



## Conclusion

Simmonds and Guttman [16] claimed: "A classical quantity associated with the electromagnetic field does not necessarily indicate the value of that quantity which will be measured. The angular momentum density of the wave was zero at the center, yet when we attempted to measure it there the classical field adjusted themselves and produced a nonzero measurement". We suggest an experiment to explain this magic trick.

## Acknowledgments and Notes

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [24] (submitted on 7 October 1999) and to Professor Timo Nieminen who drew my attention to paper [27].

Unfortunately, this experiment was rejected many times groundlessly.

**Applied Optics.** March 02, 2009

Applied Optics is not the appropriate forum for this interesting theoretical discussion. **Scott Tyo**

**Journal of Modern Optics.** July 24, 2009

I have had the "pleasure" of reviewing a large number of papers by this author on his alternative theory of optical angular momentum. At one stage, I recall, his manuscripts proudly proclaimed the long list of journals that had rejected his work. The author believes that there is an additional spin angular momentum for the photon, that is not present in standard (Maxwell-based) theory and all of his papers that I have seen are based on this, shall we say "dubious" idea. The conventional (Maxwell and Poynting - based) theory of optical angular momentum is in excellent agreement with all recent experiments and there is no need nor evidence for any correction of the type envisaged by the author. **Jonathan Marangos**

**New Journal of Physics.** August 07, 2009

We do not publish this type of article in any of our journals and so we are unable to consider your article further. **Sarah Ryder, Dr Tim Smith, Dr Elena Belsole, Rosie Walton, Dr Chris Ingle**

**Physical Review A.** August 18, 2009

The Physical Review publishes articles in which significant advances in physics are reported. Such advances must be placed in the context of recent developments in research. There is no discussion in your manuscript of how this work relates to other current physics research and adequate references to the recent research literature are lacking. Your manuscript therefore is too pedagogical for the Physical Review. **Gordon W.F. Drake**

**International Journal of Optomechatronics.** December 10, 2009

Your paper has not been considered for publication in this current form. **Hyungsuck Cho.**

**Physics Letters A.** December 26, 2009

Your theoretical paper does not contain the physical results which need an urgent publication in Physical Journal of Letters. **Vladimir M. Agranovich.**

**Journal of Modern Optics** January 13, 2010

The author has clearly failed to understand the phenomenon of the transfer of spin angular momentum to a birefringent optical element. A clear and rather straightforward account of this may be found, for example, in reference [4] for the manuscript. The author's "solution" to his "problem" is no less than to change the laws of electrodynamics, something for which there is no need and no evidence. **Jonathan Marangos**

**Optics Communications.** January 18, 2010

The author demonstrates his complete lack of understanding of the phenomenon of the transfer of angular momentum from a light beam to a birefringent element. He maintains, totally erroneously, that conventional Maxwell theory fails to account for this effect, something that is clearly explained in reference [4]. The author's "solution" to his "problem" is nothing less than a re-formulation of electromagnetism, something for which there is both no evidence and no need. The paper is just plain wrong and needs to be rejected.

The author is very proud of the fact that a previous **idiotic** paper was turned down over a hundred times. His arguments are confused and wrong; he is insulting to others whose work he does not understand. He inserts bits of referees comments into his next submission while not understanding them or learning anything. **Wolfgang Schleich**

**Optics Letters.** February 23, 2010

As we've stated in the past, we will not reconsider your work for publication in this or any other OSA journal. Please submit your paper elsewhere. Sincerely, **anonym.**

**Foundations of Physics.** May 04, 2010

I must inform you that, based on the advice received, the Editors have decided that your manuscript cannot be accepted for publication in Foundations of Physics. Below, please find the comments for your perusal. **Gerard 't Hooft**

The comments and my objections, please, see at

<http://khrapkori.wmsite.ru/ftpgetfile.php?id=45&module=files>

**American Journal of Physics.** May 25, 2010

You propose to do an experiment that you claim is easy to do, but don't do it as far as I can tell. At least I don't see any experimental data. Thus, your work would not be appropriate for an educational journal such as AJP. **Jan Tobochnik, Harvey Gould**

**Acta Physica Polonica.** September 23, 2011

The proposed experiment is useless; its result is easily predicted. Everything is based on misunderstanding. The author assumes that the absorption of photons occurs locally (where the energy density is the biggest). In fact, the process of absorption is nonlocal (best illustration: absorption of extended radiation by a small atom). **Witold Dobrowolski**

**PRA.** January 05, 2012

The author does not make an effort to put the question into a context of current research or developments. The manuscript addresses an already ten-year old partial discussion between the author (ref [11]) and Allen and collaborators (ref [12]). **M. Gaarde & G. Drake.**

**Optics Communications.** August 14, 2012

As I am unable to find willing reviewers and expect this process to take a long time, I have decided to discontinue considering your article. **Barry C. Sanders**, PhD Editor of Optics Communications

**Journal of the Optical Society of America B.** October 18, 2012

A paper is acceptable for publication in JOSA B only if there is convincing evidence that, in addition to being correct technically, it also adds a new and important result to the field. I have found that your paper does not meet this criterion for the widely researched subject of light angular momentum. **Henry Van Driel**

Please see my comments at <http://khrapkori.wmsite.ru/ftpgetfile.php?id=103&module=files>

**Advances in Optics and Photonics** October 24, 2012

You have recently submitted this work to JOSA B, where it was declined for publication. We will not reconsider this article further. Sincerely, AOP Manuscripts Office

**Journal of Optics** October 30, 2012

Your paper does not meet the criteria and thus does not warrant publication in Journal of Optics.

**Daniel Heatley, Felicity Inkpen, Claire Bedrock, Rachael Kriefman**

**Journal of Physics B** October 30, 2012

As was recently noted to you, we do not publish this type of article in any of our journals. Yours sincerely Editorial office. (**Paul Corkum** is Editor-in-Chief)

**Foundations of Physics** November 16, 2012

I regret to inform you that the editors had to conclude that this work is not suitable for publication in Foundations of Physics. **Gerard 't Hooft**

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