Geometric Cosmology

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The modified cosmological model (MCM) is explored in the context of general relativity. A flaw in the ADM positive-definiteness theorem is identified. We present an exposition of the relationship between Einstein's equations and the precessing classical oscillator. Kaluza theory is applied to the MCM and we find a logical motivation for the cylinder condition which leads to a simple mechanism for AdS/CFT.

"Any intelligent fool can make things bigger, more complex, and more violent. It takes a touch of genius – and a lot of courage – to move in the opposite direction."

∼ Einstein

The modified cosmological model (MCM) is built on a reinterpretation of the dynamical bouncing seen in loop quantum cosmology [1]. In the prevailing interpretation, the universe passes through the bounce according to some external time. When we consider the bounce as a fixed feature of spacetime and move to the reference frame of the bounce, the external time is abandoned in favor of an internal time called chiros [2]. In this reference frame, the objects doing the bouncing are interpreted as the time evolved trajectories of the processes in the bounce – one in positive time and one in negative time.

$$\widehat{LQC}|bounce\rangle = |t_+\rangle + |t_-\rangle \tag{1}$$

Initially this model of distinct spacetimes was proposed to (among other things) fix the momentum problem in the big bang theory. Before the big bang there was no momentum but for the sake of convenient argument we equate this with $p^\mu = 0$. After the big bang it is possible to maintain $p^i = 0$ with all forward motion canceling backward, left right and up down. However, the product of the big bang only moves in one direction through time leading to $p^0 \neq 0$. Thus we introduce $|t_+\rangle$ and $|t_-\rangle$ so that $p^0_+ + p^0_- = 0$.

One of the early criticisms of the MCM was that "the author clearly has no concept of the ADM mass-energy" in reference to the classic positive-definiteness theorem for the zero component of the universe's 4-momentum [3]. In good keeping with the paradigm of that era, ADM modeled the cosmos on a non-orientable Riemannian manifold to derive a surface element at spacelike infinity $dS_i \equiv 1/2\,\epsilon_{ijk}dx^j dx^k$.

A significant result in modern cosmology, namely the multipole analysis of the WMAP data, indicates that there is a so-called "good axis" in the heavens. Considering this, the cosmos may be modeled more precisely on an orientable manifold where the surface element is a symplectic two-form $dS_i \equiv 1/2\,\epsilon_{ijk}dx^j \wedge dx^k$. We use the property $dx^j \wedge dx^k = -dx^k \wedge dx^j$ to show that there is a possibility not considered by ADM.

$$dS_i^+ \equiv \frac{1}{2}\,\epsilon_{ijk}dx^j \wedge dx^k \tag{2}$$

$$dS_i^- \equiv -\frac{1}{2}\,\epsilon_{ijk}dx^j \wedge dx^k \tag{3}$$

Which of these choices is the correct one? There is no reason to give precedence to one spatial dimension over the other and an observer will never be able to distinguish the two. For this reason we invoke Schrödinger's principle of cats to conclude that an observer will occupy a linear superposition of $|t_+\rangle$ and $|t_-\rangle$.

Despite the ADM result, many cosmologists adopt the viewpoint of a zero energy universe. This view is often motivated by phenomenological equation of state calculations. For an example see [4]. It is one of the many unities contained in the MCM that both of these views are accommodated. We accept the geometric argument made by ADM with the symplectic caveats (2-3). These possibilities form a superposition $|t_\star\rangle$ with $p^0_\star = 0$ in good agreement with EOS arguments for zero energy. The observer does not observe the state of $|t_+\rangle$ or $|t_-\rangle$ and therefore EOS arguments do not apply there.

Another excellent example of why the MCM should be adopted over ΛCDM is that it reduces to Einstein's equations for a classical oscillator parameterized by $\{r, \theta\}$ [2]. To get this result the following maps were invoked.

$$f^3\psi(x_\mu) \mapsto T_{\mu\nu} \tag{4}$$

$$i\Phi^2\psi(x^+_\mu) \mapsto G_{\mu\nu} \tag{5}$$

$$\psi(x^-_\mu) \mapsto g_{\mu\nu}\Lambda \tag{6}$$

The first map (4) is well-motivated because the energy density of the vacuum is proportional to the cube of the frequencies. Likewise, the last map (6) seems well motivated in that the state of a quantum spacetime would be the metric. In what follows we will explore the motivations for putting $i\Phi^2$ into the Einstein tensor.

Duality between string theory and the MCM is one of the best reasons to adopt it in favor of ΛCDM. To demonstrate this, consider two 5D spaces $\Sigma^\pm$ spanned

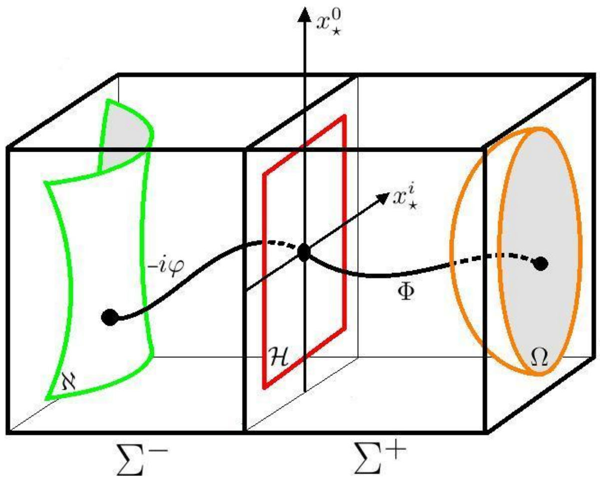

FIG. 1: A graphical representation of the MCM. Chronos flows upward and chiros flows toward the right.

by $\xi^A_\pm$ with A running from 0 to 4. Connect these spaces with a string having its behavior uniquely determined by two sets of 5D boundary conditions at its ends. In this circumstance the string effectively lives in 10D as it should. Truncate each Σ along $\xi^4 = 0$ so that ξ^4 takes on only positive values in Σ^+ and only negative in Σ^-. The resulting half spaces do not contain their respective boundaries at $\xi^4_\pm = 0$.

Join Σ^+ and Σ^- at this boundary as in figure 1. This boundary is 4D and we impose a condition that this is the flat observable universe. MCM quantum gravity is developed using three geometry basis states of constant curvature: one flat, one spherical and one hyperbolic. On each of these spacetimes we put one of three quantum vector spaces $\{\aleph, \mathcal{H}, \Omega\}$ [2].

In [1] we show that the string-like qualities of the MCM are represented with a string of length Φ in Ω (the future) and one of length $-\varphi$ in $\aleph$ (the past.) Since ξ^4_+ is always positive and ξ^4_- is always negative we will identify this dimension with string length. Having defined the flat space for $\mathcal{H}$ at $\xi^4 = 0$, we need to define spaces of constant curvature for $\aleph$ and Ω. To this end, impose a hyperboloid condition based on the string length in both Σ^+ and Σ^-.

$$\Phi^2 \;=\; -(\xi^0_+)^2 + \sum_{\alpha=1}^{4} (\xi^\alpha_+)^2 \tag{7}$$

$$-\varphi^2 \;=\; -(\xi^0_-)^2 + \sum_{\alpha=1}^{4} (\xi^\alpha_-)^2 \tag{8}$$

When we restrict $\xi^4_+ \in (0, \Phi]$ and $\xi^4_- \in [-\varphi, 0)$ the global geometry of the model will depend on Φ^2 and we see why it appears in $G_{\mu\nu}$. However, it is not yet clear why $G_{\mu\nu}$ should be complex.

Note that when a periodic boundary condition is imposed on ξ^4 it becomes identically chiros. Starting at the origin in $\mathcal{H}$ and moving in the direction of increasing chiros we reach Ω where a periodic boundary transports the trajectory to $\aleph$ before it reaches $\mathcal{H}$ again. The equivalence is illustrated by the tensor evolution operator for chiros defined in [2].

$$\hat{M}^3 : \mathcal{H} \to \Omega \to \aleph \to \mathcal{H} \tag{9}$$

$$\hat{M} := \partial_4 \tag{10}$$

The curvature of the spacetime between $\aleph$ and Ω should change smoothly implying the existence of an affine hyperboloid parameter γ such that the following conditions are satisfied.

$$\gamma^2\big|_\aleph \;= -\varphi^2 \tag{11}$$

$$\gamma^2\big|_\mathcal{H} \;= 0 \tag{12}$$

$$\gamma^2\big|_\Omega \;= \Phi^2 \tag{13}$$

We almost satisfy the conditions (11-13) when we set γ to ξ^4. Unfortunately, we do not recover the negative sign in (11). As with the Einstein tensor, we must introduce a factor of i into Σ^- to arrive at a consistent theory.

One possible method to introduce this factor is to invoke the basis vector $\hat{\imath}$ which identifies quantum states as belonging to $\aleph$. This condition is necessary to satisfy $\gamma \equiv \xi^4$ but it is not immediately clear that $\hat{\imath}$ should be extended throughout Σ^-. To develop an argument for why Σ^- should acquire a complex phase with respect to Σ^+, let there be a standard Kaluza metric g_{AB} in both Σ spaces. See [5] for an excellent review of Kaluza-Klein theories.

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix} \tag{14}$$

A scalar field ϕ appears in this metric, $g_{\alpha\beta}$ is the 4D metric tensor, A_α is the electromagnetic potential and $\kappa \equiv 4\sqrt{\pi G}$.

Assuming for now that ξ^4 is imaginary in Σ^- and real in Σ^+, the subspaces $\aleph$ and Ω are surfaces of constant ξ^4. This provides a logical motivation to invoke Kaluza's cylinder condition $\partial_4 f = 0$ for any function $f(\xi^A)|_{\aleph,\Omega}$. In solidarity with Einstein, we consider the case where there is no 5D matter-energy so that 4D physics may be motivated purely through higher-dimensional geometric considerations. The absence of higher-dimensional matter-energy sets a condition on the 5D Ricci tensor.

$$R_{AB} = 0 \tag{15}$$

Using the cylinder condition and equation (15) we may derive the following field equations in $\aleph$ and Ω where $T^{EM}_{\alpha\beta} \equiv g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}/4 - F^\gamma_\delta F_{\beta\gamma}$ and $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$.

$$
\begin{aligned}
G_{\alpha\beta} &= \frac{\kappa^2\phi^2}{2}T^{EM}_{\alpha\beta} - \frac{1}{\phi}[\nabla_\alpha(\partial_\beta\phi) - g_{\alpha\beta}\Box\phi] \quad (16)\\
\nabla^\alpha F_{\alpha\beta} &= -3\frac{\partial^\alpha\phi}{\phi}F_{\alpha\beta} \quad (17)\\
\Box\phi &= \frac{\kappa^2\phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \quad (18)
\end{aligned}
$$

If we require that the scalar field ϕ is a function of chiros only, we recover the Einstein and Maxwell equations. Note that these are only valid in subspaces such as $\aleph$ and Ω where we have have a well-motivated cylinder condition.

$$
\begin{aligned}
G_{\alpha\beta} &= 8\pi G\,\phi^2\,T^{EM}_{\alpha\beta} \quad (19)\\
\nabla^\alpha F_{\alpha\beta} &= 0 \quad (20)
\end{aligned}
$$

While $\aleph$, $\mathcal{H}$ and Ω appear disconnected in figure 1 it is important to recall that these spaces form a Gel'fand triple $\{\aleph, \mathcal{H}, \Omega\}$ where $\aleph \subset \mathcal{H} \subset \Omega$. In this picture there is a direct causal connection of the spaces. If electromagnetic potentials in $\Sigma^\pm$ are equated with the advanced and retarded potentials $A_\alpha(t_+)$ and $A_\alpha(t_-)$ of classical electromagnetism we are able to induce electromagnetism in $\mathcal{H}$ though it was never part of any 5D space.

$$
A_\alpha\big|_{\mathcal{H}} = c_1 A^+_\alpha\big|_\Omega + c_2 A^-_\alpha\big|_\aleph \quad (21)
$$

The constants c_1 and c_2 take the value 1/2 in the classical theory but presently they may be determined from higher-dimensional boundary conditions.

If our theory is robust it should also be true that the flat metric $\eta_{\mu\nu}$ in $\mathcal{H}$ is smoothly connected to the metric in Σ^+ and Σ^-. We may express the smoothness condition as follows.

$$
\lim_{\xi^4\to 0^+} g^+_{AB} + \lim_{\xi^4\to 0^-} g^-_{AB} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \quad (22)
$$

To solve this problem we mark all components of the metric in Σ^+ with a + sign and likewise for Σ^-. It is trivial to derive the following conditions for the limit $\xi^4 \to 0$.

$$
\begin{aligned}
\pm i\phi_+ &= \phi_- \quad (23)\\
i\kappa_+ &= \kappa_- \quad (24)\\
-iA^+_\alpha &= A^-_\alpha \quad (25)\\
\eta_{\mu\nu} &= g^+_{\alpha\beta} + g^-_{\alpha\beta} \quad (26)
\end{aligned}
$$

This is just the result that we have been looking for to motivate the complex factor i for the entire space Σ^- and by proxy the Einstein tensor. Note the good agreement of equations (26) and (12). Equation (22) is also satisfied with $\kappa_+ = \kappa_-$ and $A^+_\alpha = A^-_\alpha$ but we take the complex solutions for consistency.

An interesting consequence of this solution is that the electromagnetic fields in Σ^- will be imaginary. The energy is proportional to the field squared so we find a negative energy density in Σ^-. It is possible that this energy is related to the negative energy solutions of the Klein-Gordon equation which can also be derived from the Kaluza metric.

An intuitive choice of coefficients in equation (21) is $c_1 = \Phi$ and $c_2 = -i\varphi$. In this case the real coefficients will sum to unity since the electromagnetic potential in Σ^- is imaginary and negative. If the constants c_1 and c_2 can be determined from geometry alone we are able to derive full AdS/CFT duality.

We have already shown that 5D gravity induces 4D electromagnetism in $\mathcal{H}$. Now consider that there is a conformal field theory in $\mathcal{H}$ which completely specifies the potential $A^\star_\alpha$. Using an off-diagonal component of equation (22) it is possible to write the following system of two equations in two unknowns.

$$
\begin{aligned}
\lim_{\xi^4\to 0^+} \kappa_+\phi_+^2 A^+_\alpha &= -\lim_{\xi^4\to 0^-} \kappa_-\phi_-^2 A^-_\alpha \quad (27)\\
A^\star_\alpha\big|_{\mathcal{H}} &= c_1 A^+_\alpha\big|_\Omega + c_2 A^-_\alpha\big|_\aleph \quad (28)
\end{aligned}
$$

This system can be solved for $A^\pm_\alpha$ with the the aid of equations (23-25). In turn we can write the electromagnetic stress-energy tensor in Σ^+ and Σ^- as a function of $A^\star_\alpha$. Making use of the cylinder condition and equation (19) we can determine Einstein's equations on Ω and $\aleph$.

$$
G_{\alpha\beta} = 8\pi G\,\phi^2(\xi^4)\,T^{EM}_{\alpha\beta}(A^\star_\mu) \quad (29)
$$

Given this, it is a straightforward boundary value problem in Ricci flow to determine the general form of the metric in Σ. Thus up to a scalar field acting as a gauge, a conformal field theory on a 4D boundary induces gravity in a 5D space.

Once the relevant quantities are determined, the equations of motion can be derived from the least action principle. The observer takes the form of a non-Dirac delta function $\delta(\xi^4)$ which returns an undefined value at $\xi^4 = 0$ [2]. As such the observer acts as a topological obstruction to direct integration over chiros. We propose a general form for MCM action.

$$
\mathcal{S} = \int_{t_1}^{t_2} \mathcal{L}_{\mathcal{H}}\,dt + \int_{-i\varphi}^{\Phi} \delta(\xi^4)\,\mathcal{L}_\Sigma\,d\xi^4 \quad (30)
$$

The $\mathcal{L}_\Sigma$ part of this action can be computed by performing a complex rotation [2]. We have shown that the phase factor associated with this path integral is the fine structure constant.

$$\frac{1}{\alpha} = 2\pi + (\Phi\pi)^3 \tag{31}$$

The factor $\Phi\pi$ is generated by translation from one basis geometry to the next. As illustrated by equation (9), we integrate from the present to future chiral infinity, to past chiral infinity and back to the present so that three factors of $\Phi\pi$ appear.

Before discussing 2π we note a historical precedent for this process. In 10th century Baghdad it was recognized by Islamic scholars that at each step in time, a given system will be annihilated and then reformed in the advanced time state [6]. This is essentially the process defined in equation (9). Such things are beyond the scope of this article, but why did the Greeks have two words for time? It is also worthwhile to note that 20 applications of the operator $\hat{M}$ to the state $|\psi\rangle\hat{\pi}$ will generate 13 powers of Φ giving a symmetry with the 20x13 Mayan Tzolkin. If the engineering principles of time travel can be developed on MCM physics that may explain many historical anomalies.

The factor 2π is associated with the periodic boundary condition on ξ^4 and may be related to a homology on the unit circle. The numbers φ and Φ lie on the real line inside and outside the unit circle respectively so one may easily imagine some manner of rotation through 2π mapping Φ to $-i\varphi$. For instance, it is an elementary exercise to map the unit circle to a spiral through 2π which starts at φ and ends on Φ.

When a quantum phase advances by 2π we say that nothing has changed in any way whatsoever. Why should this be? The Aharonov-Bohm effect gives good reason to conclude that the phase should have some physical manifestation. However, as quantum theory stands there is no manifestation and this leads to the strange identity $2\pi i = 0$. Given the material presented here and elsewhere, it seems that quantum theory takes place in a Poincaré section of a higher-dimensional space and it is the phase that connects these regimes. If this connection exists and can be developed we may discover a map $2\pi i \mapsto 0$ which will replace $2\pi i = 0$ in the formalism.

In the quantum regime equation (30) is replaced with the operator $\hat{\Upsilon}$ [2].

$$\langle\psi|\hat{\Upsilon}|\psi\rangle = \langle\psi|\hat{U}|\psi\rangle + \langle\psi|\hat{M}^3|\psi\rangle\hat{\pi} \tag{32}$$

This form is similar to the form of an elliptic function of a complex variable $y^2 = z + z^3$ which in general will have very complex solutions containing many highly symmetric subspaces. It may be in this domain that we recover a map $\Phi \mapsto -i\varphi$ associated with a rotation of 2π. We do not treat these things with rigor but only mention them to show what is possible.

The MCM was formulated on a cyclic universe model of bangs and crunches yet it is an eternal universe model. No matter how much time passes, the bang and crunch at conformal infinity will always be infinitely far away. For clarification see [2]. In place of a cyclic cosmology we have modeled the universe as a brane flowing through chiros. To preserve the idea regarding reference frames presented in equation (1), we may also consider a fixed brane through which chiros flows. In the latter case, what appears to be equilibrium in the brane is a lower-dimensional projection of non-equilibrium chiral flow. Structures of this type have been described by Prigogine [7]. His Nobel Prize winning result was to show that irreversible processes such as the flow of chiros can lead to novel dynamic states which he calls "dissipative structures." Is it possible that our universe is one of these structures? If so, we forge a new path toward a general covariant statistical mechanics.

We have previously shown that under certain conditions the third derivative can contribute to the motion of a classical object [2]. In this case, it may be true that the most general form of physical phase space is $3N$-dimensional rather than $2N$-dimensional.

$$\omega = \int dq\, dp\, d\chi \tag{33}$$

If it is true that the third derivative can contribute to the motion, the Hamiltonian function on which the quantum theory depends so closely $H = p\dot{q} - \mathcal{L}$ cannot completely specify the motion. We have shown that third derivative contributions are scaled by the fine structure constant and very small variations in the curvature of spacetime [2]. When these corrections to Hamiltonian motion are accordingly small it is possible that deviations from computed trajectories often attributed to friction may be caused in part by an insufficiency of the Hamiltonian.

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