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Full Length Research Paper

The affect of External Electric Field on the Lasing Mechanism in the Fluid

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Abstract: In this work the plasma hydrodynamical equations are exploited to explain the physical constraints under which amplification takes place. It is shown that lasing takes place in plasma in condition of concentration of electrons is less than the equilibrium concentration. In addition the amplification transpires when the internal field is stronger than the external applied field.

Keywords: plasma; lasing; inversion

Introduction

Laser technology plays a vital role in our day life. Beside its application in telecommunication and quantum computer, laser has been used in medicine.The important role played by laser in technology motivates us and many authors to search for a new mechanism for light amplification without inversion **[1-7]**.Such a new mechanism makes it possible to discover new laser types which can be applied in new fields and technologies**[8,9].**

Many attempts are made to search for new amplification mechanism. In some of them special relativity is consumed for predicting new mechanisms without inversion. The role of polarization in amplification is also discussed in some papers**[10,11].**In this work the hydro-dynamical equations for plasma is used to find the electric susceptibility. The electromagnetic theory is used to relate susceptibility to amplification factor and the conditions under which lasing takes place is discussed.

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The Role of Susceptibility and Conductivity in Amplification

The electromagnetic wave in any medium can be described by **[12,13,14].**

$$E = E_{\circ} e^{i(kz-\omega t)} \quad (1)$$

In any polarized material the polarization vector P takes the form **[4]**

$$P = \varepsilon_0 x E \quad (2)$$

And the electric flux density assumes the following relation **[17]**

$$D = \varepsilon E = \varepsilon_0 (1+x) E = \varepsilon_0 E + P \quad (3)$$

Hence the electric permittivity ε is given by

$$\varepsilon = \varepsilon_0 (1+x) \quad (4)$$

If it happens that, the electric dipole subtends an angle $\theta = (kx - \omega t)$,with respect to the external field E the electric susceptibility can bewritten as a complex quantity as

$$P = \chi E = (\chi_1 + i\chi_2) E \quad (5)$$

Therefore,εtakes the form

$$\varepsilon_1 + i\varepsilon_2 = \varepsilon_0 + \varepsilon_0 \left(\chi_1 + i\chi_2\right) \quad (6)$$

As a consequence, the related refractive index (n) becomes a complex quantity

$$n = n_1 + i n_2 = \frac{c}{v} = c\sqrt{\mu\varepsilon} \quad (7)$$

$$n_1^2 - n_2^2 + 2n_1 n_2 i = c^2 \mu\varepsilon$$

Hence,

$$n_1^2 - n_2^2 + 2n_1 n_2 i = c^2 \mu \left(\varepsilon_1 + i\varepsilon_2\right)$$

To this end one has

$$n_1^2 - n_2^2 = c^2 \mu\varepsilon_1 = c^2 \mu\varepsilon_0 (1+\chi_1) \quad , \quad 2n_1 n_2 = c^2 \mu\varepsilon_2 = c^2 \mu\varepsilon_0 \chi_2 \quad (8)$$

Bearing in mind that, the wave number k is given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{c} n = k_1 + i k_2 = \frac{\omega}{c}(n_1 + i n_2) \quad (9)$$

Utilizing (6), (8) and (9) one gets

$$k_1 = \frac{\omega}{c} n_1 , k_2 = \frac{\omega}{c} n_2 \ , k_2 = \frac{c\varepsilon_0 \mu\omega}{2n_1} \chi_2 \quad (10)$$

By substituting equation **(9)** in equation **(10)** one finds

$$\mathrm{E} = \tilde{\mathrm{E}}_0 e^{-k_2 z} e^{i(k_1 z + \omega t)} \quad (11)$$

Accordingly, the intensity inside the matter is given by

$$I = I_0 e^{\beta z} \quad (12)$$

By comparing equations (11) and (12), the gain coefficient can be given by

$$\beta = -2k_2 = \frac{-\mu\varepsilon_0 c\omega}{n_1}\chi_2 \quad (13)$$

The gain coefficient can be related to the conductivity by using the relation between current density *J* and polarization vector P where

$$J = \frac{\partial P}{\partial t} = \varepsilon_0 \chi \frac{\partial E_0 e^{i\omega t}}{\partial t} = i\omega\varepsilon_0 \chi E = \omega\varepsilon_0 (i\chi_1 - \chi_2)E \quad (14)$$

On the other hand, the conductivity is defined by

$$J = \sigma E = (\sigma_1 + i\sigma_2)E \quad (15)$$

Comparing (14) with (15) yields

$\sigma_1 = -\omega\varepsilon_0\chi_2$ and $\sigma_2 = \omega\varepsilon_0\chi_1$ (16)

Thus according to (13) and (16),the gain coefficient takes the form

Ionic waves in plasma

$$\beta = \frac{\mu c\sigma_1}{n_1} \quad (17)$$

In the presence of electric field the equation of ions in plasma obeys the fluid equation [15]

$$mn\left(\frac{\partial v}{\partial t} + v\nabla v\right) = enE - \gamma\nabla P \quad (18)$$

With v, n, m and E standing for, velocity of ions, ions number density, fluid mass density and total external electric field applied on the system,respectively.

Where the pressure is $P = nk_B T$ in case of constant temperature, the pressure gradient takes the form [8]

$$\nabla P = k_B T\nabla n \quad (19)$$

The motion of heavy ions in plasma generates low frequency. To obtain this frequency one assumes that the density of ions is equal the density of electrons. Thus equation of fluid in electric field becomes:

$$m \cdot n\left[\frac{\partial v_i}{\partial t} + \upsilon_i \nabla \upsilon_i\right] = enE - \gamma_i \nabla P = enE - \gamma k_B T_i \nabla n \quad (20)$$

WhileP stands for the pressure of the system which opposed the external force *enE*. Thus the force exerted by this pressure apposes also the external force. The concentration gradient $-\nabla n$ is in the direction of *E*. This means that the concentration decreasesdue fields' direction.

$$E = -\nabla\phi = -\frac{\partial\phi}{\partial x} = -\frac{\partial\phi_1}{\partial x} \quad (21)$$

As the velocity of ions in the fluid is very slow, we can neglect the second order terms in equation (20), and using equation (22)we can find:

$$m \cdot n\frac{\partial \upsilon_i}{\partial t} = en\frac{\partial\phi}{\partial x} - \gamma_i k_B T_i \frac{\partial n}{\partial x} \quad (22)$$

To solve this equation, one can uses perturbation method. The perturbed terms assumed to represent standing waves in the form

$v_1 = \tilde{v}_1 e^{i(kx-\omega t)}, n_1 = \tilde{n}_1 e^{i(kx-\omega t)}, \phi = \tilde{\phi}_1 e^{i(kx-\omega t)}$ (23)

Where the equilibrium values ϕ_0 and n remain constant and v vanished [9].

Equation (22) can be solved by using perturbation method.

Where,

$$\upsilon_i \approx \upsilon_o + \upsilon_1 \approx \upsilon_1$$
$$\phi \approx \phi_0 + \phi_1$$
$$n \approx n_0 + n_1 \qquad (24)$$

Thus, equation (22) reads

$$m \cdot n_0 \frac{\partial v_1}{\partial t} = en_0 \frac{\partial \phi_1}{\partial x} - \gamma\, k_B T \frac{\partial n_1}{\partial x} \qquad (25)$$

By substituting of equations (23), (24), in equation (20) one finds:

$$-imn_0\omega\tilde{v}_1 = ien_0k\tilde{\phi}_1 - ik\gamma k_B T\tilde{n}_1 \Leftrightarrow mn_0\omega\tilde{v}_1 = k\gamma_i k_B T\tilde{n}_1 - en_0k\tilde{\phi}_1 \quad (26)$$

Hence the velocity is given by:

$$\tilde{v}_1 = \frac{(k\gamma_i k_B T_i \tilde{n}_1 - en_0 k\tilde{\phi}_1)}{mn_0\omega} \quad (27)$$

According to equations (23) and (1) the displacement x is given by

$$x = v_1 \int e^{i(kx-\omega t)} = -\frac{v_1}{i\omega} e^{i(kx-wt)} = \frac{iv_1}{\omega E_0} E_0 e^{i(kx-\omega t)} = \frac{i\tilde{v}_1}{\omega E_0} E \qquad (28)$$

On the other hand, equation (21) and (23) give the electric field intensity E as

$$\mathrm{E} = -\frac{\partial \varphi_1}{\partial x} = -ik\tilde{\varphi}_1 e^{i(kx-\omega t)} = E_0 e^{i(kx-\omega t)} \quad (29)$$

This implies

$$\mathrm{E}_0 = -ik\tilde{\varphi}_1 \ (30)$$

The polarization vector P is related to the displacement x and the density of the atoms N according to the formula

$$\mathrm{P} = e\mathrm{N}x \ (31)$$

Utilizing equation (28),in equation (31) the polarization becomes:

$$\mathrm{P} = \frac{ie\mathrm{N}\tilde{v}_1}{\omega E_0}\mathrm{E} \quad (32)$$

Comparing equation (32) with the equation (5) and utilized equation (30) the electric susceptibility χ takes the form

$$\chi = \frac{ie\mathrm{N}\tilde{v}_1}{\omega \mathrm{E}_0} = \frac{-e\mathrm{N}\tilde{v}_1}{\omega k\tilde{\varphi}_1} \quad (33)$$

Employing equation (27) equation (33)becomes:

$$\chi = \frac{\left(en_0\tilde{\varphi}_1 - k_B T\gamma\tilde{n}_1\right)}{mn_0 k\omega^2\tilde{\varphi}_1} e\mathrm{N} = \frac{eN}{m\omega^2}\left[e - \frac{k_B T\gamma\tilde{n}_1}{n_0\tilde{\varphi}_1}\right] (34)$$

The electric field described by (10) and (11) can be also considered as the fieldgenerated by an imaginary potential, entails of a real part E_1 in the direction of the external field beside an imaginary part E_1 perpendicular to the applied external field , i.e

$$\tilde{E}_0 = E_1 + iE_2 = -ik\tilde{\varphi}_1 = -ik(a_1 + ia_2) = ka_2 - ika_1, \quad (35)$$

Therefore,

$$E_1 = ka_2 \;, \quad E_2 = -ka_1 \quad (36)$$

Utilizing equation (36) in(34) and splitting χ to real and imaginary parts the equation take the form:

$$\chi_1 + i\chi_2 = \frac{eN}{m\omega^2}\left[e - \frac{k_B T\gamma\tilde{n}_1 (a_1 - ia_2)}{no\,(a_1 + ia_2)(a_1 - ia_2)} \right]$$

$$= \frac{eN}{m\omega^2}\left[e - \frac{k_B T\gamma\tilde{n}_1 a_1}{n_o (a_1^2 + a_2^2)} + \frac{ik_B T\gamma\tilde{n}_1 a_2}{n_0 (a_1^2 + a_2^2)} \right] \quad (37)$$

Thus the imaginary part ofχbecomes

$$\chi_2 = \frac{\gamma\tilde{n}_1 T k_B a_2}{n_0 m\omega^2 (a_1^2 + a_2^2)} = \left[\frac{\gamma k_B T}{n_0 m\omega^2 (a_1^2 + a_2^2)} \right] \tilde{n}_1 E_1 \quad (38)$$

Substitute equations (35) in equation (13), the gain coefficient take the form:

$$\beta = -\left[\frac{c\varepsilon_0 \mu\omega}{n_1} \right] \left[\frac{\gamma k_B T}{n_0 m\omega^2 (a_1^2 + a_2^2)} \right] \tilde{n}_1 E_1 \quad (39)$$

Conclusions

The classical electromagnetic theory and the quantum mechanical theory shows that, lasing can take place in a medium, if the concentration *n* is less than the equilibrium one. Amplification takes place when the external field is in the direction of increasing of *n*.

According to equation (39) the amplification takes place if $\beta\rangle 0$.To satisfy this condition $\tilde{n}_1 = -|\tilde{n}_1|$ if $\tilde{n}_1$ is negative. In such a case must be achieved the following condition $n = n_0 + n_1 = n_0 - |\tilde{n}_1| e^{i\theta}$. This means that, the perturbed value *n* is less than the equilibrium one n_0. This condition can be well understood as far as the equilibrium value n_0 corresponds to the ground state, while the perturbed value corresponds to the exited state, in which the lower state with the population is less populated due to the pumping process in which $|\tilde{n}_1|$ particles leave it to the upper state. Lasing takes place when these particles return back to the lower state. Thus the number of emitted photons is proportional to $|\tilde{n}_1|$. This is in complete conformity with relation (39) where β is proportional to $|\tilde{n}_1|$. Amplification can also be satisfied if E_1 in equation (39) is negative. Also the amplification increase by increasing the fluid temperature and that is true because the higher temperature lead to greater of ionization. In view of equation (20) this means that the external field should be in the direction of the increasing of the positive ions ∇n. In this, case the number of emitted photons from excited atoms increasing in the direction where n is increasing.

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