Lorentz Transformation and The Euclidian Space

Amine Benachour
a.beanchour@umc.edu.dz
Laboratoire de Physique Théorique, Faculté Des Sciences Exactes,
Université Mentouri,
Route de Aïn El-Bey,
Constantine 25000, Algeria.

Abstract

Based on the Euclidian concept of distance and velocity, we propose a thought experiment which shows that if the clocks carried by two observers in a uniform linear motion, don't indicate the same time, their relative velocities will necessarily be different.

The unification of electricity and magnetism trough Maxwell equations, was one of the greatest lat nineteen century scientific progress. Heinrich Hertz experiences, confirmed the theoretical predictions of Maxwell theory, and the electromagnetic wave nature of light, became and evidence, opening the way to a considerable technological advance, especially after the radio waves discovery. But quickly, this revolutionary progress, gave rise to the incompatibility of the electromagnetic theory with the well established principles of the classical mechanics, the theory which since Isaac Newton works, knows a great success, supported by a large number of experiments, and predictions. The origin of the problem, is the non invariance of the laws of electromagnetism, under the Galilean transformation, connecting coordinates of objects, measured by two inertial frame of reference, in uniform linear motion one relative to other, violating thereby the Galilean invariance of the physical laws. Even more, the electromagnetic theory, predicts the invariance of the speed of light, confirmed by the Michelson-Morley experiment[1], whereas in the classical relativity, the velocity of any particle or mechanical system, depends on the frame of reference, where its evolution is studied. Theses disagreements led physicists to stipulate many assumptions, like the Aether hypothesis, and it was necessary to find a new symmetry for Maxwell equations. In this context, some transformations were proposed, by W.Voigt[2], J.Thomson[3], Larmor[4, 5] , and it's from Poincare[6, 7, 8] and lorentz[9, 10, 11, 12, 13] works that Lorentz transformation emerged, as the best transformation keeping the covariance of Maxwell equations, in agreement with Michelson Morley experiment. By September of 1905, Albert Einstein published his famous article[14], and puts definitely end to all the problems due to Maxwell theory, when he showed that adopting Lorentz transformation, allows to extend the principle of relativity to all physical laws, including the electromagnetism one, if gravity isn't taken into consideration; it's the birth of special relativity theory, providing a new perception of space and time, which henceforth, play the same role by forming together, the spacetime where events occur. Later, the principle of relativity, precisely the principle of general covariance, will be extended to include gravity, in the framework of general relativity theory.

Although the special relativity theory has abolished the classical idea of the absolute time, it doesn't do so with all the fundamental concepts of the classical physic, as the velocity of an object, which remains the ratio of the change of position, and the duration of this change of position. Even if the study of the relativistic moving bodies, is performed in the spacetime, i.e. the four-dimensional Minkowski space, it doesn't prevent that in reality the relativistic body, fellows a certain parameterized trajectory, with time as parameter, in the three dimensional space, inasmuch as it's not excluded that one day it will be possible to invent machines, able to reach relativistic speeds and traveling exactly on the same path followed by an ordinary ship, an airplane, or any kind of a non relativistic vehicle, very well described by the three dimensional classical kinematic. As any new theory which aims, to replace, or to generalize another one, should also recognize the successful predictions of its predecessor, special relativity has been made, so that it reduces to classical mechanics for low velocities compared to the speed of light, and the Galilean transformation is the limit of Lorentz transformation for low velocities. As the Euclidian space is the space of the classical physics, we are able to define Lorentz transformation in the Euclidian space, as follows,

$$x^{'} = \gamma \left(x - Vt\right) \Leftrightarrow x = \gamma \left(x' + Vt'\right), \qquad (1)$$

$$y' = y, \qquad (2)$$

$$z' = z, \qquad (3)$$

$$t' = \gamma \left(t - \frac{V}{c^2}x\right) \Leftrightarrow t = \gamma \left(t^{'} + \frac{V}{c^2}x'\right), \qquad (4)$$

c is the speed of light, and $\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$, $(x(t), y(t), z(t))$, are the cartesian coordinates of the material point, at time t, relative to the inertial frame of reference $R(O, x, y, z)$, in standard configuration with the frame $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$, which is in linear uniform motion of velocity V relative to $R(O, x, y, z)$ so that, when $t = t^{'} = 0, O \equiv O^{'}$, and $(x'(t'), y'(t'), z'(t'))$ represents the coordinate of this material point relative to $R'(O', x', y', z')$. From the mathematical point of view, the material point is a geometric point, and the set of all the geometrical points ε, form an affine space, associated to an Euclidian space E, of dimension three, such as,

$$\forall\left(a,b\right)\in\varepsilon^2 \rightarrow \overrightarrow{ab}\in E, \qquad (5)$$

$$\forall\left(a,b\right)\in\varepsilon^2, \overrightarrow{ab} = -\overrightarrow{ba}, \qquad (6)$$

$$\forall\left(a,b,c\right)\in\varepsilon^3, \overrightarrow{ac} = \overrightarrow{ab} + \overrightarrow{bc}, \qquad (7)$$

$$p\in\varepsilon, \overrightarrow{v}\in E, \exists! p'\in\varepsilon : \overrightarrow{pp'} = \overrightarrow{v}. \qquad (8)$$

For all these reasons, whatever the choice of the frame of reference, at every time, position vector of the material point, must obey to the rules of the Euclidian geometry, in particular axioms(6)(7), which implies that whatever the point M, we must have

$$\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M} \Rightarrow x' = x - Vt, \qquad (9)$$

$$\overrightarrow{OO'} = -\overrightarrow{O'O} \Rightarrow OO' = O'O. \qquad (10)$$

The introduction of Lorentz factor γ in (1) constitute a deep modification of the fondamental axiom(9), the new idea (4), of the time measure depending on the frame of reference, instead of the universal time whose justification can be understood by defining time as a conventional quantization linked to the observation of a certain physical phenomenon, like a sand flow in an hourglass, or the hands rotation in a watch, unchanged if the observer motion doesn't influence the physical phenomenon itself. But on the other hand, special relativity keeps the classical definition of the material point M velocity $\overrightarrow{v} = \frac{d\overrightarrow{OM}}{dt}$, the same as the shorter distance between any two points which still to be a straight line measurable using any type of ruler as in classical mechanics, in agreement with the Euclidian geometry, making special relativity theory by the mean of Lorentz transformation, a mixture of classical fundamental concepts and new ones.

To understand the impact of such modifications and their compatibility with the Euclidean geometry, precisely the distance between two moving bodies, we propose the following thought experiment: Suppose that the observer O wants to measure the velocity of the observer $O^{'}$, and $O^{'}$ is also interested to know the speed of O. Each observer needs to use his clock and find a way to measure the distance between him and the other moving observer. Knowing that an object A at rest relative to $R(O, x, y, z)$, is situated at distance $x = OA = l$ from O, in the positive sense, O has just to wait until the passage of $O^{'}$ exactly near A, then watches his clock which will indicate a certain time say $t = T$, to conclude that $O^{'}$ velocity is

$$V = \frac{l}{T}. \qquad (11)$$

In order to measure the $O^{'}O$ distance, $O^{'}$ uses a Self-retracting tape measure, so that when O and $O^{'}$ are at rest, i.e $t = t' = 0$, and $x^{'} = x = 0$, the tape measure leading end is fixed at point O, hence by moving, $O^{'}$ will pull the retracting tape in the direction of motion, in a way that its tip pointing out zero, remains fixed at O, then just by reading the pulled linear-measurement markings, $O^{'}$ becomes able to know the distance between him and O, at every time. Now if $O^{'}$ chooses to measure this distance at time $t^{'} = T^{'}$, corresponding to the moment where he reaches A, the flexible ruler will necessarily indicates $x^{'} = l$, because the tape measure extremity indicating zero is maintained at O, and the motion is linear, so for the measuring instrument, the traveled distance, is none other than the OA length, and $O^{'}$concludes that O has moved away with velocity

$$V = \frac{l}{T'}. \qquad (12)$$

From (11), (12), we deduce that: if $T \neq T^{'}$, then

$$V \neq V^{'}, \qquad (13)$$

while theoretically, Lorentz transformation of the v_x component

$$v_x^{'} = \frac{v_x - V}{1 - \frac{V}{c^2}v_x} \Leftrightarrow v_x = \frac{v_x^{'} + V}{1 + \frac{V}{c^2}v_x^{'}}, \qquad (14)$$

predicts

$$V = V^{'}. \qquad (15)$$

To understand this discrepancy, lets $x_{o'}$, $x_o^{'}$ be the O', and O coordinates relative to R and, $R^{'}$ respectively. When O' arrives to A his clock point to $t' = T'$, while O clock indicates $t = T$, as O' is at distance $OO' = x_{o'} = l$ from O, according to (4) and (1), $T \neq T^{'}$, and $OO' = \gamma VT'$.

At A, O', see O far from him at distance $O^{'}O = \left|x_o^{'}\right|$, hence $O^{'}O = \gamma VT$, as suggested by Lorentz transformation (1). Consequently $O^{'}O \neq OO^{'}$, unlike(10), so that $\frac{O^{'}O}{T'} = \frac{OO^{'}}{T} = V$.

Special relativity tell us that the moving observers haven't the same appreciation of the interval between them, which is offset by the discordance in the time measure ensuring thereby the equality of the relative velocities, This is closely related the length contraction earlier proposed by FitzGerald[15], becoming later an essential postulate for the validity of the theory.

The main conclusion is: If we continue to adopt the ruler as a very good instrument to measure the distance between two moving objects in a uniform linear motion whatever their speed, to be consistent with special relativity, the ruler must be affected by the contraction, gets impossible by this experiment.

References

[1] A.Michelson, E.Morley, On the Relative Motion of the Earth and the Luminiferous Ether, American Journal of Science, No 203, vol 34, pp333–345, (1887).

[2] Woldemar Voigt, Ueber das Doppler'sche Princip, Göttinger Nachrichten, 41-51, Sitzung vom 8, Januar 1887, Ausgegeben am 10, März 1887.

[3] Joseph John Thomson, On the Magnetic Effects produced by Motion in the Electric Field, Philosophical Magazine, 528(170): 1–14, (1889).

[4] Joseph Larmor, On a Dynamical Theory of the Electric and Luminiferous Medium, Part 3, Relations with material media, Philosophical transactions of the Royal society of London 190: 205–300,(1897).

[5] Joseph Larmor, Aether and Matter, Cambridge University Press(1900).

[6] Henri Poincaré, La théorie de Lorentz et le principe de réaction, Archives néerlandaises des sciences exactes et naturelles 5: 252-278, (1900).

[7] Henri Poincaré , The Principles of Mathematical Physics, Congress of arts and science, universal exposition, St. Louis, 1, Boston and New York: Houghton, Mifflin and Company, pp. 604–622(1904).

[8] Henri Poincaré, Sur la dynamique de l'électron, Comptes Rendus de l'Académie des Sciences, pp1504–1508, t.140, (1905). Rendiconti del Circolo matematico di Palermo 21, pp 129–176, (1906).

[9] Hendrik Antoon Lorentz, La Théorie électromagnétique de Maxwell et son application aux corps mouvants, Archives néerlandaises des sciences exactes et naturelles 25: 363–552(1892).

[10] Hendrik Antoon Lorentz, The Relative Motion of the Earth and the Aether, Zittingsverslag Akad. v. Wet, Amsterdam1: 74(1892).

[11] Hendrik Antoon Lorentz, Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies, Leiden: E.J. Brill(1895).

[12] Hendrik Antoon Lorentz, Simplified Theory of Electrical and Optical Phenomena in Moving Systems, Proceedings of the Royal Netherlands Academy of Arts and Sciences 1: 427–442(1899).

[13] Hendrik Antoon Lorentz, Electromagnetic phenomena in a system moving with any velocity smaller than that of light, Proceedings of the Royal Netherlands Academy of Arts and Sciences 6: 809–831(1904).

[14] Albert Einstein, On the Electrodynamics of Moving Bodies, Annalen der Physik 17, pp 891–921, (1905).

[15] George Francis FitzGerald, The Ether and the Earth's Atmosphere, Science13 (328):390, (1889).