On the Gravitational Bending of Light

Was Sir Professor Dr. Arthur Stanley Eddington Right?

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Abstract. The paramount British-Led May, 29, 1919 Solar Eclipse Result of Eddington *et al.* has had tremendous if not an arcane effect in persuading scientists, philosophers and the general public, to accept Einstein's esoteric General Theory *of* Relativity (GTR) thereby "deserting" Newtonian gravitation altogether, especially in physical domains of extreme gravitation where Einstein's GTR is thought or believed to reign supreme. The *all*-crucial factor "2" predicted by Einstein's GTR has been "verified" by subsequent measurements, more so by the most impressive and precision modern technology of VLBA measurements using cosmological radio waves to within 99.998% accuracy. From within the most well accepted provinces, confines and domains of Newtonian gravitational theory, herein, we demonstrate that the gravitational to inertial mass ratio of photons in Newtonian gravitational theory where the identities of the inertial and gravitational mass are preserved, the resulting theory is very much compatible with all measurements made of the gravitational bending of light. Actually, this approach posits that these measurements of the gravitational bending of light not only confirm the gravitational bending of electromagnetic waves, but that, on a much more subtler level; rather clandestinely, these measurements are in actual fact a measurement of the gravitational to inertial mass ratio of photons. The significant 20% scatter seen in the measurements where white-starlight is used, according to the present thesis, this scatter is seen to imply that the gravitational to inertial ratio of photons may very well be variable quantity such that for radio waves, this quantity must – to within 99.998% accuracy, be unity. We strongly believe that the findings of the present reading demonstrate or hint to a much deeper reality that the gravitational and inertial mass, may – after all; not be equal as we have come to strongly believe. With great prudence, it is safe to say that, this rather disturbing (perhaps exciting) conclusion, if correct; may direct us to closely *re-examine* the validity of Einstein's central tenant – the embellished Equivalence Principle (EP), which stands as the strongest and most complete embodiment of the foundational basis of Einstein's beautiful and celebrated GTR.

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1 Introduction

The General Theory *of* Relativity (GTR) – which was published by the then 37 year old Albert Einstein (1916) after he had submitted his finished manuscript to Germany's Prussian Academy of Sciences on November 25, 1915, this theory; is held and hailed as the best paradigm of gravitation that we have at our disposal. This theory was propped to prominence by the pre-eminent British astronomer, mathematician, physicist and philosopher, Sir Prof. Dr. Stanley Arthur Eddington's esoteric Solar eclipse results of May 29, 1919. Einstein's GTR predicts that a ray of light that just barely grazing the limb of the Sun will suffer a deflection of about $1.75''$ from its otherwise straight path. The two Edinginton-led expeditions gave the result $1.98 \pm 0.18''$ and $1.61 \pm 0.11''$ (Dyson *et al.* 1920). This result was and has been taken as the first clearest indication yet, of the GTR's supremacy over Newtonian gravitation and for this reason, Eddington's Solar expedition is ranked amongst one of the single most important, esoteric and touchstone scientific measurements of 20^{th} century physics (see *e.g.* Kennefick 2009).

In its modern form, and popular understanding, the idea that a gravitational field could alter the path of light is an Einsteinian idea (see *e.g.* Overbye 2000, Pais 1983). However, in the strictest sense of the word strict itself, this idea does not originate from Einstein or Einstein's GTR, it originates from the great Sir Isaac Newton. For example, in *Query* 1 to be found, in Book III, of his great work "*Treatise of the Reflections, Refractions, Inflections & Colours of Light*"; in print-record at least, the then 62 year old great Sir Isaac Newton was perhaps the first man to scientifically excogitate and ponder about these matters when he contended:

> *"Do not Bodies act upon Light at a distance, and by their action bend its Rays, and is not this action (ceteris paribus) strongest at the least distance?"*

Given that Newton thought of light as composed of "*tiny billiard balls*"[1], it is clear from this query that Newton had the idea that massive gravitating bodies may very well bend the path of light. He never made the attempt to calculate how much the gravitational field would bend a ray of light. Judging from his own words, it strongly appears that the great Sir Isaac Newton no longer had the time he needed to perform this task because when he made his 31^{st} *Query* in Book III, he lamented that; he felt these matters should be tackled by others that will come after him because he was interrupted, and would no longer think of taking these matters into farther consideration. In his own words:

> *"When I made the foregoing Observations* [outlined in his Book II and III]*, I design'd to repeat most of them with more care and exactness, (...) But I was then interrupted, and cannot now think of taking these things into farther Consideration. (...) I shall conclude with proposing only some Queries, in order to a farther search to be made by others."*

Surely, the great Sir Isaac Newton should have had in mind the idea to calculate the expected gravitational bending of light by a massive gravitating object. It was the German astronomer and mathematician, Johann Georg *von* Soldner (1804) that made the first calculation on the bending of light by a gravitational field [2].

Soldner (1804) used Newton's theory of gravitation to make his calculation because Newton's gravitational theory was the only theory of gravitation available at the time. What he obtained is the result that for a light ray grazing the Solar limb, this ray must undergo a deflection of about $0.87''$. This calculation assumes that light particles have a mass m, and they move at the speed $c = 2.99792458 \times 10^8 \mathrm{ms}^{-1}$, so that the total kinetic energy of the light particle is $mc^2/2$ just as would be the case for an ordinary particle travelling at the speed of light.

The next to calculate the bending of light by a gravitational field was Einstein (1911). In 1907, Einstein laid down the foundations of his GTR when he formulated the *Principle of Equivalence*, a principle upon which the GTR is founded. Independently of the work of Soldner (1804), using this principle in 1911 in his paper entitled *"On the Influence of Gravitation on the Propagation of Light"*, Einstein deduced that a gravitational field must be capable of bending a ray of light. Though the reasoning and the calculation differ markedly, his [Einstein] result was essentially the same as that of Soldner (1804), that is, a light ray grazing the limb of the Sun must undergo a deflection of about $0.87''$. Einstein (1911)'s calculation did not take into account the curvature

[1] *Query 29* found in Book III of Netwon's works – pristinely demonstrates that Newton envisaged light as small hard spheres. This query reads: *"Are not the Rays of Light very small Bodies emitted from shining Substances?"*

[2] Soldner's calculation is based on Newton's corpuscular theory of light thus this calculation was perhaps not taken serious because of the fact that Newton's corpuscular theory slowly faded as the wave theory of light gained ground. The wave-particle nature of light only began to be understood in the 20^{th} century.

of space but included only the effects of mass on the time dimension of the four dimensional spacetime continuum.

Using his influence which he had gained from his earlier works made in 1905 (the Special Theory *of* Relativity *etc*), the year commonly referred to as *Einstein's Miracle Year*, and his ever growing reputation and influence in the scientific circles, Einstein wrote to a number of leading astronomers[3] of the day urging and persuading them to measure this effect (Kennefick 2009). Heeding to this call, several attempts (notably by Erwin Finlay Freundlich) to measure the deflection of starlight grazing the Solar limb during Solar eclipses were made between the years 1911 and 1915. Fortunately for Einstein and as-well for his GTR, all these attempts were thwarted by cloudy skies, logistical problems, the outbreak of the so-called First World War, amongst others.

Impatient and eager to see his result ($0.87''$) corroborated, a confident Einstein became very exasperated over the repeated failures of the experimentalists to gather-up any useful data. In his typical *style-of-confidence* that one can easily mistake for arrogance of the first kind – without an iota of doubt, Einstein was very certain that his prediction would positively be verified – all he had in his confident mind was that these experiments would confirm his result. After the 1919 Solar eclipse measurements were announced by the Eddington team, Einstein was asked by one of his students Ilse Rosenthal-Schneider, what he would have felt if his revised result ($1.75''$) had not been corroborated; disguised in a joke-like form, a very confident Einstein replied "*I would have felt sorry for the Good Lord . . . the theory is correct all the same.*" (see *e.g.* Soars 2011). This is how confident Einstein was on his ideas, even his earlier result of $0.87''$.

As is now common knowledge, legend and *lore* of *Eddington's* 1919 *Eclipse Expedition*, ironically, if any of the early experimental efforts (between the years $1911 - 1914$) had succeeded in collecting any useful data, they would have cast a serious dark-cloud on Einstein's future attempts on his GTR as he [Einstein] would have, on this occasion, been proven wrong! It was not until late in 1915, after he had completed the GTR, that he realized that his earlier prediction was incorrect by a factor of "2". This factor would prove to be decisive as it now would act as a clear landmark separating the Newtonian and Einsteinian Worlds of gravitation.

So, had the so-called First World War not intervened, it is very much likely that Einstein would never have been able to claim the $1.75''$ bending of light (at twice the Newtonian value) as a prediction of his GTR – he would have been cast into the defensive rather than the offensive. At the very least and at best, he would have been forced to explain why the observed deflection was actually consistent with the completed GTR. Surely, despite its exquisite beauty and esoteric elegance, the GTR would have then become nothing more other than an *ad hoc* and *impromptu* theory designed to explain this unfortunate state of affairs between experience and theory. Which ever the way one may see this or may want to see this, one thing that is clear to all is that, it was really lucky for Einstein that the corrected light-bending prediction was made before any Solar expeditions succeeded in making any real and meaningful measurements.

In 1919, after the so-called First World War had ended, scientific expeditions were sent to Sobral in South America and Principe in West Africa to make observations and measurements of the Solar eclipse. The reported results were deflections of $1.98 \pm 0.16''$ and $1.61 \pm 0.40''$ respectively. As already stated earlier, these results were taken as clear confirmation of Einstein's GTR prediction of $1.75''$. This success, combined with the shear mathematical complexity of Einstein non-linear equations together with the esoteric appeal to the general public of the bending light and the seemingly romantic adventure of the eclipse expeditions themselves (*i.e.*, men catching light bending under a dark sky in the day), all but contributed enormously to making Einstein a World celebrity on scale never before witnessed by any scientist in the entire history of humanity. As the last edition of the popular United States of America's *Time Magazine* of the past century graphically put it, "*It were as though humanity is now divided into two, Einstein and the rest of us.*" The name Einstein become a household name synonymous to genius of the very highest, esoteric and rare order. To the mundane and ordinary, Einstein's almost arcane genius could only be approached asymptotically in the same manner the scientist approaches the truth. For better (or for worse), this state of affairs remains to the present day.

[3]Notably German's Erwin Finlay Freundlich ($1885 - 1964$).

However, the actual combined measurements (from 1919 to 1973) of the bending of starlight show a slight but significant scatter ($\sim 19\%$) about the predicted value of $1.75''$. The scatter has been taken as indication of the experimental difficulty in the measurement process. In this reading, we suggest very strongly that this scatter may very well be a result of the variation of the gravitational to inertial mass ratio of photons. The present work comes as nothing short of a scientific exegesis to the *1919 Eddington Eclipse Result*. We are in complete agreement with the fact that Eddington *et al.*'s efforts did prove for the first time, that the path of light is altered by a gravitational field. Our *bone of contention* is whether the factor "2" vindicates Einstein's GTR and eternally puts Newtonian gravitation on the scientific crucifix?

Here in the penultimate, to cement our confidence in what we are about to present, allow us to say once more that, we herein demonstrate beyond any shadow or shred of doubt that, Newtonian gravitation actually explains this factor "2". That is, in accordance with the ideas set-forth herein, if this factor "2" emerges from the observations, it is to be interpreted as a measure of the photon's gravitational to inertial mass ratio. Further, from this same interpretation were the factor "2" is interpreted as a measure of the photon's gravitational to inertial mass ratio; applying this same interpretation to the eclipse results from $1919 - 1973$, one comes to a very interesting conclusion that the gravitational to inertial mass ratio of photons may very well be a variable quantity. In all scientific modesty, honesty and prudence, it is safe to say that, to all that seek nothing but the truth, the present reading calls for nothing short of a *rethink* of the 1919 Eddington Eclipse Result and as-well the very foundations of Einstein (1916)'s GTR.

2 Weak Equivalence Principle

As is well known, there is at least two distinct and important kinds of mass that enter Newtonian mechanics. The first is the *inertial mass* (m_i) which enters in Newton's second law of motion. As it was first stated by the great Sir Isaac Newton, this law states that the resultant of all the forces ($\boldsymbol{F}_{res}$) acting on a body is equal to the rate of change of motion of that body, *i.e.*:

$$\boldsymbol{F}_{res} = \frac{d\boldsymbol{p}}{dt} \quad \text{where,} \quad \boldsymbol{p} = m_i \boldsymbol{v}. \tag{1}$$

By motion, Newton meant the momentum $\boldsymbol{p}$ of the body in question. Momentum ($\boldsymbol{p}$) is the product of inertial mass (m_i) and the velocity ($\boldsymbol{v}$) of the body in question. In most cases considered in natural systems, the inertial mass of the object is a constant of motion, so this law is often stated as:

$$\boldsymbol{F}_{res} = m_i \boldsymbol{a} \quad \text{where,} \quad \boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}. \tag{2}$$

The vector quantity $\boldsymbol{a}$ is the acceleration of the body in question.

The second kind of mass enters Newtonian mechanics in Newton's law of universal gravitation which states that the gravitational force drawing together two objects of gravitational mass $\mathcal{M}_g$ and m_g that are separated by a distance r is:

$$\boldsymbol{F}_g = -\frac{G\mathcal{M}_g m_g}{r^2}\hat{\boldsymbol{r}}, \tag{3}$$

where ($G > 0$) is Newton's constant of universal gravitation and $\hat{\boldsymbol{r}}$ is the unit vector along the line joining the centres of mass of these objects and the negative sign is there to denote the fact that the gravitational force is a force of attraction.

As afore-stated, the weak equivalence principle due to Galileo states that test bodies fall with the same acceleration independent of their internal structure or composition: in other words, the gravitational mass appearing in (3) and inertial mass appearing (2) are the same *i.e.* $m_i \equiv m_g$. Throughout this reading, in order to distinguish between gravitational and inertial mass, we shall use the subscripts "i" and "g" respectively *i.e.* m_i and m_g. If by any chance the hypothesis $m_i \equiv m_g$ is true, then, this fact must and will come out clean from the resultant equations.

If we are to denote the ratio between the gravitational and inertial mass:

$$\frac{m_g}{m_i} = 2\gamma \quad \Longrightarrow \quad \gamma = \frac{1}{2}\frac{m_g}{m_i}, \tag{4}$$

then, the acceleration of an object in a gravitational field which is obtained by setting $\boldsymbol{F}_{res} = \boldsymbol{F}_g$ and then dividing the resultant equation throughout by m_i, *i.e.*:

$$m_i\boldsymbol{a} = -\frac{G\mathcal{M}_g m_{\rm g}}{r^2}\hat{\boldsymbol{r}} \quad \Longrightarrow \quad \boldsymbol{a} = -\frac{2\gamma G\mathcal{M}_g}{r^2}\hat{\boldsymbol{r}}. \tag{5}$$

The factor $1/2$ (or 2) in (4) has been inserted for convenience purposes. Written with this factor $1/2$, numerically speaking, γ has the same meaning as it has in Post-Parametrised Newtonian (PPN) gravitation. For example, if we are to set $\zeta = m_{\rm g}/m_i$, then, a comparison of this ζ-factor with PPN gravitation's γ-factor requires that $\zeta = 2\gamma$. So, in order that we have a direct comparison without the hindrance of this factor 2, one can deal with this once and for altime by inserting the factor $1/2$ as has been done in (4) .

Now, as already stated, under the Newtonian scheme $\gamma \equiv 1/2$. This is a fundamental assumption of Newtonian mechanics. Although he made the first attempts to verify this from experience, Newton never bothered to explain this important and crucial assumption. Judging from his writings as recorded in his great master piece *"Philosophiae Naturalis Principia Mathematica"*, Newton (1726) was a very careful man who believed in facts derived from experience. His famous statement "*Hypotheses non fingo ...*"[4] captures very well Newton as a man that strongly believes in facts of experience. Perhaps, because this fact was a fact of experience, he saw no need to explain it, but to simple take it as experience dictates. It was Einstein (1907) that made the first attempt to explain this. He [Einstein] used this as a starting point to seek his GTR. Here in, we shall drop this assumption *i.e.* $\gamma \equiv 1/2$.

To test the weak equivalence principle, one meticulously compares the accelerations of two bodies with different compositions in an external gravitational field. Such experiments are often called Eötvös experiments after Baron Ronald *von* Eötvös (1813 − 1871), the Hungarian physicist whose pioneering experiments with torsion balances provided a foundation for making these tests (*von* Eötvös 1890). These tests are assumed to vindicate Einstein's ideas on general relativity. The best test of the weak equivalence principle to date has been performed by Eric Adelberger and the Eöt-Wash collaboration at the University *of* Washington in Seattle, who have used an advanced torsion balance to compare the accelerations of various pairs of materials toward the Earth (see *e.g.* Will 2009). As afore-stated in the introductory section, their accuracy is one part to about 10^{13}. This accuracy is taken as the clearest indication yet, that γ should be unity, or equal for all material bodies in the Universe. In this case, the equivalence principle is not just a Principle *of Nature*, but a true Law *of Nature*. However, as will be seen herein, these Eötvös experiments actually measure whether or not bodies of the same mass but different compositions will have different accelerations in a gravitational field.

3 Newtonian Bending of Light Under the Assumption ($\gamma \equiv 1/2$)

It is only for instructive purposes that we go through this exercise of deriving the equation of orbit for a test particle in a Newtonian gravitational field. This derivation is found in every good textbook dealing with Classical/Newtonian Mechanics. In polar coordinates [see Figure (1) for the (r, θ, φ)-coordinate setup], the acceleration $\boldsymbol{a}$ for a two dimensional surface is given by $\boldsymbol{a} = (\ddot{r} - r\dot{\varphi}^2)\hat{\mathbf{r}} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{\boldsymbol{\varphi}}$, so that equation (5) under the assumptions ($\gamma \equiv 1/2$), gives two equations, *i.e.*:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2 = -\frac{d\Phi}{dr} = -\frac{G\mathcal{M}}{r^2}, \tag{6}$$

[4] This is a Latin statement which translates to "*I feign no hypotheses*", or "*I contrive no hypotheses*"

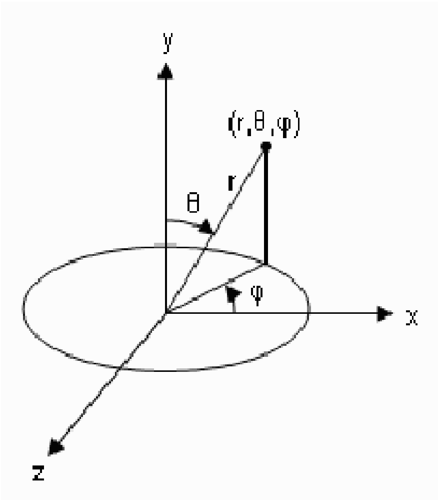

Figure (1): This figure shows a generic spherical coordinate system, with the radial coordinate denoted by r, the zenith (the angle from the North Pole; the colatitude) denoted by θ, and the azimuth (the angle in the equatorial plane; the longitude) by φ.

for the radial component. In this equation, because of the assumption that ($\gamma \equiv 1/2$), we have doped the subscript "g" from the mass "$\mathcal{M}$" since the gravitational and inertial mass are here identical. For the $\hat{\boldsymbol{\varphi}}$-component we have:

$$r\frac{d^2\varphi}{dt^2} + 2\frac{dr}{dt}\frac{d\varphi}{dt} = -\frac{1}{r}\frac{d\Phi}{d\theta}. \tag{7}$$

Now, taking equation (7) and dividing throughout by $r\dot{\varphi}$ and remembering that the specific angular momentum $J = r^2\dot{\varphi}$, we will have:

$$\frac{1}{\dot{\varphi}}\frac{d\dot{\varphi}}{dt} + \frac{2}{r}\frac{dr}{dt} = -\frac{1}{J}\frac{d\Phi}{d\varphi} \Longrightarrow \frac{1}{J}\frac{dJ}{dt} = -\frac{1}{J}\frac{d\Phi}{d\theta}, \tag{8}$$

hence:

$$\frac{dJ}{dt} = -\frac{\partial\Phi}{\partial\varphi}. \tag{9}$$

Since $\Phi = \Phi(r)$, $\partial\Phi/\partial\varphi = 0$, hence:

$$\frac{dJ}{dt} = 0. \tag{10}$$

The specific orbital angular momentum is a conserved quantity.

Now – moving on, we proceed to solve the radial component *i.e.* (6). To do this, we shall – as is usual; make the transformation $u = 1/r$. This will require us to find the expression for $\dot{r}$ and $\ddot{r}$. Doing so, one finds that:

$$\dot{r} = \frac{dr}{dt} = -J\frac{du}{d\varphi} \text{ and } \ddot{r} = \frac{d^2r}{dt^2} = -\frac{dJ}{dt}\frac{du}{d\varphi} - J^2u^2\frac{d^2u}{d\varphi^2}. \tag{11}$$

Inserting these into (6) and then dividing the resultant equation by $-u^2J$ and remembering (14) and also that $dr = -du/u^2$ and $r\dot{\varphi}^2 = u^3J^2$, one is led to:

$$\frac{d^2u}{d\varphi^2} + u = \frac{G\mathcal{M}}{J^2}. \tag{12}$$

A solution to this equation is given by $u = (1 + \epsilon \cos\varphi)/l$, *i.e.*:

$$\frac{l}{r} = 1 + \epsilon \cos\varphi, \tag{13}$$

where $l = (1+\epsilon)\mathcal{R}_{\rm min}$ is a constant and is twice the distance when or the particle in orbit when $\varphi = 90°$, ϵ is the eccentricity of the orbit and $\mathcal{R}_{\rm min}$ is the perigee distance of the orbit. Substituting (13), into (12), one obtains that the specific orbital angular momentum is given by:

$$J^2 = G\mathcal{M}l. \tag{14}$$

Since J is a conserved quantity, we can evaluate it at any given point of the orbit, the usual and more convenient point is the perigee point. At the perigee $J = v_{\rm max}\mathcal{R}_{\rm min}$. With this given as as-well that $l = (1+\epsilon)\mathcal{R}_{\rm min}$, it follows from (14) that:

$$\epsilon = \frac{\boldsymbol{v}^2_{\rm max}\mathcal{R}_{\rm min}}{G\mathcal{M}} - 1. \tag{15}$$

Now, we have to apply all the above ideas to the gravitational bending of light as is usually done or as has been done by past researchers. There is nothing new so far. In this application, light is treated as a beam composed of a stream of massive particles called photons. These photons move inside the gravitational field at a constant speed ($v_{\rm max} = c$). The path taken by light is a hyperbola [see schematic diagram (2)]. For the eccentricity of this hyperbolic orbit ($\epsilon \ggg 1$) such that we can drop the "-1" appearing in (15), so that:

$$\epsilon = \frac{c^2\mathcal{R}_{\rm min}}{G\mathcal{M}}. \tag{16}$$

On grazing the Solar limb, the distance of the ray of light is $\mathcal{R}_{\rm min} = \mathcal{R}_\odot$ and the angle of the asymptote to the hyperbole of eccentricity obtain by setting $r = \infty$ in (13). This implies that this angle is given by:

$$\Psi = \arccos\left(\frac{1}{\epsilon}\right). \tag{17}$$

The angle of deflection of the light ray δ, is shown in the Figure (2) and is such that:

$$\delta = \pi - 2\Psi. \tag{18}$$

From (17) and (16), it follows that (18) can be written as:

$$\delta_N = \pi - 2\arccos\left(\frac{G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot}\right). \tag{19}$$

Now, a Taylor expansion of the function $\arccos(x)$ gives:

$$\arccos(x) = \frac{\pi}{2} - \left(x - \frac{1}{2}\frac{x^3}{3} + \ldots + \ldots\right). \tag{20}$$

If $x \lll 1$, then the following approximation holds:

$$\arccos(x) \simeq \frac{\pi}{2} - x. \tag{21}$$

Because $(G\mathcal{M}_\odot/c^2\mathcal{R}_\odot \lll 1)$, it follows from all the above that:

$$\delta_N = \frac{2G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot} = 0.87''. \tag{22}$$

where the subscript "N" has been inserted on δ so as to identity this result as a Newtonian result. This is the famous Newtonian result that is a factor "2" less that that obtained from Einstein's GTR.

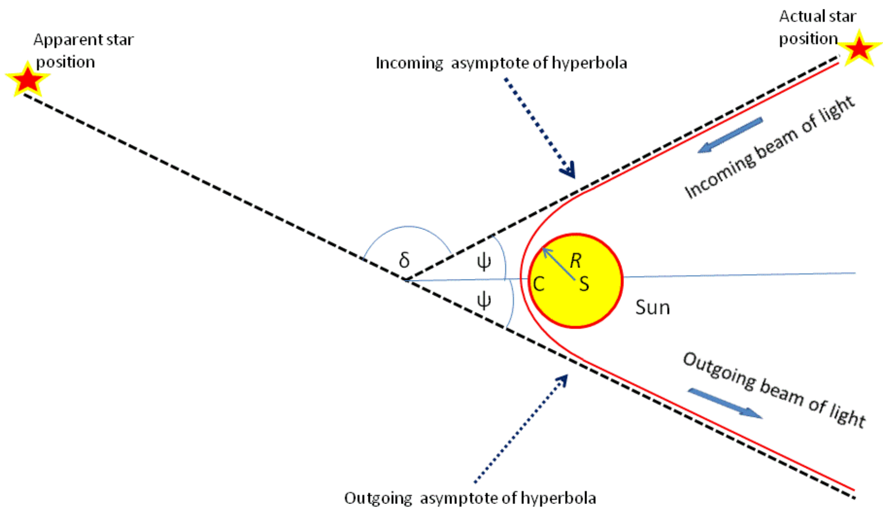

Figure (2): The light ray from a star follows an unbound hyperbolic orbit about the Sun. For deflection on grazing incidence, $\mathcal{R}$ is the Solar radius. For illustration purposes, the bending is greatly exaggerated.

We have given a full exposition of the Newtonian derivation of the $0.87''$ gravitational bending angle of the path of light by the Sun as it grazes the Solar limb. The sole purpose of this is that in Section (5), when the assumption ($\gamma \equiv 1/2$) is dropped, the reader will see clearly for themselves that Newtonian gravitation does say something about the equity of gravitational to inertial mass ratio. We do not want our reader to *quarrel* with us, but to quarrel with the bare facts before their agile and diligent eyes. So, we want (hope/wish) that this reading (to) be as lucid and as clear as it possibly can, so as to avoid any misunderstanding with our reader because the conclusions to be drawn thereof, have far reaching consequences and implications on the very foundations of physics.

4 Einsteinian Bending of Light

As stated in the introductory section, Einstein applied his newly found Equivalence Principle to the motion of light in a gravitational field. For light grazing the Solar limb, Einstein obtained the same value as that obtained from Newtonian gravitational under the assumption ($\gamma = 1/2$). After the completion of the GTR, he revised the same problem, and this time he obtained:

$$\delta_E = \frac{4G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot} = 1.75''. \quad (23)$$

We are not – as we did in the Newtonian case; going to derive this result but simple state it as we have done above. This value is twice the Newtonian value *i.e.* $\delta_E = 2\delta_N$. What this meant is that now, it was possible to experimentally differentiate Newtonian gravitation from Einsteinian gravitational. The factor two is now a very crucial arbiter.

Realizing this, (then) Professor (and not Sir) Arthur Eddington would set himself the task to vindicate Einstein's predictions. Thus, when Prof. Arthur Eddington and his Cambridge Team set out to measure the bending of light rays by the Sun – with Prof. Arthur Eddington's guidance; they set the main purpose of their expedition to conclude the truth of one of three possibilities:

1. The gravitational field of the Sun does not bend light.

2. The gravitational field of the Sun influences the bending of light according to Newtonian gravitation, with the result being a deflection of light by $0.87''$.

3. The gravitational field of the Sun influences the bending of light according to General Relativity, with the result being the deflection of light by $1.75''$.

On November 19, 1919, Eddington's expedition reported their now world famous result that posted Einstein into a scientific superstar on a scale never witnessed before for any scientist – *i.e.*, both living and dead. They reported that their observations had given the values $1.98 \pm 0.12''$ for the West African expedition, while the Principe observations yielded a value of $1.61 \pm 0.30''$. These measurements clearly showed that the first two initial possibilities could be rejected, leaving the predictions of the GTR as the most plausible.

As shown in Table I, in the years that followed, *i.e.* from 1922 up to 1973, eleven similar experiments were conducted. These measurements always recover Einstein's $1.75''$ prediction in the margins of error. Additionally, these values display a scatter in the range $1.61 \pm 0.30''$ to $2.73 \pm 0.31''$. The value $2.73 \pm 0.30''$ measured in 1936 in the then United Soviet Socialist Republic (USSR), is significantly larger than the Einsteinian value of $1.75''$. Einstein's prediction is not even recovered in the margins of error for this measurement. Even in Eddington's West African result *i.e.* $1.98 \pm 0.12''$, Einstein's prediction is not even recovered in the margins of error. The usual explanation is that these experiments are very difficult to conduct. This "*excuse*" explains that scatter and the reason why the Einsteinian $1.75''$ prediction is mostly recovered in the margins of error. Is this really the reason? Soon, we shall suggest otherwise.

About the VLBA measurements that confirm convincingly the Einsteinian $1.75''$ prediction, we say unto our reader, for now, behold. We shall come to this in Sections (8) and (9). In the next section, we shall derive our major result that will cast some doubts on the way researchers since 1919 have come to understand Eddington's result and the subsequent sister experiments.

5 Newtonian Bending of Light Under the Assumption ($\gamma \neq 1/2$)

As already stated earlier, the strongest reason for setting ($\gamma \equiv 1/2$) began with Galileo's legendary experiments at the Learning Tower *of* Pisa in Italy. He demonstrated – contrary to conventional wisdom of his day; that different masses will take the same amount of time to fall the same distance in a gravitational field. Aristotle's wisdom, which was prevalent in that day, held that heavier objects would take a shorter time while lighter objects will take a longer time to fall the same distance.

In modern times, improved experiments have been performed in a vacuum where it has been demonstrated conclusively that a light-feather and a heavier-rock will take the same amount of time to fall the same distance in a gravitational field. In a vacuum, air resistance is absent and thus has the least possible toll (if any) on the falling object. This has been taken to mean that ($\gamma = 1/2$). More elaborate, sophisticated and precision experiments known as Eötvös experiments have been performed to further test the hypothesis that ($\gamma \equiv 1/2$). These experiments find that ($2\gamma - 1 = 10^{-12}$). Practically this means that ($\gamma \equiv 1/2$). This is the position taken by the majority – if not by all physicists, that, gravitational and inertial mass are equal if not identical in nature. Einstein used this fact of experience as a starting point to construct his enduring GTR.

Before the reader proceeds, s/he must once again ask themselves the question "*Is there anything wrong in writing the Newtonian gravitational equation of motion as?*":

$$m_i \boldsymbol{a} = -\frac{G\mathcal{M}_{\rm g} m_{\rm g}}{r^2}\hat{\boldsymbol{r}}. \tag{24}$$

By so doing, all that we have done is merely preserve the identity of the gravitational and inertial mass. Clearly, the most honest answer is that – if any at all; there is no reason whatsoever to

Table I: Column (1) gives the number of the observation in the table. Column (2) and (3) gives the date and location of execution of the eclipse observations. Column (4) gives the actual measurements made, while column (5) and (6) gives the γ-factor and the ratio λ_c/λ as determined from equation (43). Column (7) gives the deviation of the measurement from Einstein's expected $1.75''$ Solar deflection.

Eclipse Data (1919 – 1973)

	Date	Location	Measurement $\delta_{\rm D}$ (1 arcsec)	γ-Factor	(λ_c/λ)	% Deviation
(1)	May 29, 1919.	Sorbal	1.98 ± 0.16^a	2.30 ± 0.20	0.66 ± 0.08	$+13.10 \pm 0.80$
(2)		Principe	1.61 ± 0.40^a	1.90 ± 0.30	0.40 ± 0.90	-8.00 ± 2.00
(3)	Sept. 21, 1922.	Austrilia	1.77 ± 0.40^b	2.00 ± 0.50	0.60 ± 0.50	$+1.10 \pm 0.30$
(4)			$1.80 \pm 0.40^{\rm c}$	2.00 ± 0.50	0.60 ± 0.50	$+2.90 \pm 0.60$
(5)			$1.72 \pm 0.15^{\rm d}$	2.00 ± 0.20	0.85 ± 0.02	-1.70 ± 0.10
(6)			$1.82 \pm 0.20^{\rm e}$	2.10 ± 0.20	0.60 ± 0.20	$+4.00 \pm 0.40$
(7)	May 9, 1929.	Sumata	2.24 ± 0.10^f	2.50 ± 0.10	0.71 ± 0.05	$+22.00 \pm 1.00$
(8)	June 19, 1936.	USSR	2.73 ± 0.31^g	3.10 ± 0.40	0.80 ± 0.06	$+56.00 \pm 6.00$
(9)		Japan	$1.70 \pm 0.40^{\rm h}$	2.00 ± 0.50	0.85 ± 0.04	-2.90 ± 0.70
(10)	May 20, 1947.	Brazil	2.01 ± 0.27^i	2.30 ± 0.30	0.70 ± 0.20	$+15.00 \pm 2.00$
(11)	Feb. 25, 1952.	Sudan	1.71 ± 0.10^j	2.00 ± 0.10	0.50 ± 0.20	-2.30 ± 0.10
(12)	June 30, 1973.	Mauritania	1.66 ± 0.19^k	1.90 ± 0.20	0.50 ± 0.40	$+5.10 \pm 0.60$
	Weighted Mean	$\longmapsto$	$\mathbf{1.92 \pm 0.05}$	$\mathbf{1.22 \pm 0.03}$	$\mathbf{0.60 \pm 0.40}$	$\langle \mathbf{S} \rangle = \mathbf{12.00 \pm 6.00}$

References: For the measurements given column (4), see: [a]Dyson *et al.* (1920); [b]Campbell *&* Trumpler (1923); [c]Campbell *&* Trumpler (1923); [d]Campbell *&* Trumpler (1923); [e]Campbell *&* Trumpler (1923); [f]Freundlich (1929); [g]Mikhailov (1940); [h]missing; [i]*van* Biesbroeck (1950); [j]*van* Biesbroeck (1953); [k]Brune *et al.* (1976), Jones (1976) reject the above equation because if ($\gamma \equiv 1/2$), this will emerge from the resultant equations of motions upon weighing this equation against the test of experience.

Surely, if the afore-stated is the case, then, it follows that there should be is no reason whatsoever to reject the final equations of motion which predict that the gravitational bending of light will depend on the γ-factor. This is about the only "modification" that we make to Newtonian gravitation. We simple say to ourself, let us identify the gravitational and inertial mass of the objects and thereafter preserve these identities as we derive the resulting equations of motion in exactly the same manner as has been done since the time of Sir Isaac Newton's derivation of the equation of the orbit of a test particle in a Newtonian gravitational field.

The result that we obtain from the resultant equations of motion have a rich mean meaning insofar as the WEP and the gravitational bending of light measurement are concerned. To dismiss this meticulous observations that Newtonian gravitation has something significant to say about γ is nothing-but justice denied. It is nothing but a deliberate denial of the truth emergent from acceptable facts. Instead of rejecting this result, we must try to comprehend it, *i.e.* what does it mean insofar as our present understanding of gravitation is concerned? This is the approach that we take, we would like to understand this result in the light of our present understanding of gravitational theories.

Now, the corresponding equation to (12) under the assumption ($\gamma \neq 1/2$) is:

$$\frac{d^2u}{d\varphi^2} + u = \frac{2\gamma G\mathcal{M}_{\rm g}}{J^2}, \tag{25}$$

and a solution to this equation is the same as that to (12) *i.e.* $u = (1 + \epsilon\cos\varphi)/l$. The *major* an *all-important* difference is the equation corresponding to (14). By substituting $u = (1 + \epsilon\cos\varphi)/l$ into (25), one interestingly and surprisingly, finds that:

$$J^2 = 2\gamma G\mathcal{M}_{\rm g}l. \tag{26}$$

This is the central result of the present reading as the main theme and conclusion to be derived herein flows from this result.

First and most important of all, notice that; in (26) the γ-factor has just entered Newtonian gravitational physics in a very significant way. This is were all the important difference comes in. However trivial this result may appear, we are certain that it is found nowhere else in the literature. It is the first time it is appearing. If not, then, its true significance has not been understood or appreciated. This reading shall demonstrate that, indeed, this result brings a whole new meaning to the equity of gravitational and inertial mass in Newtonian gravitational theory. It is imperative that the reader takes note of this. We are certain the reader does not object to the result (26). On that footing and pedestal, we do not expect the reader to object to the final findings of the reading as these finding flow smoothly from the logic thereof. Yes, the results are difficult to believe at first sight, but, they flow smoothly from the logic of the mathematics whose nomothetic physical foundations are credible. Are we to go to war with logic in the hope of emerging victorious? We think not, we must accept results deduced from logic.

The new finding (14), leads to the eccentricity now being given by:

$$\epsilon = \frac{\boldsymbol{v}_{\rm max}^2\mathcal{R}_{\rm min}}{2\gamma G\mathcal{M}_{\rm g}} - 1. \tag{27}$$

As before, for the eccentricity of the path taken by light ($\epsilon \ggg 1$) such that we can drop the "-1" appearing in (27), so that:

$$\epsilon = \frac{c^2\mathcal{R}_{\rm min}}{2\gamma G\mathcal{M}_{\rm g}}. \tag{28}$$

With the eccentricity now defined for the case $\gamma \neq 1/2$, one can go through the same algebraic exercises that lead to (19). So doing, they will arrive at a modified formula for the gravitational deflection angle. This new formula now in-cooperates the γ-factor. This formula is:

$$\delta_\gamma = \frac{4\gamma G\mathcal{M}_{\rm g}}{c^2\mathcal{R}_{\rm min}}. \quad (29)$$

where the subscript "γ" has been inserted onto δ so as to highlight that this result now incooperates the γ-factor. For electromagnetic waves grazing the Solar limb, we will have:

$$\delta_\gamma = \frac{4\gamma G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot} = 1.75''\gamma = 4.84\times 10^{-4}\gamma. \quad (30)$$

Thus, any deviation from Einstein's $1.75''$ deflection can be attributed to the γ-factor being different from unity.

6 Plausible Criticism and Rebuttals

The approach we have used in arriving at our major result (29) can be subject two major criticisms. However, this criticism can be rebutted and this is what we are going to do in this section. The first is that of the constancy of the specific orbital angular momentum and as-well the constancy of the speed of light. Combined, these two facts imply that light must orbit in circular orbits. The second is that the vantage-point (*i.e.* the expressions 14 and 26) used to arrive at the eccentricity is not the one that is traditionally used. Below, we elucidate these problems and subsequently supply the necessary rebuttals.

6.1 Constancy of Light Speed and Specific Orbital Angular Momentum

6.1.1 Problem

For the gravitational bending of light results (*i.e.* 22 *&* 23), in all the cases [*i.e.*, ($\gamma = 1/2$) and ($\gamma \neq 1/2$)], the results *i.e.* (22) *&* (23), have all been derived under the assumption that $dJ/dt \equiv 0$, which implies that $J =$ constant. Since $J = rc$ where $c =$ constant, it follows that $r =$ constant, *i.e.*, light must orbit in circular orbits. Obviously, this is unacceptable as it is at odds with physical and natural reality as we know it.

6.1.2 Rebuttal

A perfectly correct, legal and reasonable way out of these troubles would be to assume that $dJ_*/dt \equiv 0$, *i.e.* $J_* =$ constant, where J_* is the orbital angular momentum, not the specific orbital angular momentum, *i.e.* $J_* = m_i J$ where m_i is the inertial mass of the photon. It is important that we state that for as long as $dJ_*/dt \equiv 0$, the results thereof are not going to be altered.

Soldner (1804) and all subsequent researchers have applied the Newtonian result (22) without pointing out that as derived in its bare form this result implies that light must traverse in a circular orbit. As argued above, the final result is the same if one applies the correct reasoning that J_* is the conserved quantity and not J. The result that J_* is a constant implied that we must now be prepared to consider that m_i should vary with distance from the central gravitating body.

6.2 Eccentricity of the Orbit

6.2.1 Problem

Typically, the eccentricity of the orbit of a photon is calculated from the formula:

$$\epsilon^2 = 1 + \frac{2E_{\rm g}J^2}{G^2\mathcal{M}_{\rm g}^2 m_i}. \quad (31)$$

This formula is arrived at by using the kinetic energy equation, and not equation (15). The problem is that one may raise the question that in arriving at (16), we used a different formula *i.e.* (15), thus, this may be a problem. However, as we shall demonstrate shortly, this is not a problem at all.

6.2.2 Rebuttal

Typically, with (31) as given, one proceeds by substituting for the total energy $E_{\rm g}$: *i.e.*, $E_{\rm g} = m_i\boldsymbol{v}^2/2 - G\mathcal{M}_{\rm g}m_{\rm g}/r$, but in the case where $m_i\boldsymbol{v}^2/2 \ggg G\mathcal{M}_{\rm g}m_i/r$, the gravitational potential energy term $G\mathcal{M}_{\rm g}m_{\rm g}/r$ can be neglected so that $E_{\rm g} = m_i\boldsymbol{v}^2/2$. For the specific orbital angular momentum, one substitutes $J = |\boldsymbol{v}|\mathcal{R}_{\rm min}$. Putting all this information into (31), one obtains:

$$\epsilon^2 = 1 + \frac{\boldsymbol{v}^4\mathcal{R}^2_{\rm min}}{G^2\mathcal{M}^2_{\rm g}}. \tag{32}$$

Now, in the case of light $|\boldsymbol{v}| = c$, the term $\boldsymbol{v}^4\mathcal{R}^2_{\rm min}/G^2\mathcal{M}^2_{\rm g}$, is much larger than unity, so much that, the "1" on the right hand side of (32) can safely be neglected. So doing, one is led to exactly the formula as (16). So, a just demonstrated, using either (15) or (31) leads to the same result.

We just said that using either (15) or (31) leads to the same result. The truth is that this is not exactly true. In using (31), we have assumed that the kinetic energy of the photon is the same as that of a classical particle moving at a speed $|\boldsymbol{v}| = c$, that is $E_{\rm g} = m_ic^2/2$. This is obviously not correct. The correct approach would be to substitute $E_{\rm g} = m_{\rm g}c^2$. So doing, one is led to:

$$\epsilon = \frac{\sqrt{2}c^2\mathcal{R}_{\rm min}}{G\mathcal{M}_{\rm g}}. \tag{33}$$

With the eccentricity given by this formula, the corresponding formula for the deflection angle of the path of light in a gravitational field would be:

$$\delta = \frac{\delta_N}{\sqrt{2}}. \tag{34}$$

Thus, for light grazing the limb of the Sun, we will have $\delta = 0.62''$. The actual Newtonian deflection is were uses the "correct formula" for the kinetic energy of the photon, the deflection is much smaller than $0.87''$. We are not going to enter into a debate on this matter of the two deflection angles $0.62''$ and $0.87''$, we simple wanted to point this out and thereafter leave the reader pondering on these matters.

Now, for instructive purposes, we are now going to derive the formula (31), *albeit*, with the γ-factor included. It is necessary for the next section. To to this, we turn to the total energy equation, namely $E_{\rm g} = K(r)+U(r)$, where $K(r) = m_i\boldsymbol{v}^2/2$ is the kinetic energy of the particle, and $U(r) = -G\mathcal{M}_{\rm g}m_{\rm g}/r$ is the gravitational potential energy of the particle inside the gravitational field and $E_{\rm g}$ is the total energy energy content of the particle. The square of the speed is given by $\boldsymbol{v}^2 = \dot{r}^2 + r^2\dot{\varphi}^2 = \dot{r}^2 + J^2/r^2$, so that written in terms of the transformation $u = 1/r$, we have:

$$K(u) = \frac{m_iJ^2}{2}\left[\left(\frac{du}{d\varphi}\right)^2 + u^2\right] \text{ and } U(r) = -G\mathcal{M}_{\rm g}m_{\rm g}u. \tag{35}$$

Inserting these into the equation $K(u) + U(u) = E_{\rm g}$, and then dividing the resultant equation by $m_iJ^2/2$, one is led to:

$$l^2\left(\frac{du}{d\varphi}\right)^2 + l^2u^2 - 2lu = \frac{2E_{\rm g}l^2}{m_iJ^2}. \tag{36}$$

Now, applying equation (14) to the right hand side of the above equation *i.e.* $l = J^2/\gamma G\mathcal{M}_{\rm g}$, one is led to:

$$l^2\left(\frac{du}{d\varphi}\right)^2 + l^2u^2 - 2lu = \frac{2E_{\rm g}J^2}{G^2\mathcal{M}^2_{\rm g}m_i}, \tag{37}$$

and now adding "1" on both sides so that we can complete the square on the left hand side, *i.e.* $z^2 = l^2u^2 - 2lu + 1$, we will have:

$$l^2\left(\frac{du}{d\varphi}\right)^2 + l^2u^2 - 2lu + 1 = 1 + \frac{2E_{\rm g}J^2}{G^2\mathcal{M}_{\rm g}^2 m_i} = \epsilon^2. \tag{38}$$

The right hand side of this equation is a constant, and is the eccentricity of the orbit. This is what we wanted to achieve, that is, derive the eccentricity in terms of $E_{\rm g}, J$ and other physical parameters.

Now, setting $z = lu - 1$, this implies $ldu/d\varphi = dz/d\varphi$, so that the above reduces to:

$$\left(\frac{dz}{d\varphi}\right)^2 + z^2 = 1 + \frac{2E_{\rm g}J^2}{G^2\mathcal{M}_{\rm g}^2 m_i} = \epsilon^2. \tag{39}$$

This is the same formula as (31). Strictly speaking, this equation, *i.e.* (39) or (31), apply only to classical non-relativistic particle because we have used a result of classical non-relativistic physics namely that the kinetic energy is given by $K = \frac{1}{2}m_i\boldsymbol{v}^2$. The advantage of using (15) is that one does not need to know the kinetic energy of the particle in question, all they need to know is the particle's orbital angular momentum. Thus, the eccentricity formula that is derived from (15) is not restricted to classical particles but also to relativistic particles for as long as the particle's orbital angular momentum is the same in the relativistic regime – of which it is.

Now, in the event that $(\gamma \neq 1/2)$ as we have done in Section (5), the same story goes. That is, going through the same steps as has been done from (35) to (39), one is led to:

$$\epsilon^2 = 1 + \frac{2E_{\rm g}J^2}{\gamma^2 G^2\mathcal{M}_{\rm g}^2 m_i}. \tag{40}$$

Applying the same substitutions that led to the eccentricity formula in the case $(\gamma = 1/2)$, one obtains for the case $(\gamma \neq 1/2)$, the eccentricity formula (28).

7 The Scatter

If as in the cases (22) *&* (23), the new formula (30) is applicable to the motion of light, then, the γ-factor brings in a new meaning to the bending of light measurements. There is no reason whatsoever to reject (30) as it has been derived in exactly the same way as (22). The only difference has been to drop the hypothesis $m_i \equiv m_{\rm g}$. Dropping this hypothesis is not in any way a modification of Newtonian gravitation but merely taking Newtonian gravitation in its bare form without making any hypothesis about the equity of inequity of gravitational to inertial mass. we have merely preserved the identities of these masses. What distinguishes our approach is our observation that when the identities of the gravitational and inertial mass are preserved, the specific angular momentum becomes dependent on γ as given in (26). We shall now make the endeavour to associate the γ-factor with the scatter of the light bending measurements.

As a first step, let us define a measure for the deviation of the observational result $(\delta^j_{\rm D})$ from that expected from theory (δ^j_{th}) as:

$$D_j = 1 - \frac{\delta^j_{\rm D}}{\delta_{\rm th}}. \tag{41}$$

Further, let us define a measure for the scatter, that is, the overall deviation of the observational result to that expected from theory as:

$$\langle S\rangle = \sqrt{\frac{\sum_{j=1}^N D_j^2}{N}}. \tag{42}$$

Now, with the above given, the new meaning brought in by the γ-factor in the bending of light measurements is that these measurements actually measure the γ-factor for photons. That is to say, for photons grazing the limb of the Sun, we will have:

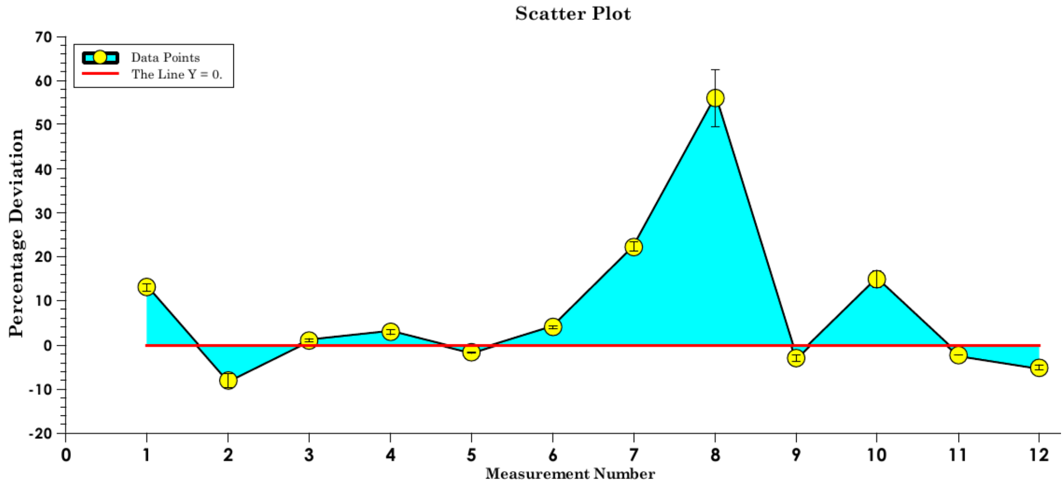

Figure (3): A scatter plot of the twelve Solar eclipse measurements presented in Table I. The scatter is measured relative to the Einsteinian value $1.75''$. Of the twelve, seven measurements have a positive deviation while five have a negative deviation. At about 56% deviation, the Japan (Sedul) measurement has the largest deviation. On average, all the twelve measurements have a scatter of $12.00 \pm 7.00\%$.

$$\gamma = \frac{\delta_\gamma}{0.87''} = 4.14 \times 10^3 \delta_\gamma. \tag{43}$$

If the above proposal is accepted, then, the scatter seen in the bending of light measurements [see column three of Table I] implies that γ is variable for these photons.

The mean in the twelve eclipse deflection measurements listed in Table I and as-well their mean percentage error is $1.89 \pm 0.11''$ and 10% respectively. For both Einsteinian and Newtonian gravitation under the assumption that $\gamma \equiv 1/2$, one has the right to calculate the mean since this value must be the same for all the twelve measurements. Judging from this mean value, the Einstein value of $1.75''$ is recovered in the error margins. However, for Newtonian gravitation where the assumption $\gamma \equiv 1/2$ has been dropped, there is no *priori* nor *posteriori* justification for assuming that γ must be constant. We simple have to take it as is. From these results, one can safely say that γ must be a variable quantity.

A value of $1.89 \pm 0.11''$ certainly favours the Einsteinian result ($1.75''$). But now, in the light of the new proposal that the assumption that $\gamma \equiv 1/2$ be dropped, the eclipse measurements no longer exclusively favour the Einsteinian result because under this new assumption, the eclipse measurements – while they measure the gravitational deflection of light, the effective measurement of these measurements according to (43), is that they measure the γ-factor for the light under observation.

7.1 Statistical Analysis

In-order to obtain an objective view on the nature of the deviation of the 12-light bending measurement presented in Table I, we shall submit these to some objective statistical analysis. Obviously, underlying our statistical analysis are some basic and fundamental assumptions. The conclusions draw thereof hinge on these assumptions.

7.1.1 Best Value

In our statistical analysis, our first port-of-call is to ask, what is the best value of Solar light bending can one derive from these 12-light bending measurement presented in Table I? The best estimate can be obtained by taking the weighted mean of the two values. For example if $(x;\ x_i + \delta x_i\ :\ i = 1, 2, \dots n)$ is set of n measurements of a constant quantity x, where x_i is the best value of for the n^{th} measurement and δx_i is its accompanying error margin, then, the best estimate of x from this set is $x_{\text{best}} = \sum w_i x_i / \sum w_i$ where w_i are the weights such that $w_i = 1/(\delta x_i)^2$ and the best estimate in the error margin δx_{best} is $\delta x_{\text{best}} = (\sum w_i)^{-1/2}$ (see *e.g.* Taylor 1982, p.150). Applying this prescription to the 12-light bending measurements in Table I, we obtain:

$$\delta_{\odot}^{Best} = 1.92 \pm 0.05''. \tag{44}$$

We shall hereafter adopt this value as the best value representing the mean value of the 12-light bending measurements. This value $1.92 \pm 0.05''$ is a significant $9.70 \pm 0.30\%$ higher than the expected Einsteinian value of $1.75''$.

7.1.2 χ^2 Statistical Analysis

If one has a sizeable set of n measurements where O_k is the k^{th} observation and E_k is the corresponding expected value, then, one can calculate χ^2 as follows:

$$\chi^2 = \frac{1}{d}\sum_{k=1}^{N}\frac{(O_k - E_k)^2}{E_k}, \tag{45}$$

where $d \in \mathbb{N} \geq 1$ is the number of degrees of freedom of the given dataset. The number of degrees of freedom $d = n - c_*$ where n is the sample size and c_* is the number of constraints required to derive or compute O_k. If we are testing whether a particular hypothesis holds, then, if $\chi^2 \leq 1$, there is no reason to believe the hypothesis under probe does not hold. If $\chi^2 > 1$, we have reasons to doubt our hypothesis. This is the so-called *null hypothesis* test or the χ^2-*test statistics* (*e.g.* Taylor 1982, pp.228–236). We shall apply this test to the 12-light bending measurements in Table I.

7.1.3 Gaussian Statistical Analysis

Whether or not the scatter in the 12-light bending measurements is a reflection or indicator of the level difficulty of the data procurement process, one fact that is clear is that, if the scatter were due to random natural causes, the distribution of the 12 measurements would be normally distributed. Inspection of Figure (4) reveals the measurements are not normally distributed. Yes, 75% of them fall within one standard deviation from the Einsteinian value $1.75''$ while 25% are above this level. Below the one 1σ-level of the Einsteinian value $1.75''$, there are no measurements. The distribution of the 12 measurements is not Gaussian. Applying a χ^2-test to ascertain whether or not the distribution is Gaussian, one finds that $\chi^2 = 2.44 > 1$, the answer of which is that the distribution is certainly not Gaussian in nature.

The standard deviation σ was computed by taking the Einsteinian value $1.75''$ as the mean *i.e.* $X = 1.75''$ where-after, it was found that $\sigma = 0.34''$. With this, the data was binned as shown in Table II. Clearly, from this table and from Figure (4), the odds are against the Newtonian value of $0.87''$ and very much in favour of the Einstein value of $1.75''$. Our major concern here is not the 75% of the observations seem to support Einstein's $1.75''$, but the 25% of the observations that are significantly well in excess of this value. What is the cause of this? This is our borne of contention. If the scatter was truly of a random statistical nature, we would expect some of the observations to fall at least below the 1σ-level of the mean. From this, one can equanimity say, there is reason to suspect that the 25% of the observations that are significantly well in excess of this value of 1σ-level of the mean may very well be something inherent in the data and not of a statistical random nature.

Table II: **Gaussian Statistical Analysis Table**

Bin, k	1	2	3	4
Range	$(x < X + \sigma)$	$(X + \sigma \leq x \leq X)$	$(X < x \leq X + \sigma)$	$(x > X + \sigma)$
	$(\delta_\odot < 1.58'')$	$(1.58'' < \delta_\odot < 1.92'')$	$(1.92'' < \delta_\odot < 2.25'')$	$(\delta_\odot > 2.25'')$
Probability, P_k	16%	34%	34%	16%
Expected Number, $E_k = NP_k$	2	4	4	2
Observed Number, O_k	0	9	1	2
Percentage (%)	0	75	17	8

7.1.4 Conclusion

If the observed scatter in the data was truly of a random statistical nature, we would expect some of the observations to fall at least below the 1σ-level of the mean 1.75%. However, this is not the case. From this – with judicious equanimity, we draw the conclusion that there is reason to suspect that the 25% of the observations that are significantly well in excess of this value of 1σ-level of the mean may very well be due to something inherent in the data; the cause of the excess may not be of a statistical random nature but systematic and inherent and due to a hitherto unbeknown phenomenon.

8 VLBA Experiments with Radio Waves

Beginning in about 1991, radio-interferometric using the Very-Long-Baseline Interferometry (VLBI) methods impressively determined that Einstein's GTR was in excellent agreement with experience to within 0.02% (Lebach *et al.* 1995). That is to say, if $\delta_{\rm D}$ is the deflection angle from measurements, and $\delta_E = 1.75''$ is the Einsteinian prediction, we can define the ratio $\gamma = \delta_{\rm D}/\delta_E$; then, the said VLBI observations gave $\gamma = 0.9996 \pm 0.0017$. This is an excellent agreement between observations and theory. If observations are to agree 100% with Einstein's theory, then $\gamma = 1$. The value $\gamma = 0.9996 \pm 0.0017$ is obviously unprecedented. This γ-value for these VBLA measurements can be given the same meaning as the γ-value as defined herein. The factor 1/2 (or 2: depends on how one envisions this factor) that we inserted in (4) has been inserted so that the γ-value for these VBLA measurements the γ-value as defined herein will have the same meaning – or at least these two γ's are comparable.

Lebach *et al.* (1995) obtained their impressive result by observing the gravitational deflection of radio waves from the strong radio source – the Quasar, 3C279. Every year in early October, the Sun passes in front of this strong radio source thus presenting interested observers with an opportunity of catch the Sun bending the radio waves from this sources. With radio waves, one does not need the eclipse to subtract the Sun from the background has happens in Eddington-type eclipse observations. Thus, this sources can be observed every year in October.

Actually, several measurements using radio waves have been conducted to test Einstein's gravitational bending of light (see *e.g.* Anderson *et al.* 2004, Shapiro *et al.* 2004, Bertotti *et al.* 2003, Robertson *et al.* 1991, Fomalont *&* Sramek 1975, Shapiro 1964). All these measurements give excellent results. They confirm Einstein's prediction to better than 1 part in 10^5. In comparison to measurements using white-light during Solar eclipses, the "improved accuracy" or the closeness of the deflection angle to that expected from the GTR may very well be that radio waves and visible light may have a different γ-values. These are matters that we will look at much more closely in the section that follows. We shall find out that electromagnetic waves may very well have different γ-values leading to a plausible explanation of not only why there is a 20% scatter seen in the twelve measurements using white-light, but also why radio waves seem to yield better results in the eyes of Einstein's GTR.

Statistical Distribution of the Solar Bending of Light Measurements

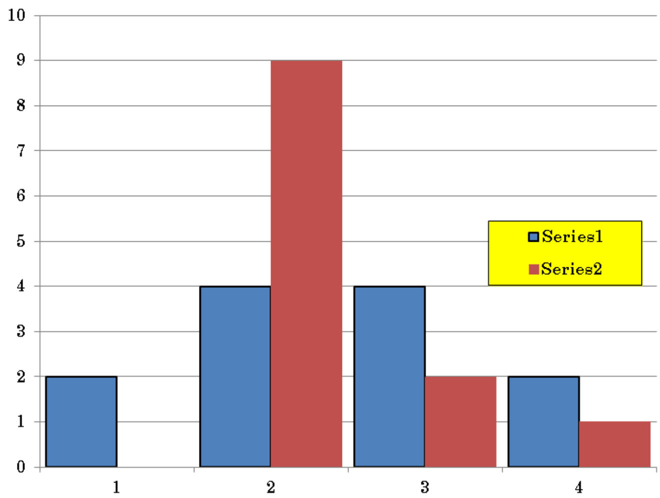

Figure (4): A histogram showing the statistical distribution of the 12 Solar bending of light measurements appearing in Table I. In the histogram, *Series* 1 is the expected Gaussian distribution if the scatter in the measurements were a result of natural randomness while *Series* 2 is the actual distribution.

9 Inertial and Gravitational Mass

In this section, we are going to derive a relationship for γ for both matter and energy (light). We will begin with that for matter and then latter for light (electromagnetic waves). We will commence our endeavours here from Einstein's relativistic energy relationship, *i.e.*:

$$E = \frac{m_0 c^2}{\sqrt{1 - \boldsymbol{v}^2/c^2}} = \Gamma m_0 c^2, \tag{46}$$

where E is the total energy of a particle, $\boldsymbol{v}$ is its speed and m_0 is its rest mass and $\Gamma = 1/\sqrt{1 - \boldsymbol{v}^2/c^2}$. This energy E is the total energy of the particle were only two kinds of energies are considered, that is, the kinetic energy and the potential energy in the form of the rest mass of the particle. If we have to consider all other forms of energy such as the thermal energy, molecular binding energy *etc*, then, $E \longmapsto E - E_{other}$ where E_{other} represents all these other various types of energies: in this case, we will have $E = m_0 c^2/\sqrt{1 - \boldsymbol{v}^2/c^2} + E_{other}$. That said, for none-relativistic speeds where $\boldsymbol{v}^2/c^2 \lll 1$, to first order approximation (46) is given by:

$$E = \frac{1}{2} m_0 \boldsymbol{v}^2 + m_0 c^2. \tag{47}$$

The term $m_0 \boldsymbol{v}^2/2$ is the usual classical kinetic energy E_K of the particle *i.e.* $E_K = m_0 \boldsymbol{v}^2/2$. This means the rest mass m_0 can be identified with the classical inertial mass of an object, *i.e.*:

$$m_0 \equiv m_i. \tag{48}$$

From the forgoing, it means we can write (47) as:

$$E = E_K + m_i c^2. \tag{49}$$

Now, the energy E is equal to mc^2 *i.e.* $E = mc^2$. The question is what is this m in the formula $E = mc^2$; is it the gravitational or inertial mass? If we have only two kinds of mass, the gravitational or inertial mass, m can only be one of these two. If this mass is the inertial mass, it would mean that the kinetic energy of any particle must be zero for all times and all situations in life since: $m_i c^2 = E_K + m_i c^2 \Longrightarrow E_K = 0$. This is obviously nonsense and *must* be *rejected forthwith* without any further deliberations. This leaves us with no choice but to identify the m in $E = mc^2$ with the gravitational mass $m_{\rm g}$, *i.e.*:

$$E_{\rm g} = m_{\rm g} c^2. \tag{50}$$

In this case where $E_{\rm g} = m_{\rm g} c^2$, the kinetic energy is the nothing but the difference between the gravitational and inertial energy of a particle *i.e.* $E_K = (m_{\rm g} - m_i)c^2$. Therefore, written with all the masses well labelled *i.e.* in-terms of the gravitational and inertial mass, (49) must sure be given:

$$E_{\rm g} = \frac{1}{2} m_i \boldsymbol{v}^2 + m_i c^2. \tag{51}$$

By substituting the correct term for the kinetic energy for light and for matter in (51), we are going to derive an expression for γ for the two cases, light and matter. For a particle with gravitational mass $m_{\rm g}$ inside the gravitational field of a particle of mass $\mathcal{M}_{\rm g}$, the total energy of the particle (51) isnow given by:

$$E_{\rm g} = E_K - \frac{G\mathcal{M}_{\rm g} m_{\rm g}}{r} + m_i c^2. \tag{52}$$

Before we embark on main task of deriving the γ-factor for matter and radiation, we shall ask a seemingly simple question which is considered by current thinking to be a settled question, that is, "Can a photon have a non-zero rest mass *i.e.* $m_0 = m_i \neq 0$?". Prevalent and conventional wisdom holds that for a photon $m_0 \equiv 0$, and, in accordance with the foregoing, this means $m_i \equiv 0$. Simple: a photon is massless. Is that so?

9.1 Massive Photon

As stated above, generally, it is agreed (perhaps believed) that a photon has no inertial mass *i.e.* $m_i = 0$. This fact is deduced from two (seemingly) immutable facts. The first is Einstein's energy-momentum equation:

$$E^2 = \boldsymbol{p}^2 c^2 + m_0^2 c^4. \tag{53}$$

The second fact is that the energy of the photon has been found from experience to be given by $E = |\boldsymbol{p}|c$ where $\boldsymbol{p}$ is the momentum of the photon under consideration. If (53) is applicable to the photon, then, the fact that for the photon we have $E = |\boldsymbol{p}|c$, it follows directly that $m_0 = 0$. We have already argued above that a comparison of non-relativistic classical physics and relativistic classical physics leads us to the fact that $m_i = m_0$. From all this, it follows that light must have zero inertial mass. If all these facts are correct, the path of light – in accordance with classical mechanics; is not supposed to be altered by a gravitational field. This question brings some contradictions. However, by applying Einstein's GTR, one is able to deduce that a zero rest mass photon will have its path altered by a gravitational field. From this fact, one might then argue that the GTR is the only theory that answers this questions satisfactorily.

From the foregoing, it is thus accepted that if an object has zero rest mass, it will move at the speed of light. Conversely, if a particle moves at the speed of light, its rest mass must vanish identically. The hidden assumption in all this reasoning is that the energies (E) in the formulae $E = |\boldsymbol{p}|c$ and $E^2 = \boldsymbol{p}^2 c^2 + m_0^2 c^4$ are identical. On a more fundamental level, there is no *priori* nor *posteriori* justification for this clandestine assumption.

Further, by taking a closer look at the wave-particle duality of matter, we here question this belief that the energies (E) in the formulae $E = |\boldsymbol{p}|c$ and $E^2 = \boldsymbol{p}^2 c^2 + m_0^2 c^4$ are identical. Our analysis rests on the notion of particle and group velocity[5] of a wave. We hold that – in principle; a particle of non-zero rest mass can move at the speed of light. To see this, if one accepts (50) and as-well that $m_0 = m_i$, then, equation (53) can be written as:

$$E_{\rm g}^2 = \boldsymbol{p}^2 c^2 + m_i^2 c^4. \tag{54}$$

If equation (54) is the energy equation for the wave-packet, then $\boldsymbol{p}$ is the momentum of the waving-particle, it is the particle momentum and not that of the wave-packet. From this, it follows that the particle velocity $\boldsymbol{v}_p$ is given by $\boldsymbol{v}_p = \boldsymbol{p}/\Gamma m_i$ where $\Gamma = 1/\sqrt{1 - \boldsymbol{v}_p^2/c^2}$ is the Einstein Γ-relativistic factor. From these simple assertions, we can – as we shall do shortly, argue that a particle does not need to have zero-inertial-mass in-order for it to move with a group velocity whose magnitude is equal to the speed of light in vacuum.

To see this, we know that for a wave-packet whose total energy is $E_{\rm g}$, the group velocity $\boldsymbol{v}_g$ of this wave-packet is given by:

$$\boldsymbol{v}_g = \frac{\partial E_{\rm g}}{\partial \boldsymbol{p}}. \tag{55}$$

Applying (55) to (54) and talking into account the afore-stated assumptions, one obtains:

$$\boldsymbol{v}_g = \left(\frac{|\boldsymbol{p}|c}{E_{\rm g}}\right) c. \tag{56}$$

Now, since for a photon $E = |\boldsymbol{p}|c$, where the energy E is assumed to be equal to $E_{\rm g}$, it follows that $|\boldsymbol{v}_g| = c$. From all this, it follows immediately from (54) that $m_i = 0$. This is not what we want. We desire a scenario where for a photon $m_i \neq 0$ and $|\boldsymbol{v}_g| = c$.

To achieve our desired end, we will question the energy formula $E = |\boldsymbol{p}|c$ for a photon. For example, we know that there are two kinds of momentums for a particle, the classical moment and the relativistic momentum. The classical momentum is given by $\boldsymbol{p}_{cl} = m_i \boldsymbol{v}$ and the relativistic momentum is given by $\boldsymbol{p} = \Gamma m_i \boldsymbol{v} = \Gamma \boldsymbol{p}_{cl}$. So, our *borne of contention* in the photon energy formula $E = |\boldsymbol{p}|c$ is what this the momentum $|\boldsymbol{p}|$ appearing in this formula? Is this the classical momentum or the relativistic momentum? If it is the relativistic momentum, we are lead nowhere into our sought for end. If we however set this momentum to be the classical momentum, we are lead to our desired result. Let us write (54) with $\boldsymbol{p} = \Gamma \boldsymbol{p}_{cl}$, *i.e.*:

$$E_{\rm g}^2 = \Gamma^2 \boldsymbol{p}_{cl}^2 c^2 + m_i^2 c^4. \tag{57}$$

so that (56) becomes:

$$\boldsymbol{v}_g = \left(\frac{\Gamma|\boldsymbol{p}_{cl}|c}{E_{\rm g}}\right) c. \tag{58}$$

This group velocity formula applies to speeds $|\boldsymbol{v}_g| < c$. Now, if the energy[6] E, of a photon is such that $E = |\boldsymbol{p}_{cl}|c = |m_i \boldsymbol{c}|c = m_i c^2$ (where $|\boldsymbol{c}| = c$), then it is possible to have $m_i \neq 0$, such that $|\boldsymbol{v}_g| = c$. What this means at the end of the day is that the velocity $\boldsymbol{v}$ in $\Gamma = 1/\sqrt{1 - \boldsymbol{v}^2/c^2}$ can not be the group velocity, it can only be the phase velocity of the wave-packet since a wave has only two associated velocities, the group and the phase velocity. From simple logic, if the velocity is not one of the two velocities, it must be the other. We therefore conclude that the velocity $\boldsymbol{v}$ in

[5]The ***group velocity*** of a wave is the velocity with which the overall shape of the wave's amplitudes – known as the modulation, wave-packet or envelope of the wave – propagates through space. The ***phase velocity*** (sometimes defined/referred to as the ***particle velocity***) of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest or trough) will appear to travel at the phase velocity.

[6]Notice that this energy E is not the total energy $E_{\rm g}$ of the photon. This would mean, the energy $\hbar\omega$ of a photon is not the total energy of the photon. It is most logical to think of or to identify $\hbar\omega$ with the kinetic energy of the photon.

Γ is the phase velocity so that $\Gamma = 1/\sqrt{1 - \boldsymbol{v}_p^2/c^2}$. From all this, it follows that for a photon, we must have $E_{\rm g}^2 = (\Gamma^2 + 1)m_i^2c^4$, so that:

$$\gamma_L = \frac{1}{2}\sqrt{\Gamma^2 + 1}. \tag{59}$$

For matter, we simple substitute $\boldsymbol{p}_{cl} = m_i\boldsymbol{v}_g$ and $E_{\rm g} = m_ic^2$ into (58), we obtain:

$$\gamma_M = \frac{1}{2}\sqrt{1 - \frac{\boldsymbol{v}_g^2}{c^2}} \implies (0 < \gamma_M \leq 1/2), \tag{60}$$

where γ_L is the γ-ratio for electromagnetic waves and γ_M is the γ-ratio for matter (waves). In the γ-value for matter $\boldsymbol{v}_g$, is the group velocity of matter, which in actual fact is the velocity that we measure of the matter particle.

Now, to find the possible range of values of γ for electromagnetic waves, we know that $\Gamma^2 \geq 1$; inserting this condition into (61) were one obtains:

$$\gamma_L \geq \frac{1}{\sqrt{2}} \simeq 0.71. \tag{61}$$

This constraint places a lower limit on the possible range of the deflection angle (δ_γ) for any gravitating object. In the case of the Sun, we will have:

$$\delta_\gamma^\odot \geq \sqrt{2}\left(\frac{2G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot}\right) = \sqrt{2}\delta_N = \frac{1}{2}\sqrt{2}\delta_E = 1.23''. \tag{62}$$

If the above ideas are correct, then, it must be possible to obtain sub-Einsteinian deflections as low as $1.23''$. Not only that, we must as-well be able to obtain super-Einsteinian gravitational deflections exceeding the $1.75''$ prediction of Einstein. From table I, seven of the twelve measurements are super-Einsteinian deflections, while, five are sub-Einsteinian deflections.

In arriving at the above result, the phase and group velocity of a photon have been assumed to be different (in magnitude) from each other. The phase and group velocity of a wave-packet are two different physical quantities. When we talk of the speed of a photon, we basically mean their group velocity. From wave theory, generally $|\boldsymbol{v}_g| \neq |\boldsymbol{v}_p|$. Bellow we shall apply the result (59) and (60) to light and matter in a gravitational field respectively. For light, it shall be seen that $|\boldsymbol{v}_g| \neq |\boldsymbol{v}_p|$ while for matter we have $|\boldsymbol{v}_g| = |\boldsymbol{v}_p|$. It appears here as though the relationship between group and particle velocity may very well distinguish between matter and waves in the wave-particle duality picture of *Nature*.

9.2 γ-Factor for Light in a Gravitational Field

9.2.1 Derivation ($|\boldsymbol{v}_p| \neq |\boldsymbol{v}_g| = c$)

If a photon is in a gravitational field, its energy-momentum relation is given by: $(E_{\rm g} - V)^2 = \Gamma^2\boldsymbol{p}_{cl}^2c^2 + m_i^2c^4$ where $V = -G\mathcal{M}_{\rm g}m_{\rm g}/r$ its gravitational potential energy of the photon in a spherically symmetric Newtonian gravitational field, it follows that:

$$\gamma_L = \frac{1}{2}\left(1 + \frac{G\mathcal{M}_{\rm g}}{rc^2}\right)^{-1}\sqrt{\Gamma^2 + 1}. \tag{63}$$

For Solar gravitational deflections, $G\mathcal{M}_{\rm g}/rc^2 \lll 1$, we will have:

$$\gamma_L = \frac{1}{2}\sqrt{1 + \left(1 - \frac{\boldsymbol{v}_p^2}{c^2}\right)^{-\frac{1}{2}}}. \tag{64}$$

Now, if we take Louis *de* Broglie's wave-particle duality relation, that is $|\boldsymbol{p}| = \hbar/\lambda$, we shall assume that the momentum $\boldsymbol{p}$ in this relation is the classical momentum of the particle, then:

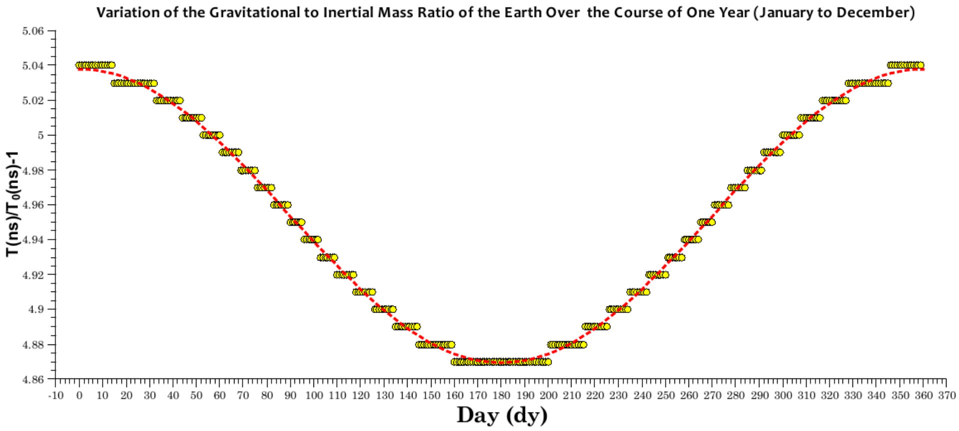

Figure (5): A graph of the expected variation of the period of a simple pendulum over the course of one year. On the x-axis, it is assumed that day 1 is the day January 1 and day 365 is December 31. In units of 10^{-9}, the y-axis represents the quantity $(\langle T\rangle - \langle T_0\rangle)/\langle T_0\rangle = \gamma^{-1/2} - 1$. The expected regime of sensitivity for the clocks required for the accuracy to measure this effect is at least better than $0.01\eta s$. This sensitivity is well within the capabilities of current technology. In all its aspects, this experiment, if conducted, it will have to be accurate on a level of at least 10^{-11}%; for example, the length L and the gravitational field strength g_E, at the Earth's surface will have to be known at this level of accuracy.

$$\gamma_L = \frac{1}{2}\sqrt{1 + \left(1 - \frac{\lambda_c^2}{\lambda^2}\right)^{-\frac{1}{2}}} = \frac{1}{2}\sqrt{1 + \left(1 - \frac{f^2}{f_c^2}\right)^{-\frac{1}{2}}}, \tag{65}$$

where λ_c and f_c are the Compton wavelength and frequency of the photon respectively. From (65), it follows that $\gamma_L = \gamma_L(\lambda, \lambda_c) = \gamma_L(f, f_c)$.

9.2.2 Why the "Excellent" VLBA Measurements?

Before we go on to derive the γ-relationship for matter, we are going to take this opportunity to give a plausible explanation for the excellent results obtained for radio waves and the seemingly poor results from eclipse measurements. The first thing we should take note of that Eclipse measurements make use of white-light while VLBA make use of nearly monochromatic radio waves. For electromagnetic waves like radio waves which confirm Einstein's GTR, according to the present idea, these waves are such that $\gamma_L \simeq 1$, the magnitude of their phase velocity is approximately $\sqrt{6}c/3 \simeq 0.82c$ which leads to $\gamma_L \simeq 1$. As for white-light as is the case in Eclipse observations, an explanation for a varying γ_L leading to a scatter in δ may very well be that this white-light is coming from stars emitting at different wavelengths. Since their temperatures certainly varies, their emitted wavelength must vary leading to a variable deflection angle as evidenced by the 20% scatter.

9.3 γ-Factor for Matter in a Gravitational Field

9.3.1 Derivation ($|\boldsymbol{v}_p| = |\boldsymbol{v}_g|$)

Taking the equation $(E_{\rm g} - V)^2 = \boldsymbol{p}^2c^2 + m_i^2c^4$ to first order approximation in $E_{\rm g}$, one finds that for an ordinary classical particle:

$$E_{\rm g} = \frac{1}{2}m_i\boldsymbol{v}_p^2 - \frac{G\mathcal{M}_{\rm g}m_{\rm g}}{r} + m_ic^2. \tag{66}$$

Now, given that $\boldsymbol{v}_p^2 = 2\gamma_M G\mathcal{M}_{\rm g}/r$, it follows that after dividing (66) throughout by m_ic^2 and rearranging, one obtains:

$$\gamma_M = \frac{1}{2}\left(1 + \frac{G\mathcal{M}_g}{rc^2}\right)^{-1} \simeq \frac{1}{2}\left(1 - \frac{G\mathcal{M}_g}{rc^2}\right), \tag{67}$$

One can show that (67) can be derived from (60) under the assumption that $|\boldsymbol{v}_g| = |\boldsymbol{v}_p|$. From this we conclude that for matter we must have $|\boldsymbol{v}_g| = |\boldsymbol{v}_p|$. Since we have already argued that for light $|\boldsymbol{v}_g| \neq |\boldsymbol{v}_p|$, the relationship between the group and particle velocity must play a central role in deciding the difference between matter and light.

9.3.2 Proposal to Measure γ for Matter

We put forward a proposal to test the hypothesis here set-forth that γ may vary depending on the particle's position in a gravitational field of a massive central gravitating body. Our proposal is to use the very same experiment that the great Sir Isaac Newton used to measure this quantity *i.e.*, the simple pendulum experiment. The period of a simple pendulum of length L under and a variable γ_M is given by:

$$T = 2\pi\sqrt{\frac{L}{2\gamma_M g_E}} \simeq 2\pi\sqrt{\frac{L}{g_E}}\left(1 + \frac{1}{2}\frac{G\mathcal{M}_g}{rc^2}\right), \tag{68}$$

where g_E is the gravitational field strength at the Earth's surface. From the above, we can write $\langle T\rangle = \langle T_0\rangle + \langle \delta T_0\rangle$, where:

$$\langle T_0\rangle = 2\pi\sqrt{\frac{L}{g_E}} \quad \text{and} \quad \frac{\langle T\rangle - \langle T_0\rangle}{\langle T_0\rangle} = \frac{1}{2}\frac{G\mathcal{M}_g}{rc^2}. \tag{69}$$

In the above, $\langle T\rangle$ is the average value of the period of the pendulum as measured in the laboratory. Likewise, the value $\langle T_0\rangle$ is the average value of $2\pi\sqrt{L/g_E}$ as measured in the laboratory. Now given that $1/r = (1+\epsilon\cos\varphi)/l$ where ϵ is the eccentricity of the Earth's orbit and $l = (1-\epsilon^2)\mathcal{R}_{\min}$ is the semi-luctus rectum of this orbit: $\mathcal{R}_{\min}$ is the minimum radial distance of closest approach of the Earth to the Sun. From the given information, it follows that:

$$\frac{\langle T\rangle - \langle T_0\rangle}{\langle T_0\rangle} = \frac{G\mathcal{M}_g(1+\epsilon\cos\varphi)}{2lc^2} = \frac{1}{\sqrt{\gamma}} - 1. \tag{70}$$

If a seasonal variation in $(\langle T\rangle - \langle T_0\rangle)/\langle T_0\rangle$ is found as shown in figure (5), it would be a clear indicator of the correctness of the present ideas. The greatest difficulty would be in the accuracy of the measurements, one would require a clock that can measure time to an accuracy of 12 significant figures. To see this, lets make a crude but accurate calculation. We know that $G = 6.667\times10^{-11}\,\mathrm{kg^{-1}m^3s^{-2}}$, on average $l = 1.49\times10^{11}\,\mathrm{m}$, $c = 2.99792458\times10^{8}\,\mathrm{m/s}$ and taking $\mathcal{M}_g = \mathcal{M}_\odot^g = 1.99\times10^{30}\,\mathrm{kg}$, so that $\langle\delta T_0\rangle/\langle T_0\rangle = 5.00\times10^{-9}$. It follows that in-order to measure the period of the pendulum in a way that will yield results, one has to be able to measure this period to the $(9+3)^{th} = 12^{th}$ significant figure.

For example, let us take a simple pendulum of length $L = 1.00000000000\,\mathrm{m}$ and further, let us assume that gravitational field strength at the Earth's surface $g_E = 9.80000000000\,\mathrm{ms^{-2}}$, such a pendulum will have a period of:

$$\langle T\rangle = 2.00708993\overline{\mathbf{310}}\,s. \tag{71}$$

In this time interval, the most important figures are the last three numbers in bold and with an over-bar. These are the figures that will determine the correctness of our assertion because over the course of a year (preferably from January to December), these three figures are expected to vary in a way conforming to our assertion set-forth here-above. So, the proposed experiment must have the capacity to measure time durations to an accuracy of at least $0.01\eta s$.This sensitivity is well within the capabilities of current technology and thus it should be possible to conduct this experiment.

10 Status of Einstein's Principle of Equivalence

At the very heart and nimbus of Einstein's GTR is Einstein's seemingly sacrosanct *Equivalence Principle* (EP); this is an idea that came to Einstein in a melodramatic 1907 epiphany, that is, two years after he had developed the STR (Einstein 1907). Einstein wanted to extend the successful Principle of Relativity to include accelerated reference systems. The Principle of Relativity is a principle upon which his STR is founded. This principle holds that *Physical Laws* hold good in *all* inertial reference systems. To accomplish this, Einstein reasoned from the standpoint of the well known and clearly evident fact that a person in free-fall in a gravitational field does not feel their own weight. This idea led him to the melodramatic conclusion (hypothesis) that the inertial and gravitational phenomenon are intimately linked. In Einstein's own words in his 1907 review article on his STR, his exact word read:

> "... consider two systems S_1 and S_2 ... Let S_1 be accelerated in the direction of its $x-axis$, and let g be the (temporally constant) magnitude of that acceleration. S_2 shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration −g in the direction of the $x-axis$. As far as we know, the physical laws with respect to S_1 do not differ from those with respect to S_2; this is based on the fact that all bodies are equally accelerated in a gravitational field. At our present state of experience we have thus no reason to assume that the systems S_1 and S_2 differ from each other in any respect ... we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system."

Off cause, the great Sir Isaac Newton was the first to show a link between the gravitational and inertial phenomenon, but Einstein was the first to go further and deeper than Newton on the issue when he reasoned that the gravitational phenomenon was not only linked to inertial phenomenon, but to the geometry of spacetime; in his conclusion – ultimately, mass curves spacetime and a curved spacetime tells one of the amount and distribution of mass giving rise to this curvature. What set Einstein apart from Newton on this issue is his seeming bizarre assertion that gravity "does not exist" with respect to a freely falling system of reference; Einstein elevated this idea to the status of a universal Principle *of* Nature applicable to all matter everywhere all the time, *i.e.* across all spacetime. Einstein called this principle – which has been embellished over the years – the *Equivalence Principle*.

Put in simple terms, the EP states that gravity and acceleration are equivalent physical phenomenon. In an environment free from gravitation, one can generate a gravitational field by using inertial acceleration, and conversely, the effects of gravitational acceleration are the same as those generated from an inertial acceleration. Interwoven and intertwine in Einstein's EP are three separate principles (see *e.g.* Will 2009): that is, (1) The WEP, and (2) the principles of Local Lorentz Invariance (LLI) and (3) Local Position Invariance (LPI). More explicitly:

1. **WEP:** Test bodies fall with the same acceleration independently of their internal structure or composition. This is the WEP first set into motion by Galileo's famous experiment at the Learning Tower *of* Pisa in Italy.

2. **LLI:** The outcome of any local non-gravitational experiment is independent of the velocity and acceleration of the freely-falling reference system in which it is performed. This is the Local Lorentz Invariance principle.

3. **LPI:** The outcome of any local non-gravitational experiment is independent of where and when in the Universe it is performed. This is the Local Position Invariance.

The EP naturally casts the gravitational field into a metric theory (see *e.g.* Will 2009), thus justifying Einstein's GTR.

Now, for as long as all material bodies have the same value of γ at a given point (r, θ, φ) in a gravitational field, they will fall at the same rate, thus, Galileo's falling bodies experiments at the Learning Tower of Pisa in Italy, will hold exactly leading to the same conclusion everywhere in the Universe. The present ideas suggest that for matter $\gamma = \gamma(r)$ [or more generally $\gamma = \gamma(r, \theta, \varphi)$], so all material bodies have the same value of γ at a given point in a gravitational field. This result comes about because we find that for matter, the group and the particle velocities lead to material particles to have the same value of γ at a given point in a gravitational field. The same is not true for light because for light the group and particle velocities are not equal thus leading to photons to have different values of γ which depend on their wavelength. The consequence is that the otherwise straight path of light of different wavelengths will suffer different bending angles in a gravitational field.

For both matter and radiation, in the local neighbour of a freely falling cabin in a gravitational field, the LLI and the LPI principles will hold exactly. For an observer in a closed freely falling cabin, they will not be able to distinguish whether they are in a gravitational field or they are experiencing an inertia generated acceleration. They will however be able to tell that their closed freely falling cabin is undergoing an acceleration by measuring the deflection of light rays of different wavelength.

Now, given the violation of the WEP by radiation, the question arises "Does this mean that Einstein's EP does not hold any longer?" Further, "Does this mean that Einstein's GTR is not correct?" We strongly believe that Einstein's GTR requires the survival of only the LLI and the LPI principles. These two principles *i.e.* the LLI and the LPI principles should be sufficient to uphold the Principle of Relativity. In its depth and breath, the EP's ultimate endeavour is to uphold the LLI and the LPI principles, that is:

> *...for as long as the outcome of any local non-gravitational experiment is independent of the velocity and acceleration of the freely-falling reference system in which it is performed ...*

Einstein's GTR is safe. Actually, in-order to in-cooperate the new development, there is a way out. Einstein's bare GTR will have to be transformed into a conformal theory of gravitation much akin to Weyl (1918, 1927*a*,7)'s conformal theory of gravitation *i.e.*, in Riemann geometry, the metric $g_{\mu\nu}$ will have to be transformed into $2\gamma g_{\mu\nu}$: $g_{\mu\nu} \longmapsto 2\gamma g_{\mu\nu}$, so that the line element of spacetime ds now becomes:

$$ds^2 = 2\gamma g_{\mu\nu} dx^\mu dx^\nu, \tag{72}$$

where x^μ is the four position in spacetime. Under such a setting, γ is now a scalar field. The geodesic equation in such kind of a spacetime is:

$$\frac{d^2x^\lambda}{ds^2} - 2\gamma \left(\Gamma^\lambda_{\mu\nu} + \mathrm{W}^\lambda_{\mu\nu}\right) \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \tag{73}$$

where $\Gamma^\lambda_{\mu\nu}$ is the usual Christophel three symbol in Reimann geometry and:

$$\mathrm{W}^\lambda_{\mu\nu} = \frac{1}{2}\left(\delta^\lambda_\nu \partial_\mu + \delta^\lambda_\mu \partial_\nu - g_{\mu\nu}\partial^\lambda\right) \ln \gamma, \tag{74}$$

is the corresponding *Weyl* conformal affine connection. To first order approximation, in the weak field approximation where the *Weyl* conformal affine connection $\mathrm{W}^\lambda_{\mu\nu} \sim 0$, (73) reproduces the equation of motion (25) which leads directly to the main result (29) of the present work. Obviously, there is need for more work than has been conducted here in-order to get these ideas on a firm pedestal.

11 Discussion, Conclusion and Recommendations

11.1 General Discussion

At any rate, we do not doubt that the path of light (or an electromagnetic wave) is altered by a gravitational field nor do we doubt that experience as revealed by the gravitational deflection of radio waves strongly and clearly favours Einstein's GTR. Our *bone of contention* is what this deflection really measures? As presented herein, other than a confirmation that gravity does indeed alter the path of light, we have asked a rather disturbing question: "Are these experiments not also a measure of the gravitational to inertial mass ratio of photons? Why do we have a scatter ($\sim 20\%$) that is one and half times the average margin of error ($\sim 13\%$) of all the twelve experiments from 1919 to 1973?" If anything, these (seemingly) simple questions need solid answers. This is what we have tried to provide herein.

The findings of this reading if correct, then, it is without doubt that they have serious implications on the foundations of physics, at the very least, they call for nothing short of a *rethink* on our understanding of these foundations. For example, "Truly, does the all-embellished WEP hold universally? That is, does it hold for all matter and radiation?" In the light of what has been presented, this is just but one of the many deep, important foundational questions that come to the deeply enquiring mind seeking understanding of the most fundamental Laws of *Nature*. In our modest and humble of opinion, we strongly believe that we have shown herein that, from its own internal logic and consistency, Newton's gravitational theory strongly suggests that the WEP may not hold, especially with electromagnetic radiation.

In short, the main thrust of this work has been to rise the point that the γ-factor (as defined herein) may be used in Newtonian gravitation to furnish any deficiency that may arise from measurements of the deflection of light by a gravitational field. Solar gravitational deflection of light is not constrained to Einstein's $1.75''$ deflection but any arbitrary amount depending on the photon's γ-value. Naturally, unwillingly and involuntarily, this rises the rather contentious and polemical question of "*What one is to make of Sir Professor Eddington's efforts ... and as-well, what one is to make of similar efforts?*"

Are these efforts to be understood as confirming Einstein's $1.75''$ deflection? or are they to be understood as confirming the a gravitational field does alter the path of an electromagnetic wave in such a manner that this deflection angle is a direct measure of the photon's gravitational to inertial mass ration γ as defined herein? Are we to be content with the 20% scatter about Einstein's $1.75''$ as reflective of the level difficutly in the measurements or are we to understand this scatter as reflective of a variable γ that deviates from the expected $\gamma = 1/2$?

As tabled in Table I, the average value for the displacement of the stellar images from the Sobral observations was $1.98 \pm 0.12''$, and the Principe observations yielded a value of $1.61 \pm 0.30''$. These measurements clearly showed that two of the initial possibilities could be rejected, leaving the predictions of the GTR as the most plausible. It is now common knowledge that this expedition determined Einstein's findings to be correct to within 8 to 34%. In 1995 the accuracy was greatly improved. It was determined by radio-interferometric (Very-Long-Baseline Interferometry; VLBI) methods that and these measurements indicated that Einstein's GTR was correct to within 0.02% (Lebach *et al.* 1995). The "improved accuracy" or the closeness of the deflection angle to that expected from the GTR may very well be that radio waves and visible light may have a different γ-values. These are matters that will require to be looked at much more closely in-order to establish the truth.

If, truly, it occurs that $\gamma \neq 1/2$, invariably, this means, once the WEP is found wanting, so is Einstein's GTR, it must be found wanting too. Is this the case? The answer is yes, Einstein's GTR will be found in a wanting-state. To see this, for a minute, imagine or suppose one is inside a freely falling cabin and in this cabin, they hold two relatively small masses that have differing γ's. Further, the γ of the cabin itself is different from that of the two masses – lets suppose it is much less than that of the two masses. Clearly, these two masses if let loose, they will not remain at rest relative to the freely falling cabin, but will begin to move. The mass with a much higher γ will be seen to accelerate toward the fall of the cabin much faster than the other mass with a lower

γ. The freely falling cabin is no longer an inertial reference system. But, if the γ of the cabin and the two masses are the same, the cabin will behave like an inertial reference system exactly as Einstein (1907) imagined.

In the light of the present reading – unless off-cause one has sufficiently reasonable ground to do so; one can not arbitrarily set ($\gamma = 1/2$), because, the very fact that the bending angle, as procured from experience, is not compatible with the setting ($\gamma = 1/2$) as evidenced by the significant scatter in the observational measurements of δ_{D}, this may very well be that the value of γ that restores agreement between experience and theory is the γ-value of the photons under observation.

In-closing, allow us to say that if we were asked whether or not the WEP holds, without hesitation, our answer to this is that "*...it is very possible that this principle – as commonly understood i.e. were γ is the same for all material bodies everywhen and everywhere; may not hold as we have come to believe ...*" The justification is what we have presented herein. In the light of this, we do not know what interesting answer our reader(s) would give to this same question. As to whether Einstein's GTR falls apart or not, we would say no, it does not fall apart as it has been argued herein that the violation of the WEP in its popular understanding, this does not translate nor entail a violation of the EP by matter. However, Einstein's GTR will need to be revisited if the γ-factor is accepted or shown to actually explain the scatter in the deflection measurements because Einstein's GTR will have to successfully stand to these data. This will certainly require a modification of the GTR. In the final two subsections, we give in a succinct a manner as is possible, our conclusions drawn thereof and recommendations.

11.2 Conclusion

Assuming the correctness (or acceptability) of the thesis set-forth herein, we hereby make the following conclusions:

1. As is widely believed, the eclipse results of the Solar gravitational bending of light carried by Eddington and his team in 1919 and all subsequent eclipse measurement results from 1922 to 1973; these measurements do not point to Einstein's GTR being a superior theory to Newtonian gravitational theory, because, according to the Newtonian gravitational theory under the WEP violation, the eclipse results of the Solar gravitational bending of light actually measure the γ-factor for light and the deflection angle. To predict the deflection angle, one will need to know forehand that value of γ for light. There is no *priori* justification to set this value to unity thus, from these eclipse measurements, one can safely say that is strongly appears that γ varies markedly for white-light while for radio wave waves this ratio appears to be constant.

2. The excellent results of the VLBA measurements leading to researchers to fervently claim that these excellent measurements vindicate Einstein's GTR; this may very well suggest that for these waves $|\boldsymbol{v}_p| \simeq \sqrt{6}c/3$; in this way, *i.e.* $|\boldsymbol{v}_p| \simeq \sqrt{6}c/3$, we will have $\gamma \simeq 1$, which would for the Sun mean $\delta_D \sim 1.75''$.

3. The poor eclipse measurements leading to researchers to suggest that these poor results are a reflection of the *level difficulty* in the measurement process itself; these results, may very well suggest that the gravitational to inertial ratio of photons depends on the wavelength. Since the white-light grazing the Solar limb comes from light sources emitting in varying frequency range in the visible part of the electromagnetic spectrum, these photons will have different γ-ratios, leading to varying deflection angles. The variation of the deflection angle is perhaps what we observe as a 20% scatter in the twelve Eclipse measurements so far made from 1919 right up to 1973. This scatter is a significant 1.46 times the average margin of

error of the twelve Eclipse measurements.

4. It strongly appears that the WEP may very well not hold. However, its violation does not lead to a violation of the Local Position Invariance which is necessary for the survival of Einsten's GTR. Consequently, Einstein's GTR holds, *albeit*, it will have to be modified so that is takes into account the hypothesised positional variation of γ. We have briefly shown herein that a conformal metric theory of gravitation akin to that of Weyl can replace Einstein's GTR with relative ease. They resulting theory needs to be worked out properly to put it on a firm footing and pedestal.

5. A crucial test to the present ideas is to measure the gravitational to inertial mass of the Earth *via* the proposed pendulum experiment like that performed by Sir Isaac Newton. These experiments will however have to be performed with a level accuracy of the order of one part in one hundred billion. A positive result would mean the WEP does not hold as currently understood that the ratio of the gravitational to inertia mass of all material bodies in the Universe is identical. Expected to be observed in this experiment is a seasonal variation of γ-ratio for material that is *in-situ* on Earth.

11.3 Recommendations

Assuming the correctness (acceptability/plausibleness) of the ideas laid down herein, we hereby put forward the following recommendations:

1. There is need for renewed efforts to measure much more accurately the gravitational deflection of light at every eclipse opportune in the future in-order to ascertain for sure that Einstein's GTR exclusively explains this phenomenon. It must be demanded of these precision measurements that the deflection angle must not vary markedly from Einstein's prediction. Any scatter in these results only support the thesis set-forth herein that γ is not unity and that is varies depending on the light mixture.

2. In the light of the new ideas set-forth herein, the need for a thorough reanalysis of all the available eclipse data can not be overstated. Such a study needs to treat the deflection of each star individually and not what has been happening where the individually measured deflections of stars belonging to a set of given a eclipse data set, one averages the deflection of all the stars on the plate after which a single value for the deflection for that dataset is given as representative of the overall Solar gravitational deflection of light. The thrust of such a study would be to unearth the true nature of the scatter of these deflections about the predicted Einsteinian value of $1.75''$.

3. To avoid and reduce – if not eliminate public scepticism; in the true spirit of science, raw data from the eclipse measurements must be published so as to allow and afford able members of the general public to verify for themselves the true nature and extend of Solar gravitational deflection. Eddington – and some latter eclipse observers, have discarded some of the plates with the simple remark that "*these plates were bad, so bad that no useful data could be derived from them*". The general public must be given the opportunity to judge for themselves whether or not these plates are really bad such that no useful data can be derived from them. This is important for the progress of science and as-well for progress on this matter of the Solar gravitational bending of light *vis* whether or not these measurements vindicate 100% Einstein and place his GTR on a distinctively superior pedestal to Newton's gravitational theory.

Given that currently, Einstein's GTR is an all-embellished and seemingly sacrosanct touchstone of gravitation – at this juncture in its history and development; could there be something Einsteinian gravitation can learn from its predecessor *i.e.* Newtonian gravitation? We pause or ask this question to our dear reader because we solemnly hold that, in the light of the new ideas presented herein, a thorough non-biased publishing of raw data will certainly lead to a scenario where Einstein's GTR may learn one or two things from Newtonian gravitation. As to ourself on this matter – *i.e.* on Einstein's GTR learning one or two things from Newtonian gravitation; confidently, without an iota or dot of doubt; we think, believe and hold that:

"*Yes, there is.*"

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