Could $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM be a part of TGD?

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January 1, 2013

Abstract

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. These states should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states. So called massless extremals (MEs) allow massless solutions of the modified Dirac equation for right-handed neutrino in the interior of space-time surface, and this seems to be case quite generally in Minkowskian signature for preferred extremals. This suggests that particle represented as magnetic flux tube structure with two wormhole contacts sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM emerges at the level of space-time geometry. The following arguments inspired by the article of Nima Arkani-Hamed et al about twistorial scattering amplitudes suggest a more detailed physical interpretation of the possible SUSY associated with the right-handed neutrinos.

The fact that right handed neutrinos have only gravitational interaction suggests a radical re-interpretation of SUSY: no SUSY breaking is needed since it is very difficult to distinguish between mass degenerate spartners of ordinary particles. In order to distinguish between different spartners one must be able to compare the gravitomagnetic energies of spartners in slowly varying external gravimagnetic field: this effect is extremely small.

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1 Introduction

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2 Scattering amplitudes and the positive Grassmannian

The work of Nima Arkani-Hamed and others represents something which makes me very optimistic and I would be happy if I could understand the horrible technicalities of their work. The article Scattering Amplitudes and the Positive Grassmannian by Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, and Trnka [B1] summarizes the recent situation in a form, which should be accessible to ordinary physicist. Lubos has already discussed the article. The following considerations do not relate much to the main message of the article (positive Grassmannians) but more to the question how this approach could be applied in TGD framework.

2.1 All scattering amplitudes have on shell amplitudes for massless particles as building bricks

The key idea is that all planar amplitudes can be constructed from on shell amplitudes: all virtual particles are actually real. In zero energy ontology I ended up with the representation of TGD analogs of Feynman diagrams using only mass shell massless states with both positive and negative energies. The enormous number of kinematic constraints eliminates UV and IR divergences and also the description of massive particles as bound states of massless ones becomes possible.

In TGD framework quantum classical correspondence requires a space-time correlate for the on mass shell property and it indeed exists. The mathematically ill-defined path integral over all 4-surfaces is replaced with a superposition of preferred extremals of Kähler action analogous to Bohr orbits, and one has only a functional integral over the 3-D ends at the light-like boundaries of causal diamond (Euclidian/Minkowskian space-time regions give real/imaginary Chern-Simons exponent to the vacuum functional). This would be obviously the deeper principle behind on mass shell representation of scattering amplitudes that Nima and others are certainly trying to identify. This principle in turn reduces to general coordinate invariance at the level of the world of classical worlds.

Quantum classical correspondence and quantum ergodicity would imply even stronger condition: the quantal correlation functions should be identical with classical correlation functions for any preferred extremal in the superposition: all preferred extremals in the superposition would be statistically equivalent [K2]. 4-D spin glass degeneracy of Kähler action however suggests that this is is probably too strong a condition applying only to building bricks of the superposition.

Minimal surface property is the geometric counterpart for masslessness and the preferred extremals are also minimal surfaces: this property reduces to the generalization of complex structure at space-time surfaces, which I call Hamilton-Jacobi structure for the Minkowskian signature of the induced metric. Einstein Maxwell equations with cosmological term are also satisfied.

2.2 Massless extremals and twistor approach

The decomposition $M^4 = M^2 \times E^2$ is fundamental in the formulation of quantum TGD, in the number theoretical vision about TGD, in the construction of preferred extremals, and for the vision about generalized Feynman diagrams. It is also fundamental in the decomposition of the degrees of string to longitudinal and transversal ones. An additional item to the list is that also the states appearing in thermodynamical ensemble in p-adic thermodynamics correspond to four-momenta in M^2 fixed by the direction of the Lorentz boost. In twistor approach to TGD the possibility to decompose also internal lines to massless states at parallel space-time sheets is crucial.

Can one find a concrete identification for $M^2 \times E^2$ decomposition at the level of preferred extremals? Could these preferred extremals be interpreted as the internal lines of generalized Feynman diagrams carrying massless momenta? Could one identify the mass of particle predicted by p-adic thermodynamics with the sum of massless classical momenta assignable to two preferred extremals of this kind connected by wormhole contacts defining the elementary particle?

Candidates for this kind of preferred extremals indeed exist. Local $M^2 \times E^2$ decomposition and light-like longitudinal massless momentum assignable to M^2 characterizes "massless extremals" (MEs, "topological light rays"). The simplest MEs correspond to single space-time sheet carrying a conserved light-like M^2 momentum. For several MEs connected by wormhole contacts the longitudinal massless momenta are not conserved anymore but their sum defines a time-like conserved four-momentum: one has a bound states of massless MEs. The stable wormhole contacts binding MEs together possess Kähler magnetic charge and serve as building bricks of elementary particles. Particles are necessary closed magnetic flux tubes having two wormhole contacts at their ends and connecting the two MEs.

The sum of the classical massless momenta assignable to the pair of MEs is conserved even when they exchange momentum. Quantum classical correspondence requires that the conserved classical rest energy of the particle equals to the prediction of p-adic mass calculations. The massless momenta assignable to MEs would naturally correspond to the massless momenta propagating along the internal lines of generalized Feynman diagrams assumed in zero energy ontology. Masslessness of virtual particles makes also possible twistor approach. This supports the view that MEs are fundamental for the twistor approach in TGD framework.

2.3 Scattering amplitudes as representations for braids whose threads can fuse at 3-vertices

Just a little comment about the content of the article. The main message of the article is that non-equivalent contributions to a given scattering amplitude in $\mathcal{N} = 4$ SYM represent elements of the group of permutations of external lines - or to be more precise - decorated permutations which replace permutation group S_n with $n!$ elements with its decorated version containing $2^n n!$ elements. Besides 3-vertex the basic dynamical process is permutation having the exchange of neighboring lines as a generating permutation completely analogous to fundamental braiding. BFCW bridge has interpretation as a representations for the basic braiding operation.

This supports the TGD inspired proposal (TGD as almost topological QFT) that generalized Feynman diagrams are in some sense also knot or braid diagrams allowing besides braiding operation also two 3-vertices [K1]. The first 3-vertex generalizes the standard stringy 3-vertex but with totally different interpretation having nothing to do with particle decay: rather particle travels along two paths simultaneously after $1 \to 2$ decay. Second 3-vertex generalizes the 3-vertex of ordinary Feynman diagram (three 4-D lines of generalized Feynman diagram identified as Euclidian space-time regions meet at this vertex). The main idea is that in TGD framework knotting and braiding emerges at two levels.

1. At the level of space-time surface string world sheets at which the induced spinor fields (except right-handed neutrino [K2]) are localized due to the conservation of electric charge can form 2-knots and can intersect at discrete points in the generic case. The boundaries of strings world sheets at light-like wormhole throat orbits and at space-like 3-surfaces defining the ends of the space-time at light-like boundaries of causal diamonds can form ordinary 1-knots, and get linked and braided. Elementary particles themselves correspond to closed loops at the ends of space-time surface and can also get knotted (possible effects are discussed in [K1]).

2. One can assign to the lines of generalized Feynman diagrams lines in M^2 characterizing given causal diamond. Therefore the 2-D representation of Feynman diagrams has concrete physical interpretation in TGD. These lines can intersect and what suggests itself is a description of non-planar diagrams (having this kind of intersections) in terms of an algebraic knot theory. A natural guess is that it is this knot theoretic operation which allows to describe also non-planar

diagrams by reducing them to planar ones as one does when one constructs knot invariant by reducing the knot to a trivial one. Scattering amplitudes would be basically knot invariants.

"Almost topological" has also a meaning usually not assigned with it. Thurston's geometrization conjecture stating that geometric invariants of canonical representation of manifold as Riemann geometry, defined topological invariants, could generalize somehow. For instance, the geometric invariants of preferred extremals could be seen as topological or more refined invariants (symplectic, conformal in the sense of 4-D generalization of conformal structure). If quantum ergodicity holds true, the statistical geometric invariants defined by the classical correlation functions of various induced classical gauge fields for preferred extremals could be regarded as this kind of invariants for sub-manifolds. What would distinguish TGD from standard topological QFT would be that the invariants in question would involve length scale and thus have a physical content in the usual sense of the word!

3 Could $\mathcal{N}$ =2 or $\mathcal{N}$ = 4 SUSY have something to do with TGD?

$\mathcal{N}$ = 4 SYM has been the theoretical laboratory of Nima and others. $\mathcal{N}$ = 4 SYM is definitely a completely exceptional theory, and one cannot avoid the question whether it could in some sense be part of fundamental physics. In TGD framework right handed neutrinos have remained a mystery: whether one should assign space-time SUSY to them or not. Could they give rise to something resembpling $\mathcal{N}$ = 2 or $\mathcal{N}$ = 4 SUSY with fermion number conservation?

3.1 Earlier results

My latest view is that *fully* covariantly constant right-handed neutrinos decouple from the dynamics completely. I will repeat first the earlier arguments which consider only fully covariantly constant right-handed neutrinos.

1. $\mathcal{N}$ = 1 SUSY is certainly excluded since it would require Majorana property not possible in TGD framework since it would require superposition of left and right handed neutrinos and lead to a breaking of lepton number conservation. Could one imagine SUSY in which both MEs between which particle wormhole contacts reside have $\mathcal{N}$ = 2 SUSY which combine to form an $\mathcal{N}$ = 4 SUSY?

2. Right-handed neutrinos which are covariantly constant right-handed neutrinos in both M^4 degrees of freedom cannot define a non-trivial theory as shown already earlier. They have no electroweak nor gravitational couplings and carry no momentum, only spin.

   The fully covariantly constant right-handed neutrinos with two possible helicities at given ME would define representation of SUSY at the limit of vanishing light-like momentum. At this limit the creation and annihilation operators creating the states would have vanishing anticommutator so that the oscillator operators would generate Grassmann algebra. Since creation and annihilation operators are hermitian conjugates, the states would have zero norm and the states generated by oscillator operators would be pure gauge and decouple from physics. This is the core of the earlier argument demonstrating that $\mathcal{N}$ = 1 SUSY is not possible in TGD framework: LHC has given convincing experimental support for this belief.

3.2 Could massless right-handed neutrinos covariantly constant in CP_2 degrees of freedom define $\mathcal{N}$ = 2 or $\mathcal{N}$ = 4 SUSY?

Consider next right-handed neutrinos, which are covariantly constant in CP_2 degrees of freedom but have a light-like four-momentum. In this case fermion number is conserved but this is consistent with $\mathcal{N}$ = 2 SUSY at both MEs with fermion number conservation. $\mathcal{N}$ = 2 SUSYs could emerge from $\mathcal{N}$ = 4 SUSY when one half of SUSY generators annihilate the states, which is a basic phenomenon in supersymmetric theories.

1. At space-time level right-handed neutrinos couple to the space-time geometry - gravitation - although weak and color interactions are absent. One can say that this coupling forces them

to move with light-like momentum parallel to that of ME. At the level of space-time surface right-handed neutrinos have a spectrum of excitations of four-dimensional analogs of conformal spinors at string world sheet (Hamilton-Jacobi structure).

For MEs one indeed obtains massless solutions depending on longitudinal M^2 coordinates only since the induced metric in M^2 differs from the light-like metric only by a contribution which is light-like and contracts to zero with light-like momentum in the same direction. These solutions are analogs of (say) left movers of string theory. The dependence on E^2 degrees of freedom is holomorphic. That left movers are only possible would suggest that one has only single helicity and conservation of fermion number at given space-time sheet rather than 2 helicities and non-conserved fermion number: two real Majorana spinors combine to single complex Weyl spinor.

2. At imbedding space level one obtains a tensor product of ordinary representations of $\mathcal{N}=2$ SUSY consisting of Weyl spinors with opposite helicities assigned with the ME. The state content is same as for a reduced $\mathcal{N}=4$ SUSY with four $\mathcal{N}=1$ Majorana spinors replaced by two complex $\mathcal{N}=2$ spinors with fermion number conservation. This gives 4 states at both space-time sheets constructed from ν_R and its antiparticle. Altogether the two MEs give 8 states, which is one half of the 16 states of $\mathcal{N}=4$ SUSY so that a degeneration of this symmetry forced by non-Majorana property is in question.

3.3 Is the dynamics of $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM possible in right-handed neutrino sector?

Could $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM be a part of quantum TGD? Could TGD be seen a fusion of a degenerate $\mathcal{N}=4$ SYM describing the right-handed neutrino sector and string theory like theory describing the contribution of string world sheets carrying other leptonic and quark spinors? Or could one imagine even something simpler?

What is interesting that the net momenta assigned to the right handed neutrinos associated with a pair of MEs would correspond to the momenta assignable to the particles and obtained by p-adic mass calculations. It would seem that right-handed neutrinos provide a representation of the momenta of the elementary particles represented by wormhole contact structures. Does this mimircry generalize to a full duality so that all quantum numbers and even microscopic dynamics of defined by generalized Feynman diagrams (Euclidian space-time regions) would be represented by right-handed neutrinos and MEs? Could a generalization of $\mathcal{N}=4$ SYM with non-trivial gauge group with proper choices of the ground states helicities allow to represent the entire microscopic dynamics?

Irrespective of the answer to this question one can compare the TGD based view about supersymmetric dynamics with what I have understood about $\mathcal{N}=4$ SYM.

1. In the scattering of MEs induced by the dynamics of Kähler action the right-handed neutrinos play a passive role. Modified Dirac equation forces them to adopt the same direction of four-momentum as the MEs so that the scattering reduces to the geometric scattering for MEs as one indeed expects on basic of quantum classical correspondence. In ν_R sector the basic scattering vertex involves four MEs and could be a re-sharing of the right-handed neutrino content of the incoming two MEs between outgoing two MEs respecting fermion number conservation. Therefore $\mathcal{N}=4$ SYM with fermion number conservation would represent the scattering of MEs at quantum level.

2. $\mathcal{N}=4$ SUSY would suggest that also in the degenerate case one obtains the full scattering amplitude as a sum of permutations of external particles followed by projections to the directions of light-like momenta and that BCFW bridge represents the analog of fundamental braiding operation. The decoration of permutations means that each external line is effectively doubled. Could the scattering of MEs can be interpreted in terms of these decorated permutations? Could the doubling of permutations by decoration relate to the occurrence of pairs of MEs?

   One can also revert these questions. Could one construct massive states in $\mathcal{N}=4$ SYM using pairs of momenta associated with particle with integer label k and its decorated copy with label $k+n$? Massive external particles obtained in this manner as bound states of massless ones could solve the IR divergence problem of $\mathcal{N}=4$ SYM.

3. The description of amplitudes in terms of leading singularities means picking up of the singular contribution by putting the fermionic propagators on mass shell. In the recent case it would give the inverse of massless Dirac propagator acting on the spinor at the end of the internal line annihilating it if it is a solution of Dirac equation.

   The only way out is a kind of cohomology theory in which solutions of Dirac equation represent exact forms. Dirac operator defines the exterior derivative d and virtual lines correspond to non-physical helicities with $d\Psi \neq 0$. Virtual fermions would be on mass-shell fermions with non-physical polarization satisfying $d^2\Psi = 0$. External particles would be those with physical polarization satisfying $d\Psi = 0$, and one can say that the Feynman diagrams containing physical helicities split into products of Feynman diagrams containing only non-physical helicities in internal lines.

4. The fermionic states at wormhole contacts should define the ground states of SUSY representation with helicity +1/2 and -1/2 rather than spin 1 or -1 as in standard realization of $\mathcal{N} = 4$ SYM used in the article. This would modify the theory but the twistorial and Grassmannian description would remain more or less as such since it depends on light-likeneness and momentum conservation only.

3.4 3-vertices for sparticles are replaced with 4-vertices for MEs

In $\mathcal{N} = 4$ SYM the basic vertex is on mass-shell 3-vertex which requires that for real light-like momenta all 3 states are parallel. One must allow complex momenta in order to satisfy energy conservation and light-likeness conditions. This is strange from the point of view of physics although number theoretically oriented person might argue that the extensions of rationals involving also imaginary unit are rather natural.

The complex momenta can be expressed in terms of two light-like momenta in 3-vertex with one real momentum. For instance, the three light-like momenta can be taken to be p, k, and $p - ka$ with $k = ap_R$. Here p (incoming momentum) and p_R are real light-like momenta satisfying $p \cdot p_R = 0$ but with opposite sign of energy, and a is complex number. What is remarkable that also the negative sign of energy is necessary also now.

In zero energy ontology the situation is different due to the presence of a pair of MEs, the allowance of negative energies in internal lines, and the fact that scattering is of sparticles is induced by that of MEs. In the simplest vertex a massive external particle with non-parallel MEs carrying non-parallel light-like momenta can decay to a pair of MEs with light-like momenta. This can be interpreted as 4-ME-vertex rather than 3-vertex (say) BFF so that complex momenta are not needed. For an incoming boson identified as wormhole contact the vertex can be seen as BFF vertex.

To obtain space-like momentum exchanges one must allow negative sign of energy and one has strong conditions coming from momentum conservation and light-likeness which allow non-trivial solutions since the basically 4-vertices are in question. This reduces dramatically the number of graphs. Note that one can also consider vertices in which three pairs of MEs join along their ends so that 6 MEs (analog of 3-bson vertex) would be involved.

3.5 Is SUSY breaking possible or needed?

It is difficult to imagine the breaking of the proposed kind of SUSY in TGD framework, and the first guess is that all these 4 super-partners of particle have identical masses. p-Adic thermodynamics does not distinguish between these states and the only possibility is that the p-adic primes differ for the spartners. But is the breaking of SUSY really necessary? Can one really distinguish between the 8 different states of a given elementary particle using the recent day experimental methods?

1. In electroweak and color interactions the spartners behave in an identical manner classically. The coupling of right-handed neutrinos to space-time geometry however forces the right-handed neutrinos to adopt the same direction of four-momentum as MEs has. Could some gravitational effect allow to distinguish between spartners? This would be trivially the case if the p-adic mass scales of spartners would be different. Why this should be the case remains however an open question.

2. In the case of unbroken SUSY only spin distinguishes between spartners. Spin determines statistics and the first naive guess would be that bosonic spartners obey totally different atomic physics allowing condensation of selectrons to the ground state. Very probably this is not true: the right-handed neutrinos are delocalized to 4-D MEs and other fermions correspond to wormhole contact structures and 2-D string world sheets.

   The coupling of the spin to the space-time geometry seems to provide the only possible manner to distinguish between spartners. Could one imagine a gravimagnetic effect with energy splitting proportional to the product of gravimagnetic moment and external gravimagnetic field B? If gravimagnetic moment is proportional to spin projection in the direction of B, a non-trivial effect would be possible. Needless to say this kind of effect is extremely small so that the unbroken SUSY might remain undetected.

3. If the spin of sparticle be seen in the classical angular momentum of ME as quantum classical correspondence would suggest then the value of the angular momentum might allow to distinguish between spartners. Also now the effect is extremely small.

Theoretical Physics

[B1] Nima Arkani-Hamed et al. Scattering amplitides and the positive Grassmannian. `http://arxiv.org/pdf/1212.5605v1.pdf`.

Books related to TGD

[K1] M. Pitkänen. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. `http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html#knotstgd`, 2006.

[K2] M. Pitkänen. The Recent Vision About Preferred Extremals and Solutions of the Modified Dirac Equation. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. `http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html#dirasvira`, 2012.