The Alcubierre Warp Drive using Lorentz Boosts according to the Harold White Spacetime Metric potential θ.

Fernando Loup *
Daniel Rocha †

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. The first of these solutions was discovered by the Mexican mathematician Miguel Alcubierre in 1994.The Alcubierre warp drive seems to be very attractive because allows interstellar space travel at arbitrarily large speeds avoiding the time dilatation and mass increase paradoxes of Special Relativity. However it suffers from a very serious drawback:Interstellar space is not empty:It is fulfilled with photons and particle dusts and a ship at superluminal speeds would impact these obstacles in highly energetic collisions disrupting the warp field and placing the astronauts in danger.This was pointed out by a great number of authors like Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne, Barcelo,Finazzi and Liberati.
In order to travel significant interstellar distances in reasonable amounts of time a ship would need to attain 200 times the speed of light but according to Clark,Hiscock and Larson the impact between the ship and a single photon of Cosmic Background Radiation(COBE)would release an amount of energy equal to the photosphere of a star like the Sun.And how many photons of COBE we have per cubic centimeter of space?This serious problem seems to have no solution at first sight.
However some years ago Harold White from NASA appeared with an idea that may well solve this problem:According to him the ship never surpass the speed of light but the warp field generates a Lorentz Boost resulting in an apparent superluminal speed as seen by the astronauts on-board the ship and on the Earth while the warp bubble is always below the light speed with the ability to manoeuvre against these obstacles avoiding the lethal collisions.
In this work we examine the feasibility of the White idea using clear mathematical arguments and we arrived at the conclusion that Harold White is correct.

*Residencia de Estudantes Universitas-Lisboa-Portugal-email:spacetimeshortcut@yahoo.com
†INPI- Instituto Nacional da Propriedade Industrial-Rio de Janeiro-Brazil-email:danieldiniz@gmail.com

1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of the Mexican mathematician Miguel Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1]).

However the major drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons of Cosmic Background Radiation(COBE).

According to Clark,Hiscock and Larson a single collision between a ship and a COBE photon would release an amount of energy equal to the photosphere of a star like the Sun.(see pg 11 in [2]).And how many photons of COBE we have per cubic centimeter of space??

These highly energetic collisions would pose a very serious threat to the astronauts as pointed out by McMonigal,Lewis and O'Byrne (see pg 10 in [4]).

Another problem:these highly energetic collisions would raise the temperature of the warp bubble reaching the Hawking temperature as pointed out by Barcelo,Finazzi and Liberati.(see pg 6 in in [5]).At pg 9 they postulate that all future spaceships cannot bypass 99 percent of the light speed.

However some years ago in 2003 Harold "Sonny" White from NASA Lyndon B.Johnson Space Center Houston Texas proposed a different idea. According to him a ship leaves the Earth and achieves a subluminal velocity of $0,1c$.The spacetime metric is then engineered to produce a Lorentz Boost resulting in an apparent speed of 10c as seen by observers on Earth and observers on-board the ship.However the warp bubble moves always with $0,1c$ allowing manoeuvres against interstellar obstacles.(see pg 6 in [8],pg 5 in [9])).

Without the Lorentz Boost the ship would take 43 years to reach Proxima Centauri however according to White the Lorentz Boost is like a film played in "fast forwarding mode".It "accelerates" the clocks on Earth and on-board the ship so both observers "sees" an apparent speed of 10c and the completion of the journey in $4,3$ months and not in 43 years.The Lorentz Boost is like a "jump into the Future".Both Earth and ship "sees" 43 years passing in $4,3$ months.This may sounds unbelievable but the mathematics employed by Harold White is entirely correct although accessible only to advanced readers.[2]

The purpose of this work is to demonstrate the idea of White proving that the "fast-forwarding" is entirely correct and giving all the needed mathematical demonstrations in a formalism designed to reach an audience of wide scope:beginners or intermediate students.

[1] do not violates Relativity

[2] He presented his results in a resumed way.A beginner or intermediate student would not figure out the White idea at first sight.

2 The Alcubierre Warp Drive using Lorentz Boosts according to the Harold White Spacetime Metric potential θ

In this work we use the Geometrized System of Units in which $c = G = 1$

Spacetime metrics in General Relativity are often written in the following form:
(see eq 7.13 pg 175 in [6])(signature (-,+,+,+))

$$ds^2 = -e^{-2a(t,r)}dt^2 + e^{2b(t,r)}dr^2 + r^2 d\Omega^2 \tag{1}$$

changing the signature to (+,-,-,-) we have:

$$ds^2 = e^{-2a(t,r)}dt^2 - e^{2b(t,r)}dr^2 - r^2 d\Omega^2 \tag{2}$$

defining $\theta = -a$ we have:

$$ds^2 = e^{2\theta(t,r)}dt^2 - e^{2b(t,r)}dr^2 - r^2 d\Omega^2 \tag{3}$$

The expression above is very important as we will shall see later.

The central ides of White is the apparent velocity vs defined as (see pg 6 in [8],pg 5 and 17 in [9]).

$$vs = v_{eff} = v_i \times \gamma \tag{4}$$

In the expression above $vs = v_{eff}$ is the apparent speed seen by the Earth and observers on-board the spaceship (10c) while v_i is the real speed of the warp bubble$(0, 1c)$.
γ is the Lorentz Boost generated by the g_{00} component of spacetime metrics with the form similar or resembling eq 3[3].
From above we can easily see that the real velocity $v_i << 1$ but due to the Lorentz Boost γ we can have $vs >> 1$ or $v_{eff} >> 1$.
The Alcubierre warp drive metric is defined by the following expression(signature(+,-,-,-)):(see eq 8 pg 4 in [1],eq 1 pg 2 in [8] and [9])

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 \tag{5}$$

expanding the metric we have:

$$[dx - vsf(rs)dt]^2 = dx^2 - 2vsf(rs)dxdt + vs^2 f(rs)^2 dt^2 \tag{6}$$

Rearranging the terms we arrive at the final form of the Alcubierre warp drive metric which is given by:

$$ds^2 = [1 - vs^2 f(rs)^2]dt^2 + 2vsf(rs)dxdt - dx^2 \tag{7}$$

The term $[1 - vs^2 f(rs)^2]$ is very important:it is the g_{00} component of the Alcubierre metric.It appears in eqs 10 and 11 pg 7 in [8],pg 5 and 23 in [9]

[3]see Lorentz Transformations in Wikipedia:The Free Encyclopedia

Writing together the Alcubierre metric in expanded form and eq 3 we have:

$$ds^2 = [1 - vs^2 f(rs)^2]dt^2 + 2vsf(rs)dxdt - dx^2 \tag{8}$$

$$ds^2 = e^{2\theta(t,r)}dt^2 - e^{2b(t,r)}dr^2 - r^2 d\Omega^2 \tag{9}$$

the term in g_{00} is given by(pg 23 in [9]):

$$e^{2\theta(t,r)} = [1 - vs^2 f(rs)^2] \tag{10}$$

The White spacetime metric potential θ and its respective Lorentz Boost are given by(see pg 7 eqs 10 and 11 in [8],pg 5 and 23 in [9]):[4].

$$\theta = \frac{1}{2}\ln[|1 - vs^2 f(rs)^2|] \tag{11}$$

$$\gamma = \cosh(\theta) \tag{12}$$

Note the fact that we used $|1 - vs^2 f(rs)^2|$ and not $1 - vs^2 f(rs)^2$.This is due to the chosen signature (+,-,-,-).With a signature (-,+,+,+) the term would be $vs^2 f(rs)^2 - 1$. Since the apparent speed $vs >> 1$ we cannot have a logarithm of a negative number.The modulus fills the gap between both signatures.

We derived the Lorentz Boost using the apparent velocity vs.In order to find the real velocity v_i of the warp bubble we must use:

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\frac{1}{2}\ln[|1 - vs^2 f(rs)^2|]))} \tag{13}$$

Now we must examine the most important point of view of the White idea:The concept that allows a warp bubble with a real velocity of $v_i = 0,1$c being seen with an apparent speed of $vs = 10$c by observers on-board the ship and on Earth: The concept of the "fast-forwarding film".

$f(rs)$ is the Alcubierre shape function defined as:(see eq 6 pg 4 in [1])

$$f(rs) = \frac{\tanh[@(rs + R)] - \tanh[@(rs - R)]}{2\tanh(@R)} \tag{14}$$

According to Alcubierre we have 3 possible values for $f(rs)$:(see eq 7 pg 4 in [1])

- 1)- inside the warp bubble where the ship resides $f(rs) = 1$
- 2)- outside the warp bubble where Earth resides $f(rs) = 0$
- 3)- in the Alcubierre warped region (warp bubble walls) $1 > f(rs) > 0$

Note that this is the situation seen by two observers A and B placed on Earth watching the ship passing by them with an apparent velocity of $10c$ due to the "fast forwarding film" concept.

[4]again see Lorentz Transformations in Wikipedia:The Free Encyclopedia

We already know that

$$vs = v_i \times \gamma = v_i \cosh(\theta) = v_i \cosh(\frac{1}{2} \ln[|1 - vs^2 f(rs)^2|]) \tag{15}$$

$$v_i = \frac{vs}{\gamma} = \frac{vs}{\cosh(\theta))} = \frac{vs}{\cosh(\frac{1}{2} \ln[|1 - vs^2 f(rs)^2|]))} \tag{16}$$

When the observers on Earth A and B "looks" to the ship both "sees" the $f(rs) = 1$ inside the warp bubble.Hence inside the warp bubble the White potential and Lorentz Boost are given by:

$$\theta = \frac{1}{2} \ln[|1 - vs^2|] \tag{17}$$

$$\gamma = \cosh(\theta) \tag{18}$$

Both sees the ship in a "fast forwarding" apparent velocity of $vs = v_i \times \gamma$

Although the real speed v_i of the warp bubble is $0,1$c taking 43 years to reach Proxima Centauri both A and B "sees" the ship with an apparent velocity of 10c taking only $4,3$ months to complete the journey due to the "fast forwarding film" concept.

A and B "sees" the clocks in the ship "accelerated" due to the "fast-forwarding" completing 43 years in $4,3$ months.A leap into the Future.[5]

But outside the warp bubble $f(rs) = 0$ where the observers A and B resides. Hence outside the warp bubble the White potential and Lorentz Boost are given by:

$$\theta = \frac{1}{2} \ln[|1|] = 0 \tag{19}$$

$$\gamma = \cosh(\theta) = 1 \tag{20}$$

Note that outside the warp Bubble the Lorentz Boost is 1.This means to say that when A looks to B and vice-versa both are normal.No "fast-forwarding" outside the warp bubble.

[5] we know this is unbelievable.but White mathematics is correct

Now we must analyze the point of view of 2 observers C and D on-board the ship looking to the Earth,looking to Proxima Centauri ..and looking to themselves,

From the Alcubierre metric

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 \tag{21}$$

Remembering that the ship is a co-moving coordinates frame as defined by:(see pg 2 in [7],pg 4 in [3])

$$dx = dx' + vsdt \tag{22}$$

Rewriting the Alcubierre metric for co-moving coordinates frame we have:

$$ds^2 = dt^2 - [dx' + vsdt - vsf(rs)dt]^2 \tag{23}$$

$$ds^2 = dt^2 - [dx' + vs(1 - f(rs))dt]^2 \tag{24}$$

$f(rs)$ is the Alcubierre shape function seen by Earth observers.Ship observers in a co-moving coordinates frame sees the shape function as $g(rs)$:

$$g(rs) = 1 - f(rs) \tag{25}$$

$$ds^2 = dt^2 - [dx' + vsg(rs))dt]^2 \tag{26}$$

Expanding the metric we have:

$$[dx' + vsg(rs))dt]^2 = dx'^2 + 2vsg(rs)dx'dt + vs^2g(rs)^2dt^2 \tag{27}$$

Rearranging the terms we arrive at the final form of the Alcubierre warp drive metric for the co-moving coordinates frame which is given by:

$$ds^2 = [1 - vs^2g(rs)^2]dt^2 - 2vsg(rs)dx'dt - dx'^2 \tag{28}$$

the term in g_{00} is now given by:

$$e^{2\theta(t,r)} = [1 - vs^2g(rs)^2] \tag{29}$$

Note that(eq 10 pg 4 in [3]).

$$[1 - vs^2g(rs)^2] = 1 - vs^2(1 - f(rs))^2 \tag{30}$$

The White spacetime metric potential θ and its respective Lorentz Boost are given by:

$$\theta = \frac{1}{2}\ln[|1 - vs^2g(rs)^2|] \tag{31}$$

$$\gamma = \cosh(\theta) \tag{32}$$

But the most important thing is the fact that due to

$$g(rs) = 1 - f(rs) \tag{33}$$

the observers C and D on-board the ship "sees" the opposite of what is being "seen" from Earth..or Proxima Centauri.

- 1)- inside the warp bubble where the ship resides $g(rs) = 0$
- 2)- outside the warp bubble where Earth resides $g(rs) = 1$
- 3)- in the Alcubierre warped region (warp bubble walls) $1 > g(rs) > 0$

Inside the warp bubble from the point of view of C and D the White potential and Lorentz Boost are given by:

$$\theta = \frac{1}{2}\ln[|1|] = 0 \tag{34}$$

$$\gamma = \cosh(\theta) = 1 \tag{35}$$

Note that inside the warp Bubble the Lorentz Boost is 1.This means to say that when C looks to D and vice-versa both are normal.No "fast-forwarding" inside the warp bubble.

But when C or D looks the Earth they "sees" a White spacetime metric potential θ and its respective Lorentz Boost given by:

$$\theta = \frac{1}{2}\ln[|1 - vs^2 g(rs)^2|] \tag{36}$$

$$\gamma = \cosh(\theta) \tag{37}$$

Note that due to this Lorentz Boost both observers C and D "sees" the regions outside the warp bubble with the same "fast-forwarding" as the Earth "sees" the ship.Both observers "sees" the Earth moving away from them at 10c and Proxima Centauri moving close to them also at 10c.

Both C and D "sees" Earth and Proxima Centauri in a "fast forwarding" "accelerated" clock rate and also making 43 years in $4,3$ months.Both "sees" the same "leap" into the Future

3 Conclusion

In this work we demonstrated that the Harold White concept of the "fast-forwarding" concept is entirely correct. Both Earth and ship makes "leaps" into the Future. This may somewhat sounds to be unbelievable but perhaps White is the correct solution to solve the problem of the interstellar navigation.

4 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[6]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[7]

5 Remarks

Reference [8] "A Discussion Of Space-Time Metric Engineering" by Harold "Sonny" White of NASA Lyndon B.Johnson Space Center Houston Texas is available only to subscribers of the peer-review scientific journal General Relativity and Gravitation. Due to fortunate circumstances we got access to an original copy of the manuscript.

Reference [9] "Warp Field Mechanics 101" by Harold "Sonny" White of NASA Lyndon B.Johnson Space Center Houston Texas is available at NASA Technical Reports Server (NTRS)[8] however we can provide a copy in PDF Acrobat reader for those interested.

[6]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[7]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[8]browse Google for NASA Technical Report Server-20110015936

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