Reduced Total Energy Requirements for the Natario Warp Drive Spacetime using Heaviside Step Functions as Analytical Shape Functions.

Fernando Loup *
Daniel Rocha †

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the warp drive violates all the known energy conditions because the stress energy momentum tensor(the right side of the Einstein Field Equations) for the Einstein tensor G_{00} is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the Quantum Field Theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself.The major drawback concerning negative energies for the warp drive are the huge negative energy density requirements to sustain a stable warp bubble configuration. Ford and Pfenning computed these negative energy and concluded that at least 10 times the mass of the Universe is required to sustain a warp bubble configuration. However both Alcubierre and Natario warp drives as members of the same family of the Einstein Field Equations requires the so-called shape functions in order to be mathematically defined. We present in this work two new shape functions for the Natario warp drive spacetime based on the Heaviside step function and one of these functions allows arbitrary superluminal speeds while keeping the negative energy density at "low" and "affordable" levels.We do not violate any known law of quantum physics and we maintain the original geometry of the Natario warp drive spacetime We also discuss briefly Horizons and infinite Doppler blueshifts.

*Residencia de Estudantes Universitas-Lisboa-Portugal-email:spacetimeshortcut@yahoo.com
†INPI- Instituto Nacional da Propriedade Industrial-Rio de Janeiro-Brazil-email:danieldiniz@gmail.com

1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of large negative energy requirements enough to sustain the warp bubble.While from a classical point of view negative energy densities are forbidden the Quantum Field Theory allows the existence of such energies but the major problem affecting the Quantum Field Theory negative energies for the warp drive are the so called Quantum Inequalities(QI):The QI restricts the time we can observe a negative energy density which means to say that as large the amount of negative energy density is then the time when this amount of energy can exists to be observed becomes incredible small.This time is known as the sampling time which is inversely proportional to the magnitude of the negative energy density amount.

Ford and Pfenning computed the QI for the Alcubierre warp drive using a Planck Length[2] Scale shape function and arrived at the conclusion that the sampling time is incredible small and approximately around the order of 10^{-33} seconds[3]. This means to say that the negative energy according to them can exists for only 10^{-33} seconds making the warp drive an impracticable form of transport because for an interstellar trip for example a given star at 20 light years away with a warp drive speed of 200 times faster than light some months are required.to complete the journey but if the warp bubble can exists for only 10^{-33} seconds then the warp drive is impossible for such an interstellar trip. Ford and Pfenning also computed the negative energy density requirements for the Alcubierre warp drive and they arrived at the conclusion that in order to sustain a warp bubble able to perform interstellar travel the amount of negative energy density is of about 10 times the mass of the universe(see pg 10 in [3])

Again they concluded that the warp drive is impossible.

However they performed all the calculations for the Alcubierre warp drive and not the Natario one.
This means to say that while the Alcubierre warp drive is physically impossible the possibility or impossibility of Natario warp drive is still an open quest to be solved by modern science in the future.

[1] do not violates Relativity

[2] 10^{-35} meters.See Wikipedia The free Encyclopedia

[3] Consider a warp bubble with a 100 meters of radius a thickness Δ defined by eq 23 pg 9 in [3] moving at 200 times light speed.Look to eqs 19 pg 8 and eq 20 pg 9 in [3].Insert the thickness and the speed of the bubble in these equations and the sampling time of 10^{-33} seconds will appear.

In this work we follow the line of reason of Ford and Pfenning and we compute the total energy requirements for the Natario warp drive.Warp drives are defined mathematically by the so-called shape functions.This means to say the functions that defines the mathematical structure for the geometry of the warp drive spacetimes.While in the Alcubierre warp drive the shape function $f(rs)$ gives 1 in one region(inside the warp bubble) and 0 in another region(outside the bubble) and $1 > f(rs) > 0$ in a third region(warp bubble walls)(eq 7 pg 4 in [1] or top of pg 4 in [2]),in the Natario warp drive the shape function $n(rs)$ gives 0 in one region(inside the bubble),$\frac{1}{2}$ in another region(outside the bubble) and $0 < n(rs) < \frac{1}{2}$ in a third region(warp bubble walls)(pg 5 in [2]).

We will demonstrate in this work the fact that the request or requirements of negative energy densities able to sustain the geometry of the warp drive are driven by the mathematical form of the shape function and in a last analysis the form of the derivatives or integrals of the shape function.

However Natario point out in his work an important statement:

Any function that gives 0 in one region and $\frac{1}{2}$ in another region can be regarded as a valid shape function for the Natario warp drive spacetime.

We will demonstrate in this work that some particular choices for the shape function will produce completely different mathematical results when compared between each other and some shape functions are better than other ones.

Ford and Pfenning arrived at the result of 10 tines the mass of the Universe for the Alcubierre warp drive due to the particular choice they adopted for the shape function.A different shape function would produce different results.And remember that their shape function was not analytical[4] in all points of the trajectory.What would happen with eq 4 pg 3 in [3] when $rs = R - \frac{\Delta}{2}$ or $rs = R + \frac{\Delta}{2}$??.

All the studies about analytical shape functions for the warp drive are being carried over the Alcubierre shape function or functions constructed over the Alcubierre model.We introduce in this work the Heaviside step functions $h(rs)$ as valid analytical shape functions for the Natario warp drive.Dividing the Heaviside step function that gives 0 in one region,1 in another region and $1 > h(rs) > 0$ in a third region by 2 we get exactly the requested behavior of a valid Natario shape function.

In this work we introduce two Heaviside step functions as valid analytical Natario shape functions and we demonstrate that while one of these functions still produces huge amounts of negative energy,the other keeps the Natario warp drive with "low" and "affordable" amounts of negative energy even at large superluminal speeds.

[4] a function in order to be analytical must be continuous and differentiable in every point of the trajectory

This work is divided in 4 sections:

- 1)-Basic Concepts of the Natario Warp Drive Spacetime:Warp Drive with Zero Expansion
- 2)-The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive
- 3)-The Heaviside Step Functions as valid Analytical Shape Functions for the Natario Warp Drive Spacetime
- 4)-Horizon and Infinite Doppler Blueshifts in both Alcubierre and Natario Warp Drive Spacetimes

The first section is a rigorous mathematical description of the basic properties of the Natario warp drive spacetime.Our goal is to produce an auto-contained or a self-contained paper with rigorous mathematical formalism on this subject.Anyone with knowledge on differential forms can almost[5] follow the study(or derive)the geometrical properties of the Natario spacetime using only [6]the material presented here.

The second section illustrates the problem of the negative energy requirements in the warp drive spacetime and why some scientists are skeptic about this concept.We compute the total energy density requirements for a spaceship travelling at 200 times light speed and even without Ford and Pfenning we also arrive at an unphysical result.But in the end we show that the mathematical form of the derivative of the shape function is very important to low or ameliorate these energy density requirements.

The third section is the most important section of this work.It presents 2 Heaviside step functions as valid analytical shape functions for the Natario warp drive spacetime.This material is new and was never covered before by all the existing published or e-print literature available about warp drives[7].The first choosen of these functions still produces a pathological result but the other produces very interesting results and as a matter of fact even at 200 times light speed these functions keeps the negative energy density at arbitrarily low levels.

The fourth section is a brief discussion of the other two problems that affects the warp drive geometry:Horizons(causally disconnected portions of spacetime) and Doppler blueshifts outlining the fact that the Natario warp drive behaves slightly different when compared to its Alcubierre counterpart.

In this work the Eulerian element is defined as $r^2 = rs^2 = [(x - xs)^2 + y^2 + z^2]$ or $r = rs$ according to pg 4 in [1] except in section 4.

[5]we outline the word almost

[6]of course for the complex tensor calculus(e.g extrinsic curvatures etc) a computer program like the GrTensorII would be required

[7]as far as we can tell the Heaviside step function as an analytical shape function for the Natario warp drive never appeared before neither in the well-known peer-review scientific publications nor in the known available e-print servers eg. arXiv,HAL or viXra.We are simply confirming a fact!

2 Basic Concepts of the Natario Warp Drive Spacetime:Warp Drive with Zero Expansion:

In 2001 the Portuguese mathematician Jose Natario introduced a new warp drive spacetime defined using the canonical basis of the Hodge Star in spherical coordinates defined as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r \sin\theta d\varphi) \tag{1}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial \theta} \sim rd\theta \sim (r \sin\theta d\varphi) \wedge dr \tag{2}$$

$$e_\varphi \equiv \frac{1}{r \sin\theta}\frac{\partial}{\partial \varphi} \sim r \sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{3}$$

We consider here the following pedagogical approach

$$x = r\cos\theta \tag{4}$$

$$dx = d(r\cos\theta) \tag{5}$$

$$X = vs \tag{6}$$

X is known as the shift vector and in this case depicts the speed of the warp drive vs
Introducing the Natario definition for the warp drive according to the following statement(pg 4 in [2]):

- 1)-Any Natario Vector nX generates a warp drive spacetime if $nX = 0$ for a small value of $|x|$ defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ for a large value of $|x|$ defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble(pg 5 in [2]).

Explaining the definition better:A given Natario vector nX generates a Natario warp drive spacetime if and only if satisfies these conditions stated below:

- 1)-A Natario Vector nX being $nX = 0$ for a small value of $|x|$(interior of the warp bubble)
- 2)-A Natario Vector $nX = -Xdx$ or $nX = Xdx$ for a large value of $|x|$(exterior of the warp bubble)
- 3)-A shift vector X depicting the speed of the warp bubble being $X = 0$(interior of the warp dubble) while $X = vs$ seen by distant observers(exterior of the warp bubble).

Applying the Natario equivalence between spherical and cartezian coordinates as shown below(pg 5 in [2]):

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2 \sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{7}$$

We would get the following expression(pg 5 in [2])[8]

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s f(r)\cos\theta\mathbf{e}_r + v_s(2f(r)+rf'(r))\sin\theta\mathbf{e}_\theta \tag{8}$$

From now on we will use this pedagogical approach that gives results practically similar the ones depicted in the original Natario vector shown above:

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s f(r)\cos\theta dr + v_s(2f(r)+rf'(r))r\sin\theta d\theta \tag{9}$$

In order to make the definition of the Natario warp drive holds true we need for the Natario vector nX a continuous Natario shape function being $f(r) = \frac{1}{2}$ for large r(outside the warp bubble) and $f(r) = 0$ for small r(inside the warp bubble) while being $0 < f(r) < \frac{1}{2}$ in the walls of the warp bubble(pg 5 in [2])

To avoid confusion with the Alcubierre shape function $f(rs)$ (pg 4 in [1])we will redefine the Natario shape function as $n(r)$ and the Natario vector as shown below

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r)+rn'(r))r\sin\theta d\theta \tag{10}$$

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r)+r[\frac{dn(r)}{dr}])r\sin\theta d\theta \tag{11}$$

The Natario Vector $nX = -vs(t)dx = 0$ vanishes inside the warp bubble because inside the warp bubble there are no motion at all because $dx = 0$ or $n(r) = 0$ or $X = 0$ while being $nX = -vs(t)dx \neq 0$ not vanishing outside the warp bubble because $n(r)$ do not vanish.Then an external observer would see the warp bubble passing by him with a speed defined by the shift vector $X = -vs(t)$ or $X = vs(t)$.

Redefining the Natario vector nX as being the rate-of-strain tensor of Fluid Mechanics as shown below(pg 5 in [2]):

$$nX = X^r\mathbf{e}_r + X^\theta\mathbf{e}_\theta + X^\varphi\mathbf{e}_\varphi \tag{12}$$

Applying the extrinsic curvature for the shift vectors contained in the Natario vector nX above we would get the following results(pg 5 in [2]):

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{13}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial\theta} + \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{14}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial\varphi} + \frac{X^r}{r} + \frac{X^\theta\cot\theta}{r} = v_s n'(r)\cos\theta \tag{15}$$

$$K_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\theta}{r}\right) + \frac{1}{r}\frac{\partial X^r}{\partial\theta}\right] = v_s\sin\theta\left(n'(r) + \frac{r}{2}n''(r)\right) \tag{16}$$

$$K_{r\varphi} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\varphi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial X^r}{\partial\varphi}\right] = 0 \tag{17}$$

[8]for a complete mathematical demonstration see Appendices B and C

$$K_{\theta\varphi} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{X^{\varphi}}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial X^{\theta}}{\partial\varphi}\right] = 0 \qquad (18)$$

Examining the first three results we can clearly see that(pg 5 in [2]):

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \qquad (19)$$

The expansion of the normal volume elements in the Natario warp drive is Zero.

A warp drive with zero expansion.

The spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular)(pg 5 in [2]).

The energy density in the Natario warp drive is given by the following expression(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(r))^2\cos^2\theta + \left(n'(r) + \frac{r}{2}n''(r)\right)^2\sin^2\theta\right]. \qquad (20)$$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dn(r)}{dr})^2\cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2}\frac{d^2n(r)}{dr^2}\right)^2\sin^2\theta\right]. \qquad (21)$$

This energy density is negative and depends on the configuration of the the Natario shape function $n(r)$ or its derivatives..In order to generate the warp drive as a dynamical spacetime large outputs of energy are needed due to the factor vs^2 and this is a critical issue unless we use very low derivatives of the Natario warp drive continuous shape function $n(r)$.

3 The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{22}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression:

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{23}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[9]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{24}$$

In the equatorial plane:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{25}$$

Note that in the above expressions the warp drive speed vs appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!

Our Earth have a mass[10] of about $6 \times 10^{24} kg$ and multiplying this by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3 \times 10^6)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R^3_{Earth}$ we would find the value of $4,77 \times 10^{20} \frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20} \frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

[9] see Appendix A

[10] see Wikipedia:The free Encyclopedia

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical(see title and pg 10 in [3])!!"

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \right] \tag{26}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of 10^{48} ameliorating the negative energy requirements to sustain the warp drive.

In the next section we will present the Heaviside step function as an analytical shape function for the Natario warp drive.

4 The Heaviside Step Functions as valid Analytical Shape Functions for the Natario Warp Drive Spacetime

We already know that a valid Natario analytical shape function $n(rs)$ in order to be satisfied demands two points:

- 1)-Must be continuous and differentiable in every point of the trajectory
- 2)-Must have the value of 0 inside the warp bubble,$\frac{1}{2}$ outside the warp bubble and $0 < n(rs) < \frac{1}{2}$ in the warp bubble walls

Examining the definition requirements of the continuous form of the Heaviside step function $h(r)$[11] we can see that these requirements must obey the following conditions:

- 1)-Must be continuous and differentiable in every point of the trajectory
- 2)-Must have the value of 0 in one region ,1 in another region and $0 < h(r) < 1$ in a third region

From the statements above anyone can see that if we divide the Heaviside step function by 2 we will get exactly the description of the Natario shape function[12].

Before going further in the definitions of the Heaviside step functions we must explain the parameter k that will appear in these definitions and we must change the Eulerian element in order to adapt the Heaviside step function as a Natario shape function.

As higher the parameter k is as sharp is the transition from 0 to 1.From the point of view of the Natario shape function the transition from the region where $n(rs) = 0$ to the region where $n(rs) = \frac{1}{2}$ is so sharp meaning a warp bubble with very thin walls.The region where $0 < n(rs) < \frac{1}{2}$ is very thin.

In this section the Eulerian element is defined as $r = rs = \sqrt{[(x-xs)^2 + y^2 + z^2]} - R$.
Considering the motion in the radial direction then $\theta = 0$ and $\cos(\theta) = 1$.
This is equivalent to the motion in the equatorial plane of the warp bubble which means to say that $y^2 + z^2 = 0$.
The Eulerian element then becomes: $r = rs = \sqrt{[(x-xs)^2]} - R$ or $r = rs = (x-xs) - R$.
Considering also $xs = 0$ the center of the bubble we have $r = rs = x - R$.Note that being R the radius of the bubble a constant this do not affect the dr.We prefer to write $r = rs = p - R$ in order to do not confuse the the x coordinate.

Considering a warp bubble with $R = 1000$ meters of radius large enough to contains a spaceship and considering that we are examining the region between $p = 999$ meters and $p = 1000,17$ meters with an increase rate of $0,01$ we can now present the Heaviside step functions.

[11]Source:Wikipedia The Free Encyclopedia
[12]"it fits like a glove"

The first of these Heaviside step functions gives the value of 0 from $p = 999$ meters to $p = 999,99$ meters.At $p = 1000$ meters the value of $0,5$ and from $p = 1000,01$ and beyond the function returns the value of 1 as expected by a Heaviside step function.

Remarkably the value of k being $k = 10^{300}$ or $k = 10^{50}$ do not affect the result.This can be confirmed by any calculation program available in any personal computer(eg. Excel,OpenOffice etc).

- 1)-The function is:

$$h(r) = \frac{1}{2} + [\frac{1}{2}]\tanh(kr) \tag{27}$$

- 2)-Its derivative is:

$$h'(r) = [\frac{1}{2}]\frac{k}{\cosh(kr)^2} \tag{28}$$

- 3)-The square of the derivative is:

$$h'(r)^2 = [\frac{1}{4}]\frac{k^2}{\cosh(kr)^4} \tag{29}$$

Integrating[13] we get:

$$\int (h'(r)^2)dr = \int ([\frac{1}{4}]\frac{k^2}{\cosh(kr)^4})dr \tag{30}$$

$$\int (h'(r)^2)dr = [\frac{1}{4}][(k^2)(\frac{2\tanh(kr)}{3k} + \frac{\tanh(kr)sech(kr)^2}{3k}) \tag{31}$$

$$\int (h'(r)^2)dr = [\frac{1}{4}][(k^2)(\frac{2\tanh(kr)}{3k} + \frac{\tanh(kr)}{3k\cosh(kr)^2}) \tag{32}$$

Note that we integrated in cartezian coordinates because Alcubierre wrote his metric using hyperbolic functions in cartezian coordinates(see eqs 6 to 8 pg 4 in [1]). Using the volume element of spherical coordinates which is $(2\pi)(r^2)dr$ the expression would be more complicated because we would need to integrate hyperbolic functions in spherical coordinates but wether in cartezian or spherical coordinates both expressions have one thing in common.The parameter k is in the upper part of the fraction and considering that a Natario shape function would simply be the division by 2 of this step function the integration of the energy density using this function would not ameliorate the 10^{48} arising from the speed 200 times faster than light.

As a matter of fact this function makes it worst due to the parameter k of ($k = 10^{300}$ or $k = 10^{50}$)[14].

[13] we use the on-line analytical integrator available at http://www.wolframalpha.com/

[14] if this function do not fits our purposed then why waste time writing a lengthly expression in spherical coordinates when want only outline the parameter k in the upper part of the fraction that appears in both expressions ??

The second of these Heaviside step functions gives the value of $1,6 \times 10^{-12}$ from $p = 999$ meters to $p = 999,99$ meters.At $p = 1000$ meters the value of $0,5$ and from $p = 1000,01$ and beyond the function returns the value of $0,999999999...$ as expected by a Heaviside step function when k approaches ∞.

Like as the previous function the value of k being $k = 10^{300}$ or $k = 10^{50}$ do not affect the result.This can be confirmed by any calculation program available in any personal computer(eg. Excel,OpenOffice etc).Also both functions changed its values by $0,5$ exactly at 1000 meters.

We were limited by the numerical precision of Excel or OpenOffice but we have no doubts that with values of k even bigger we would get results even closer to 0 or even closer to 1.

- 1)-The function is:

$$h(r) = \lim_{k->\infty}[\frac{1}{2} + \frac{1}{\pi}\arctan(kr)] \tag{33}$$

- 2)-Its derivative is:

$$h'(r) = \lim_{k->\infty}[\frac{1}{\pi}\frac{k}{[1+(kr)^2]}] \tag{34}$$

- 3)-The square of the derivative is:

$$h'(r)^2 = \lim_{k->\infty}[\frac{1}{\pi^2}\frac{k^2}{[1+(kr)^2]^2}] \tag{35}$$

Integrating in spherical coordinates[15] we get:

$$\int(h'(r)^2)(2\pi)(r^2))dr = \int(\lim_{k->\infty}[\frac{1}{\pi^2}\frac{k^2}{[1+(kr)^2]^2}])(2\pi)(r^2))dr \tag{36}$$

$$\int(\lim_{k->\infty}[\frac{1}{\pi^2}\frac{k^2}{[1+(kr)^2]^2}])(2\pi)(r^2))dr = \int(\lim_{k->\infty}[\frac{2}{\pi}\frac{k^2}{[1+(kr)^2]^2}])(r^2))dr \tag{37}$$

$$\int(\lim_{k->\infty}[\frac{2}{\pi}\frac{k^2}{[1+(kr)^2]^2}])(r^2))drs = \lim_{k->\infty}[\frac{2}{\pi}(\frac{\arctan(kr) - \frac{kr}{1+(kr)^2}}{2k})] \tag{38}$$

The result above is very important:it is the core(or the nucleus) and the main reason of existence of this work.

Wether k being $k = 10^{300}$ or $k = 10^{50}$,k appears in the lower part of the fraction.Since the Natario shape function is the Heaviside step function divided by 2 any reader can see that this function can obliterate the factor 10^{48} arising from the speed 200 times faster than light.As larger is k as lower is the energy.

[15]we use the on-line analytical integrator available at http://www.wolframalpha.com/

This was the main reason of this work:to demonstrate that the energy density requirements depends on the form of the shape function and some shape functions are better than other ones.

So a warp drive cannot be ruled out due to a pathological result if we can change the shape function to achieve better results.

While the first Heaviside step function renders the Natario warp drive impossible,the second makes it perfectly possible

5 Horizon and Infinite Doppler Blueshifts in both Alcubierre and Natario Warp Drive Spacetimes

According to pg 6 in [2] warp drives suffers from the pathology of the Horizons and according to pg 8 in [2] warp drive suffer from the pathology of the infinite Doppler Blueshifts that happens when a photon sent by an Eulerian observer to the front of the warp bubble reaches the Horizon.This would render the warp drive impossible to be physically feasible.

The Horizon occurs in both spacetimes.This means to say that the Eulerian observer cannot signal the front of the warp bubble wether in Alcubierre or Natario warp drive because the photon sent to signal will stop in the Horizon..The solution for the Horizon problem must be postponed until the arrival of a Quantum Gravity theory that encompasses both General Relativity and Non-Local Quantum Entanglements of Quantum Mechanics-

The infinite Doppler Blueshift happens in the Alcubierre warp drive but not in the Natario one.This means to say that Alcubierre warp drive is physically impossible to be achieved but the Natario warp drive is perfectly physically possible to be achieved.

Consider the negative energy density distribution in the Alcubierre warp drive spacetime(see eq 8 pg 6 in [3])[16]:

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{39}$$

And considering again the negative energy density in the Natario warp drive spacetime(see pg 5 in [2])[17]:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{40}$$

In pg 6 in [2] a warp drive with a x-axis only is considered.In this case for the Alcubierre warp drive $[y^2+z^2]=0$ and the negative energy density is zero but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{41}$$

The Alcubierre shape function $f(rs)$ is defined as being 1 inside the warp bubble and 0 outside the warp bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region according to eq 7 pg 4 in [1] or top of pg 4 in [2].

Expanding the quadratic term in eq 8 pg 4 in [1] and solving eq 8 for a null-like interval $ds^2 = 0$ we will have the following equation for the motion of the photon sent to the front[18]:

$$\frac{dx}{dt} = vsf(rs) - 1 \tag{42}$$

[16] $f(r_s)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c = G = 1$

[17] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

[18] The coordinate frame for the Alcubierre warp drive as in [1] is the remote observer outside the ship

Inside the Alcubierre warp bubble $f(rs) = 1$ and $vsf(rs) = vs$.Outside the warp bubble $f(rs) = 0$ and $vsf(rs) = 0$.

Somewhere inside the Alcubierre warped region when $f(rs)$ starts to decrease from 1 to 0 making the term $vsf(rs)$ decreases from vs to 0 and assuming a continuous behavior then in a given point $vsf(rs) = 1$ and $\frac{dx}{dt} = 0$.The photon stops,A Horizon is established.This due to the fact that there are no negative energy density in the front of the Alcubierre warp drive in the x-axis to deflect the photon.

Now taking the components of the Natario vector defined in the top of pg 5 in [2] and inserting these components in the first equation of pg 2 in [2] and solving for the same null-like interval $ds^2 = 0$ considering only radial motion we will get the following equation for the motion of the photon sent to the front[19]:

$$\frac{dx}{dt} = 2vsn(rs) - 1 \tag{43}$$

The Natario shape function $n(rs)$ is defined as being 0 inside the warp bubble and $\frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region according to pg 5 in [2].

Inside the Natario warp bubble $n(rs) = 0$ and $2vsn(rs) = 0$.Outside the warpb bubble $n(rs) = \frac{1}{2}$ and $2vsn(rs) = vs$. Somewhere inside the Natario warped region $n(rs)$ starts to increase from 0 to $\frac{1}{2}$ making the term $2vsn(rs)$ increase from 0 to vs and assuming a continuous behavior then in a given point we would have a $2vsn(rs) = 1$ and a $\frac{dx}{dt} = 0$ The photon would stops.A Horizon would be established.

However when the photon reaches the beginning of the Natario warped region it suffers a deflection by the negative energy density in front of the Natario warp drive because this negative energy is not null.So in the case of the Natario warp drive the photon never reaches the Horizon and the Natario warp drive never suffer from the pathology of the infinite Doppler Blueshift due to a different distribution of energy density when compared to its Alcubierre counterpart.

Adapted from the negative energy in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

[19]The coordinate frame for the Natario warp drive as in [2] is the ship frame observer in the center of the warp bubble $xs = 0$

6 Conclusion

In this work we demonstrated that the energy density requirements for the Natario warp drive depends on the form of the shape function.We adopted a new point of view introducing the Heaviside step functions in the study of the Natario warp drive spacetime.This was never done before. Since one of these Heaviside step functions allows low energy requirements and since the Natario warp drive can bypass the physical problem of the infinite Doppler Blueshift then the Natario warp drive can be regarded as a valid candidate for interstellar space travel. In this work we mentioned many times our interstellar trip of 20 light years at 200 times faster than light.It was inspired by the first planet discovered in the habitable zone of another star:the Gliese 581 at 20 light years away.Given the huge number of planets in the habitable zones of their parent stars example:Kepler-22 at 600 light years away or Kepler-47 at 4950 light years away these planets can only be accessible to our space exploration if faster than light space travel could ever be developed.

7 Appendix A:The Natario Warp Drive Negative Energy Density in Cartezian Coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[20]:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \cos^2\theta + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 \sin^2\theta \right] \tag{44}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 (\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 (\frac{y}{rs})^2 \right] \tag{45}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \right] \tag{46}$$

[20] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

8 Appendix B:Differential Forms,Hodge Star and the Mathematical Demonstration of the Natario Vectors $nX = -vsdx$ and $nX = vsdx$ for a constant speed vs

This appendix is being written for novice or newcomer students on Warp Drive theory still not acquainted with the methods Natario used to arrive at the final expression of the Natario vector nX

The Canonical Basis of the Hodge Star in spherical coordinates can be defined as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \sim r^2 \sin\theta(d\theta \wedge d\varphi) \tag{47}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \sim r\sin\theta(d\varphi \wedge dr) \tag{48}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \sim r(dr \wedge d\theta) \tag{49}$$

From above we get the following results

$$dr \sim r^2 \sin\theta(d\theta \wedge d\varphi) \tag{50}$$

$$rd\theta \sim r\sin\theta(d\varphi \wedge dr) \tag{51}$$

$$r\sin\theta d\varphi \sim r(dr \wedge d\theta) \tag{52}$$

Note that this expression matches the common definition of the Hodge Star operator * applied to the spherical coordinates as given by(pg 8 in [5]):

$$*dr = r^2 \sin\theta(d\theta \wedge d\varphi) \tag{53}$$

$$*rd\theta = r\sin\theta(d\varphi \wedge dr) \tag{54}$$

$$*r\sin\theta d\varphi = r(dr \wedge d\theta) \tag{55}$$

Back again to the Natario equivalence between spherical and cartezian coordinates(pg 5 in [2]):

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2 \sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2 \sin^2\theta d\varphi\right) \tag{56}$$

Look that

$$dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \tag{57}$$

Or

$$dx = d(r\cos\theta) = \cos\theta dr - \sin\theta r d\theta \tag{58}$$

Applying the Hodge Star operator * to the above expression:

$$*dx = *d(r\cos\theta) = \cos\theta(*dr) - \sin\theta(*rd\theta) \tag{59}$$

$$*dx = *d(r\cos\theta) = \cos\theta[r^2\sin\theta(d\theta\wedge d\varphi)] - \sin\theta[r\sin\theta(d\varphi\wedge dr)] \tag{60}$$

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] - [r\sin^2\theta(d\varphi\wedge dr)] \tag{61}$$

We know that the following expression holds true(see pg 9 in [4]):

$$d\varphi\wedge dr = -dr\wedge d\varphi \tag{62}$$

Then we have

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] + [r\sin^2\theta(dr\wedge d\varphi)] \tag{63}$$

And the above expression matches exactly the term obtained by Natario using the Hodge Star operator applied to the equivalence between cartezian and spherical coordinates(pg 5 in [2]).

Now examining the expression:

$$d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{64}$$

We must also apply the Hodge Star operator to the expression above

And then we have:

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{65}$$

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \sim \frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] + \frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] \tag{66}$$

According to pg 10 in [4] the term $\frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] = 0$

This leaves us with:

$$\frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] \sim \frac{1}{2}r^2 2\sin\theta\cos\theta(d\theta\wedge d\varphi) + \frac{1}{2}\sin^2\theta 2r(dr\wedge d\varphi) \tag{67}$$

Because and according to pg 10 in [4]:

$$d(\alpha+\beta) = d\alpha + d\beta \tag{68}$$

$$d(f\alpha) = df\wedge\alpha + f\wedge d\alpha \tag{69}$$

$$d(dx) = d(dy) = d(dz) = 0 \tag{70}$$

From above we can see for example that

$$*d[(\sin^2\theta)d\varphi] = d(\sin^2\theta)\wedge d\varphi + \sin^2\theta \wedge dd\varphi = 2sin\theta\cos\theta(d\theta\wedge d\varphi) \tag{71}$$

$$*[d(r^2)d\varphi] = 2rdr\wedge d\varphi + r^2\wedge dd\varphi = 2r(dr\wedge d\varphi) \tag{72}$$

And then we derived again the Natario result of pg 5 in [2]

$$r^2\sin\theta\cos\theta(d\theta\wedge d\varphi) + r\sin^2\theta(dr\wedge d\varphi) \tag{73}$$

Now we will examine the following expression equivalent to the one of Natario pg 5 in [2] except that we replaced $\frac{1}{2}$ by the function $f(r)$:

$$*d[f(r)r^2\sin^2\theta d\varphi] \tag{74}$$

From above we can obtain the next expressions

$$f(r)r^2 * d[(\sin^2\theta)d\varphi] + f(r)\sin^2\theta * [d(r^2)d\varphi] + r^2\sin^2\theta * d[f(r)d\varphi] \tag{75}$$

$$f(r)r^2 2sin\theta\cos\theta(d\theta\wedge d\varphi) + f(r)\sin^2\theta 2r(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{76}$$

$$2f(r)r^2 sin\theta\cos\theta(d\theta\wedge d\varphi) + 2f(r)r\sin^2\theta(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{77}$$

Comparing the above expressions with the Natario definitions of pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta)\wedge(r\sin\theta d\varphi) \sim r^2\sin\theta(d\theta\wedge d\varphi) \tag{78}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi)\wedge dr \sim r\sin\theta(d\varphi\wedge dr) \sim -r\sin\theta(dr\wedge d\varphi) \tag{79}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr\wedge(rd\theta) \sim r(dr\wedge d\theta) \tag{80}$$

We can obtain the following result:

$$2f(r)\ cos\theta[r^2 sin\theta(d\theta\wedge d\varphi)] + 2f(r)\ sin\theta[r\sin\theta(dr\wedge d\varphi)] + f'(r)r\sin\theta[r\sin\theta(dr\wedge d\varphi)] \tag{81}$$

$$2f(r)\ cos\theta e_r - 2f(r)\ sin\theta e_\theta - rf'(r)\sin\theta e_\theta \tag{82}$$

$$*d[f(r)r^2\sin^2\theta d\varphi] = 2f(r)\ cos\theta e_r - [2f(r) + rf'(r)]\sin\theta e_\theta \tag{83}$$

Defining the Natario Vector as in pg 5 in [2] with the Hodge Star operator * explicitly written :

$$nX = vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{84}$$

$$nX = -vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{85}$$

We can get finally the latest expressions for the Natario Vector nX also shown in pg 5 in [2]

$$nX = 2vs(t)f(r)\ cos\theta e_r - vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{86}$$

$$nX = -2vs(t)f(r)\ cos\theta e_r + vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{87}$$

With our pedagogical approaches

$$nX = 2vs(t)f(r)\ cos\theta dr - vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{88}$$

$$nX = -2vs(t)f(r)\ cos\theta dr + vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{89}$$

9 Appendix C:Mathematical Demonstration of the Natario Warp Drive Equation for a constant speed vs

The warp drive spacetime according to Natario is defined by the following equation but we changed the metric signature from $(-,+,+,+)$ to $(+,-,-,-)$(pg 2 in [2])

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{90}$$

where X^i is the so-called shift vector.This shift vector is the responsible for the warp drive behavior defined as follows(pg 2 in [2]):

$$X^i = X, Y, Z \curvearrowright i = 1, 2, 3 \tag{91}$$

The warp drive spacetime is completely generated by the Natario vector nX(pg 2 in [2])

$$nX = X^i \frac{\partial}{\partial x^i} = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}, \tag{92}$$

Defined using the canonical basis of the Hodge Star in spherical coordinates as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r \sin\theta d\varphi) \tag{93}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \tag{94}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{95}$$

Redefining the Natario vector nX as being the rate-of-strain tensor of fluid mechanics as shown below(pg 5 in [2]):

$$nX = X^r \mathbf{e}_r + X^\theta \mathbf{e}_\theta + X^\varphi \mathbf{e}_\varphi \tag{96}$$

$$nX = X^r dr + X^\theta r d\theta + X^\varphi r\sin\theta d\varphi \tag{97}$$

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{98}$$

$$X^i = r, \theta, \varphi \curvearrowright i = 1, 2, 3 \tag{99}$$

We are interested only in the coordinates r and θ according to pg 5 in [2])

$$ds^2 = dt^2 - (dr - X^r dt)^2 - (rd\theta - X^\theta dt)^2 \tag{100}$$

$$(dr - X^r dt)^2 = dr^2 - 2X^r dr dt + (X^r)^2 dt^2 \tag{101}$$

$$(rd\theta - X^{\theta}dt)^2 = r^2 d\theta^2 - 2X^{\theta} r d\theta dt + (X^{\theta})^2 dt^2 \tag{102}$$

$$ds^2 = dt^2 - (X^r)^2 dt^2 - (X^{\theta})^2 dt^2 + 2X^r dr dt + 2X^{\theta} r d\theta dt - dr^2 - r^2 d\theta^2 \tag{103}$$

$$ds^2 = [1 - (X^r)^2 - (X^{\theta})^2]dt^2 + 2[X^r dr + X^{\theta} r d\theta]dt - dr^2 - r^2 d\theta^2 \tag{104}$$

making $r = rs$ we have the Natario warp drive equation:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs} drs + X^{\theta} rs d\theta]dt - drs^2 - rs^2 d\theta^2 \tag{105}$$

According with the Natario definition for the warp drive using the following statement(pg 4 in [2]):any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble

The expressions for X^{rs} and X^{θ} are given by:(see pg 5 in [2])

$$nX \sim -2v_s n(rs)\cos\theta \mathbf{e}_{rs} + v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{106}$$

$$nX \sim 2v_s n(rs)\cos\theta \mathbf{e}_{rs} - v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{107}$$

$$nX \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{108}$$

$$nX \sim 2v_s n(rs)\cos\theta drs - v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{109}$$

But we already know that the Natario vector nX is defined by(pg 2 in [2]):

$$nX = X^{rs} drs + X^{\theta} rs d\theta \tag{110}$$

Hence we should expect for:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{111}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{112}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{113}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{114}$$

10 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[21]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[22]

11 Legacy

This work is dedicated as a tribute to memory of the English mathematician Oliver Heaviside.Although self-student without an university degree and ill-casted or outcasted by the great majority of the scientific community of his time,his contributions prevailed and survived to our days.He adapted complex numbers to the study of electrical circuits, invented mathematical techniques to the solution of differential equations (later found to be equivalent to Laplace transforms), reformulated Maxwell's field equations in terms of electric and magnetic forces and energy flux, and independently co-formulated vector analysis. Although at odds with the scientific establishment for most of his life, Heaviside changed the face of mathematics and science for years to come.[23] [24] Without the Heaviside step function the Natario warp drive would perhaps remain forever as an unphysical mathematical curiosity and impossible to be achieved from a real physical point of view.

[21] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[22] "Ideas And Opinions" Einstein compilation, ISBN $0 - 517 - 88440 - 2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[23] quoted from Wikipedia The Free Encyclopedia

[24] all these contributions from a man that do not have a standard university degree ranks Oliver Heaviside among the greatest mathematicians that ever lived.And perhaps the greatest of all!!!

Figure 1: **A Portrait of Oliver Heaviside-one of the greatest mathematicians that ever lived. (**18**-May-**1850**,London,England,**03**-February-**1925**,Devon,England**). (Source:Wikipedia The Free Encyclopedia)

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