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NEUTROSOPHIC SUPER MATRICES AND QUASI SUPER MATRICES

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PREFACE

In this book authors study neutrosophic super matrices. The concept of neutrosophy or indeterminacy happens to be one the powerful tools used in applications like FCMs and NCMs where the expert seeks for a neutral solution. Thus this concept has lots of applications in fuzzy neutrosophic models like NRE, NAM etc. These concepts will also find applications in image processing where the expert seeks for a neutral solution.

Here we introduce neutrosophic super matrices and show that the sum or product of two neutrosophic matrices is not in general a neutrosophic super matrix.

Another interesting feature of this book is that we introduce a new class of matrices called quasi super matrices; these matrices are the larger class which contains the class of super matrices. These class of matrices lead to more partition of $n \times m$ matrices where $n > 1$ and $m > 1$, where m and n can also be equal. Thus this concept cannot be defined on usual row matrices or column matrices.

These matrices will play a major role when studying a problem which needs multi fuzzy neutrosophic models.

This book is organized into four chapters. Chapter one is introductory in nature. Chapter two introduces the notion of neutrosophic super matrices and describes operations on them. In chapter three the major product of super row vectors and column vectors are defined and are extended to bisuper vectors.

The final chapter introduces the new concept of quasi super matrices and suggests some open problems.

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W.B.VASANTHA KANDASAMY
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Chapter One

INTRODUCTION

In this chapter we just indicate the concepts which we have used and the place of reference of the same.

Throughout this book I will be denote the indeterminate such that $I^2 = I$. $\langle Z \cup I\rangle$, $\langle Q \cup I\rangle$, $\langle R \cup I\rangle$, $\langle C \cup I\rangle$, $\langle Z_n \cup I\rangle$ and $\langle C(Z_n) \cup I\rangle$ denotes the neutrosophic ring [3-4, 7]. For the definition and properties of super matrices please refer [2, 5]. The notion of bimatrices is introduced in [6].

Here in this book the notion of neutrosophic super matrices are introduced and operations on them are performed.

We also can use the entries of these super neutrosophic matrice from rings; $(\langle Z \cup I\rangle)$ (g) = $\{a + bg \mid a, b \in \langle Z \cup I\rangle$ and g is the new element such that $g^2 = 0\}$. Z can be replaced by R or Q or Z_n or C and we call them as dual number rings [11].

$(\langle Q \cup I \rangle)(g_1) = \{a + bg_1 \mid a, b \in \langle Q \cup I \rangle$ where g_1 is a new element such that $g_1^2 = g_1\}$. We call $(\langle Q \cup I \rangle)(g_1)$ as special dual like number neutrosophic rings. Here also Q can be replaced by R or Z or C or Z_n.

$(\langle R \cup I \rangle)(g_2) = \{a + bg \mid a, b \in \langle R \cup I \rangle$, g_2 the new element such that $g_2^2 = -g_2\}$ is the special quasi dual number neutrosophic ring. We can replace R by Q or Z or Z_n or C and still we get special quasi dual number neutrosophic rings.

Finally we can also use $(\langle Z \cup I \rangle)(g_1, g_2, g_3) = \{a_1 + a_2g_1 + a_3g_2 + a_4g_3 \mid a_i \in \langle Z \cup I \rangle; 1 \leq i \leq 4; g_1, g_2, g_3$ are new elements such that $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$ with $g_ig_j = (0$ or g_i or $g_j)$, $i \neq j; 1 \leq i, j \leq 3\}$ the special mixed neutrosophic dual number rings [11-13].

Finally in this book the new notion of quasi super matrices is introduced and their properties are studied. This new concept of quasi super matrices can be used in the construction of fuzzy-neutrosophic models.

Chapter Two

NEUTROSOPHIC SUPER MATRICES

In this chapter we for the first time introduce and study the notion of super neutrosophic matrices and their properties.

A super matrix A in which the element indeterminacy I is present is called a super neutrosophic matrix. To be more precise we will define these notions.

DEFINITION 2.1: *Let* $X = \{(x_1\ x_2 \ldots x_r \mid x_{r+1} \ldots x_t \mid x_{t+1} \ldots.\ x_n)$ *where* $x_i \in \langle R \cup I \rangle$*; R reals* $1 \leq i \leq n)$ *be a super row matrix which will be known as a super row neutrosophic matrix.*

Thus all super row matrices in general are not super row neutrosophic matrix.

We illustrate this situation by an example.

***Example 2.1*:** Let X = (3 I 1 –4 | 20 –5 | 2I 0 1 | 7 2 1 4), X is a super row neutrosophic matrix.

All super row matrices are not super row neutrosophic matrices.

For take A = (3 1 0 1 1 | 25 – 27 | 0 1 2 3 4), A is only a super row matrix and is not a super row neutrosophic matrix.

Example 2.2: Let A = (I 3I 1 | –I 5I | 2I 0 3I 2 1 | 0 3); A is a super row neutrosophic matrix.

Example 2.3: Let
A = (0.3I, 0, 1 | 0.7, $\sqrt{2}$ I, 0 I + $\sqrt{7}$ | –8I, 0.9I + 2); A is a super row neutrosophic matrix.

DEFINITION 2.2: *Let*

$$Y = \left(\begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_{r_1} \\ \hline \vdots \\ \hline y_{t+1} \\ \vdots \\ y_n \end{array} \right) \text{ where } y_i \in \langle R \cup I \rangle;\ 1 \leq i \leq n.$$

Y is a super column matrix known as the super column neutrosophic matrix.

We illustrate this situation by some examples.

Example 2.4: Let

$$Y = \left[\begin{array}{c} 2 \\ I \\ 3+4I \\ \hline 0 \\ 1 \\ 0.3I \end{array} \right]$$

be a super column matrix. Y is clearly a neutrosophic super column matrix (super neutrosophic column matrix).

***Example 2.5*: Let**

$$Y = \left[\begin{array}{c} 3I \\ -2 \\ \hline 0 \\ 1 \\ -5 \\ \hline I \\ 0.4 \\ \sqrt{5} \\ 6+I \end{array}\right],$$

be a super neutrosophic column vector / matrix.

It is pertinent to mention here that in general all super column matrices are not super neutrosophic column matrices. This is proved by an example.

***Example 2.6*: Let**

$$A = \left[\begin{array}{c} 3 \\ 0 \\ 1 \\ \hline -31 \\ 4 \\ 0.5 \\ \hline \sqrt{2} \\ 1 \end{array}\right],$$

be a super column matrix which is clearly not a super neutrosophic column matrix.

We now proceed onto define the notion of neutrosophic super square matrix.

DEFINITION 2.3: *Let*

$$A = (a_{ij}) = \left[\begin{array}{cc|c|c} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \hline \vdots & & & \vdots \\ \hline \vdots & & & \vdots \\ \hline a_{n1} & \cdots & \cdots & a_{nn} \end{array}\right]$$

be a n × n super square matrix where $a_{ij} \in \langle R \cup I \rangle$. A is called the neutrosophic super square matrix of order n × n.

We illustrate this by an example.

Example 2.7: Let

$$A = \left[\begin{array}{ccc|cc|c} 2 & I & 0 & 5 & -7 & 2 \\ I & 1 & 3 & 0 & I & 2I \\ 0 & 1 & I & 3 & I & 7 \\ \hline -2 & 5 & 0 & 2 & 0 & 6 \\ 1 & 3 & 5 & 3 & I & 7 \\ 1 & 1 & 1 & -I & 1 & I \end{array}\right]$$

be a 6×6 super square matrix. Clearly as the entries of A are from ⟨R ∪ I⟩. A is neutrosophic super square matrix.

***Example 2.8*:** Let

$$A = \left[\begin{array}{cc|ccc|cc} 3 & 0 & I & 0 & 1 & -I & 5I \\ \hline I & 1 & 0 & 1 & 5I & 0 & 1 \\ 2 & -3I & I & 0 & 0.3 & 2 & 3 \\ \hline 1 & 2 & 1 & I & 0 & I & 2I \\ 3 & 4 & -2I & 2 & -3I & 3 & 4 \\ 5I & 6 & 0 & 1 & 3 & 5I & 6I \\ \hline 1 & 3I & 5 & 6 & 7 & 8 & 9I \end{array}\right],$$

A is a 7×7 neutrosophic square super matrix.

All super square matrices need not in general be a neutrosophic square super matrices.

We illustrate this by an example.

***Example 2.9*:** Let

$$A = \left[\begin{array}{cc|ccc} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 0 \\ 0 & 9 & 8 & 7 & 6 \\ \hline 5 & 4 & 3 & 2 & 1 \\ 2 & 4 & 8 & 0 & 7 \end{array}\right]$$

A is just a square super matrix and is not a square neutrosophic super matrix.

Now we proceed onto define a rectangular super matrix.

DEFINITION 2.4: *Let*

$$M = (m_{ij}) = \left[\begin{array}{ccc|c|c} m_{11} & m_{12} & m_{13} & \cdots & m_{1n} \\ \hline m_{21} & m_{22} & m_{23} & \cdots & m_{2n} \\ \vdots & \vdots & & & \vdots \\ m_{p-11} & m_{p-12} & m_{p-13} & \cdots & m_{p-1n} \\ m_{p1} & m_{p2} & m_{p3} & \cdots & m_{pn} \end{array}\right]$$

be a $p \times n$ $(p \neq n)$ rectangular super matrix where $m_{ij} \in \langle R \cup I \rangle$. Clearly M is a rectangular neutrosophic $p \times n$ super matrix.

We illustrate this by an example.

Example 2.10: Let

$$A = \left[\begin{array}{cc|ccc|cccc} 3 & 0 & I & 6 & 5 & 4I & 3 & 2I & 1 \\ \hline 5 & 2 & 0 & I & 2 & 3I & 4 & 5I & 6 \\ 1 & I & 5 & 0 & I & 2 & 3I & 4 & 5I \\ I & 0 & 1 & 1 & 0 & I & 1 & I & 2 \end{array}\right]$$

A is a 4×9 rectangular neutrosophic super matrix.

***Example 2.11*:** Let

$$A = \left[\begin{array}{ccc|cc} 3 & I & 1 & 2 & 0 \\ I & 4 & I & 5 & -3I \\ \hline 1 & 2 & 3I & I & 4 \\ 4I & 5 & 6 & 0 & -2I \\ 7 & 8I & 9 & 3 & 7 \\ \hline 0 & 1 & 2I & 8 & -9I \end{array}\right]$$

be a 6×5 rectangular super neutrosophic matrix.

All super rectangular matrices in general need not be a neutrosophic super rectangular matrices.

We illustrate this by an example.

Example 2.12: Let

$$A = \left[\begin{array}{ccc|ccc|cc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 0 & 9 & 8 & 7 & 6 & 5 & 4 \\ \hline 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 & 9 & 0 & 1 & 2 \end{array}\right]$$

be a 4 × 8 super rectangular matrix. Clearly A is not a super neutrosophic rectangular matrix.

Thus we can say as in case of supermatrices if we take any neutrosophic matrix and partition it, we would get the super neutrosophic matrix.

We can as in case of supermatrices define a neutrosophic super matrix is one in which one or more of its elements are themselves simple neutrosophic matrices. The order of the neutrosophic super matrix is defined in the same way as that of a neutrosophic matrix.

We have to define addition and subtraction of neutrosophic super matrices.

We cannot add in general two neutrosophic matrices A and B of same natural order say m × n.

We shall first illustrate this by an example.

***Example 2.13*:** Let

$$A = \left[\begin{array}{cc|ccc|c} 3 & 5 & 6 & 7 & 8 & 1 \\ 0 & 1 & 2 & I & 3 & 4 \\ \hline I & 0 & 1 & 0 & 0 & I \\ 1 & 2 & 3 & 4 & I & 0 \\ 0 & I & 1 & 2 & 3 & 4 \\ 0 & 1 & I & 3 & 0 & 1 \\ \hline 0 & 1 & 1 & 0 & I & 0 \end{array}\right] \text{ and } B = \left[\begin{array}{ccc|ccc} 5 & 3 & I & 2 & 0 & 6 \\ 0 & 1 & 0 & I & 1 & 2 \\ I & 0 & 1 & 2 & 0 & I \\ \hline 0 & I & 6 & 0 & I & 0 \\ I & 2 & 3 & 4 & 0 & 5 \\ 0 & 1 & 2 & 3 & I & 0 \\ 6 & I & 7 & 2 & 0 & I \end{array}\right]$$

be two super neutrosophic matrices of natural order 7×6. Clearly we cannot add A with B though as simple matrices A and B can be added.

Thus for two super matrices to have a compatible addition it is not sufficient they should be of same natural order they should necessarily be of same natural order but satisfy further conditions. This condition we shall define as matrix order of a super matrix.

DEFINITION 2.5: *Let*

$$A = \left[\begin{array}{c|c|c} a_{11} & \cdots & a_{1n} \\ \hline \vdots & & \vdots \\ \hline a_{m1} & \cdots & a_{mn} \end{array}\right]$$

be a neutrosophic super matrix where each a_{ij} is a simple neutrosophic matrix. $1 \le i \le m$ and $1 \le j \le n$.

The natural order of A is $t \times s$; clearly $t \ge m$ and $s \ge n$ that is A has t rows and s columns.

The matrix order of the neutrosophic super matrix A is $m \times n$. i.e., it has m number of matrix rows and n number of *matrix columns. The number rows in each simple neutrosophic matrix a_{ip}, i fixed is the same, p may vary; $1 \leq p \leq n$. Likewise for a_{kj}; j fixed, $1 \leq k \leq m$; the number columns is the same.*

We now give examples of these.

Example 2.14: Let

$$A = \left[\begin{array}{ccc|cccc|cc} 3 & I & 1 & 2 & I & 1 & 0 & 2 & -I \\ 0 & 2 & 0 & 0 & 0 & I & 2 & 1 & 0 \\ \hline 1 & 0 & I & 1 & 1 & I & 3 & 1 & 2 \\ 2 & 1 & 1 & 3 & I & 2 & 1 & 3 & 4 \\ 4 & I & 2 & 0 & 0 & I & 1 & 5 & 6I \\ \hline I & I & 0 & 1 & 0 & 1 & 0 & I & 3I \end{array}\right]$$

$$= \left[\begin{array}{c|c|c} a_{11} & a_{12} & a_{13} \\ \hline a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right]$$

where $a_{11} = \begin{bmatrix} 3 & I & 1 \\ 0 & 2 & 0 \end{bmatrix}$, $a_{12} = \begin{bmatrix} 2 & I & 1 & 0 \\ 0 & 0 & I & 2 \end{bmatrix}$, $a_{13} = \begin{bmatrix} 2 & -I \\ 1 & 0 \end{bmatrix}$,

$a_{21} = \begin{bmatrix} 1 & 0 & I \\ 2 & 1 & 1 \\ 4 & I & 2 \end{bmatrix}$, $a_{22} = \begin{bmatrix} 1 & 1 & I & 3 \\ 3 & I & 2 & 1 \\ 0 & 0 & I & 1 \end{bmatrix}$, $a_{23} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6I \end{bmatrix}$,

$a_{31} = [I\ I\ 0]$, $a_{32} = [1\ 0\ 1\ 0]$ and $a_{33} = [I\ 3\ I]$ are the neutrosophic submatrices of A also known as are the component matrices of A.

The natural order of A is 6×9 and the matrix order of A is 3×3.

Example 2.15: Let

$$A = \left[\begin{array}{cc|ccc} 3 & I & 0 & 1 & 2 \\ 0 & 0 & I & 1 & 0 \\ \hline 1 & 1 & 1 & 2 & 3 \\ I & 0 & 4 & 5 & 6I \\ \hline 1 & I & 0 & I & 3 \end{array}\right] = \left[\begin{array}{c|c} a_{11} & a_{12} \\ \hline a_{21} & a_{22} \\ \hline a_{31} & a_{32} \end{array}\right]$$

be super neutrosophic matrix. The natural order of A is 5×5 and the matrix order of A is 3×2.

Now having seen example of matrix order of a neutrosophic super matrix we now proceed on to define addition of two super neutrosophic matrices.

DEFINITION 2.6: *Let A and B be two neutrosophic super matrices of same natural order say m × n. Let A and B be a neutrosophic super matrices of matrix order t × s where t ≤ m and s ≤ n.*

We can say A+B the sum of the super neutrosophic matrices A and B be defined if and only if each simple matrix a_{ij} in A is of same order for each b_{ij} in B. $1 \le i \le t$; $1 \le j \le s$;

i.e., if $A = \left[\begin{array}{c|c|c|c} a_{11} & a_{12} & \cdots & a_{1s} \\ \hline \vdots & & & \vdots \\ \hline a_{t1} & a_{t2} & \cdots & a_{ts} \end{array}\right]$ *and* $B = \left[\begin{array}{c|c|c|c} b_{11} & b_{12} & \cdots & b_{1s} \\ \hline \vdots & & & \vdots \\ \hline b_{t1} & b_{t2} & \cdots & b_{ts} \end{array}\right]$

where a_{ij} and b_{ij} are simple matrices. Here order of a_{ij} = order of b_{ij}; $1 \le i \le s$ and $1 \le j \le t$.

We illustrate this by some examples.

***Example 2.16*:** Let

$$A = \left[\begin{array}{c|c|c|c} a_{11} & a_{12} & a_{13} & a_{14} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} \end{array}\right] \text{ where } a_{11} = \begin{bmatrix} 3 & I & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad a_{12} = \begin{bmatrix} I \\ 2 \end{bmatrix},$$

$$a_{13} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, a_{14} = \begin{bmatrix} 1 & 1 & 1 & I \\ 0 & 0 & I & 0 \end{bmatrix}, \; a_{21} = [\,0\ 1\ I\,]\ , a_{22} = [3],$$

$a_{23} = [1\ I]$ and $a_{24} = [5\ 1\ I\ 0\ ]$.

The natural order of A is 3×10.

$$A = \left[\begin{array}{ccc|c|cc|cccc} 3 & I & 0 & I & 1 & 0 & 1 & 1 & 1 & I \\ 1 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & I & 0 \\ \hline 0 & 1 & I & 3 & 1 & I & 5 & 1 & I & 0 \end{array}\right].$$

The matrix order of A is 2×4.

Take a neutrosophic super matrix B of natural order 3×10 and matrix order 2×4 given by

$$B = \left[\begin{array}{ccc|c|cc|cccc} 0 & 1 & I & 0 & 1 & 0 & 1 & 0 & 1 & I \\ 1 & 0 & 1 & 1 & 2 & 1 & 0 & 1 & I & 0 \\ \hline I & 0 & 1 & 2 & 1 & I & 1 & 0 & 5 & 2 \end{array}\right]$$

$$= \left[\begin{array}{c|c|c|c} b_{11} & b_{12} & b_{13} & b_{14} \\ \hline b_{21} & b_{22} & b_{23} & b_{24} \end{array}\right].$$

Since natural order of matrix a_{ij} = natural order of b_{ij}, for each i and j, $1 \leq i \leq 2$ and $1 \leq j \leq 4$ and as A and B have same natural order as well as matrix order. We can add A and B.

$$A+B=\left[\begin{array}{ccc|c|cc|ccccc} 3 & I+1 & I & I & 2 & 0 & 2 & 1 & 2 & 2 & I \\ 2 & 0 & 2 & 3 & 2 & 2 & 0 & 1 & 2 & I & 0 \\ \hline I & 15 & 1+I & 5 & 2 & 2I & 6 & 1 & 5 & +I & 2 \end{array}\right].$$

We see A+B is also a neutrosophic super matrix of natural order 3×10 and matrix order 2×4.

It is important to mention that even if A and B are two neutrosophic super matrices of same natural order and same matrix order yet the sum of A and B may not be defined.

This is established by an example.

Example 2.17: Let

$$A=\left[\begin{array}{cc|ccc|cc} I & 0 & 1 & 2 & 3 & I & 0 \\ 1 & I & 0 & 3 & 2 & 1 & I \\ \hline 0 & 1 & I & 2 & 3 & 4 & 0 \\ 5 & 0 & 1 & I & 0 & I & 1 \\ 2 & 3 & 4 & 0 & I & 1 & I \end{array}\right]=\left[\begin{array}{c|c|c} a_{11} & a_{12} & a_{13} \\ \hline a_{21} & a_{22} & a_{23} \end{array}\right] \text{ and}$$

$$B=\left[\begin{array}{ccc|cc|cc} 5 & I & 3 & 3 & I & 1 & I \\ 0 & 1 & 4 & 2 & 0 & I & 0 \\ \hline I & 3 & 2 & 1 & 2 & 1 & I \\ 1 & 2 & 3 & 3 & 4 & 1 & 0 \\ 0 & I & 1 & 5 & I & 1 & I \end{array}\right]=\left[\begin{array}{c|c|c} b_{11} & b_{12} & b_{13} \\ \hline b_{21} & b_{22} & b_{23} \end{array}\right]$$

be two super neutrosophic matrices of natural order 5×7 and matrix order 2×3. Clearly A+B, i.e., the sum of A with B is not defined as $a_{11} \neq b_{11}$, $a_{12} \neq b_{12}$, $a_{21} \neq b_{21}$ and $a_{22} \neq b_{22}$. Hence the claim.

Thus we can restate the definition of the sum of two neutrosophic super matrices as follows.

DEFINITION 2.7: *Let A and B be two super neutrosophic matrices of same natural order say $m \times n$ and matrix order $s \times t$. The addition of A with B (B with A) is defined if and only if the natural order of each a_{ij} equal to natural order of b_{ij} for every $1 \leq i \leq s$ and $1 \leq j \leq t$.*

$$\text{i.e., } A = \left(\begin{array}{c|c|c} a_{11} & \cdots & a_{1t} \\ \hline \vdots & & \vdots \\ \hline a_{s1} & \cdots & a_{st} \end{array}\right) \text{ and } B = \left(\begin{array}{c|c|c} b_{11} & \cdots & b_{1t} \\ \hline \vdots & & \vdots \\ \hline b_{s1} & \cdots & b_{st} \end{array}\right)$$

where a_{ij} and b_{ij} are simple matrices or the component submatrices of A and B respectively; $1 \leq i \leq s$ and $1 \leq j \leq t$.

On similar lines the subtraction or difference between two neutrosophic super matrices can be defined.

Another important fact about the addition of neutrosophic super matrices is that sum of two neutrosophic super matrices need not in general give way to a neutrosophic super matrix it can be only a super matrix.

We illustrate this by an example.

Example 2.18: Let

$$A = \left[\begin{array}{ccc|cc} 1 & 0 & 5 & I & 2 \\ 1 & 2 & 3 & 4 & I \\ I & 0 & 1 & 2 & 3 \\ \hline 6 & 7 & 8 & 9 & 0 \end{array}\right] \text{ and } B = \left[\begin{array}{ccc|cc} 1 & 6 & 0 & -I & 1 \\ 0 & 1 & 0 & 2 & -I \\ -I & 2 & 1 & 1 & 0 \\ \hline 0 & 1 & 2 & 1 & 2 \end{array}\right]$$

be two super neutrosophic matrices of same natural and matrix order.

Since each of the corresponding component matrices are equal their sum is defined.

$$\text{Now A+B} = \left[\begin{array}{ccc|cc} 2 & 6 & 5 & 0 & 3 \\ 1 & 3 & 3 & 6 & 0 \\ 0 & 2 & 2 & 3 & 3 \\ \hline 6 & 8 & 10 & 10 & 2 \end{array}\right].$$

We see A+B is only a super matrix and is not a neutrosophic super matrix.

Now we can have the following type of super neutrosophic matrices.

DEFINITION 2.8: *Let*

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

be a type A super vector where each V_i's are column subvectors; $1 \leq i \leq n$. If some of the V_i's are neutrosophic column vectors then we call V to be a type A neutrosophic super vector or neutrosophic super vector of type A.

We illustrate this by some examples.

***Example 2.19*:** Let

$$V = \left[\begin{array}{c} 3 \\ I \\ 0 \\ \hline 1 \\ 1 \\ 0 \\ 4 \\ \hline I \end{array}\right] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix},$$

V is a neutrosophic super vector of type A.

***Example 2.20*:** Let

$$V = \left[\begin{array}{c} I \\ \hline 2 \\ 1 \\ 0 \\ \hline 1 \\ 1 \\ I \\ \hline 1 \\ I \end{array}\right] = \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array}\right],$$

V is a again a neutrosophic super vector of type A. All super vectors of type A need not be neutrosophic super vector of type A.

We illustrate this by an example.

***Example 2.21*:** Let

$$V = \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ \hline 4 \\ 5 \\ 6 \\ \hline 9 \\ 8 \\ 0 \\ 1 \end{array}\right] = \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \end{array}\right]$$

be a super vector of type A. Clearly V is not a neutrosophic super vector of type A.

***Example 2.22*:** Let

$$V = \left[\begin{array}{c} 3 \\ 0 \\ 1 \\ 4 \\ \hline 1 \\ 2 \\ \hline 9 \\ 8 \\ 7 \end{array}\right] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

be a super vector of type A and is not a neutrosophic supervector.

Let $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ be a super vector of type A.

The transpose of V is again a super vector which is a row supervector given by

$$V^t = [3\ 0\ 1\ 4 \mid 1\ 2 \mid 9\ 7\ 8] = \begin{bmatrix} V_1^t & V_2^t & V_3^t \end{bmatrix}.$$

We now define neutrosophic super vector of type B.

DEFINITION 2.9: *Let $T = [a_{11} \mid a_{12} \mid \ldots \mid a_{1n}]$ be a neutrosophic super vector where a_{ij} is a $m \times m_i$ matrix $1 \leq i \leq n$. T is a neutrosophic super vector of type B. We call the transpose of this neutrosophic super vector of type B to be also a neutrosophic super vector of type B.*

We illustrate this by the following examples.

***Example 2.23*:** Let

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \left[\begin{array}{ccc} 0 & 2 & 1 \\ 5 & I & 0 \\ 1 & 1 & 2 \\ \hline I & 0 & 1 \\ 0 & 1 & 3 \\ \hline 0 & I & 0 \\ 1 & 0 & 3 \\ 1 & 1 & 4 \\ I & 2 & 5 \end{array}\right],$$

A is a neutrosophic super vector of type B.

***Example 2.24*:** Let

$$A = \left[\begin{array}{ccccc} 2 & 1 & 3 & 5 & 6 \\ I & 0 & 1 & 2 & 3 \\ 4 & I & 5 & 6 & 7 \\ \hline 0 & 0 & I & 0 & 1 \\ 1 & 0 & 0 & 2 & 3 \\ 0 & I & 4 & 5 & 0 \\ 7 & 0 & 3 & 1 & 1 \\ \hline 3 & 0 & 1 & 1 & I \\ I & 6 & 1 & 0 & 0 \\ 0 & 2 & 5 & I & 0 \\ 1 & I & 0 & 1 & 1 \\ 2 & 3 & 6 & 1 & 2 \\ 3 & I & 0 & 1 & 3 \end{array}\right] = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}.$$

A is a neutrosophic super vector of type B.

We wish to say super vector of type B are in general not neutrosophic.

We illustrate this situation by an example.

***Example 2.25*:** Let

$$A = \left[\begin{array}{ccc} 3 & 1 & 5 \\ 0 & 2 & 1 \\ \hline 3 & 1 & 1 \\ 4 & 0 & 2 \\ 1 & 1 & 6 \\ 7 & 8 & 9 \\ \hline 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix},$$

A is only a supervector of type B which is not a neutrosophic super vector of type B.

We define the transpose of a neutrosophic supervector of type B.

DEFINITION 2.10: *Let*

$$A = \begin{bmatrix} A_1 \\ \vdots \\ A_n \end{bmatrix}$$

be a neutrosophic super vector of type B.

The transpose of A denoted by

$$A^t = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_b \end{bmatrix}^t = \begin{bmatrix} A_1^t & A_2^t & \cdots & A_n^t \end{bmatrix}$$

here each A_i is a neutrosophic matrix which has same number of columns. A is also a neutrosophic super vector of type B.

We illustrate this by the following example.

***Example 2.26*:** Let

$$A = \left[\begin{array}{ccc} 3 & 0 & I \\ 1 & 2 & 3 \\ 9 & 8 & 7 \\ \hline 7 & 6 & 0 \\ 8 & 9 & 4 \\ 1 & 4 & 0 \\ 2 & 7 & 1 \\ 3 & 9 & 0 \\ 5 & 2 & I \\ \hline 1 & 2 & 3 \end{array}\right] = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

be a neutrosophic supervector of type B.

$$A^t = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}^t = \begin{bmatrix} A_1^t & A_2^t & A_3^t \end{bmatrix}$$

$$= \left[\begin{array}{ccc|cccccc|c} 3 & 1 & 9 & 7 & 8 & 1 & 2 & 3 & 5 & 1 \\ 0 & 2 & 8 & 6 & 9 & 4 & 7 & 9 & 2 & 2 \\ I & 3 & 7 & 0 & 4 & 0 & 1 & 0 & I & 3 \end{array}\right]$$

is again a neutrosophic super vector of type B.

As in case of neutrosophic super matrices we can define the sum of two neutrosophic super vectors of type A and type B.

DEFINITION 2.11: *Let*

$$A = \left[\begin{array}{c} a_1 \\ \hline \vdots \\ \hline \vdots \\ a_{n-1} \\ a_n \end{array}\right] = \begin{bmatrix} A_1 \\ \vdots \\ A_m \end{bmatrix}$$

be a neutrosophic supervector of type A.

Let the natural order of A be $n \times 1$.

$$\text{Suppose } B = \left[\begin{array}{c} b_1 \\ \hline \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline b_n \end{array}\right] = \begin{bmatrix} B_1 \\ \vdots \\ B_m \end{bmatrix}$$

B is also a neutrosophic super vector of type A of same natural order say $n \times 1$.

The sum of A and B denoted by A+B is defined if and only if the natural order of A_i is equal to the natural order of B_i for every i, i = 1,2,…, m i.e., both the matrices A and B have same matrix order.

We illustrate this by the following example.

***Example 2.27*:** Let

$$A = \left[\begin{array}{c} 0 \\ 1 \\ I \\ \hline 0 \\ 2 \\ \hline 3 \\ I \end{array}\right] = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \text{ and } B = \left[\begin{array}{c} I \\ 0 \\ 1 \\ \hline 1 \\ 1 \\ \hline 0 \\ 2 \end{array}\right] = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

be two neutrosophic super matrices of type A.

$$A + B = \left[\begin{array}{c} 0 \\ 1 \\ I \\ \hline 0 \\ 2 \\ \hline 3 \\ I \end{array}\right] + \left[\begin{array}{c} I \\ 0 \\ 1 \\ \hline 1 \\ 1 \\ \hline 0 \\ 2 \end{array}\right] = \left[\begin{array}{c} I \\ 0 \\ I+1 \\ \hline 1 \\ 3 \\ \hline 3 \\ 2+I \end{array}\right].$$

Clearly A+B is also a neutrosophic super vector of type A same natural order and matrix order that of A and B.

We can define also the subtraction of neutrosophic super vector of type A. Now A–B is calculated.

$$A - B = \left[\begin{array}{c} 0 \\ 1 \\ I \\ \hline 0 \\ 2 \\ \hline 3 \\ I \end{array}\right] - \left[\begin{array}{c} I \\ 0 \\ 1 \\ \hline 1 \\ 1 \\ \hline 0 \\ 2 \end{array}\right] = \left[\begin{array}{c} -I \\ 1 \\ I-1 \\ \hline -1 \\ 1 \\ \hline 3 \\ I-2 \end{array}\right].$$

We see A–B is also a neutrosophic super vector of type A of same natural order and matrix order as that of A and B.

However we wish to state that sum of difference of two neutrosophic supervector of type A need not always give way to neutrosophic super vector of type A it can also be only a supervector of type A.

We illustrate this situation by an example.

***Example 2.28*:** Let

$$A = \left[\begin{array}{c} I \\ 1 \\ 0 \\ 2 \\ \hline 1 \\ I \\ 0 \\ \hline 2 \\ 3 \end{array}\right] \text{ and } B = \left[\begin{array}{c} -I \\ 1 \\ 0 \\ 1 \\ \hline 4 \\ -I \\ 2 \\ \hline 1 \\ 0 \end{array}\right]$$

be two neutrosophic super vector of type A.

$$A + B = \begin{bmatrix} I \\ 1 \\ 0 \\ 2 \\ \hline 1 \\ I \\ 0 \\ \hline 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -I \\ 1 \\ 0 \\ 1 \\ \hline 4 \\ -I \\ 2 \\ \hline 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \\ \hline 5 \\ 0 \\ 2 \\ \hline 3 \\ 3 \end{bmatrix}$$

is only a super vector of type A and not a neutrosophic super vector of type A.

We just indicate by an example how the minor product of type A neutrosophic super vectors is carried out.

Example 2.29: Let

$$A = [3\ I\ 0 \mid 1\ 2\ 1\ 4 \mid 0\ I]\ [A_1\ A_2\ A_3] \text{ and}$$

$$B = \begin{bmatrix} 0 \\ 1 \\ I \\ \hline 0 \\ 1 \\ 2 \\ 0 \\ \hline 2 \\ 0 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}.$$

The minor product of AB = [3 I 0 | 1 2 1 4 | 0 I] $\begin{bmatrix} 0 \\ 1 \\ I \\ \hline 0 \\ 1 \\ 2 \\ 0 \\ \hline 2 \\ 0 \end{bmatrix}$

$$= [3\ I\ 0] \begin{bmatrix} 0 \\ 1 \\ I \end{bmatrix} + [1\ 2\ 1\ 4] \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} + [0\ I] \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \{3\times0 + I\times1 + 0\times I\} + \{1\times0 + 2\times1 + 1\times2 + 4\times0\} + \{0\times2 + I\times0\}$$
$$= \{0 + I + 0\} + \{0 + 2 + 2 + 0\} + [0 + 0]$$
$$= I + 4 + 0 = I + 4.$$

Now in general minor product of A and B, two neutrosophic super vectors of type A may not be defined. If A is the neutrosophic super row vector of type A with A = $[A_1 \ldots A_t]$ with natural order 1 × n and matrix order 1 × t then if B is a neutrosophic column super vector of type B then the minor product of A with B = $[B_1 \ldots B_t]^t$ is defined if and only if natural order of B is n × 1 and the matrix order of B is t × 1 with natural order of each B_i in B is the same as the transpose of the natural order of each A_i in A for i = 1, 2, …, t or equivalently the natural order of A_i in A is equal to the order of the transpose of the natural order of B_i in B for each i = 1, 2, …, t.

Thus we see the minor product of a neutrosophic super row vector A with the transpose of the A is well defined for all the conditions required for the compatibility of the minor product of neutrosophic super vectors of type A are satisfied.

We will illustrate this situation by some examples before we proceed onto define more types of products on these neutrosophic super vectors of different types.

***Example 2.30*:** Let

$$A = [1\ 0\ I\ 2 \mid 0\ 0\ 1 \mid 2\ 3\ I\ 0\ 1]$$

be a neutrosophic super vector of type A of natural order 1×12 and matrix order 1×3.

$$\text{Clearly } A^t = \left[\begin{array}{c} 1 \\ 0 \\ I \\ 2 \\ \hline 0 \\ 0 \\ 1 \\ \hline 2 \\ 3 \\ I \\ 0 \\ 1 \end{array}\right]$$

is also a neutrosophic super vector of type A for which AA^t, the matrix minor product is well defined.

$$AA^t = [1\ 0\ I\ 2 \mid 0\ 0\ 1 \mid 2\ 3\ I\ 0\ 1] \left[\begin{array}{c} 1 \\ 0 \\ I \\ 2 \\ \hline 0 \\ 0 \\ 1 \\ \hline 2 \\ 3 \\ I \\ 0 \\ 1 \end{array}\right]$$

$$= [\ 1\ 0\ I\ 2] \begin{bmatrix} 1 \\ 0 \\ I \\ 2 \end{bmatrix} + [0\ 0\ 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + [2\ 3\ I\ 0\ 1] \begin{bmatrix} 2 \\ 3 \\ I \\ 0 \\ 1 \end{bmatrix}$$

$$= \{1\times1 + 0\times0 + I\times I + 2\times2\} + \{0\times0 + 0\times0 + 1\times1\} + \{2\times2 + 3\times3 + I\times I + 0\times0 + 1\times1\}$$

$$= \{1+0+I+4\} + \{0+0+1\} + \{4+9+I+0+1\}$$

$$= \{5+I\} + \{1\} + \{14 + I\}$$

$$= 20 + 2I.$$

Thus we have the following theorems the proof of which is left as an exercise for the reader.

THEOREM 2.1: *Let A = [A_1 A_2 … A_t] be a neutrosophic super row vector of natural order $1\times n$ and matrix order $1\times t$. Then AA^t the minor product of AA^t exists (well defined).*

THEOREM 2.2: *Let A be a neutrosophic super row vector of type A. If B denotes the matrix A without partition then $AA^t = BB^t$.*

Now for the same type A neutrosophic super vector A we define the major product.

DEFINITION 2.12: *Let*

$$A = \begin{bmatrix} A_1 \\ \vdots \\ A_t \end{bmatrix} \text{ and } B = [B_1 \ldots B_s]$$

be two neutrosophic super vectors of type A.

The major product of A with B is defined by

$$AB = \begin{bmatrix} A_1 \\ \vdots \\ A_t \end{bmatrix} [B_1 \ldots B_s]$$

$$= \begin{bmatrix} A_1B_1 & A_1B_2 & \cdots & A_1B_s \\ A_2B_1 & A_2B_2 & \cdots & A_2B_s \\ \vdots & \vdots & & \vdots \\ A_tB_s & A_tB_s & \cdots & A_tB_s \end{bmatrix}.$$

Here each A_i and B_j are matrices for $1 \leq i \leq t$ and $1 \leq j \leq s$.

We illustrate this situation by the following example.

***Example 2.31*:** Let

$$A = \left[\begin{array}{c} 3 \\ 2 \\ I \\ \hline 2 \\ 0 \\ \hline I \\ 4 \end{array}\right] \text{ and } B = [2\ 6\ 0 \mid 1\ I]$$

be two neutrosophic super vectors of type A.

$$AB = \left[\begin{array}{c} 3 \\ 2 \\ I \\ \hline 2 \\ 0 \\ \hline I \\ 4 \end{array}\right] [2\ 6\ 0 \mid 1\ I]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix} 3 \\ 2 \\ I \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \end{bmatrix} & \begin{bmatrix} 3 \\ 2 \\ I \end{bmatrix} \begin{bmatrix} 1 & I \end{bmatrix} \\ \hline \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & I \end{bmatrix} \\ \hline \begin{bmatrix} I \\ 4 \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \end{bmatrix} & \begin{bmatrix} I \\ 4 \end{bmatrix} \begin{bmatrix} 1 & I \end{bmatrix} \end{array}\right]$$

$$= \left[\begin{array}{ccc|cc} 6 & 18 & 0 & 3 & 3I \\ 4 & 12 & 0 & 2 & 2I \\ 2I & 6I & 0 & I & I \\ \hline 4 & 12 & 0 & 2 & 2I \\ 0 & 0 & 0 & 0 & 0 \\ \hline 2I & 6I & 0 & I & I \\ 8 & 24 & 0 & 4 & 4I \end{array}\right].$$

***Example 2.32*:** Let

$$A = \left[\begin{array}{c} 0 \\ 1 \\ I \\ \hline 2 \\ 3 \\ 4 \\ 1 \\ \hline I \\ 2 \\ 3 \end{array}\right] \text{ and}$$

B = [0 1 | I 2 0 | 3 4 0 | I 0] be any two neutrosophic super vectors of type A.

We obtain the major product of the super column vector A with the super row vector B of the two neutrosophic matrices.

$$AB = \left[\begin{array}{c} 0 \\ 1 \\ I \\ \hline 2 \\ 3 \\ 4 \\ 1 \\ \hline I \\ 2 \\ 3 \end{array}\right] [0\ 1 \mid I\ 2\ 0 \mid 3\ 4\ 0 \mid I\ 0]$$

$$= \left[\begin{array}{c|c|c|c}
\begin{bmatrix} 0 \\ 1 \\ I \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ I \end{bmatrix}\begin{bmatrix} I & 2 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ I \end{bmatrix}\begin{bmatrix} 3 & 4 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ I \end{bmatrix}\begin{bmatrix} I & 0 \end{bmatrix} \\ \hline
\begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} I & 2 & 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 4 & 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} I & 0 \end{bmatrix} \\ \hline
\begin{bmatrix} I \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} & \begin{bmatrix} I \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} I & 2 & 0 \end{bmatrix} & \begin{bmatrix} I \\ 2 \\ 0 \end{bmatrix}\begin{bmatrix} 3 & 4 & 0 \end{bmatrix} & \begin{bmatrix} I \\ 2 \\ 0 \end{bmatrix}\begin{bmatrix} I & 0 \end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{cc|ccc|ccc|cc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & I & 2 & 0 & 3 & 4 & 0 & I & 0 \\
0 & I & I & 2I & 0 & 3I & 4I & 0 & I & 0 \\
\hline
0 & 2 & 2I & 4 & 0 & 6 & 8 & 0 & 2I & 0 \\
0 & 3 & 3I & 6 & 0 & 9 & 12 & 0 & 3I & 0 \\
0 & 4 & 4I & 8 & 0 & 12 & 16 & 0 & 4I & 0 \\
0 & 1 & I & 2 & 0 & 3 & 4 & 0 & I & 0 \\
\hline
0 & I & I & 2I & 0 & 3I & 4I & 0 & I & 0 \\
0 & 2 & 2I & 4 & 0 & 6 & 8 & 0 & 2I & 0 \\
0 & 3 & 3I & 6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right].$$

Major product of two neutrosophic super vectors of type A is illustrated below by the following example.

***Example 2.33*:** Let

$$A = \left[\begin{array}{c} 2 \\ 3 \\ I \\ \hline 1 \\ 2 \\ 0 \\ \hline 0 \\ 1 \\ 1 \end{array}\right] \text{ and } B = [3\ 0\ 1 \mid 0\ I\ 1]$$

be any two neutrosophic super vectors of type A.

$$BA = \begin{bmatrix} [3\ \ 0\ \ 1]\begin{bmatrix}2\\3\\I\end{bmatrix} & [3\ \ 0\ \ 1]\begin{bmatrix}1\\2\\0\end{bmatrix} & [3\ \ 0\ \ 1]\begin{bmatrix}0\\1\\1\end{bmatrix} \\ [0\ \ I\ \ 1]\begin{bmatrix}2\\3\\I\end{bmatrix} & [0\ \ I\ \ 1]\begin{bmatrix}1\\2\\0\end{bmatrix} & [0\ \ I\ \ 1]\begin{bmatrix}0\\1\\1\end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 2+0\times 3+1\times I & 3\times 1+0\times 2+1\times 0 & 3\times 0+0\times 1+1\times 1 \\ 0\times 2+I\times 3+1\times I & 0\times 1+I\times 2+1\times 0 & 0\times 0+I\times 1+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6+0+I & 3+0+0 & 0+0+1 \\ 0+3I+I & 0+2I+0 & 0+I+1 \end{bmatrix}$$

$$= \begin{bmatrix} 6+I & 3 & 1 \\ 4I & 2I & 1+I \end{bmatrix}.$$

Now we proceed onto define the minor product of neutrosophic super vectors of type B.

We only illustrate this by numerical examples for the reader to understand the minor product without any confusion as it will involve lot of notations and symbols repelling non mathematicians away from using these matrices.

***Example 2.34*:**

Let $A = \left[\begin{array}{cc|c|ccc} 2 & 1 & 3 & 0 & 1 & I \\ 0 & 0 & 0 & 1 & 0 & 0 \\ I & 0 & I & 0 & 0 & 1 \end{array}\right]$ and $B = \left[\begin{array}{cc} 0 & 1 \\ I & 2 \\ \hline 1 & 0 \\ \hline 3 & 1 \\ 1 & 0 \\ 2 & 1 \end{array}\right]$ be any two

neutrosophic super vectors of type B.

The minor product of AB is defined as follows:

$$AB = \left[\begin{array}{cc|c|ccc} 2 & 1 & 3 & 0 & 1 & I \\ 0 & 0 & 0 & 1 & 0 & 0 \\ I & 0 & I & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cc} 0 & 1 \\ I & 2 \\ \hline 1 & 0 \\ \hline 3 & 1 \\ 1 & 0 \\ 2 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 2 & 1 \\ 0 & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ I & 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ I \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & I \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I & 4 \\ 0 & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 0 \\ I & 0 \end{bmatrix} + \begin{bmatrix} 1+2I & I \\ 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3I & 4+I \\ 3 & 1 \\ 2+I & 1+I \end{bmatrix}.$$

We from the numerical illustration observe the following facts.

1. The minor products of neutrosophic super vectors of type B is always a simple matrix. It may be a simple neutrosophic matrix or may not be a simple neutrosophic matrix.

2. If $A = [A_1 \ldots A_t]$ and $B = \begin{bmatrix} B_1 \\ \vdots \\ B_t \end{bmatrix}$ are neutrosophic supervectors of type B, the minor product AB is defined if and only if

(a) the number of submatrices in A and B are equal.

(b) The product A_iB_i is compatible, i.e., defined for each i, $1 \leq i \leq t$.

(c) The resultant simple matrix is of order m×n where m is the number of rows of A_i and n is the number of columns of B_i which is the same for each A_i and each B_i i.e., number of rows of A is the same as that of A_i and the number of columns of B is the same as that of each B_i.

We give yet another numerical illustration to make one understand the working.

Example 2.35:

$$\text{Let } A = \left[\begin{array}{cccc|cc|ccc} 0 & 0 & I & 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & I & 0 & 0 & 1 & 2 \\ I & 1 & 0 & 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \text{ and}$$

$$B = \left[\begin{array}{ccccc} 2 & 0 & 1 & I & 3 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 1 & 2 & 0 & 1 & 1 \\ \hline 0 & 1 & 2 & 3 & I \\ 1 & 2 & 0 & 0 & 1 \\ \hline 1 & 0 & I & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 1 \end{array}\right]$$

be two neutrosophic super vectors of type B.

We now find the minor product of A with B.

$$AB = \left[\begin{array}{cccc|cc|ccc} 0 & 0 & I & 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & I & 0 & 0 & 1 & 2 \\ I & 1 & 0 & 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \left[\begin{array}{ccccc} 2 & 0 & 1 & I & 3 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 1 & 2 & 0 & 1 & 1 \\ \hline 0 & 1 & 2 & 3 & I \\ 1 & 2 & 0 & 0 & 1 \\ \hline 1 & 0 & I & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 0 & 0 & I & 1 \\ 1 & 0 & 1 & 1 \\ I & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & I & 3 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 1 & 2 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ I & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & I \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & I & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & I & 1+I & 1 \\ 3 & 2 & 1+I & I+2 & 4 \\ 2I+1 & 1 & I & I & 3I \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 & 3 & I \\ 0 & I & 2I & 3I & I \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 4 & 0 & 2I & +1 & 21 \\ 8 & 2 & 1 & 2 & 3 \\ 5 & 1 & 2I & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 3I+3 & 2I6 & I+2 \\ 11 & 4+I & 3I+2 & 4I+4 & I+7 \\ 2I+7 & 4 & 3I & I+3 & 3I+2 \end{bmatrix}.$$

Thus we see if A is a neutrosophic super vector of type B. the transpose of A is denoted by A^t then AA^t is just neutrosophic simple matrix or a simple matrix.

We will illustrate this by an example.

***Example 2.36*:** Let

$$A = \left[\begin{array}{ccc|cc|cccc} 5 & 1 & 2 & 4 & 1 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 & 1 & I & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 & 0 & 0 & I & 0 & 0 \\ 2 & 0 & 0 & I & 0 & 1 & 0 & 1 & 0 \\ I & 1 & 5 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

be a neutrosophic super vector of type B. We now find the minor product of AA^t.

$$AA^t = \left[\begin{array}{ccc|cc|cccc} 5 & 1 & 2 & 4 & 1 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 & 1 & I & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 & 0 & 0 & I & 0 & 0 \\ 2 & 0 & 0 & I & 0 & 1 & 0 & 1 & 0 \\ I & 1 & 5 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \left[\begin{array}{ccccc} 5 & 0 & 1 & 2 & I \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 3 & 0 & 5 \\ \hline 4 & 1 & 2 & I & 0 \\ 1 & 1 & 0 & 0 & 1 \\ \hline 1 & I & 0 & 1 & 0 \\ 2 & 1 & I & 0 & 1 \\ 3 & 2 & 0 & 1 & 0 \\ 4 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 5 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \\ 2 & 0 & 0 \\ I & 1 & 5 \end{bmatrix} \begin{bmatrix} 5 & 0 & 1 & 2 & I \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 3 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & 1 \\ 2 & 0 \\ I & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & 2 & I & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$+\begin{bmatrix} 1 & 2 & 3 & 4 \\ I & 1 & 2 & 3 \\ 0 & I & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & I & 0 & 1 & 0 \\ 2 & 1 & I & 0 & 1 \\ 3 & 2 & 0 & 1 & 0 \\ 4 & 3 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 1 & 11 & 10 & 5I+11 \\ 1 & 1 & 0 & 0 & 1 \\ 11 & 0 & 10 & 2 & I+15 \\ 0 & 0 & 2 & 4 & 2I \\ 5I+11 & 1 & I+15 & 2I & I+26 \end{bmatrix} + \begin{bmatrix} 17 & 5 & 8 & 4I & 1 \\ 5 & 2 & 2 & I & 1 \\ 8 & 2 & 4 & 2I & 0 \\ 4I & I & 2I & I & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$+\begin{bmatrix} 30 & 20+I & 2I & 4 & 614+I \\ I+20 & 14+I & I & I+2 & 4 \\ 2I & I & I & 0 & I \\ 4 & I+2 & 0 & 2 & 0 \\ 6 & 4 & I & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 77 & 26+I & 19+2I & 14+4I & 5I+18 \\ I+26 & 17+I & 2+I & 2I+2 & 6 \\ 19+2I & 2+I & 14+I & 2+2I & 2I+15 \\ 14+4I & 2I+2 & 2+2I & 6+I & 2I \\ 5I+18 & 6 & 2I+15 & 2I & I+29 \end{bmatrix}.$$

We see AA^t is a symmetric simple neutrosophic matrix. It is always a square simple matrix. This simple matrix may be neutrosophic or may not be neutrosophic but it is always symmetric.

We give yet another numerical illustration.

Example 2.37:

Let A be a neutrosophic supervector of type B given by

$$A = \left[\begin{array}{cc|c|cccc|ccccc|ccc} 4 & 1 & 0 & 2 & 1 & 3 & I & 1 & 0 & 5 & 2 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 1 & 2 & 0 & 0 & 1 & 0 & I & 0 & 6 & 0 & 0 \\ 5 & 3 & I & 3 & 0 & 1 & 2 & 1 & 0 & 1 & 0 & 1 & I & 1 & 0 \\ 1 & I & 0 & 1 & 0 & 3 & 0 & 0 & I & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$AA^t = \left[\begin{array}{cc|c|cccc|ccccc|ccc} 4 & 1 & 0 & 2 & 1 & 3 & I & 1 & 0 & 5 & 2 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 1 & 2 & 0 & 0 & 1 & 0 & I & 0 & 6 & 0 & 0 \\ 5 & 3 & I & 3 & 0 & 1 & 2 & 1 & 0 & 1 & 0 & 1 & I & 1 & 0 \\ 1 & I & 0 & 1 & 0 & 3 & 0 & 0 & I & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 4 & 2 & 5 & 1 \\ 1 & 0 & 3 & I \\ \hline 0 & 1 & I & 0 \\ \hline 2 & 0 & 3 & 1 \\ 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 3 \\ I & 0 & 2 & 0 \\ \hline 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & I \\ 5 & 0 & 1 & 0 \\ 2 & I & 0 & 0 \\ 1 & 0 & 1 & 1 \\ \hline 0 & 6 & I & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 0 \\ 5 & 3 \\ 1 & I \end{bmatrix} \begin{bmatrix} 4 & 2 & 5 & 1 \\ 1 & 0 & 3 & I \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ I \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & I & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 1 & 3 & I \\ 0 & 1 & 2 & 0 \\ 3 & 0 & 1 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 & 1 \\ 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 3 \\ I & 0 & 2 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 5 & 2 & 1 \\ 0 & 1 & 0 & I & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & I & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & I \\ 5 & 0 & 1 & 0 \\ 2 & I & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 0 \\ I & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 6 & I & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 8 & 23 & 4+I \\ 8 & 4 & 10 & 2 \\ 23 & 10 & 34 & 5+3I \\ 4+I & 2 & 5+3I & 1+I \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & I & 0 \\ 0 & I & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 17 & 8 & 9+2I & 11 \\ 7 & 5 & 2 & 6 \\ 9+2I & 2 & 14 & 6 \\ 11 & 6 & 6 & 10 \end{bmatrix}$$

$$+ \begin{bmatrix} 31 & 2I & 7 & 1 \\ 2I & 1+I & 0 & I \\ 7 & 0 & 3 & 1 \\ 1 & I & 1 & 1+I \end{bmatrix} + \begin{bmatrix} 5 & 0 & 1 & 2 \\ 0 & 36 & 6I & 0 \\ 1 & 6I & 1+I & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 67+I & 15+2I & 40+2I & 18+I \\ 15+2I & 11+I & 12+7I & 8+I \\ 40+2I & 12+7I & 52+2I & 12+3I \\ 18+I & 8+I & 12+3I & 13+2I \end{bmatrix}.$$

AA^t is clearly a neutrosophic square symmetric matrix of order 4.

Now we proceed on to illustrate major product of neutrosophic super vectors of type B by an example.

Example 2.38**:** Let

$$A = \left[\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 2 & 1 \\ \hline I & 0 & 1 \\ \hline 0 & 1 & 2 \\ 6 & 0 & I \\ 1 & 0 & 2 \\ 3 & 1 & 0 \\ I & 0 & 0 \end{array}\right] \text{ and } B = \left[\begin{array}{cc|c|ccccc} 1 & 0 & 1 & 3 & 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 0 & 0 & 2 \\ 0 & I & 1 & 0 & 0 & 1 & 0 & I \end{array}\right]$$

be two neutrosophic super vectors of type B. To find

$$AB = \left[\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 2 & 1 \\ \hline I & 0 & 1 \\ \hline 0 & 1 & 2 \\ 6 & 0 & I \\ 1 & 0 & 2 \\ 3 & 1 & 0 \\ I & 0 & 0 \end{array}\right] \left[\begin{array}{cc|c|ccccc} 1 & 0 & 1 & 3 & 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 0 & 0 & 2 \\ 0 & I & 1 & 0 & 0 & 1 & 0 & I \end{array}\right]$$

$$= \left[\begin{array}{c|c|c}
\begin{bmatrix}3&1&0\\1&2&1\end{bmatrix}\begin{bmatrix}1&0\\2&1\\0&I\end{bmatrix} & \begin{bmatrix}3&1&0\\1&2&1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} & \begin{bmatrix}3&1&0\\1&2&1\end{bmatrix}\begin{bmatrix}3&0&1&2&1\\1&1&0&0&2\\0&0&1&0&I\end{bmatrix} \\ \hline
\begin{bmatrix}I&0&1\end{bmatrix}\begin{bmatrix}1&0\\2&1\\0&I\end{bmatrix} & \begin{bmatrix}I&0&1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} & \begin{bmatrix}I&0&1\end{bmatrix}\begin{bmatrix}3&0&1&2&1\\1&1&0&0&2\\0&0&1&0&I\end{bmatrix} \\ \hline
\begin{bmatrix}0&1&2\\6&0&I\\1&0&2\\3&1&0\\I&0&0\end{bmatrix}\begin{bmatrix}1&0\\2&1\\0&I\end{bmatrix} & \begin{bmatrix}0&1&2\\6&0&I\\1&0&2\\3&1&0\\I&0&0\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} & \begin{bmatrix}0&1&2\\6&0&I\\1&0&2\\3&1&0\\I&0&0\end{bmatrix}\begin{bmatrix}3&0&1&2&1\\1&1&0&0&2\\0&0&1&0&I\end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{cc|c|ccccc}
5 & 1 & 3 & 10 & 1 & 3 & 6 & 5 \\
5 & 2+I & 2 & 5 & 2 & 2 & 2 & 4+I \\ \hline
I & I & I+1 & 3I & 0 & I+1 & 2I & 2I \\ \hline
2 & 1+2I & 2 & 1 & 1 & 2 & 0 & 2+2I \\
6 & I & 6+I & 18 & 0 & 6+I & 12 & 6+I \\
1 & 2I & 3 & 3 & 0 & 3 & 2 & 1+2I \\
5 & 1 & 3 & 10 & 1 & 3 & 6 & 5 \\
I & 0 & I & 3I & 0 & I & 2I & I
\end{array}\right].$$

AB is clearly a neutrosophic super matrix. It may be a super matrix not necessarily neutrosophic.

Now we illustrate the major product of neutrosophic super vector of type B by an example.

***Example 2.39*:**

Consider the two neutrosophic super vectors.

$$A = \left[\begin{array}{cccc} 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & I & 0 & 0 \\ \hline I & 0 & 1 & 0 \\ 0 & 0 & 1 & I \\ \hline 1 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \text{ and } B = \left[\begin{array}{ccc|cc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & I \\ 0 & I & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 1 \end{array}\right].$$

The major product of the neutrosophic super vector A with the neutrosophic super vector B is described in the following:

$$AB = \left[\begin{array}{cccc} 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & I & 0 & 0 \\ \hline I & 0 & 1 & 0 \\ 0 & 0 & 1 & I \\ \hline 1 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|cc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & I \\ 0 & I & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 1 \end{array}\right]$$

$$
= \left[\begin{array}{c|c|c}
\begin{bmatrix}2&1&0&1\\0&1&1&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&I&0\\0&0&1\\0&0&0\end{bmatrix} &
\begin{bmatrix}2&1&0&1\\0&1&1&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&0\\0&I\end{bmatrix} &
\begin{bmatrix}2&1&0&1\\0&1&1&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}0&1&0&I\\I&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}I&0&1&0\\0&0&1&I\end{bmatrix}\begin{bmatrix}1&0&0\\0&I&0\\0&0&1\\0&0&0\end{bmatrix} &
\begin{bmatrix}I&0&1&0\\0&0&1&I\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&0\\0&I\end{bmatrix} &
\begin{bmatrix}I&0&1&0\\0&0&1&I\end{bmatrix}\begin{bmatrix}0&1&0&I\\I&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}1&I&0&0\\0&0&0&I\\1&0&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&I&0\\0&0&1\\0&0&0\end{bmatrix} &
\begin{bmatrix}1&I&0&0\\0&0&0&I\\1&0&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&0\\0&I\end{bmatrix} &
\begin{bmatrix}1&I&0&0\\0&0&0&I\\1&0&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1&0&I\\I&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}
\end{array}\right]
$$

$$
= \left[\begin{array}{ccc|cc|cccc}
2 & I & 0 & 2 & I & I & 2 & 0 & 2I+1 \\
0 & I & 1 & 0 & 0 & I & 0 & 1 & 0 \\
0 & I & 0 & 0 & 0 & I & 0 & 0 & 0 \\ \hline
I & 0 & 1 & I & 0 & 0 & I & 1 & I \\
0 & 0 & 1 & 0 & I & 0 & 0 & 1 & I \\ \hline
1 & I & 0 & 1 & 0 & I & 1 & 0 & I \\
0 & 0 & 0 & 0 & I & 0 & 0 & 0 & I \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & I \\
0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 1
\end{array}\right].
$$

We see AB is also a neutrosophic super matrix.

Now we illustrate by examples the major product of a neutrosophic super vector of type B with its transpose.

Example 2.40: Let

$$A = \left[\begin{array}{ccc} 3 & 0 & 1 \\ 0 & I & 0 \\ 1 & 0 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 0 & I \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ I & 0 & 1 \\ 0 & I & 0 \end{array}\right]$$

be a neutrosophic super vector of type B.

Now the transpose of A denoted by

$$A^t = \left[\begin{array}{ccc|cc|ccccc} 3 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 1 & 0 & I \\ 1 & 0 & 0 & 0 & I & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$AA^t = \left[\begin{array}{ccc} 3 & 0 & 1 \\ 0 & I & 0 \\ 1 & 0 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 0 & I \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ I & 0 & 1 \\ 0 & I & 0 \end{array}\right] \left[\begin{array}{ccc|cc|ccccc} 3 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 1 & 0 & I \\ 1 & 0 & 0 & 0 & I & 1 & 0 & 0 & 1 & 0 \end{array}\right] =$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix}\begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix} & \begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&I\end{bmatrix} & \begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&0&1&0&I\\1&0&0&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}1&0&0\\0&0&I\end{bmatrix}\begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix} & \begin{bmatrix}1&0&0\\0&0&I\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&I\end{bmatrix} & \begin{bmatrix}1&0&0\\0&0&I\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&0&1&0&I\\1&0&0&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\I&0&1\\0&I&0\end{bmatrix}\begin{bmatrix}3&0&1\\0&I&0\\1&0&0\end{bmatrix} & \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\I&0&1\\0&I&0\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&I\end{bmatrix} & \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\I&0&1\\0&I&0\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&0&1&0&I\\1&0&0&1&0\end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{ccc|cc|ccccc}
10&0&3&3&I&1&3&0&3I+1&0\\
0&I&0&0&0&0&0&1&0&I\\
3&0&1&1&0&0&0&0&I&0\\ \hline
3&0&1&1&0&0&1&0&I&0\\
I&0&0&0&I&I&0&0&I&0\\ \hline
1&0&0&0&I&1&0&0&1&0\\
3&0&1&1&0&0&1&0&I&0\\
0&I&0&0&0&0&0&1&0&I\\
3I+1&0&I&I&I&1&I&0&I+1&0\\
0&I&0&0&0&0&0&I&0&I
\end{array}\right].$$

We see in the first place AA^T is also a neutrosophic super matrix. Further AA^t happens to be a symmetric neutrosophic super matrix.

Thus as in case of usual matrices we get in case of neutrosophic row super vector A of type B, AA^T happens to be a symmetric neutrosophic super matrix.

We illustrate this yet by an example as we are more interested in the reader to apply and use it, that is why we are avoiding the notational difficult way of expressing it as a definition.

***Example 2.41*:** Let

$$A = \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

be a neutrosophic super vector of type B.

$$A^t = \left[\begin{array}{cccc} 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 \\ 1 & 0 & I & 1 \\ 0 & 1 & 0 & 0 \end{array}\right].$$

Now we find

$$A^tA = \left[\begin{array}{cccc} 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 \\ 1 & 0 & I & 1 \\ 0 & 1 & 0 & 0 \end{array}\right] \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix} 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ I & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & I \\ 1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & I & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & I & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ I & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & I & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & I & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}\right]$$

$$= \left[\begin{array}{ccccc|ccc} I & 0 & 0 & 0 & 0 & I & I & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & I & I & 0 & I \\ 0 & 1 & 0 & 2 & 1 & 0 & 1+I & 0 \\ 0 & 0 & I & 1 & 2 & 1 & I & 1 \\ \hline I & 0 & I & 0 & 1 & 2 & 1 & 1 \\ I & 1 & 0 & 1+I & I & 1 & 2+I & 0 \\ 0 & 0 & I & 0 & 1 & 1 & 0 & 1 \end{array}\right].$$

We see clearly the neutrosophic super matrix is symmetric.

We get yet another example.

***Example 2.42*:**

$$\text{Let } B = \left[\begin{array}{ccccc} 3 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & I \\ 1 & 0 & 0 & 0 & 1 \\ \hline 0 & I & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

be a neutrosophic super vector of type B.

$$B^t = \left[\begin{array}{ccc|c|cc|cc} 3 & 0 & 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & I & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & I & 1 & 0 & I & 0 & 0 & 0 \end{array}\right]$$

Consider

$$BB^t = \left[\begin{array}{ccccc} 3 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & I \\ 1 & 0 & 0 & 0 & 1 \\ \hline 0 & I & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right] \left[\begin{array}{ccc|c|cc|cc} 3 & 0 & 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & I & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & I & 1 & 0 & I & 0 & 0 & 0 \end{array}\right]$$

$$
\left[\begin{array}{c|c}
\begin{bmatrix}3&0&1&1&0\\0&0&0&1&I\\1&0&0&0&1\end{bmatrix}\begin{bmatrix}3&0&1\\0&0&0\\1&0&0\\1&1&0\\0&I&1\end{bmatrix} & \begin{bmatrix}3&0&1&1&0\\0&0&0&1&I\\1&0&0&0&1\end{bmatrix}\begin{bmatrix}0\\I\\0\\0\\0\end{bmatrix} \\ \hline
\begin{bmatrix}0&I&0&0&0\end{bmatrix}\begin{bmatrix}3&0&1\\0&0&0\\1&0&0\\1&1&0\\0&I&1\end{bmatrix} & \begin{bmatrix}0&I&0&0&0\end{bmatrix}\begin{bmatrix}0\\I\\0\\0\\0\end{bmatrix} \\ \hline
\begin{bmatrix}2&0&0&0&I\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}3&0&1\\0&0&0\\1&0&0\\1&1&0\\0&I&1\end{bmatrix} & \begin{bmatrix}2&0&0&0&I\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}0\\I\\0\\0\\0\end{bmatrix} \\ \hline
\begin{bmatrix}0&1&1&0&0\\1&0&0&1&0\end{bmatrix}\begin{bmatrix}3&0&1\\0&0&0\\1&0&0\\1&1&0\\0&I&1\end{bmatrix} & \begin{bmatrix}0&1&1&0&0\\1&0&0&1&0\end{bmatrix}\begin{bmatrix}0\\I\\0\\0\\0\end{bmatrix}
\end{array}\right]
$$

$$\left[\begin{array}{c|c}
\begin{bmatrix}3&0&1&1&0\\0&0&0&1&I\\1&0&0&0&1\end{bmatrix}\begin{bmatrix}2&0\\0&0\\0&1\\0&0\\I&0\end{bmatrix} & \begin{bmatrix}3&0&1&1&0\\0&0&0&1&I\\1&0&0&0&1\end{bmatrix}\begin{bmatrix}0&1\\1&0\\1&0\\0&1\\0&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&I&0&0&0\end{bmatrix}\begin{bmatrix}2&0\\0&0\\0&1\\0&0\\I&0\end{bmatrix} & \begin{bmatrix}0&I&0&0&0\end{bmatrix}\begin{bmatrix}0&1\\1&0\\1&0\\0&1\\0&0\end{bmatrix} \\ \hline
\begin{bmatrix}2&0&0&0&I\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}2&0\\0&0\\0&1\\0&0\\I&0\end{bmatrix} & \begin{bmatrix}2&0&0&0&I\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}0&1\\1&0\\1&0\\0&1\\0&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&1&1&0&0\\1&0&0&1&0\end{bmatrix}\begin{bmatrix}2&0\\0&0\\0&1\\0&0\\I&0\end{bmatrix} & \begin{bmatrix}0&1&1&0&0\\1&0&0&1&0\end{bmatrix}\begin{bmatrix}0&1\\1&0\\1&0\\0&1\\0&0\end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{ccc|c|cc|cc}
11 & 1 & 3 & 0 & 6 & 1 & 1 & 4 \\
1 & 1+I & I & 0 & I & 0 & 0 & 1 \\
3 & I & 2 & 0 & 2+I & 0 & 0 & 1 \\ \hline
0 & 0 & 0 & I & 0 & 0 & I & 0 \\ \hline
6 & I & 2+I & 0 & 4+I & 0 & 0 & 2 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline
1 & 0 & 0 & I & 0 & 1 & 2 & 0 \\
4 & 1 & 1 & 0 & 2 & 0 & 0 & 2
\end{array}\right].$$

It is easily verified BB^T is again a neutrosophic super matrix which is symmetric.

Example 2.43:

$$\text{Let } A = \left[\begin{array}{cc} 2 & 0 \\ 0 & I \\ \hline 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{array}\right] \text{ and } B = \left[\begin{array}{cc|ccccc} 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

be a neutrosophic super vectors of type B.

$$\text{Now } AB = \left[\begin{array}{cc} 2 & 0 \\ 0 & I \\ \hline 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{array}\right] \left[\begin{array}{cc|ccccc} 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix} 2 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}\right]$$

$$= \left[\begin{array}{cc|ccccc} 0 & 2 & 2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

We see AB is a super matrix. Clearly AB is not a neutrosophic super matrix.

Thus it is pertinent to mention here that the major product of two neutrosophic super vectors of type B need not in general be a neutrosophic super matrix; it can only be a super matrix.

This is seen from example.

Now we proceed onto show by a numerical example how the minor product of two super neutrosophic matrices which has a compatible multiplication is carried out.

***Example 2.44*:**

Let A and B be any two super neutrosophic matrices, where

$$A = \left[\begin{array}{c|cc|ccc} 1 & 0 & I & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & I \\ \hline I & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ \hline 3 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \quad B = \left[\begin{array}{cccc|cc|ccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & I & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & I & I & 0 & I & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$AB = \left[\begin{array}{c|cc|ccc} 1 & 0 & I & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & I \\ \hline I & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ \hline 3 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \left[\begin{array}{cccc|cc|ccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & I & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & I & I & 0 & I & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c} 1 \\ 2 \\ \hline I \\ 0 \\ 0 \\ 0 \\ \hline 3 \end{array}\right] [1\ 0\ 0\ 1 \mid 0\ 0 \mid 0\ I\ 0] +$$

$$\left[\begin{array}{cc} 0 & I \\ 1 & 0 \\ \hline 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{cccc|cc|ccc} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & I & I & 0 & I & 0 & 0 \end{array}\right]$$

$$+\left[\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & I \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & I & 0 \\ \hline 1 & 0 & 1 \end{array}\right]\left[\begin{array}{cccc|cc|ccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$=\left[\begin{array}{c|c|c} \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} & \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 0 & I & 0 \end{pmatrix} \\ \hline \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{pmatrix} 0 & I & 0 \end{pmatrix} \\ \hline (3)\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} & (3)\begin{pmatrix} 0 & 0 \end{pmatrix} & (3)\begin{pmatrix} 0 & I & 0 \end{pmatrix} \end{array}\right]$$

$$+\left[\begin{array}{c|c|c} \begin{bmatrix} 0 & I \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & I \end{bmatrix} & \begin{bmatrix} 0 & I \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ I & 0 \end{bmatrix} & \begin{bmatrix} 0 & I \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ I & 0 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & I \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ I & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ I & 0 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & I \end{bmatrix} & \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ I & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ I & 0 & 0 \end{bmatrix} \end{array}\right]$$

$$
+\left[\begin{array}{c|c|c}
\begin{bmatrix}0&1&0\\1&0&I\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&0\end{bmatrix} & \begin{bmatrix}0&1&0\\1&0&I\end{bmatrix}\begin{bmatrix}0&1\\0&0\\1&0\end{bmatrix} & \begin{bmatrix}0&1&0\\1&0&I\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\0&I&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&0\end{bmatrix} & \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\0&I&0\end{bmatrix}\begin{bmatrix}0&1\\0&0\\1&0\end{bmatrix} & \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\\0&I&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}1&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&0\end{bmatrix} & \begin{bmatrix}1&0&1\end{bmatrix}\begin{bmatrix}0&1\\0&0\\1&0\end{bmatrix} & \begin{bmatrix}1&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}
\end{array}\right]
$$

$$
=\left[\begin{array}{cccc|cc|ccc}
1&0&0&1&0&0&0&I&0\\
2&0&0&2&0&0&0&2I&0\\ \hline
I&0&0&I&0&0&0&I&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\ \hline
3&0&0&3&0&0&0&3I&0
\end{array}\right]+\left[\begin{array}{cccc|cc|ccc}
I&0&0&I&I&0&I&0&0\\
0&1&0&0&0&1&0&1&0\\ \hline
0&1&0&0&0&1&0&1&0\\
1&1&0&I&I&1&I&1&0\\
1&0&0&I&I&0&I&0&0\\
0&0&0&0&0&0&0&0&0\\ \hline
1&0&0&I&I&0&I&0&0
\end{array}\right]
$$

$$
+\left[\begin{array}{cccc|cc|ccc}
0&1&0&1&0&0&0&1&0\\
1&0&I&0&I&1&1+I&0&I\\ \hline
0&0&1&0&1&0&1&0&1\\
1&0&0&0&0&1&1&0&0\\
0&1&0&1&0&0&0&1&0\\
0&I&0&I&0&0&0&I&0\\ \hline
1&0&1&0&1&1&2&0&1
\end{array}\right]
$$

$$= \left[\begin{array}{cccc|cc|ccc} 1+I & 1 & 0 & 2+I & I & 0 & I & 1+I & 0 \\ 3 & 1 & I & 2 & I & 2 & 1+I & 2I+1 & I \\ \hline I & 1 & 1 & I & 1 & 1 & 1 & 1+I & 1 \\ 2 & 1 & 0 & I & I & 2 & 1+I & 1 & 0 \\ 1 & 1 & 0 & 1+I & I & 0 & I & 1 & 0 \\ 0 & I & 0 & I & 0 & 0 & 0 & I & 0 \\ \hline 5 & 0 & 1 & 3+I & 1+I & 1 & 2+I & 3I & 1 \end{array}\right].$$

Thus the minor product is again a super neutrosophic matrix.

We give yet another illustration.

Example 2.45:

Let A and B be two neutrosophic super matrices where

$$A = \left[\begin{array}{c|cc|cccc} 1 & 0 & 1 & 1 & 2 & 0 & 0 \\ 0 & I & 0 & 0 & 1 & 0 & I \\ \hline I & 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 & 0 & 1 \\ 1 & I & 0 & 1 & I & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & I & 0 \\ \hline 1 & 1 & 2 & 0 & 1 & I & 1 \end{array}\right] \text{ and}$$

$$B = \left[\begin{array}{cc|cccc|c} 0 & 1 & 1 & I & 0 & 0 & I \\ \hline 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ I & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & I & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & I & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$AB = \left[\begin{array}{c|cc|cccc} 1 & 0 & 1 & 1 & 2 & 0 & 0 \\ 0 & I & 0 & 0 & 1 & 0 & I \\ \hline I & 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & I & 0 & 1 & 0 & 1 \\ 1 & I & 0 & 1 & I & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & I & 0 \\ \hline 1 & 1 & 2 & 0 & 1 & I & 1 \end{array}\right] \left[\begin{array}{cc|cccc|c} 0 & 1 & 1 & I & 0 & 0 & I \\ \hline 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ I & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & I & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & I & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c} 1 \\ 0 \\ \hline I \\ 0 \\ 1 \\ 0 \\ \hline 1 \end{array}\right] \left[\begin{array}{cc|cccc|c} 0 & 1 & 1 & I & 0 & 0 & I \end{array}\right] + \left[\begin{array}{cc} 0 & 1 \\ I & 0 \\ \hline 1 & 2 \\ 0 & I \\ I & 0 \\ 0 & 1 \\ \hline 1 & 2 \end{array}\right] \left[\begin{array}{cc|cccc|c} 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ I & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$+ \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & I \\ \hline 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ \hline 0 & 1 & I & 1 \end{array}\right] \left[\begin{array}{cc|cccc|c} 1 & 0 & I & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & I & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Here the usual product of each submatrices are performed and we see in this minor product of neutrosophic super matrices each cell is compatible with usual product of submatrices.

$$= \left[\begin{array}{c|c|c}
\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & I & 0 & 0\end{bmatrix} & \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\end{bmatrix} \\ \hline
\begin{bmatrix}I\\0\\1\\0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}I\\0\\1\\0\end{bmatrix}\begin{bmatrix}1 & I & 0 & 0\end{bmatrix} & \begin{bmatrix}I\\0\\1\\0\end{bmatrix}\begin{bmatrix}1\end{bmatrix} \\ \hline
\begin{bmatrix}1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}1\end{bmatrix}\begin{bmatrix}1 & I & 0 & 0\end{bmatrix} & \begin{bmatrix}1\end{bmatrix}
\end{array}\right] +$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix}0 & 1\\I & 0\end{bmatrix}\begin{bmatrix}0 & 1\\I & 0\end{bmatrix} & \begin{bmatrix}0 & 1\\I & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix} & \begin{bmatrix}0 & 1\\I & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \\ \hline
\begin{bmatrix}1 & 2\\0 & I\\I & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\I & 0\end{bmatrix} & \begin{bmatrix}1 & 2\\0 & I\\I & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix} & \begin{bmatrix}1 & 2\\0 & I\\I & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \\ \hline
\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}0 & 1\\I & 0\end{bmatrix} & \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix} & \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}
\end{array}\right] +$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix}1&2&0&0\\0&1&0&I\end{bmatrix}\begin{bmatrix}1&0\\0&1\\1&1\\0&0\end{bmatrix} & \begin{bmatrix}1&2&0&0\\0&1&0&I\end{bmatrix}\begin{bmatrix}I&1&0&0\\0&0&1&0\\0&0&I&0\\0&0&0&1\end{bmatrix} & \begin{bmatrix}1&2&0&0\\0&1&0&I\end{bmatrix}\begin{bmatrix}1\\0\\I\\1\end{bmatrix} \\ \hline
\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}1&0\\0&1\\1&1\\0&0\end{bmatrix} & \begin{bmatrix}1&0&1&0\\0&1&0&1\\1&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}I&1&0&0\\0&0&1&0\\0&0&I&0\\0&0&0&1\end{bmatrix} & \begin{bmatrix}1&0&1&0\\0&1&0&1\\1&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}1\\0\\I\\1\end{bmatrix} \\ \hline
\begin{bmatrix}0&1&I&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\\1&1\\0&0\end{bmatrix} & \begin{bmatrix}0&1&I&1\end{bmatrix}\begin{bmatrix}I&1&0&0\\0&0&1&0\\0&0&I&0\\0&0&0&1\end{bmatrix} & \begin{bmatrix}0&1&I&1\end{bmatrix}\begin{bmatrix}1\\0\\I\\1\end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{cc|cccc|c}
0&1&1&I&0&0&I\\
0&0&0&0&0&0&0\\ \hline
0&I&I&I&0&0&I\\
0&0&0&0&0&0&0\\
0&1&1&I&0&0&I\\
0&0&0&0&0&0&0\\ \hline
0&1&1&I&0&0&I
\end{array}\right] + \left[\begin{array}{cc|cccc|c}
I&0&0&1&0&0&0\\
0&I&I&0&I&0&I\\ \hline
2I&1&1&2&1&0&1\\
I&0&0&I&0&0&0\\
0&I&I&0&I&0&I\\
I&0&0&1&0&0&0\\ \hline
2I&1&1&2&1&0&1
\end{array}\right]$$

$$+ \left[\begin{array}{cc|cccc|c}
1&2&I&1&2&0&1\\
0&1&0&0&1&I&I\\ \hline
2&1&I&1&I&0&1+I\\
0&1&0&0&1&1&1\\
1&I&I&1&I&0&1\\
I&I&0&0&I&0&I\\ \hline
I&1+I&0&0&1+I&1&1+I
\end{array}\right]$$

$$= \left[\begin{array}{cc|cccc|c} 1+I & 3 & 1+I & 2+I & 2 & 0 & 1+I \\ 0 & 1+I & I & 0 & 1+I & I & 2I \\ \hline 2+2I & I+2 & 2I+1 & I+3 & 1+I & 0 & 2+2I \\ I & 1 & 0 & I & 1 & 1 & 1 \\ 1 & 1+2I & 1+2I & I+1 & 2I & 0 & 2I+1 \\ 2I & I & 0 & 1 & I & 0 & I \\ \hline 3I & 3+I & 2 & 2+I & 2+I & 1 & 2+2I \end{array}\right].$$

We see AB is again a neutrosophic super matrix under the minor product of neutrosophic matrices.

Now we show the minor product of a neutrosophic super matrix with its special transpose can be defined.

Let us first illustrate by a few examples the special transpose of a super neutrosophic matrix.

***Example 2.46*:**

$$\text{Let } A = \left[\begin{array}{cc|ccccc} 2 & 1 & 3 & 1 & 2 & I & 0 \\ 1 & I & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & I & 1 & 1 & 0 & 2 \\ \hline 3 & I & 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ I & 1 & 1 & 0 & I & 0 & 0 \end{array}\right]$$

be a neutrosophic super matrix.

Now the special transpose of A denoted by A^t is given by a neutrosophic matrix.

$$A^t = \left[\begin{array}{ccc|c|cc} 2 & 1 & 0 & 3 & 0 & I \\ 1 & I & 1 & I & 1 & 1 \\ \hline 3 & 0 & I & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 & I \\ I & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \end{array}\right].$$

Example 2.47:

$$\text{Let } A = \left[\begin{array}{ccccc|cc|cccc} 2 & 1 & 0 & 0 & 0 & 3 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 5 & 2 & I & I & 1 & 0 & 1 & 2 & I \\ 7 & 8 & 9 & 0 & 7 & 1 & 2 & I & 0 & 1 & 2 \\ 0 & I & 0 & 1 & 0 & I & 6 & 0 & 1 & 2 & 0 \\ \hline 1 & 0 & I & 0 & 1 & 2 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & I & I & 1 & 2 & 7 \\ \hline 1 & 0 & 0 & 1 & 1 & 0 & 2 & 1 & 0 & 2 & 3 \\ 0 & I & 1 & 0 & I & 1 & I & 0 & I & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 3 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

be a super neutrosophic matrix.

To find the transpose of the neutrosophic super matrix A we take the transpose of each of the submatrices in A.

$$A^t = \left[\begin{array}{cccc|cc|ccc} 2 & 0 & 7 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 8 & I & 0 & 0 & 0 & I & 0 \\ 0 & 5 & 9 & 0 & I & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & I & 7 & 0 & 1 & 0 & 1 & I & 2 \\ \hline 3 & I & 1 & I & 2 & 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 6 & 1 & I & 2 & I & 1 \\ \hline 3 & 0 & I & 0 & 1 & I & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 2 & 1 & 0 & I & 1 \\ 1 & 2 & 1 & 2 & 3 & 2 & 2 & 1 & 0 \\ 0 & I & 2 & 0 & 0 & 7 & 3 & 2 & 1 \end{array}\right];$$

clearly A^t is also a neutrosophic super matrix.

A^t will be known as the special transpose of A.

Clearly if A is a neutrosophic row super vector of type B then the special transpose of A, A^t is a neutrosophic column super vector of type B.

Similarly if A is a neutrosophic column super vector of type B the A^t the special transpose of A is a neutrosophic row super vector of type B.

We illustrate this by a few examples.

Example 2.48:

Let A = [2 1 0 | 1 1 1 I 0 | 0 3 I 0] be a neutrosophic super row vector then

$$A^t = \left[\begin{array}{c} 2 \\ 1 \\ 0 \\ \hline 1 \\ 1 \\ 1 \\ I \\ 0 \\ \hline 0 \\ 3 \\ I \\ 0 \end{array}\right], \quad A^t \text{ is a neutrosophic super column vector.}$$

Example 2.49:

$$\text{Let } A = \left[\begin{array}{c} 3 \\ 6I \\ 1 \\ 2 \\ 0 \\ \hline 1 \\ 1 \\ I \\ 0 \\ \hline 2 \\ I \end{array}\right] \text{ a neutrosophic super column vector.}$$

Then the special transpose of A; $A^t = [3\ 6\ I\ 1\ 2\ 0 \mid 1\ 1\ I\ 0 \mid 2\ I\ ]$. Clearly A^t is a neutrosophic super row vector.

***Example 2.50*:**

$$\text{Let } A = \left[\begin{array}{cc|ccc|cccc} 3 & 1 & 0 & 1 & I & 3 & 1 & 2 & I \\ 1 & 2 & 1 & 0 & 4 & 0 & 1 & 2 & 3I \\ I & 0 & I & 2 & 3 & 4I & 2 & 0 & 1 \\ 0 & 1 & 4 & 3I & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

be a neutrosophic super row vector of type B.

Then A^t the special transpose of A.

That is if $A = [A_1\ A_2\ A_3]$ we see $A^t = \left[A_1^t \quad A_2^t \quad A_3^t\right]$ and the row vectors are arranged as column vectors so that the number of columns of A is equal to the number of rows of A^t.

$$A^t = \left[\begin{array}{cccc} 3 & 1 & I & 0 \\ 1 & 2 & 0 & 1 \\ \hline 0 & 1 & I & 4 \\ 1 & 0 & 2 & 3I \\ I & 4 & 3 & 0 \\ \hline 3 & 0 & 4I & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 2 & 0 & 0 \\ I & 3I & 1 & 1 \end{array}\right]$$

is neutrosophic super column vector of type B.

***Example 2.51*:** Let

$$A = \left[\begin{array}{ccccc} 0 & I & 1 & 2 & 0 \\ \hline 1 & 0 & 0 & I & 0 \\ 1 & 0 & 1 & 0 & I \\ \hline 2 & 3 & 4 & 5 & 0 \\ 0 & 1 & 0 & I & 2 \\ -1 & 0 & 0 & 1 & 5I \\ \hline -7I & 0 & 2 & 1 & -3 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ I & I & I & I & 3 \\ -2 & 0 & 4 & 1 & 0 \\ 5 & 1 & 2 & 3 & I \end{array}\right]$$

be a neutrosophic super column vector of type B. The special transpose of A is

$$A^t = \left[\begin{array}{c|cc|ccc|cccccc} 0 & 1 & 1 & 2 & 0 & -1 & -7I & 0 & 1 & I & -2 & 5 \\ I & 0 & 0 & 3 & 1 & 0 & 0 & 1 & 1 & I & 0 & 1 \\ 1 & 0 & 1 & 4 & 0 & 0 & 2 & 0 & 1 & I & 4 & 2 \\ 2 & I & 0 & 5 & I & 1 & 1 & 1 & 1 & I & 1 & 3 \\ 0 & 0 & I & 0 & 2 & 5I & -3 & 1 & 1 & 3 & 0 & I \end{array}\right].$$

We see A^t is a neutrosophic super row vector of type B.

We now illustrate the minor product of the neutrosophic super matrices A with its transpose.

***Example 2.52*:**

$$\text{Let } A = \left[\begin{array}{ccc|cc|ccccc} 3 & 0 & 1 & 1 & 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & I & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & I & 0 & 0 & I & 1 & 0 & 1 \\ 4 & 1 & 0 & 2 & 1 & I & 0 & 0 & 1 & 2 \\ \hline 0 & 1 & 0 & 3 & 0 & 0 & 0 & I & 0 & 1 \\ \hline 0 & 1 & 1 & 1 & 2 & 1 & 0 & 2 & 0 & 1 \\ I & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 & I & 0 & 0 & I & 0 \end{array}\right]$$

be a neutrosophic super matrix.

To find the minor product of A with A^t.

$$A^t = \left[\begin{array}{cccc|c|ccc} 3 & 0 & 1 & 4 & 0 & 0 & I & 1 \\ 0 & I & 0 & 1 & 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & I & 2 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 2 & 1 & 0 \\ \hline 0 & 1 & 0 & I & 0 & 1 & 0 & I \\ 1 & 0 & I & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & I & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & I \\ 1 & 0 & 1 & 2 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$AA^t = \left[\begin{array}{ccc|cc|ccccc}
3 & 0 & 1 & 1 & 0 & 0 & 1 & 2 & 0 & 1 \\
0 & I & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & I & 0 & 0 & I & 1 & 0 & 1 \\
4 & 1 & 0 & 2 & 1 & I & 0 & 0 & 1 & 2 \\
\hline
0 & 1 & 0 & 3 & 0 & 0 & 0 & I & 0 & 1 \\
\hline
0 & 1 & 1 & 1 & 2 & 1 & 0 & 2 & 0 & 1 \\
I & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 2 & 0 & 1 & 0 & I & 0 & 0 & I & 0
\end{array}\right]$$

$$\left[\begin{array}{cccc|c|ccc}
3 & 0 & 1 & 4 & 0 & 0 & I & 1 \\
0 & I & 0 & 1 & 1 & 1 & 0 & 2 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\hline
1 & 0 & I & 2 & 3 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 2 & 1 & 0 \\
\hline
0 & 1 & 0 & I & 0 & 1 & 0 & I \\
1 & 0 & I & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 1 & 0 & I & 2 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & I \\
1 & 0 & 1 & 2 & 1 & 1 & 0 & 0
\end{array}\right] =$$

$$\left[\begin{array}{ccc}
3 & 0 & 1 \\
0 & I & 0 \\
1 & 0 & 1 \\
4 & 1 & 0 \\
\hline
0 & 1 & 0 \\
\hline
0 & 1 & 1 \\
I & 0 & 0 \\
1 & 2 & 0
\end{array}\right]
\left[\begin{array}{cccc|c|ccc}
3 & 0 & 1 & 4 & 0 & 0 & I & 1 \\
0 & I & 0 & 1 & 1 & 1 & 0 & 2 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0
\end{array}\right] +
\left[\begin{array}{cc}
1 & 0 \\
0 & 1 \\
I & 0 \\
2 & 1 \\
\hline
3 & 0 \\
\hline
1 & 2 \\
0 & 1 \\
1 & 0
\end{array}\right]
\left[\begin{array}{cccc|c|ccc}
1 & 0 & I & 2 & 3 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 2 & 1 & 0
\end{array}\right]$$

$$
+\left[\begin{array}{ccccc}
0 & 1 & 2 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 \\
0 & I & 1 & 0 & 1 \\
I & 0 & 0 & 1 & 2 \\
\hline
0 & 0 & I & 0 & 1 \\
\hline
1 & 0 & 2 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 \\
I & 0 & 0 & I & 0
\end{array}\right]
\left[\begin{array}{cccc|c|ccc}
0 & 1 & 0 & I & 0 & 1 & 0 & I \\
1 & 0 & I & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 1 & 0 & I & 2 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & I \\
1 & 0 & 1 & 2 & 1 & 1 & 0 & 0
\end{array}\right]
$$

$$
=\left[\begin{array}{c|c|c}
\begin{bmatrix} 3 & 0 & 1 \\ 0 & I & 0 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 4 \\ 0 & I & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} &
\begin{bmatrix} 3 & 0 & 1 \\ 0 & I & 0 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} &
\begin{bmatrix} 3 & 0 & 1 \\ 0 & I & 0 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & I & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} \\
\hline
\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 4 \\ 0 & I & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} &
\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} &
\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & I & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} \\
\hline
\begin{bmatrix} 0 & 1 & 1 \\ I & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 4 \\ 0 & I & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} &
\begin{bmatrix} 0 & 1 & 1 \\ I & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} &
\begin{bmatrix} 0 & 1 & 1 \\ I & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & I & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}
\end{array}\right]
$$

$$+\left[\begin{array}{c|c|c}
\begin{bmatrix}1&0\\0&1\\I&0\\2&1\end{bmatrix}\begin{bmatrix}1&0&I&2\\0&1&0&1\end{bmatrix} & \begin{bmatrix}1&0\\0&1\\I&0\\2&1\end{bmatrix}\begin{bmatrix}3\\0\end{bmatrix} & \begin{bmatrix}1&0\\0&1\\I&0\\2&1\end{bmatrix}\begin{bmatrix}1&0&1\\2&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}3&0\end{bmatrix}\begin{bmatrix}1&0&I&2\\0&1&0&1\end{bmatrix} & \begin{bmatrix}3&0\end{bmatrix}\begin{bmatrix}3\\0\end{bmatrix} & \begin{bmatrix}3&0\end{bmatrix}\begin{bmatrix}1&0&1\\2&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}1&2\\0&1\\1&0\end{bmatrix}\begin{bmatrix}1&0&I&2\\0&1&0&1\end{bmatrix} & \begin{bmatrix}1&2\\0&1\\1&0\end{bmatrix}\begin{bmatrix}3\\0\end{bmatrix} & \begin{bmatrix}1&2\\0&1\\1&0\end{bmatrix}\begin{bmatrix}1&0&1\\2&1&0\end{bmatrix}
\end{array}\right]+$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix}0&1&2&0&1\\1&0&0&1&0\\0&I&1&0&1\\I&0&0&1&2\end{bmatrix}\begin{bmatrix}0&1&0&I\\1&0&I&0\\2&0&1&0\\0&1&0&1\\1&0&1&2\end{bmatrix} & \begin{bmatrix}0&1&2&0&1\\1&0&0&1&0\\0&I&1&0&1\\I&0&0&1&2\end{bmatrix}\begin{bmatrix}0\\0\\I\\0\\1\end{bmatrix} & \begin{bmatrix}0&1&2&0&1\\1&0&0&1&0\\0&I&1&0&1\\I&0&0&1&2\end{bmatrix}\begin{bmatrix}1&0&I\\0&1&0\\2&0&0\\0&1&I\\1&0&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&0&I&0&1\end{bmatrix}\begin{bmatrix}0&1&0&I\\1&0&I&0\\2&0&1&0\\0&1&0&1\\1&0&1&2\end{bmatrix} & \begin{bmatrix}0&0&I&0&1\end{bmatrix}\begin{bmatrix}0\\0\\I\\0\\1\end{bmatrix} & \begin{bmatrix}0&0&I&0&1\end{bmatrix}\begin{bmatrix}1&0&I\\0&1&0\\2&0&0\\0&1&I\\1&0&0\end{bmatrix} \\ \hline
\begin{bmatrix}1&0&2&0&1\\0&1&0&1&0\\I&0&0&I&0\end{bmatrix}\begin{bmatrix}0&1&0&I\\1&0&I&0\\2&0&1&0\\0&1&0&1\\1&0&1&2\end{bmatrix} & \begin{bmatrix}1&0&2&0&1\\0&1&0&1&0\\I&0&0&I&0\end{bmatrix}\begin{bmatrix}0\\0\\I\\0\\1\end{bmatrix} & \begin{bmatrix}1&0&2&0&1\\0&1&0&1&0\\I&0&0&I&0\end{bmatrix}\begin{bmatrix}1&0&I\\0&1&0\\2&0&0\\0&1&I\\1&0&0\end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{cccc|c|ccc} 10 & 0 & 4 & 12 & 0 & 1 & 3I & 3 \\ 0 & I & 0 & I & I & I & 0 & 2I \\ 4 & 0 & 2 & 4 & 0 & 1 & I & 1 \\ 12 & I & 4 & 17 & 1 & 1 & 4I & 6 \\ \hline 0 & I & 0 & 1 & 1 & 1 & 0 & 2 \\ \hline 1 & I & 1 & 1 & 1 & 2 & 0 & 2 \\ 3I & 0 & I & 4I & 0 & 0 & I & I \\ 3 & 2I & 1 & 6 & 2 & 2 & I & 5 \end{array}\right] + \left[\begin{array}{cccc|c|ccc} 1 & 0 & I & 2 & 3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 2 & 1 & 0 \\ I & 0 & I & 2I & 3I & I & 0 & I \\ 2 & 1 & 2I & 5 & 6 & 4 & 1 & 2 \\ \hline 3 & 0 & 3I & 6 & 9 & 3 & 0 & 3 \\ \hline 1 & 2 & I & 4 & 3 & 5 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & I & 2 & 3 & 1 & 0 & 1 \end{array}\right]$$

$$+ \left[\begin{array}{cccc|c|ccc} 6 & 0 & 3+I & 2 & 2I+1 & 5 & 1 & 0 \\ 0 & 2 & 0 & 1+I & 0 & 1 & 1 & 2I \\ I+3 & 0 & 2+I & 2 & 1+I & 3 & I & 0 \\ 2 & 1+I & 2 & I+5 & 2 & I+2 & 1 & 2I \\ \hline 2I+1 & 0 & I+1 & 2 & 1+I & 2I+1 & 0 & 0 \\ \hline 5 & 1 & 3 & I+2 & 2I+1 & 6 & 0 & I \\ 1 & 1 & I & 1 & 0 & 0 & 2 & I \\ 0 & 2I & 0 & 2I & 0 & I & I & 2I \end{array}\right]$$

$$= \left[\begin{array}{cccc|c|ccc} 17 & 0 & 7+2I & 16 & 4+2I & 7 & 3I+1 & 4 \\ 0 & 3+I & 0 & 2+2I & I & 3+I & 2 & 4I \\ 7+2I & 0 & 4+2I & 6+2I & 4I+1 & 4+I & 2I & 1+I \\ 16 & 2+2I & 6+2I & I+27 & 9 & I+7 & 4I+2 & 8+2I \\ \hline 4+2I & I & 4I+1 & 9 & 11+I & 5+2I & 0 & 5 \\ \hline 7 & 3+I & 4+I & I+7 & 5+2I & 13 & 2 & 3+I \\ 3I+1 & 2 & 2I & 4I+2 & 0 & 2 & 3+I & 2I \\ 4 & 4I & 1+I & 8+2I & 5 & 3+I & 2I & 6+2I \end{array}\right].$$

It is clear that AA^t is a neutrosophic super symmetric matrix.

Thus minor product of a matrix A with its special transpose A^t gives a super symmetric matrix.

It is pertinent to observe A is not a symmetric neutrosophic matrix.

Hence A^t is also not a neutrosophic symmetric super matrix how ever the product is a symmetric super neutrosophic super matrix.

We give yet another example.

Example 2.53:

$$\text{Let } A = \left[\begin{array}{ccc|cccc|cccc}
3 & 0 & I & 0 & 4 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & I \\
6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\hline
7 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & I & 0 & 0 & I & 0 & 0 & 0 \\
0 & 1 & I & 0 & 0 & I & 0 & 0 & I & 0 & 0 \\
\hline
4 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & I & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 2 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & I & 0 & 1 & 0 & 0 & 0 & 1
\end{array}\right]$$

be a super matrix.

To find the minor product of A with

$$A^t = \left[\begin{array}{ccc|ccccc|cccc}
3 & 0 & 6 & 7 & 1 & 0 & 1 & 0 & 4 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\
I & 0 & 0 & 0 & 0 & 1 & 1 & I & 0 & 0 & 1 & 0 \\
\hline
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & I & 1 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & I & 0 & 1 & 0 & 0 & I \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 0 & 0 & I & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 1 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & I & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}\right]$$

product of A with A^t.

$$AA^t = \left[\begin{array}{ccc|cccc|cccc}
3 & 0 & I & 0 & 4 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & I \\
6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\hline
7 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & I & 0 & 0 & I & 0 & 0 & 0 \\
0 & 1 & I & 0 & 0 & I & 0 & 0 & I & 0 & 0 \\
\hline
4 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & I & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 2 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & I & 0 & 1 & 0 & 0 & 0 & 1
\end{array}\right]$$

$$\left[\begin{array}{ccc|ccccc|cccc}
3 & 0 & 6 & 7 & 1 & 0 & 1 & 0 & 4 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\
I & 0 & 0 & 0 & 0 & 1 & 1 & I & 0 & 0 & 1 & 0 \\
\hline
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & I & 1 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & I & 0 & 1 & 0 & 0 & I \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 0 & 0 & I & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 1 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & I & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}\right] =$$

$$\left[\begin{array}{ccc}
3 & 0 & 1 \\
0 & 1 & 0 \\
6 & 0 & 0 \\
\hline
7 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
1 & 0 & 1 \\
0 & 1 & I \\
\hline
4 & 0 & 0 \\
1 & 0 & 0 \\
0 & 2 & 1 \\
1 & 0 & 0
\end{array}\right]
\left[\begin{array}{ccc|ccccc|cccc}
3 & 0 & 6 & 7 & 1 & 0 & 1 & 0 & 4 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\
I & 0 & 0 & 0 & 0 & 1 & 1 & I & 0 & 0 & 1 & 0
\end{array}\right] +$$

$$
\left[\begin{array}{cccc}
0 & 4 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\hline
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
\hline
0 & 1 & 0 & I \\
I & 0 & 0 & 1 \\
1 & 0 & I & 0 \\
0 & I & 0 & 1
\end{array}\right]
\left[\begin{array}{ccc|ccccc|cccc}
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & I & 1 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & I & 0 & 1 & 0 & 0 & I \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 0 & 0 & I & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 1 & 0 & 1
\end{array}\right]
$$

$$
+\left[\begin{array}{cccc}
0 & 1 & 0 & 1 \\
0 & 1 & 0 & I \\
0 & 0 & 0 & 1 \\
\hline
0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
I & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
\hline
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]
\left[\begin{array}{ccc|ccccc|cccc}
0 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & I & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]
$$

$$= \left[\begin{array}{c|c|c}
\begin{bmatrix}3&0&1\\0&1&0\\6&0&0\end{bmatrix}\begin{bmatrix}3&0&6\\0&1&0\\1&0&0\end{bmatrix} &
\begin{bmatrix}3&0&1\\0&1&0\\6&0&0\end{bmatrix}\begin{bmatrix}7&1&0&1&0\\1&0&0&0&1\\0&0&1&1&I\end{bmatrix} &
\begin{bmatrix}3&0&1\\0&1&0\\6&0&0\end{bmatrix}\begin{bmatrix}4&1&0&1\\0&0&2&0\\0&0&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}7&1&0\\1&0&0\\0&0&1\\1&0&1\\0&1&I\end{bmatrix}\begin{bmatrix}3&0&6\\0&1&0\\1&0&0\end{bmatrix} &
\begin{bmatrix}7&1&0\\1&0&0\\0&0&1\\1&0&1\\0&1&I\end{bmatrix}\begin{bmatrix}7&1&0&1&0\\1&0&0&0&1\\0&0&1&1&I\end{bmatrix} &
\begin{bmatrix}7&1&0\\1&0&0\\0&0&1\\1&0&1\\0&1&I\end{bmatrix}\begin{bmatrix}4&1&0&1\\0&0&2&0\\0&0&1&0\end{bmatrix} \\ \hline
\begin{bmatrix}4&0&0\\1&0&0\\0&2&1\\1&0&0\end{bmatrix}\begin{bmatrix}3&0&6\\0&1&0\\1&0&0\end{bmatrix} &
\begin{bmatrix}4&0&0\\1&0&0\\0&2&1\\1&0&0\end{bmatrix}\begin{bmatrix}7&1&0&1&0\\1&0&0&0&1\\0&0&1&1&I\end{bmatrix} &
\begin{bmatrix}4&0&0\\1&0&0\\0&2&1\\1&0&0\end{bmatrix}\begin{bmatrix}4&1&0&1\\0&0&2&0\\0&0&1&0\end{bmatrix}
\end{array}\right] +$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix}0&4&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\4&0&0\\0&1&0\\0&0&1\end{bmatrix} &
\begin{bmatrix}0&4&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&1&1&0&0\\1&0&0&I&0\\0&1&0&0&I\\0&0&1&0&0\end{bmatrix} &
\begin{bmatrix}0&4&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&I&1&0\\1&0&0&I\\0&0&I&0\\I&1&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}1&1&0&0\\1&0&1&0\\1&0&0&1\\0&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}0&0&0\\4&0&0\\0&1&0\\0&0&1\end{bmatrix} &
\begin{bmatrix}1&1&0&0\\1&0&1&0\\1&0&0&1\\0&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}1&1&1&0&0\\1&0&0&I&0\\0&1&0&0&I\\0&0&1&0&0\end{bmatrix} &
\begin{bmatrix}1&1&0&0\\1&0&1&0\\1&0&0&1\\0&I&0&0\\0&0&I&0\end{bmatrix}\begin{bmatrix}0&I&1&0\\1&0&0&I\\0&0&I&0\\I&1&0&1\end{bmatrix} \\ \hline
\begin{bmatrix}0&1&0&I\\I&0&0&1\\1&0&I&0\\0&I&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\4&0&0\\0&1&0\\0&0&1\end{bmatrix} &
\begin{bmatrix}0&1&0&I\\I&0&0&1\\1&0&I&0\\0&I&0&1\end{bmatrix}\begin{bmatrix}1&1&1&0&0\\1&0&0&I&0\\0&1&0&0&I\\0&0&1&0&0\end{bmatrix} &
\begin{bmatrix}0&1&0&I\\I&0&0&1\\1&0&I&0\\0&I&0&1\end{bmatrix}\begin{bmatrix}0&I&1&0\\1&0&0&I\\0&0&I&0\\I&1&0&1\end{bmatrix}
\end{array}\right] +$$

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix}0&1&0&1\\0&1&0&I\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\1&1&0\\0&0&0\\1&I&1\end{bmatrix} &
\begin{bmatrix}0&1&0&1\\0&1&0&I\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&1&1&0&I\\1&0&1&0&0\\1&0&0&0&0\end{bmatrix} &
\begin{bmatrix}0&1&0&1\\0&1&0&I\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&0&1&1\\1&1&0&0\\0&1&1&0\\I&0&0&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\1&1&0\\0&0&0\\1&I&1\end{bmatrix} &
\begin{bmatrix}0&0&1&1\\1&1&0&0\\0&1&1&0\\I&0&0&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&1&1&0&I\\1&0&1&0&0\\1&0&0&0&0\end{bmatrix} &
\begin{bmatrix}0&0&1&1\\1&1&0&0\\0&1&1&0\\I&0&0&0\\0&I&0&0\end{bmatrix}\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \\ \hline
\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\1&1&0\\0&0&0\\1&I&1\end{bmatrix} &
\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&1&0&I&0\\0&1&1&0&I\\1&0&1&0&0\\1&0&0&0&0\end{bmatrix} &
\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&0\\1&0&0&0\end{bmatrix}
\end{array}\right]
$$

$$
= \left[\begin{array}{ccc|ccccc|cccc}
10&0&18&21&3&1&4&I&12&3&1&3\\
0&1&0&1&0&0&0&1&0&0&2&0\\
18&0&36&42&6&0&6&0&24&6&0&6\\ \hline
21&1&42&50&7&0&7&1&28&7&2&7\\
3&0&6&7&1&0&1&0&4&1&0&1\\
1&0&0&0&0&1&1&I&0&0&1&0\\
4&0&6&7&1&1&2&I&4&1&1&1\\
I&1&0&1&0&I&I&1+I&0&0&2+I&0\\ \hline
12&0&24&28&4&0&4&0&16&4&0&4\\
3&0&6&7&1&0&1&0&4&1&0&1\\
1&2&0&2&0&1&1&2+I&0&0&5&0\\
3&0&6&7&1&0&1&0&4&1&0&1
\end{array}\right] +
$$

$$\left[\begin{array}{ccc|ccccc|cccc}
16 & 0 & 0 & 4 & 0 & 0 & 4I & 0 & 4 & 0 & 0 & 4I \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 0 & 0 & I & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & I & 1 & 0 & 1 \\
\hline
4 & 0 & 0 & 2 & 1 & 1 & I & 0 & 1 & I & 1 & I \\
0 & 1 & 0 & 1 & 2 & 1 & 0 & I & 0 & I & 1+I & 0 \\
0 & 0 & 1 & 1 & 1 & 2 & 0 & 0 & I & 1+I & 1 & 1 \\
4I & 0 & 0 & I & 0 & 0 & I & 0 & I & 0 & 0 & I \\
0 & I & 0 & 0 & I & 0 & 0 & I & 0 & 0 & I & 0 \\
\hline
4 & 0 & I & 1 & 0 & I & I & 0 & 1+I & I & 0 & 2I \\
0 & 0 & 1 & I & I & 1+I & 0 & 0 & I & 1+I & I & 1 \\
0 & I & 0 & 1 & 1+I & 1 & 0 & I & 0 & I & 2I & 0 \\
4I & 0 & 1 & I & 0 & 1 & I & 0 & 2I & 1 & 0 & 2I
\end{array}\right] +$$

$$\left[\begin{array}{ccc|ccccc|cccc}
2 & 1+I & 1 & 1 & 1 & 1 & 0 & I & 1 & 0 & 1 & 0 \\
1+I & 2I & I & I & 1 & 1 & 0 & I & I & 0 & 1 & 0 \\
1 & I & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
1 & I & 1 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 2 & 1 & I & I & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 2 & 0 & I & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & I & 0 & I & 0 & 0 & I & 0 & 0 \\
I & I & 0 & 0 & I & I & 0 & I & 0 & 0 & I & 0 \\
\hline
1 & I & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & I & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & I & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] =$$

$$\left[\begin{array}{ccc|ccccc|cccc}
28 & 1+I & 19 & 26 & 4 & 2 & 4+4I & 2I & 17 & 3 & 2 & 3+4I \\
1+I & 2+2I & I & 1+I & 2 & 1 & 0 & 2I+1 & I & 0 & 3+I & 0 \\
19 & I & 38 & 43 & 6 & 1 & 6 & 0 & 25+I & 7 & 0 & 7 \\
\hline
26 & 1+I & 43 & 54 & 8 & 2 & 7+I & 1 & 30 & 7+I & 3 & 7+I \\
4 & 2 & 6 & 8 & 5 & 2 & 1+I & 2I & 4 & 2+I & 2+I & 1 \\
2 & 1 & 1 & 2 & 2 & 5 & 1 & 2I & I & 1+I & 3 & 1 \\
4+4I & 0 & 6 & 7+I & 1+I & 1 & 2+2I & I & 4+I & 1+I & 13 & 1+I \\
2I & 2I+1 & 0 & 1 & 2I & 2I & I & 1+3I & 0 & 0 & 2+3I & 0 \\
\hline
17 & I & 25+I & 30 & 4 & I & 4+I & 0 & 18+I & 4+I & 0 & 4+2I \\
3 & 0 & 7 & 7+I & 2+I & 1+I & 1+I & 0 & 4+I & 3+I & I & 2 \\
2 & 3+I & 0 & 3 & 2+I & 3 & 13 & 2+3I & 0 & I & 6+2I & 0 \\
3+4I & 0 & 7 & 7+I & 1 & 1 & 1+I & 0 & 4+2I & 2 & 0 & 1+2I
\end{array}\right]$$

We see the minor product of a neutrosophic super matrix A with its transpose A^t is a symmetric neutrosophic super matrix.

The entries in these super matrices can be replaced from the dual number neutrosophic rings, special dual like number neutrosophic rings, special quasi dual number neutrosophic rings and special mixed dual number neutrosophic rings.

Chapter Three

SUPER BIMATRICES AND THEIR GENERALIZATION

In this chapter we introduce the notion of super bimatrices or equivalently will be known as bisuper matrices. The study of bimatrices has been carried out in [6]. Here we define super bimatrices and generalize them.

This chapter has two sections. Section one defines super bimatrices and enumerates a few of their properties. In section two the generalization of super bimatrices (bisupermatrices) to super n-matrices (n-super matrices) is carried out ($n > 2$) when $n = 2$ this structure will be known as bisupermatrices or superbimatrices. When $n = 3$ we call it as trisupermatrix or supertrimatrix. This can be made into a neutrosophic superbimatrix by taking the entries from $\langle Z \cup I\rangle$ or $\langle Q \cup I\rangle$ or $\langle R \cup I\rangle$ or $\langle C \cup I\rangle$ or $\langle Z_n \cup I\rangle$.

3.1 Super bimatrices and their properties

In this section we introduce the new notion of super bimatrices (bisuper matrices) and enumerate a few of its algebraic properties and operations on them.

DEFINITION 3.1.1 : *Let $A = A_1 \cup A_2$ where A_1 and A_2 are super row vectors of type A, $A_1 = (a_1, a_2, \ldots, a_r \mid a_{r+1} \ldots \mid \ldots \mid a_m \ldots a_n)$ and $A_2 = (b_1, \ldots, b_s \mid b_{s+1} \ldots \mid \ldots \mid b_t \ldots b_r)$. Then we call $A = A_1 \cup A_2$ to be a super birow vectors (bisuper row vectors or super row bivectors) of type A if and only if A_1 and A_2 are distinct in atleast one of the partitions.*

We illustrate this situation by some examples.

Example 3.1.1: Let $A = A_1 \cup A_2 = (3\ 0\ 1 \mid 1\ 1\ 0\ 4 \mid -5 \mid 7\ 8\ 3\ 1\ 2) \cup (1\ 0 \mid 3\ 1\ 2 \mid 1\ 1\ 0\ 4 \mid 6\ 1\ 0)$. A is a super row bivector (bisuper row vector) of type A.

Example 3.1.2: Let $A = A_1 \cup A_2 = (0\ 0 \mid 0\ 0\ 0 \mid \ldots \mid 0\ 0\ 0) \cup (0\ 0\ 0\ 0\ 0 \mid 0\ 0\ 0\ 0\ 0 \mid 0\ 0)$, A is a super row bivector of type A also known as the super row zero bivector of type A.

Example 3.1.3: Let $A = A_1 \cup A_2 = (1\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1\ 1 \mid 1\ 1) \cup (1\ 1 \mid 1\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1\ 1\ 1)$; A is a super row bivector of type A. Infact A is also know as the super row unit bivector of type A or bisuper row unit vector of type A.

Example 3.1.4 : Let $A = A_1 \cup A_2 = (3\ 1\ 0 \mid 4\ 5 \mid 1\ 1\ 1\ 2\ 5) \cup (3\ 0\ 1 \mid 4\ 5 \mid 1\ 1\ 1\ 2\ 5)$; A is not a super row bivector for A_1 and A_2 are not distinct. If $A = A_1 \cup A_2 = (3 \mid 1\ 0\ \ 4\ 5 \mid 1\ 1\ 1\ 2\ 5) \cup (3\ 0 \mid 1\ 4\ 5\ 1 \mid 1\ 1\ 1\ 2\ 5)$ then A is a super row bivector of type A.

Now we proceed on to define the new notion of bilength of the super row bivector of type A.

DEFINITION 3.1.2: *Let $A = A_1 \cup A_2 = (a_1 \ldots \mid \ldots. \mid \ldots \mid a_r \ldots a_n) \cup (b_1 \ldots \mid \ldots. \mid \ldots \mid b_s \ldots b_n)$ be a super row bivector of type A where both A_1 and A_2 are of length n but they are distinct in the partitions. We say A is of bilength n and denote the bilength by (n, n).*

If $A = A_1 \cup A_2 = (a_1 \ldots | \ldots. | \ldots | a_r \ldots a_n) \cup (b_1 \ldots | \ldots. | \ldots | b_s \ldots b_n)$ *be a super row bivector of type A* $(m \neq n)$ *then we say A is of bilength (m, n).*

We illustrate this by some examples.

Example 3.1.5: Let $A = A_1 \cup A_2 = (1\ 0\ 2\ 3 \mid 5\ 6\ 7\ 8\ 9 \mid 2\ 1) \cup (0\ 1\ 2\ 0 \mid 1\ 0\ 9\ 2\ 7 \mid 5\ 3)$ be a super birow vector of type A. We say A is of bilength (11, 11), i.e., this super birow vector has same bilength.

Example 3.1.6: Let

$$A = A_1 \cup A_2 = (0\ 1\ 0 \mid 2\ 5\ 3 \mid 1\ 1\ 0) \cup (3\ 1\ 2\ 0 \mid 2\ 1 \mid 5)$$

be a super birow vector of type A. A is of bilength (9,7). We see this super row bivector has different bilength for its component super row vectors A_1 and A_2.

We can define the notion of super row bivector equivalently in this form.

DEFINITION 3.1.3: *Let* $A = A_1 \cup A_2$, *if* A_1 *and* A_2 *are two distinct super row vectors of type A, then we call A to be a super row bivector of type A.*

Now we proceed onto define the notion of super column bivector of type A (super bicolumn vector or bisuper column vector of type A).

DEFINITION 3.1.4: *Let* $A = A_1 \cup A_2$ *where* A_1 *and* A_2 *are super column vectors of type A which are distinct. Then we say A is a super column bivector of type A or bisuper column vector of type A or super bicolumn vector of type A.*

We illustrate this situation by the following examples.

Example 3.1.7: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{c} 3 \\ 1 \\ 0 \\ \hline 1 \\ 1 \\ 1 \\ \hline 0 \\ 7 \end{array}\right] \cup \left[\begin{array}{c} 5 \\ \hline 7 \\ -7 \\ 2 \\ 4 \\ 0 \\ \hline 3 \\ 1 \\ 2 \end{array}\right]$$

be a super bicolumn vector of type A. Clearly A_1 and A_2 are distinct.

Example 3.1.8: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \end{array}\right] \cup \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

be a super column bivector of type A. We say A is the zero super column bivector of type A or super column zero vector of type A.

Example 3.1.9: Let

$$A = \left[\begin{array}{c} 1 \\ 1 \\ 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \end{array}\right] \cup \left[\begin{array}{c} 1 \\ 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \hline 1 \\ 1 \\ 1 \end{array}\right] = A_1 \cup A_2.$$

A is a super column bivector of type A known as the super column unit bivector of type A.

DEFINITION 3.1.5: *Let*

$$A = A_1 \cup A_2 = \left[\begin{array}{c} a_1 \\ \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline a_m \end{array}\right] \cup \left[\begin{array}{c} b_1 \\ \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline b_n \end{array}\right]$$

be a super column bivector of type A. If $(m \neq n)$ *then we say A is of super bilength* (m, n)*.*

If in $A = A_1 \cup A_2$ *where* A_1 *and* A_2 *are distinct super column vectors, if the length of* A_1 *is the same as that of* A_2 *then we say A is a super column bivector of length n and denote the bilength by* (n, n)*.*

Example 3.1.10: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{c} 3 \\ 0 \\ 2 \\ 5 \\ \hline 1 \\ 3 \\ 0 \\ \hline 1 \\ 1 \\ 0 \\ 4 \end{array}\right] \cup \left[\begin{array}{c} 2 \\ 1 \\ \hline 0 \\ 6 \\ 5 \\ 7 \\ 8 \\ 9 \\ \hline 1 \\ 4 \\ 0 \end{array}\right]$$

be a super column bivector of type A. A is of bilength (11,11), i.e., both A_1 and A_2 are of length 11. Clearly A_1 and A_2 are distinct super column vectors.

Example 3.1.11: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{c} 3 \\ 1 \\ 0 \\ 1 \\ 3 \\ \hline 1 \\ 2 \\ \hline 1 \\ 1 \\ 5 \\ 7 \end{array}\right] \cup \left[\begin{array}{c} -3 \\ 4 \\ 0 \\ 2 \\ \hline 7 \\ 5 \\ 8 \\ 9 \\ 1 \\ \hline 10 \end{array}\right]$$

be a super column bivector of type A. We say A is of bilength (11,10). Clearly A_1 and A_2 are of different lengths.

These new type of super birow vectors and super bicolumn vectors will be useful in computer networking, in coding / cryptography and in fuzzy models.

Now we proceed on to define the transpose of a super row bivector and super column bivector of type A.

DEFINITION 3.1.6: *Let $A = A_1 \cup A_2$ be a super row bivector of type A. We define the transpose of A as the transpose of each of the super row vectors A_1 and A_2, i.e.,*

$A^t = (A_1 \cup A_2)^t = A_1^t \cup A_2^t$. Clearly the transpose of a super row bivector is a super column bivector of type A.

Likewise if $A = A_1 \cup A_2$ is a super column bivector of type A then the transpose of A is a super row bivector of type A, i.e., A^t is the union of the transpose of the column vectors A_1 and A_2 respectively. We denote $A^t = (A_1 \cup A_2)^t = A_1^t \cup A_2^t$.

We shall illustrate this by the following example.

Example 3.1.12: Let $A = A_1 \cup A_2$

$$= (0\ 1\ 2\ 3 \mid 4\ 3\ 1 \mid 7\ 0\ 3\ 5\ 6\ 8) \cup (3\ 0 \mid 2\ 5\ 4\ 1 \mid 0\ 0\ 7\ 2\ 5)$$

be a super row bivector of type A.

The transpose of A denoted by

$$A^t = \left[\begin{array}{c} 0\\1\\2\\3\\ \hline 4\\3\\1\\ \hline 7\\0\\3\\5\\6\\8 \end{array}\right] \cup \left[\begin{array}{c} 3\\0\\ \hline 2\\5\\4\\1\\ \hline 0\\0\\7\\2\\5 \end{array}\right] = A_1^t \cup A_2^t .$$

It is clear that A^t is a super column bivector or super bicolumn vector of type A.

Example 3.1.13: Let A =

$$A_1 \cup A_2 = \left[\begin{array}{c} 3\\0\\1\\2\\5\\6\\ \hline 1\\0\\3\\ \hline 7\\4\\5\\9 \end{array}\right] \cup \left[\begin{array}{c} 1\\3\\0\\ \hline 5\\6\\7\\8\\9\\ \hline 3\\-1\\2\\5\\4 \end{array}\right]$$

be a super column bivector of type A. The transpose of A denoted by $A^t = A_1^t \cup A_2^t$ = [3 0 1 2 5 6 | 1 0 3 | 7 4 5 9] ∪ [1 3 0 | 5 6 7 8 9 | 3 – 1 2 5 4].

It is easily verified that A^t is a super row bivector.

Now having seen the transpose when is the addition and multiplication of these super bivectors of type A is possible.

DEFINITION 3.1.7: *Let $A = A_1 \cup A_2$ and $B = B_1 \cup B_2$ be two super row (column) bivectors of type A. We say A and B are similar or "identical in structure" (identical in structure does not mean they are identical) if the following conditions are satisfied.*

1. *The bilength of A and the bilength of B are the same i.e., if bilength of $A = A_1 \cup A_2$ is (m,n) then the bilength of $B = B_1 \cup B_2$ is also (m,n).*

2. *The partition of the row (column) vector A_i is identical with that of the row (column) vector B_i, true for i=1,2.*

We illustrate this by the following examples.

Example 3.1.14: Let $A = A_1 \cup A_2$

= [3 1 5 6 | 0 2 3 | 1 5 – 2 3 1 5] ∪ [8 0 | 1 2 3 | 7 | 5 6 7 8]

be a super row bivector.

B = [0 4 0 1 | 6 – 12 | 3 0 1 1 0 7] ∪ [3 5 | 0 1 0 | 8 | 3 0 1 5]

be another super row bivector. We can say A and B are identical in structure.

We see bilength of A is (13,10) and that of B is (13,10), they are equal.

Example 3.1.15: Let A =

$$A_1 \cup A_2 = \begin{bmatrix} 3 \\ 0 \\ \frac{1}{5} \\ 7 \\ 8 \\ \frac{1}{8} \\ 0 \end{bmatrix} \cup \begin{bmatrix} 8 \\ \frac{1}{3} \\ 4 \\ 5 \\ 7 \\ \frac{1}{2} \\ 0 \\ 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ \frac{1}{0} \\ 1 \\ 2 \\ \frac{3}{4} \\ 5 \end{bmatrix} \cup \begin{bmatrix} 0 \\ \frac{2}{1} \\ 2 \\ 3 \\ 4 \\ \frac{5}{9} \\ 8 \\ 7 \end{bmatrix} = B_1 \cup B_2 .$$

Clearly the super column bivectors A and B are identical in structure and we see the bilength of A is (9,10) and that of B is (9,10) i.e., the bilength of A and B are the same.

Thus we see if two super bivectors of type A are identical in structure they also enjoy the property that they have same bilength. Only to make things clear we mention it as a separate property though the concept identical in structure covers the equal bilength.

However it is pertinent to mention here that equal bilength will not in general imply they are identical in structure. But identical in structure will always imply they have same bilength.

Now we proceed onto define the notion of addition of super bivectors of type A. When we say super bivectors of type A it implies the super bivector can be a sub birow vector of type A or super bicolumn vector of type A; i.e., the term super bivector of type A includes both row bivector as well as column bivector.

DEFINITION 3.1.8: *Let $A = A_1 \cup A_2$ and $B = B_1 \cup B_2$ be any two super row bivectors which are identical in structure. Then addition or subtraction of A with B denoted by*

$A \pm B = (A_1 \pm A_2) \cup (B_1 \pm B_2)$ is well defined and $A_i \pm B_i$ is carried out component wise and $A \pm B$ is again a super birow vector which are identical in structure with A and B.

On similar lines we can define the addition or subtraction of A with B in case of super column bivectors A and B which are identical in structure.

We illustrate this by the following examples.

Example 3.1.16: Let

$$A = \left[\begin{array}{c} 3 \\ 0 \\ 1 \\ 2 \\ \hline 5 \\ 7 \\ 3 \\ \hline 4 \\ 5 \end{array}\right] \cup \left[\begin{array}{c} 2 \\ \hline -1 \\ 0 \\ 3 \\ 4 \\ \hline 7 \\ 2 \\ 0 \end{array}\right] = A_1 \cup A_2 \text{ and } B = \left[\begin{array}{c} 4 \\ 1 \\ -1 \\ 2 \\ \hline 3 \\ 0 \\ 1 \\ \hline 4 \\ -1 \end{array}\right] \cup \left[\begin{array}{c} 0 \\ \hline 1 \\ 2 \\ 3 \\ -1 \\ \hline 0 \\ 2 \\ -5 \end{array}\right] = B_1 \cup B_2$$

be two super column bivectors which are identical in structure. We find A+B the sum of A with B to be $(A_1 + B_1) \cup (A_2 + B_2)$

$$= \left[\begin{array}{c} 7 \\ 1 \\ 0 \\ 4 \\ \hline 8 \\ 7 \\ 4 \\ \hline 8 \\ 4 \end{array}\right] \cup \left[\begin{array}{c} 2 \\ \hline 0 \\ 2 \\ 6 \\ 3 \\ \hline 7 \\ 4 \\ -5 \end{array}\right] = A+B.$$

Clearly A+B is again a super column bivector which are identical in structure with A and B.

Now $A - B = A_1 - B_1 \cup A_2 - B_2$

$$= \left[\begin{array}{c} -1 \\ -1 \\ 2 \\ 0 \\ \hline 2 \\ 7 \\ 2 \\ \hline 0 \\ 6 \end{array}\right] \cup \left[\begin{array}{c} 2 \\ \hline -2 \\ -2 \\ 0 \\ 5 \\ \hline 7 \\ 0 \\ 5 \end{array}\right].$$

We see A – B is again super row bivector of type A identical in structure with A and B. Thus we see the property of identical in structure is preserved under both addition and subtraction.

In view of this we have the following interesting result.

THEOREM 3.1.1: *Let S = {A = $A_1 \cup A_2$ / the collection of all super row bivectors of type A which are identical in structures with A having its entries from Q or R or C or Z_n or $C(Z_n)$ or Z(g), $Z(g_1)$ or $Z(g_2)$ ($g^2 = 0$ is a new element $g_1^2 = g_1$ is a new element and $g_2^2 = -g_2$ a new element n is such that, $2 \leq n \leq \infty$}. Then S is a group under the operation of addition.*

The proof is left as an exercise for the reader.

THEOREM 3.1.2: *Let B = {B = $B_1 \cup B_2$ / the collection of all super column bivectors of type A which are identical in structure of type A with B having its entries from Q or R or C or Z_n or $C(Z_n)$ or Z(g) or $Z(g_1)$ or $Z(g_2)$ ($g^2 = 0$ is a new element $g_1^2 = g_1$ is a new element and $g_2^2 = -g_2$ a new element; n is such that, $2 \leq n \leq \infty$}. Then B is a group under the operation of addition of the super column bivectors.*

This proof is also simple left as an exercise for the reader.

Now we define the product of a super row bivector of type A with a super column bivector of type A where the multiplication is compatible.

We first show how the minor product of two super vectors are defined whenever the multiplication happens to be compatible.

This is illustrated by some examples.

Example 3.1.17:

Let x = [3 0 1 | 2 3 4 5] and y = [1 0 0 | 2 3 1 6]^t.

The minor product of the super vectors x with y is given by

$$xy = [3\ 0\ 1 \mid 2\ 3\ 4\ 5]\begin{bmatrix}1\\0\\0\\\hline 2\\3\\1\\6\end{bmatrix}$$

$$= [3\ 0\ 1]\begin{bmatrix}1\\0\\0\end{bmatrix} + [2\ 3\ 4\ 5]\begin{bmatrix}2\\3\\1\\6\end{bmatrix}$$

$= 3 + \{4 + 9 + 4 + 30\}$
$= 3+47 = 50.$

Example 3.1.18: Let

$$x = [0\ 1\ 2\ 3 \mid 4\ 5 \mid 7\ 8\ 9\ 0\ 1\ 2\ 3 \mid 8] \text{ and } y = \begin{bmatrix}6\\-1\\2\\3\\\hline -4\\5\\\hline 7\\-8\\9\\6\\-1\\2\\-3\\\hline -8\end{bmatrix}$$

be two super vectors

$$xy = [0\ 1\ 2\ 3 \mid 4\ 5 \mid 7\ 8\ 9\ 0\ 1\ 2\ 3 \mid 8]\left[\begin{array}{c} 6 \\ -1 \\ 2 \\ 3 \\ \hline -4 \\ 5 \\ \hline 7 \\ -8 \\ 9 \\ 6 \\ -1 \\ 2 \\ -3 \\ \hline -8 \end{array}\right]$$

$$= [0\ 1\ 2\ 3]\begin{bmatrix} 6 \\ -1 \\ 2 \\ 3 \end{bmatrix} + [4\ 5]\begin{bmatrix} -4 \\ 5 \end{bmatrix} + [7\ 8\ 9\ 0\ 1\ 2\ 3]\begin{bmatrix} 7 \\ -8 \\ 9 \\ 6 \\ -1 \\ 2 \\ -3 \end{bmatrix} + [8]\,[-8]$$

$$= \{0-1 + 4+9\} + \{-16+25\} + \{49 - 64 + 81+0-1+4-9\} + \{-64\}$$

$$= 12 + 9 + 60 - 64 = 17.$$

DEFINITION 3.1.9: *Let*

$$x = [V_1\ V_2 \ldots V_n] \text{ and } y = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}$$

be two super vectors of type A.

$$xy = [V_1\ V_2 \ldots V_n] \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}$$

$$= V_1 W_1 + V_2 W_2 + \ldots + V_n W_n.$$

Note:

The compatability of each V_i with W_i is well defined for $i = 1, 2, \ldots, n$, this is illustrated by the following examples.

Example 3.1.19: Let

$$x = [1\ 2\ 0\ 1\ 5 \mid 3\ 2\ 0 \mid 7\ 0 - 1\ 2\ 3\ 4\ 8 \mid 8\ 0\ 1\ 4]$$

be a super row vector.

$$x^t = \left[\begin{array}{c} 1 \\ 2 \\ 0 \\ 1 \\ 5 \\ \hline 3 \\ 2 \\ 0 \\ \hline 7 \\ 0 \\ -1 \\ 2 \\ 3 \\ 4 \\ 8 \\ \hline 8 \\ 0 \\ 1 \\ 4 \end{array}\right]$$

$$xx^t = [1\ 2\ 0\ 1\ 5] \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 5 \end{bmatrix} + [\ 3\ 2\ 0] \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} +$$

$$[7\ 0\ {-1}\ 2\ 3\ 4\ 8]\begin{bmatrix}7\\0\\-1\\2\\3\\4\\8\end{bmatrix} + [8\ 0\ 1\ 4]\begin{bmatrix}8\\0\\1\\4\end{bmatrix}$$

$$= \{1 + 4 + 0 + 1 + 25\} + \{9 + 4 + 0\} + \{49 + 0 + 1 + 4 + 9 + 16 + 64\} + \{64 + 0 + 1 + 16\}$$

$$= \{31 + 13 + 143 + 81\}$$

$$= 268.$$

Thus we can define the product in case of super row vector x with x^t which is as follows:

$$\text{If } x = [V_1\ V_2\ \ldots\ V_n] \text{ then } x^t = \begin{bmatrix}V_1^t\\V_2^t\\\vdots\\V_n^t\end{bmatrix}.$$

$$\text{Now } xx^t = [V_1\ V_2\ \ldots\ V_n]\begin{bmatrix}V_1^t\\V_2^t\\\vdots\\V_n^t\end{bmatrix}.$$

$$= V_1V_1^t + V_2V_2^t + \ldots + V_nV_n^t.$$

We illustrate this by an example.

Example 3.1.20: Let

x = [0 1 | 3 4 0 1 | –3 2 1 6 7 8 9 | 3 2 1 4 5] then

$$x^t = \left[\begin{array}{c} 0 \\ 1 \\ \hline 3 \\ 4 \\ 0 \\ 1 \\ \hline -3 \\ 2 \\ 1 \\ 6 \\ 7 \\ 8 \\ 9 \\ \hline 3 \\ 2 \\ 1 \\ 4 \\ 5 \end{array}\right]$$

where x^t is a transpose of x and x is a super row vector.

$$xx^t = [0\,1\,|\,3\,4\,0\,1\,|\,-3\,2\,1\,6\,7\,8\,9\,|\,3\,2\,|\,1\,4\,5]\left[\begin{array}{c} 0 \\ 1 \\ \hline 3 \\ 4 \\ 0 \\ 1 \\ \hline -3 \\ 2 \\ 1 \\ 6 \\ 7 \\ 8 \\ 9 \\ \hline 3 \\ 2 \\ \hline 1 \\ 4 \\ 5 \end{array}\right]$$

$$= [0\ 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} + [3\ 4\ 0\ 1]\begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix} + [-3\ 2\ 1\ 6\ 7\ 8\ 9]\begin{bmatrix} -3 \\ 2 \\ 1 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

$$+ [3\ 2\ 1\ 4\ 5]\begin{bmatrix}3\\2\\1\\4\\5\end{bmatrix}$$

$$= \{0 + 1\} + \{9 + 16 + 0 + 1\} + \{9 + 4 + 1 + 36 + 49 + 64 + 81\} + \{9 + 4 + 1 + 16 + 25\}$$

$$= 1 + 26 + 244 + 55$$

$$= 326.$$

Now we proceed onto define the notion of major product of two super vectors.

We shall only illustrate this by simple examples.

Example 3.1.21: Let

$$x = \begin{bmatrix}0\\1\\2\\\hline 4\\5\\7\\8\\\hline 1\\4\end{bmatrix} \text{ and } y = [1\ 2 \mid 0\ 3\ 4\ 5 \mid 8\ 9\ 2\ 1\ 0]$$

be two super vectors of type A.

The major product of x with y denoted by

$$xy = \left[\begin{array}{c} 0 \\ 1 \\ 2 \\ \hline 4 \\ 5 \\ 7 \\ 8 \\ \hline 1 \\ 4 \end{array}\right] [1\ 2 \mid 0\ 3\ 4\ 5 \mid 8\ 9\ 2\ 1\ 0]$$

$$= \left[\begin{array}{c|c|c} \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}0 & 3 & 4 & 5\end{bmatrix} & \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}8 & 9 & 2 & 1 & 0\end{bmatrix} \\ \hline \begin{bmatrix}4\\5\\7\\8\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}4\\5\\7\\8\end{bmatrix}\begin{bmatrix}0 & 3 & 4 & 5\end{bmatrix} & \begin{bmatrix}4\\5\\7\\8\end{bmatrix}\begin{bmatrix}8 & 9 & 2 & 1 & 0\end{bmatrix} \\ \hline \begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}0 & 3 & 4 & 5\end{bmatrix} & \begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}8 & 9 & 2 & 1 & 0\end{bmatrix} \end{array}\right]$$

$$= \left[\begin{array}{cc|cccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 3 & 4 & 5 & 8 & 9 & 2 & 1 & 0 \\ 2 & 4 & 0 & 6 & 8 & 10 & 16 & 18 & 4 & 2 & 0 \\ \hline 4 & 8 & 0 & 12 & 16 & 20 & 32 & 36 & 8 & 4 & 0 \\ 5 & 10 & 0 & 15 & 20 & 25 & 40 & 45 & 10 & 5 & 0 \\ 7 & 14 & 0 & 21 & 28 & 35 & 56 & 63 & 14 & 7 & 0 \\ 8 & 16 & 0 & 24 & 32 & 40 & 72 & 81 & 18 & 9 & 0 \\ \hline 1 & 2 & 0 & 3 & 4 & 5 & 8 & 9 & 2 & 1 & 0 \\ 4 & 8 & 0 & 12 & 16 & 20 & 32 & 36 & 8 & 4 & 0 \end{array}\right]$$

Example 3.1.22: Let

$$x = \begin{bmatrix} 1 \\ 4 \\ \hline 3 \\ 7 \\ 0 \\ 2 \\ 5 \\ 1 \\ \hline 0 \\ 1 \\ -1 \\ 2 \\ 3 \\ 4 \\ 7 \\ 0 \end{bmatrix} \text{ and } y = [\,3\;4\;0\,|\,2\;1\;0\;1\;3\,|\,1\;2\;1\;4\,|\,1\,]$$

be two super vector.

The major product of x with y denoted xy =

$$\left[\begin{array}{c} 1 \\ 4 \\ \hline 3 \\ 7 \\ 0 \\ 2 \\ 5 \\ 1 \\ \hline 0 \\ 1 \\ -1 \\ 2 \\ 3 \\ 4 \\ 7 \\ 0 \end{array}\right] \left[\begin{array}{ccc|ccccc|cccc|c} 3 & 4 & 0 & 2 & 1 & 0 & 1 & 3 & 1 & 2 & 1 & 4 & 1 \end{array}\right]$$

We see for major product to be defined we need not have

(1) The natural order of the super column vector to be equal to the transpose of the super row vector.

(2) The number of partitions of x need not be equal to the number of partitions of y.

$$= \left(\begin{array}{c|c|c|c}
\begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}3 & 4 & 0\end{bmatrix} & \begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1 & 3\end{bmatrix} & \begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}1 & 2 & 1 & 4\end{bmatrix} & \begin{bmatrix}1\\4\end{bmatrix}1 \\ \hline
\begin{bmatrix}3\\7\\0\\2\\5\\1\end{bmatrix}\begin{bmatrix}3 & 4 & 0\end{bmatrix} & \begin{bmatrix}3\\7\\0\\2\\5\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1 & 3\end{bmatrix} & \begin{bmatrix}3\\7\\0\\2\\5\\1\end{bmatrix}\begin{bmatrix}1 & 2 & 1 & 4\end{bmatrix} & \begin{bmatrix}3\\7\\0\\2\\5\\1\end{bmatrix}1 \\ \hline
\begin{bmatrix}0\\1\\-1\\2\\3\\4\\7\\0\end{bmatrix}\begin{bmatrix}3 & 4 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\-1\\2\\3\\4\\7\\0\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1 & 3\end{bmatrix} & \begin{bmatrix}0\\1\\-1\\2\\3\\4\\7\\0\end{bmatrix}\begin{bmatrix}1 & 2 & 1 & 4\end{bmatrix} & \begin{bmatrix}0\\1\\-1\\2\\3\\4\\7\\0\end{bmatrix}1
\end{array}\right)$$

$$= \left[\begin{array}{ccc|ccccc|cccc|c}
3 & 4 & 0 & 2 & 1 & 0 & 1 & 3 & 1 & 2 & 1 & 4 & 1 \\
12 & 16 & 0 & 8 & 4 & 0 & 4 & 12 & 4 & 8 & 4 & 16 & 4 \\ \hline
9 & 12 & 0 & 6 & 3 & 0 & 3 & 9 & 3 & 6 & 3 & 12 & 3 \\
21 & 25 & 0 & 14 & 7 & 0 & 7 & 21 & 7 & 14 & 7 & 28 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 8 & 0 & 4 & 2 & 0 & 2 & 6 & 2 & 4 & 2 & 8 & 2 \\
15 & 20 & 0 & 10 & 5 & 0 & 5 & 15 & 5 & 10 & 5 & 20 & 5 \\
3 & 4 & 0 & 2 & 1 & 0 & 1 & 3 & 1 & 2 & 1 & 4 & 1 \\ \hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 4 & 0 & 2 & 1 & 0 & 1 & 3 & 1 & 2 & 1 & 4 & 1 \\
-3 & -4 & 0 & -2 & -1 & 0 & -1 & -3 & -1 & -2 & -1 & -4 & -1 \\
6 & 8 & 0 & 4 & 2 & 0 & 2 & 6 & 2 & 4 & 2 & 8 & 2 \\
9 & 12 & 0 & 6 & 3 & 0 & 3 & 9 & 3 & 6 & 3 & 12 & 3 \\
12 & 16 & 0 & 8 & 4 & 0 & 4 & 12 & 4 & 8 & 4 & 16 & 4 \\
21 & 28 & 0 & 14 & 7 & 0 & 7 & 21 & 7 & 14 & 7 & 28 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

Clearly the major product of two super vectors is a super matrix.

Now we show by examples the structure of a major product of a super column vector with its transpose.

Example 3.1.23: Let

$$A = \left[\begin{array}{c} 3 \\ 0 \\ 1 \\ 5 \\ \hline -1 \\ 2 \\ 3 \\ 7 \\ 0 \\ 2 \\ 1 \\ \hline 5 \\ 0 \\ 1 \end{array}\right]$$

be a super column vector. To find the major product of A with A^t.

Here $A^t = [3\ 0\ 1\ 5 \mid -1\ 2\ 3\ 7\ 0\ 2\ 1 \mid 5\ 0\ 1]$

Now $AA^t = \left[\begin{array}{c} 3 \\ 0 \\ 1 \\ 5 \\ \hline -1 \\ 2 \\ 3 \\ 7 \\ 0 \\ 2 \\ 1 \\ \hline 5 \\ 0 \\ 1 \end{array}\right]$ [3 0 1 5 | –1 2 3 7 0 2 1 | 5 0 1]

$$= \left[\begin{array}{c|c|c} \begin{bmatrix} 3 \\ 0 \\ 1 \\ 5 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 5 \end{bmatrix} & \begin{bmatrix} 3 \\ 0 \\ 1 \\ 5 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 & 7 & 0 & 2 & 1 \end{bmatrix} & \begin{bmatrix} 3 \\ 0 \\ 1 \\ 5 \end{bmatrix}\begin{bmatrix} 5 & 0 & 1 \end{bmatrix} \\ \hline \begin{bmatrix} -1 \\ 2 \\ 3 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 5 \end{bmatrix} & \begin{bmatrix} -1 \\ 2 \\ 3 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 & 7 & 0 & 2 & 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 2 \\ 3 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 5 & 0 & 1 \end{bmatrix} \\ \hline \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 & 5 \end{bmatrix} & \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 & 7 & 0 & 2 & 1 \end{bmatrix} & \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 5 & 0 & 1 \end{bmatrix} \end{array}\right]$$

$$
= \left[\begin{array}{cccc|ccccccc|ccc}
9 & 0 & 3 & 15 & -3 & 6 & 9 & 21 & 0 & 6 & 3 & 15 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 5 & -1 & 2 & 3 & 7 & 0 & 2 & 1 & 5 & 0 & 1 \\
15 & 0 & 5 & 25 & -5 & 10 & 15 & 35 & 0 & 10 & 5 & 25 & 0 & 5 \\
\hline
-3 & 0 & -1 & -5 & 1 & -2 & -3 & -7 & 0 & -2 & -1 & -5 & 0 & -1 \\
6 & 0 & 2 & 10 & -2 & 4 & 6 & 14 & 0 & 4 & 2 & 10 & 0 & 2 \\
9 & 0 & 3 & 15 & -3 & 6 & 9 & 21 & 0 & 6 & 3 & 15 & 0 & 3 \\
21 & 0 & 7 & 35 & -7 & 14 & 21 & 49 & 0 & 14 & 7 & 35 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 2 & 10 & -2 & 4 & 6 & 14 & 0 & 4 & 2 & 10 & 0 & 2 \\
3 & 0 & 1 & 5 & -1 & 2 & 3 & 7 & 0 & 2 & 1 & 5 & 0 & 1 \\
\hline
15 & 0 & 5 & 25 & -5 & 10 & 15 & 35 & 0 & 10 & 5 & 25 & 0 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 5 & -1 & 2 & 3 & 7 & 0 & 2 & 1 & 5 & 0 & 1
\end{array}\right]
$$

We see the major product of A with its transpose is a symmetric super matrix.

The symmetry in them occurs in a very special way the main diagonal component is a symmetric matrix where as the other components are transpose of each other; this can be observed from the above super matrix.

Example 3.1.24: Let

$$A = \left[\begin{array}{c} 3 \\ \hline 1 \\ 2 \\ \hline 0 \\ 1 \\ 4 \\ \hline 1 \\ 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\right]$$

be the super column vector.

To find the major product of A with its transpose.

$$A^t = [3 \mid 1\ 2 \mid 0\ 1\ 4 \mid 10\ 2\ 3\ 4\ 5\ 6].$$

We see the major product of A and A^t results in a special symmetric super matrix whose natural order as well as the matrix order is a square matrix.

We find AA^t. $AA^t = \left[\begin{array}{c} 3 \\ \hline 1 \\ 2 \\ \hline 0 \\ 1 \\ 4 \\ \hline 1 \\ 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\right]$ [3 | 1 2 | 0 1 4 | 10 2 3 4 5 6].

$$= \left[\begin{array}{c|c|c|c}
3\times 3 & 3(1\ \ 2) & 3(0\ \ 1\ \ 4) & 3(1\ \ 0\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6) \\ \hline
\begin{bmatrix}1\\2\end{bmatrix}3 & \begin{bmatrix}1\\2\end{bmatrix}(1\ \ 2) & \begin{bmatrix}1\\2\end{bmatrix}(0\ \ 1\ \ 4) & \begin{bmatrix}1\\2\end{bmatrix}(1\ \ 0\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6) \\ \hline
\begin{bmatrix}0\\1\\4\end{bmatrix}3 & \begin{bmatrix}0\\1\\4\end{bmatrix}(1\ \ 2) & \begin{bmatrix}0\\1\\4\end{bmatrix}(0\ \ 1\ \ 4) & \begin{bmatrix}0\\1\\4\end{bmatrix}(1\ \ 0\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6) \\ \hline
\begin{bmatrix}1\\0\\2\\3\\4\\5\\6\end{bmatrix}3 & \begin{bmatrix}1\\0\\2\\3\\4\\5\\6\end{bmatrix}(1\ \ 2) & \begin{bmatrix}1\\0\\2\\3\\4\\5\\6\end{bmatrix}(0\ \ 1\ \ 4) & \begin{bmatrix}1\\0\\2\\3\\4\\5\\6\end{bmatrix}(1\ \ 0\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6)
\end{array}\right]$$

$$= \left[\begin{array}{c|cc|ccc|ccccccc}
9 & 3 & 6 & 0 & 3 & 12 & 3 & 0 & 6 & 9 & 12 & 15 & 18 \\
\hline
3 & 1 & 2 & 0 & 1 & 4 & 1 & 0 & 2 & 3 & 4 & 5 & 6 \\
6 & 2 & 4 & 0 & 2 & 8 & 2 & 0 & 4 & 6 & 8 & 10 & 12 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 2 & 0 & 1 & 4 & 1 & 0 & 2 & 3 & 4 & 5 & 6 \\
12 & 4 & 8 & 0 & 4 & 16 & 4 & 0 & 8 & 12 & 16 & 20 & 24 \\
\hline
3 & 1 & 2 & 0 & 1 & 4 & 1 & 0 & 2 & 3 & 4 & 5 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 4 & 0 & 2 & 8 & 2 & 0 & 4 & 6 & 8 & 10 & 12 \\
9 & 3 & 6 & 0 & 3 & 12 & 3 & 0 & 6 & 9 & 12 & 15 & 18 \\
12 & 4 & 8 & 0 & 4 & 16 & 4 & 0 & 8 & 12 & 16 & 20 & 24 \\
15 & 5 & 10 & 0 & 5 & 20 & 5 & 0 & 10 & 15 & 20 & 25 & 30 \\
18 & 6 & 12 & 0 & 6 & 24 & 6 & 0 & 12 & 18 & 24 & 30 & 36
\end{array}\right]$$

It is easily observed that AA^t is a super symmetric matrix.

This type of minor and major products in case of super vectors are extended to super bivectors which will be exhibited by one or two examples.

Example 3.1.25: Let

$$A = [0\ 1\ 2\ 1 \mid 3\ 4 \mid 0\ 2\ 1] \cup [1\ 0\ 0\ 0\ 1\ 1 \mid 0\ 1\ 2 \mid 4\ 1] \text{ and}$$

$$B = \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \hline 0 \\ 1 \\ \hline 1 \\ 0 \\ 1 \end{array}\right] \cup \left[\begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ \hline 1 \\ 2 \\ 3 \\ \hline 0 \\ 1 \end{array}\right]$$

be two super bivectors. The minor product of $A = A_1 \cup A_2$ with $B = B_1 \cup B_2$ is given by

$$AB = (A_1 \cup A_2)\,(B_1 \cup B_2)$$

$$= A_1\,B_1 \cup A_2\,B_2$$

$$= [0\;1\;2\;1 \mid 3\;4 \mid 0\;2\;1] \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \hline 0 \\ 1 \\ \hline 1 \\ 0 \\ 1 \end{array}\right] \cup [1\;0\;0\;0\;1\;1 \mid 0\;1\;2 \mid 4\;1] \left[\begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ \hline 1 \\ 2 \\ 3 \\ \hline 0 \\ 1 \end{array}\right]$$

$$= \left\{ [0\ \ 1\ \ 2\ \ 1]\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + [3\ \ 4]\begin{bmatrix}0\\1\end{bmatrix} + [0\ \ 2\ \ 1]\begin{bmatrix}1\\0\\1\end{bmatrix} \right\} \cup$$

$$\left\{ [1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1]\begin{bmatrix}0\\1\\1\\1\\0\\0\end{bmatrix} + [0\ \ 1\ \ 2]\begin{bmatrix}1\\2\\3\end{bmatrix} + [4\ \ 1]\begin{bmatrix}0\\1\end{bmatrix} \right\}$$

$$= \{12 + 4 + 1\} \cup \{0 + 8 + 1\}$$

$$= \{17\} \cup \{9\}.$$

We give yet another example.

Example 3.1.26: Let

$$A = [0\ 1\ 2\ 3\ 4 \mid 5\ 6 \mid 7 \mid 8\ 9\ 0\ 1\ 2\ 3\ 4] \cup [0\ 1\ 2 \mid 0\ 1 \mid 3\ 0\ 1\ 5]$$

and

$$B = \left[\begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ 5 \\ \hline 1 \\ 0 \\ \hline 2 \\ \hline 1 \\ 0 \\ 1 \\ 0 \\ 2 \\ \hline -1 \\ 0 \end{array}\right] \cup \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ \hline 5 \\ 0 \\ \hline 2 \\ 1 \\ 0 \\ 2 \end{array}\right]$$

be two super bivectors.

To find the minor product of A with B.

(In case of minor product we see each of the component submatrix of the row super matrix is multiplied by the corresponding component submatrix of the column super matrix we get the resultant to be a singleton bimatrix, which is clearly not a super matrix).

$$AB = [0\ 1\ 2\ 3\ 4 \mid 5\ 6 \mid 7 \mid 8\ 9\ 0\ 1\ 2\ 3\ 4] \left[\begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ 5 \\ \hline 1 \\ 0 \\ \hline 2 \\ \hline 1 \\ 0 \\ 1 \\ 0 \\ 2 \\ \hline -1 \\ 0 \end{array}\right] \cup$$

$$[0\ 1\ 2 \mid 0\ 1 \mid 3\ 0\ 1\ 5] \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ \hline 5 \\ 0 \\ \hline 2 \\ 1 \\ 0 \\ 2 \end{array}\right]$$

$$= \left\{ \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7 \times 2 + \begin{bmatrix} 8 & 9 & 0 & 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 2 \\ -1 \\ 0 \end{bmatrix} \right\}$$

$$\cup \left\{ \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix} \right\}$$

$$= \{24 + 5 + 14 + 9\} \cup \{0 + 0 + 16\} = \{52\} \cup \{16\}.$$

Now we proceed onto illustrate the major product of two super bivectors in an analogous way as we have done for super vector by some examples.

Example 3.1.27: Let A =

$$\left[\begin{array}{c} 0 \\ 1 \\ 2 \\ \hline 3 \\ 1 \\ \hline 4 \\ 5 \\ 6 \\ 7 \\ 0 \\ 1 \end{array} \right] \cup \left[\begin{array}{c} 1 \\ 0 \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline 0 \\ 1 \\ 5 \\ \hline 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array} \right]$$

and B = [1 0 | 2 4 3 1 | 0 1 2 3 0 1 0] ∪ [3 1 2 0 | 1 1 2 0 0 | 3 0 | 2 5 1 0 1 1] be two super bivectors. To find the major product of these two super bivectors.

$$AB = (A_1 \cup A_2)\,(B_1 \cup B_2)$$
$$= A_1 B_1 \cup A_2 B_2$$

$$= \left[\begin{array}{c} 0 \\ 1 \\ 2 \\ \hline 3 \\ 1 \\ \hline 4 \\ 5 \\ 6 \\ 7 \\ 0 \\ 1 \end{array}\right] [\,1\ 0 \mid 2\ 4\ 3\ 1 \mid 0\ 1\ 2\ 3\ 0\ 1\ 0\,] \cup$$

$$\left[\begin{array}{c} 1 \\ 0 \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline 0 \\ 1 \\ 5 \\ \hline 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}\right] [\,3\ 1\ 2\ 0 \mid 1\ 1\ 2\ 0\ 0 \mid 3\ 0 \mid 2\ 5\ 1\ 0\ 1\ 1\,]$$

$$
= \left[\begin{array}{c|c|c}
\begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}2 & 4 & 3 & 1\end{bmatrix} & \begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}0 & 1 & 2 & 3 & 0 & 1 & 0\end{bmatrix} \\
\hline
\begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}2 & 4 & 3 & 1\end{bmatrix} & \begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}0 & 1 & 2 & 3 & 0 & 1 & 0\end{bmatrix} \\
\hline
\begin{bmatrix}4\\5\\6\\7\\0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} & \begin{bmatrix}4\\5\\6\\7\\0\\1\end{bmatrix}\begin{bmatrix}2 & 4 & 3 & 1\end{bmatrix} & \begin{bmatrix}4\\5\\6\\7\\0\\1\end{bmatrix}\begin{bmatrix}0 & 1 & 2 & 3 & 0 & 1 & 0\end{bmatrix}
\end{array}\right] \cup
$$

$$
\left[\begin{array}{c|c|c|c}
\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}3 & 1 & 2 & 0\end{bmatrix} & \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 1 & 2 & 0 & 0\end{bmatrix} & \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}3 & 0\end{bmatrix} & \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}2 & 5 & 1 & 0 & 1 & 1\end{bmatrix} \\
\hline
\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}3 & 1 & 2 & 0\end{bmatrix} & \begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}1 & 1 & 2 & 0 & 0\end{bmatrix} & \begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}3 & 0\end{bmatrix} & \begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}2 & 5 & 1 & 0 & 1 & 1\end{bmatrix} \\
\hline
\begin{bmatrix}0\\1\\5\end{bmatrix}\begin{bmatrix}3 & 1 & 2 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\5\end{bmatrix}\begin{bmatrix}1 & 1 & 2 & 0 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\5\end{bmatrix}\begin{bmatrix}3 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\5\end{bmatrix}\begin{bmatrix}2 & 5 & 1 & 0 & 1 & 1\end{bmatrix} \\
\hline
\begin{bmatrix}0\\1\\0\\1\end{bmatrix}\begin{bmatrix}3 & 1 & 2 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 2 & 0 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix}\begin{bmatrix}3 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix}\begin{bmatrix}2 & 5 & 1 & 0 & 1 & 1\end{bmatrix}
\end{array}\right]
$$

$$
= \left[\begin{array}{cc|cccc|ccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 4 & 3 & 1 & 0 & 1 & 2 & 3 & 0 & 1 & 0 \\
0 & 2 & 4 & 8 & 6 & 2 & 0 & 2 & 4 & 6 & 0 & 2 & 0 \\
\hline
0 & 3 & 6 & 12 & 9 & 3 & 0 & 3 & 6 & 9 & 0 & 3 & 0 \\
0 & 1 & 2 & 4 & 3 & 1 & 0 & 1 & 2 & 3 & 0 & 1 & 0 \\
\hline
0 & 4 & 8 & 16 & 12 & 4 & 0 & 4 & 8 & 12 & 0 & 4 & 0 \\
0 & 5 & 10 & 20 & 15 & 5 & 0 & 5 & 10 & 15 & 0 & 5 & 0 \\
0 & 6 & 12 & 24 & 18 & 6 & 0 & 6 & 12 & 18 & 0 & 6 & 0 \\
0 & 7 & 14 & 28 & 21 & 7 & 0 & 7 & 14 & 21 & 0 & 7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 4 & 3 & 1 & 0 & 1 & 2 & 3 & 0 & 1 & 0
\end{array}\right] \cup
$$

$$
\left[\begin{array}{cccc|ccccc|cc|cccccc}
3 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 3 & 0 & 2 & 5 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
3 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 3 & 0 & 2 & 5 & 1 & 0 & 1 & 1 \\
6 & 2 & 4 & 0 & 2 & 2 & 4 & 0 & 0 & 6 & 0 & 4 & 10 & 2 & 0 & 2 & 2 \\
9 & 3 & 6 & 0 & 3 & 3 & 6 & 0 & 0 & 9 & 0 & 6 & 15 & 3 & 0 & 3 & 3 \\
12 & 4 & 8 & 0 & 4 & 4 & 8 & 0 & 0 & 12 & 0 & 8 & 20 & 4 & 0 & 4 & 4 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 3 & 0 & 2 & 5 & 1 & 0 & 1 & 1 \\
15 & 5 & 10 & 0 & 5 & 5 & 10 & 0 & 0 & 15 & 0 & 10 & 25 & 5 & 0 & 5 & 5 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 3 & 0 & 2 & 5 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 3 & 0 & 2 & 5 & 1 & 0 & 1 & 1
\end{array}\right]
$$

Thus the major product of two super bivectors is a super bimatrix.

Now we give one more numerical illustration of the major product of two superbivectors

Example 3.1.28: Let

$$A = \left[\begin{array}{c} 3 \\ 1 \\ 0 \\ \hline 1 \\ 1 \\ 1 \\ 2 \\ \hline 0 \\ 0 \\ 0 \\ 6 \\ 1 \\ 0 \\ 0 \\ \hline 2 \\ 1 \end{array}\right] \cup \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ 0 \\ 1 \\ 0 \\ 1 \\ \hline 0 \\ 2 \\ 3 \\ \hline 2 \\ 1 \\ \hline 5 \end{array}\right]$$

$= A_1 \cup A_2$ and $B = B_1 \cup B_2 = [3\ 1\ 0\ 0 \mid 1\ 2\ 0\ 5\ 6\ 7 \mid 2\ 1\ 0] \cup [1\ 0\ 1 \mid 2\ 3\ 4\ 0\ 5 \mid 1\ 1\ 0\ 0\ 0\ 2\ 1\ 0\ 3]$ be two super bivectors to find the major product of A with B.

In case of the major product of a column superbimatrix with a row row super bimatrix we get the resultant to be a super bimatrix rectangular square.

$$AB = (A_1 \cup A_2)\ (B_1 \cup B_2) = A_1 B_1 \cup A_2 B_2$$

$$= \begin{bmatrix} 3 \\ 1 \\ 0 \\ \hline 1 \\ 1 \\ 1 \\ 2 \\ \hline 0 \\ 0 \\ 0 \\ 6 \\ 1 \\ 0 \\ 0 \\ \hline 2 \\ 1 \end{bmatrix} [3\,1\,0\,0 \mid 1\,2\,0\,5\,6\,7 \mid 2\,1\,0] \cup \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 2 \\ 3 \\ 2 \\ 1 \\ 5 \end{bmatrix} [1\,0\,1 \mid 2\,3\,4\,0\,5 \mid 1\,1\,0\,0\,0\,2\,1\,0\,3]$$

$$\left[\begin{array}{c|c|c}
\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 & 5 & 6 & 7 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \\ \hline
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 & 5 & 6 & 7 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \\ \hline
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 & 5 & 6 & 7 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \\
\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 & 5 & 6 & 7 \end{bmatrix} & \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \end{bmatrix}
\end{array}\right]$$

$$
\subset \left[\begin{array}{c|c|c}
\begin{bmatrix}1\\2\\3\\0\\1\\0\\1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\end{bmatrix} & \begin{bmatrix}1\\2\\3\\0\\1\\0\\1\end{bmatrix}\begin{bmatrix}2 & 3 & 4 & 0 & 5\end{bmatrix} & \begin{bmatrix}1\\2\\3\\0\\1\\0\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3\end{bmatrix} \\
\hline
\begin{bmatrix}0\\2\\3\end{bmatrix}\begin{bmatrix}1 & 0 & 1\end{bmatrix} & \begin{bmatrix}0\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4 & 0 & 5\end{bmatrix} & \begin{bmatrix}0\\2\\3\end{bmatrix}\begin{bmatrix}1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3\end{bmatrix} \\
\hline
\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\end{bmatrix} & \begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}2 & 3 & 4 & 0 & 5\end{bmatrix} & \begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3\end{bmatrix} \\
\hline
5\begin{bmatrix}1 & 0 & 1\end{bmatrix} & 5\begin{bmatrix}2 & 3 & 4 & 0 & 5\end{bmatrix} & 5\begin{bmatrix}1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3\end{bmatrix}
\end{array}\right]
$$

$$
= \left[\begin{array}{cccc|cccccc|ccc}
9 & 3 & 0 & 0 & 3 & 6 & 0 & 15 & 18 & 21 & 6 & 3 & 0 \\
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0 \\
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0 \\
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0 \\
6 & 2 & 0 & 0 & 2 & 4 & 0 & 10 & 12 & 14 & 4 & 2 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
18 & 6 & 0 & 0 & 6 & 12 & 0 & 30 & 36 & 42 & 12 & 6 & 0 \\
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
6 & 2 & 0 & 0 & 2 & 4 & 0 & 10 & 12 & 14 & 4 & 2 & 0 \\
3 & 1 & 0 & 0 & 1 & 2 & 0 & 5 & 6 & 7 & 2 & 1 & 0
\end{array}\right] \subset
$$

$$
\left[\begin{array}{ccc|ccccc|ccccccccc}
1 & 0 & 1 & 2 & 3 & 4 & 0 & 5 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\
2 & 0 & 2 & 4 & 6 & 8 & 0 & 10 & 2 & 2 & 0 & 0 & 0 & 4 & 2 & 0 & 6 \\
3 & 0 & 3 & 6 & 9 & 12 & 0 & 15 & 3 & 3 & 0 & 0 & 0 & 6 & 3 & 0 & 9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 2 & 3 & 4 & 0 & 5 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 2 & 3 & 4 & 0 & 5 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 2 & 4 & 6 & 8 & 0 & 10 & 2 & 2 & 0 & 0 & 0 & 6 & 2 & 0 & 6 \\
3 & 0 & 3 & 6 & 9 & 12 & 0 & 15 & 3 & 3 & 0 & 0 & 0 & 6 & 3 & 0 & 9 \\
\hline
2 & 0 & 2 & 4 & 6 & 8 & 0 & 10 & 2 & 2 & 0 & 0 & 0 & 4 & 2 & 0 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 0 & 5 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\
\hline
5 & 0 & 5 & 10 & 15 & 20 & 0 & 25 & 5 & 5 & 0 & 0 & 0 & 10 & 5 & 0 & 15
\end{array}\right]
$$

We see AB the minor product of the two super bivectors is a super bimatrix.

Now we proceed on to illustrate the major product of a super column vector with its transpose, by some numerical examples.

We see the major product of a column super vector with its transpose which is a row super vector get the resultant to be a square symmetric super matrix.

Example 3.1.29: Let

$$A = \left[\begin{array}{c} 2\\ 1\\ 0\\ 1\\ \hline 3\\ 5\\ 0\\ 1\\ 0\\ \hline 3\\ 2\\ 1\\ \hline 7 \end{array}\right]$$

be a super column vector. To find the major product of A with its transpose $A^t = [2\ 1\ 0\ 1\ |\ 3\ 5\ 0\ 1\ 0\ |\ 3\ 2\ 1\ |\ 7]$.

$$AA^t = \left[\begin{array}{c} 2\\ 1\\ 0\\ 1\\ \hline 3\\ 5\\ 0\\ 1\\ 0\\ \hline 3\\ 2\\ 1\\ \hline 7 \end{array}\right] [2\ 1\ 0\ 1\ |\ 3\ 5\ 0\ 1\ 0\ |\ 3\ 2\ 1\ |\ 7]$$

$$
= \left[\begin{array}{c|c|c|c}
\begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}3 & 5 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}3 & 2 & 1\end{bmatrix} & \begin{bmatrix}2\\1\\0\\1\end{bmatrix}7 \\
\hline
\begin{bmatrix}3\\5\\0\\1\\0\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}3\\5\\0\\1\\0\end{bmatrix}\begin{bmatrix}3 & 5 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}3\\5\\0\\1\\0\end{bmatrix}\begin{bmatrix}3 & 2 & 1\end{bmatrix} & \begin{bmatrix}3\\5\\0\\1\\0\end{bmatrix}7 \\
\hline
\begin{bmatrix}3\\2\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}3\\2\\1\end{bmatrix}\begin{bmatrix}3 & 5 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}3\\2\\1\end{bmatrix}\begin{bmatrix}3 & 2 & 1\end{bmatrix} & \begin{bmatrix}3\\2\\1\end{bmatrix}7 \\
\hline
7\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & 7\begin{bmatrix}3 & 5 & 0 & 1 & 0\end{bmatrix} & 7\begin{bmatrix}3 & 2 & 1\end{bmatrix} & [7][7]
\end{array}\right]
$$

$$
= \left[\begin{array}{cccc|ccccc|ccc|c}
4 & 2 & 0 & 2 & 6 & 10 & 0 & 2 & 0 & 6 & 4 & 2 & 14 \\
2 & 1 & 0 & 1 & 3 & 5 & 0 & 1 & 0 & 3 & 2 & 1 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 3 & 5 & 0 & 1 & 0 & 3 & 2 & 1 & 7 \\
\hline
6 & 3 & 0 & 3 & 9 & 15 & 0 & 3 & 0 & 9 & 6 & 3 & 21 \\
10 & 5 & 0 & 5 & 15 & 25 & 0 & 5 & 0 & 15 & 10 & 5 & 35 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 3 & 5 & 0 & 1 & 0 & 3 & 2 & 1 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
6 & 3 & 0 & 3 & 9 & 15 & 0 & 3 & 0 & 9 & 6 & 3 & 21 \\
4 & 2 & 0 & 2 & 6 & 10 & 0 & 2 & 0 & 6 & 4 & 2 & 14 \\
2 & 1 & 0 & 1 & 3 & 5 & 0 & 1 & 0 & 3 & 2 & 1 & 0 \\
\hline
14 & 7 & 0 & 7 & 21 & 35 & 0 & 7 & 0 & 21 & 14 & 7 & 49
\end{array}\right].
$$

It is easily verified the major product of a super column vector with its transpose is a super matrix.

Having just seen the major product of a super column vector with its transpose now we proceed on to illustrate the major product of a super column bivector with its transpose by a numerical example in an analogous way.

Example 3.1.30: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{c} 2 \\ 1 \\ 0 \\ 1 \\ \hline 3 \\ 1 \\ 0 \\ \hline 5 \\ 7 \\ 1 \\ 2 \\ 0 \\ 1 \end{array}\right] \cup \left[\begin{array}{c} 1 \\ 2 \\ \hline 3 \\ 0 \\ 1 \\ 4 \\ \hline 0 \\ 5 \\ 1 \\ \hline 7 \\ 3 \end{array}\right]$$

be a super column bivector.

$A^t = A_1^t \cup A_2^t = [2\ 1\ 0\ 1 \mid 3\ 1\ 0 \mid 5\ 7\ 1\ 2\ 0\ 1] \cup [1\ 2 \mid 3\ 0\ 1\ 4 \mid 0\ 5\ 1 \mid 7\ 3]$.

The major biproduct of $AA^t = (A_1 \cup A_2)\ (A_1^t \cup A_2^t)$ which will result in a symmetric square super bimatrix.

$$= A_1A_1^t \cup A_2A_2^t = \left[\begin{array}{c} 2 \\ 1 \\ 0 \\ 1 \\ \hline 3 \\ 1 \\ 0 \\ \hline 5 \\ 7 \\ 1 \\ 2 \\ 0 \\ 1 \end{array}\right] [2\ 1\ 0\ 1 \mid 3\ 1\ 0 \mid 5\ 7\ 1\ 2\ 0\ 1] \cup$$

$$\left[\begin{array}{c} 1 \\ 2 \\ \hline 3 \\ 0 \\ 1 \\ 4 \\ \hline 0 \\ 5 \\ 1 \\ \hline 7 \\ 3 \end{array}\right] [1\ 2 \mid 3\ 0\ 1\ 4 \mid 0\ 5\ 1 \mid 7\ 3]$$

$$= \left[\begin{array}{c|c|c}
\begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}3 & 1 & 0\end{bmatrix} & \begin{bmatrix}2\\1\\0\\1\end{bmatrix}\begin{bmatrix}5 & 7 & 1 & 2 & 0 & 1\end{bmatrix} \\ \hline
\begin{bmatrix}3\\1\\0\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}3\\1\\0\end{bmatrix}\begin{bmatrix}3 & 1 & 0\end{bmatrix} & \begin{bmatrix}3\\1\\0\end{bmatrix}\begin{bmatrix}5 & 7 & 1 & 2 & 0 & 1\end{bmatrix} \\ \hline
\begin{bmatrix}5\\7\\1\\2\\0\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 1\end{bmatrix} & \begin{bmatrix}5\\7\\1\\2\\0\\1\end{bmatrix}\begin{bmatrix}3 & 1 & 0\end{bmatrix} & \begin{bmatrix}5\\7\\1\\2\\0\\1\end{bmatrix}\begin{bmatrix}5 & 7 & 1 & 2 & 0 & 1\end{bmatrix}
\end{array}\right]$$

$$\cup \left[\begin{array}{c|c|c|c}
\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}3 & 0 & 1 & 4\end{bmatrix} & \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}0 & 5 & 1\end{bmatrix} & \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}7 & 3\end{bmatrix} \\ \hline
\begin{bmatrix}3\\0\\1\\4\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}3\\0\\1\\4\end{bmatrix}\begin{bmatrix}3 & 0 & 1 & 4\end{bmatrix} & \begin{bmatrix}3\\0\\1\\4\end{bmatrix}\begin{bmatrix}0 & 5 & 1\end{bmatrix} & \begin{bmatrix}3\\0\\1\\4\end{bmatrix}\begin{bmatrix}7 & 3\end{bmatrix} \\ \hline
\begin{bmatrix}0\\5\\1\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}0\\5\\1\end{bmatrix}\begin{bmatrix}3 & 0 & 1 & 4\end{bmatrix} & \begin{bmatrix}0\\5\\1\end{bmatrix}\begin{bmatrix}0 & 5 & 1\end{bmatrix} & \begin{bmatrix}0\\5\\1\end{bmatrix}\begin{bmatrix}7 & 3\end{bmatrix} \\ \hline
\begin{bmatrix}7\\3\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} & \begin{bmatrix}7\\3\end{bmatrix}\begin{bmatrix}3 & 0 & 1 & 4\end{bmatrix} & \begin{bmatrix}7\\3\end{bmatrix}\begin{bmatrix}0 & 5 & 1\end{bmatrix} & \begin{bmatrix}7\\3\end{bmatrix}\begin{bmatrix}7 & 3\end{bmatrix}
\end{array}\right]$$

$$
= \left[\begin{array}{cccc|ccc|cccccc}
4 & 2 & 0 & 2 & 6 & 2 & 0 & 10 & 14 & 2 & 4 & 0 & 2 \\
2 & 1 & 0 & 1 & 3 & 1 & 0 & 5 & 7 & 1 & 2 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 3 & 1 & 0 & 5 & 7 & 1 & 2 & 0 & 1 \\
\hline
6 & 3 & 0 & 3 & 9 & 3 & 0 & 15 & 21 & 3 & 6 & 0 & 3 \\
2 & 1 & 0 & 1 & 3 & 1 & 0 & 5 & 7 & 1 & 2 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
10 & 5 & 0 & 5 & 15 & 5 & 0 & 25 & 35 & 5 & 10 & 0 & 5 \\
14 & 7 & 0 & 7 & 21 & 7 & 0 & 35 & 49 & 7 & 14 & 0 & 7 \\
2 & 1 & 0 & 1 & 3 & 1 & 0 & 5 & 7 & 1 & 2 & 0 & 1 \\
4 & 2 & 0 & 2 & 6 & 2 & 0 & 10 & 14 & 2 & 4 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 3 & 1 & 0 & 5 & 7 & 1 & 2 & 0 & 1
\end{array}\right]
$$

$$
\cup \left[\begin{array}{cc|cccc|ccc|cc}
1 & 2 & 3 & 0 & 1 & 4 & 0 & 5 & 1 & 7 & 3 \\
2 & 4 & 6 & 0 & 2 & 8 & 0 & 10 & 2 & 14 & 6 \\
\hline
3 & 6 & 9 & 0 & 3 & 12 & 0 & 15 & 3 & 21 & 9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 3 & 0 & 1 & 4 & 0 & 5 & 1 & 7 & 3 \\
4 & 8 & 12 & 0 & 4 & 16 & 0 & 20 & 4 & 28 & 12 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 10 & 15 & 0 & 5 & 20 & 0 & 25 & 5 & 35 & 15 \\
1 & 2 & 3 & 0 & 1 & 4 & 0 & 5 & 1 & 7 & 3 \\
\hline
7 & 14 & 21 & 0 & 7 & 28 & 0 & 35 & 7 & 49 & 21 \\
3 & 6 & 9 & 0 & 3 & 12 & 0 & 15 & 3 & 21 & 9
\end{array}\right].
$$

We see the major product of a super column bivector with its transpose a super bimatrix which is symmetric.

Example 3.1.31: Let

$$A = \left[\begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \\ \hline 2 \\ 1 \\ 3 \\ \hline 0 \\ 1 \\ 3 \\ 7 \\ 5 \\ 1 \end{array}\right] \cup \left[\begin{array}{c} 7 \\ 3 \\ 6 \\ 0 \\ -1 \\ 2 \\ 1 \\ \hline 0 \\ 0 \\ 3 \\ \hline 3 \\ 1 \\ 1 \end{array}\right]$$

$= A_1 \cup A_2$ be a super column bivector. $A^t = A_1^t \cup A_2^t$ = [1 0 1 0 | 2 1 3 | 0 1 3 7 5 1] $\cup$ [7 3 6 0 –1 2 1 | 0 0 3 | 3 1 1]; we find $AA^t = (A_1 \cup A_2)\,(A_1^t \cup A_2^t) = A_1A_1^t \cup A_2A_2^t$

$$= \left[\begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \\ \hline 2 \\ 1 \\ 3 \\ \hline 0 \\ 1 \\ 3 \\ 7 \\ 5 \\ 1 \end{array}\right] \left[\begin{array}{cccc|ccc|cccccc} 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{c} 7 \\ 3 \\ 6 \\ 0 \\ -1 \\ 2 \\ 1 \\ \hline 0 \\ 0 \\ 3 \\ \hline 3 \\ 1 \\ 1 \end{array}\right] [7\ 3\ 6\ 0\ -1\ 2\ 1 \mid 0\ 0\ 3 \mid 3\ 1\ 1]$$

$$= \left[\begin{array}{c|c|c}
\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\end{bmatrix}\begin{bmatrix}2 & 1 & 3\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\end{bmatrix}\begin{bmatrix}0 & 1 & 3 & 7 & 5 & 1\end{bmatrix} \\ \hline
\begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}2 & 1 & 3\end{bmatrix} & \begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}0 & 1 & 3 & 7 & 5 & 1\end{bmatrix} \\ \hline
\begin{bmatrix}0\\1\\3\\7\\5\\1\end{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix} & \begin{bmatrix}0\\1\\3\\7\\5\\1\end{bmatrix}\begin{bmatrix}2 & 1 & 3\end{bmatrix} & \begin{bmatrix}0\\1\\3\\7\\5\\1\end{bmatrix}\begin{bmatrix}0 & 1 & 3 & 7 & 5 & 1\end{bmatrix}
\end{array}\right]$$

$$\cup \left(\begin{array}{c|c|c} \begin{bmatrix} 7 \\ 3 \\ 6 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 7 & 3 & 6 & 0 & -1 & 2 & 1 \end{bmatrix} & \begin{bmatrix} 7 \\ 3 \\ 6 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 \end{bmatrix} & \begin{bmatrix} 7 \\ 3 \\ 6 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \end{bmatrix} \\ \hline \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 7 & 3 & 6 & 0 & -1 & 2 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \end{bmatrix} \\ \hline \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 7 & 3 & 6 & 0 & -1 & 2 & 1 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \end{bmatrix} \end{array} \right)$$

$$= \left[\begin{array}{cccc|ccc|cccccc} 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 2 & 0 & 4 & 2 & 6 & 0 & 2 & 6 & 14 & 10 & 2 \\ 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \\ 3 & 0 & 3 & 0 & 6 & 3 & 9 & 0 & 3 & 9 & 21 & 15 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \\ 3 & 0 & 3 & 0 & 6 & 3 & 9 & 0 & 3 & 9 & 21 & 15 & 3 \\ 7 & 0 & 7 & 0 & 14 & 7 & 21 & 0 & 7 & 21 & 49 & 35 & 7 \\ 5 & 0 & 5 & 0 & 10 & 5 & 15 & 0 & 5 & 15 & 35 & 25 & 5 \\ 1 & 0 & 1 & 0 & 2 & 1 & 3 & 0 & 1 & 3 & 7 & 5 & 1 \end{array} \right]$$

$$\cup \left[\begin{array}{ccccccc|ccc|ccc}
49 & 21 & 42 & 0 & -7 & 14 & 7 & 0 & 0 & 21 & 21 & 7 & 7 \\
21 & 9 & 18 & 0 & -3 & 6 & 3 & 0 & 0 & 9 & 9 & 3 & 3 \\
42 & 18 & 24 & 0 & -6 & 12 & 6 & 0 & 0 & 18 & 18 & 6 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-7 & -3 & -6 & 0 & 1 & -2 & -1 & 0 & 0 & -3 & -3 & -1 & -1 \\
14 & 6 & 12 & 0 & -2 & 4 & 2 & 0 & 0 & 6 & 6 & 2 & 2 \\
7 & 3 & 6 & 0 & -1 & 2 & 1 & 0 & 0 & 3 & 3 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
21 & 9 & 18 & 0 & -3 & 6 & 3 & 0 & 0 & 9 & 9 & 3 & 3 \\
\hline
21 & 9 & 18 & 0 & -3 & 6 & 3 & 0 & 0 & 9 & 9 & 3 & 3 \\
7 & 3 & 6 & 0 & -1 & 2 & 1 & 0 & 0 & 3 & 3 & 1 & 1 \\
7 & 3 & 6 & 0 & -1 & 2 & 1 & 0 & 0 & 3 & 3 & 1 & 1
\end{array}\right]$$

It is easily seen AA^t is a symmetric super bimatrix.

3.2 Operations on Super bivectors

In this section we define the notion of super row bivector of type B and super column bivector of type B. We define the basic operations of addition, subtraction, minor and major product whenever such operations are compatible. This concept is made easy to understand by numerous examples.

Now we proceed onto define the new notion of super row bivector of type B and super column bivector of type B.

DEFINITION 3.2.1: *Let $A = A_1 \cup A_2$ where A_1 and A_2 are distinct super row vectors of type B then we call A to be a super row bivector of type B (super birow vector of type B).*

We first illustrate this by an example.

Example 3.2.1: Let

$$A = \left[\begin{array}{ccc|cc|ccccc} 3 & 0 & 1 & 3 & 1 & 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 2 & 0 \\ 5 & 2 & 0 & 2 & 0 & 3 & 0 & 4 & 0 & 5 \end{array}\right]$$

$$\cup \left[\begin{array}{ccccc|ccc|cc} 1 & 0 & 1 & 2 & 3 & 3 & 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 2 & 2 & 3 \\ 0 & 5 & 1 & 2 & 0 & 1 & 1 & 4 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \end{array}\right] =$$

$A_1 \cup A_2$. A is a super row bivector of type B.

Example 3.2.2: Let

$$A = \left[\begin{array}{ccc|ccccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 3 & 1 & 1 & 2 & 3 & 4 \\ 3 & 1 & 5 & 5 & 0 & 1 & 0 & 2 & 0 & 1 & 2 & 3 \end{array}\right] \cup$$

$$\left[\begin{array}{cc|ccccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$= A_1 \cup A_2$; we see A is a super row bivector of type B.

Example 3.2.3: Let $A = A_1 \cup A_2 =$

$$\left[\begin{array}{c|ccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|cc|ccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right];$$

A is a super row bivector of type B. We see A is a zero super row bivector of type B.

Example 3.2.4: Let $A = A_1 \cup A_2 =$

$$\left[\begin{array}{cc|ccc|c|ccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

be a super row bivector of type B.

We call this as a super unit row bivector of type B or super row unit bivector or bisuper unit vector of type B.

We now proceed onto define the notion of super column bivector of type B.

DEFINITION 3.2.2: *Let $A = A_1 \cup A_2$ where A_1 and A_2 are distinct super column vectors of type B. We call A to be a super column bivector of type B.*

We illustrate this by the following examples.

Example 3.2.5: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{ccccc} 3 & 0 & 1 & 4 & 5 \\ 2 & 1 & 0 & 3 & 7 \\ \hline 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ \hline 1 & 2 & 3 & 0 & 1 \\ 4 & 0 & 5 & 1 & 2 \\ 3 & 7 & 2 & 0 & 0 \end{array}\right] \cup \left[\begin{array}{ccc} 3 & 0 & 1 \\ 4 & 5 & 8 \\ 7 & 6 & 7 \\ \hline 8 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \\ \hline 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 1 & 2 \end{array}\right];$$

A is a super column bivector of type B.

We give yet another example before we proceed to give examples of the notion of super zero column bivector and super column unit bivector of type B.

Example 3.2.6: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ \hline 3 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 \end{array}\right] \cup \left[\begin{array}{cc} 3 & 1 \\ 2 & 1 \\ 1 & 2 \\ 1 & 3 \\ \hline 3 & 0 \\ 1 & 0 \\ 1 & 1 \\ \hline 0 & 1 \\ 5 & 1 \end{array}\right];$$

A is a super column bivector of type B.

Example 3.2.7: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \cup \left[\begin{array}{cc} 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}\right];$$

A is defined to be a super zero column bivector of type B.

Example 3.2.8: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \cup \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right];$$

A is not a super zero column bivector of type B as $A_1 = A_2$.

Example 3.2.9: Let $A = A_1 \cup A_2 =$

$$\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cup \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right];$$

A is a super zero column bivector of type B.

Example 3.2.10:

$$\text{Let } A = \left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array}\right] \cup \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right];$$

A is a super unit column bivector of type B.

Now as in case of super row bivectors of type B we can define the transpose of a super row bivector of type B. It is easy to see that the transpose of a super row bivector of type B is a super column bivector of type B and vice versa.

We shall illustrate this situation by some simple examples.

Example 3.2.11: Let $A = A_1 \cup A_2 =$

$$\left[\begin{array}{cccc|c|ccccc} 3 & 1 & 2 & 0 & 3 & 0 & 1 & 0 & 1 & 3 \\ 1 & 0 & 1 & 1 & 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 5 & 1 & 2 & 3 & 4 & 5 \end{array}\right] \cup$$

$$\left[\begin{array}{cc|c|cccc} 3 & 1 & 1 & 0 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 & 5 & 6 & 7 \\ 1 & 1 & 2 & 6 & 0 & 1 & 1 \end{array}\right]$$

be a super row bivector of type B.

Now transpose of A denotes

by $A^t = (A_1 \cup A_2)^t = A_1^t \cup A_2^t$ $$= \left[\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \\ \hline 3 & 4 & 5 \\ \hline 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \\ 3 & 0 & 5 \end{array}\right] \cup \left[\begin{array}{cccc} 3 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 2 \\ \hline 0 & 1 & 0 & 6 \\ 3 & 2 & 5 & 0 \\ 4 & 3 & 6 & 1 \\ 5 & 4 & 7 & 1 \end{array}\right].$$

It is easily seen that A^t is a super column bivector of type B.

Example 3.2.12: Let

$$A = \left[\begin{array}{ccc} 0 & 1 & 3 \\ 1 & 1 & 1 \\ \hline 0 & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 0 & 1 \\ \hline 3 & 0 & 1 \\ 0 & 8 & 4 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right] \cup \left[\begin{array}{cc} 2 & 1 \\ 3 & 5 \\ 6 & 7 \\ 0 & 1 \\ \hline 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ \hline 0 & 1 \\ 2 & 3 \\ 4 & 5 \\ 6 & 7 \\ 8 & 9 \\ 1 & 2 \\ 0 & 3 \end{array}\right]$$

be a super column bivector of type B.

The transpose of A denoted by A^t =

$$A_1^t \cup A_2^t = \left[\begin{array}{cc|ccc|cccccc} 0 & 1 & 0 & 4 & 7 & 3 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 5 & 0 & 0 & 8 & 1 & 1 & 0 & 1 \\ 3 & 1 & 1 & 6 & 1 & 1 & 4 & 1 & 0 & 1 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccc|ccccccc} 2 & 3 & 6 & 0 & 1 & 3 & 5 & 0 & 2 & 4 & 6 & 8 & 1 & 0 \\ 1 & 5 & 7 & 1 & 2 & 4 & 6 & 1 & 3 & 5 & 7 & 9 & 2 & 3 \end{array}\right].$$

Clearly A^t is a super row bivector of type B.

Now as in case of type A super row (column) bivectors we can define identical in structure super row (column) bivectors of type B.

DEFINITION 3.2.3: *Let $A = A_1 \cup A_2$ where A_1 and A_2 are distinct i.e., $A = A_1 \cup A_2$ is a super row bivector of type B. Let $B = B_1 \cup B_2$ be a super row bivector of type B.*

We say A and B similar or "identical in structure" if the length of A_i and B_i are the same and the number of rows in both A_i and B_i are equal and the partitions in A_i and B_i are identical for i=1,2.

By the term partitions are identical we mean if in A_i the first partition is say between the r and $(r+1)^{th}$ column then in B_i also the first partition will be only between the r and $(r+1)^{th}$ column and the number of partition carried out in A_i will be the same as that carried out in B_i, i=1,2.

We will illustrate this by some simple examples.

Example 3.1.13: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{cccc|cccccc|cc} 3 & 0 & 1 & 2 & 3 & 7 & 1 & 2 & 0 & 1 & 3 & 2 \\ 1 & 1 & 0 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 5 \\ 0 & 0 & 0 & 5 & 1 & 1 & 0 & 0 & 0 & 1 & 7 & 8 \end{array}\right]$$

$$\cup \left[\begin{array}{c|cc|ccccc} 2 & 3 & 2 & 3 & 1 & 2 & 5 & 4 \\ 3 & 1 & 0 & 0 & 2 & 3 & 1 & 0 \\ 1 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 7 & 1 & 1 & 2 & 3 & 7 \end{array}\right] \text{and}$$

$$B = \left[\begin{array}{cccc|cccccc|cc} 1 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 3 & 4 & 2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 5 & 6 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 7 & 0 & 0 & 0 & 0 & 8 & 1 & 0 \end{array}\right] \cup$$

$$\left[\begin{array}{c|cc|ccccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

be two super row vectors of type B. We see A and B are "identical in structure". For the bilength of A and B is (12, 8).

Also the partitions are identical in B_i and A_i, i=1,2. The number of rows in A_i equal to the number of rows in B_i, i=1,2.

Example 3.2.14: Let

$$A = A_1 \cup A_2 = \left[\begin{array}{cc|cccc|ccc} 2 & 0 & 0 & 1 & 1 & 3 & 3 & 7 & 8 \\ 1 & 1 & 6 & 2 & 1 & 5 & 1 & 2 & 3 \end{array}\right] \cup$$

$$\left[\begin{array}{ccc|ccccccc|c} 5 & 7 & 8 & 3 & 8 & 8 & 0 & 0 & 5 & 1 & 3 \\ 9 & 1 & 2 & 4 & 1 & 1 & 6 & 8 & 1 & 5 & 7 \\ 0 & 3 & 7 & 3 & 0 & 0 & 7 & 4 & 2 & 7 & 8 \end{array}\right] \text{ and}$$

$$B = B_1 \cup B_2 = \left[\begin{array}{cc|cccc|ccc} 1 & 3 & 5 & 6 & 7 & 8 & 4 & 3 & 8 \\ 2 & 4 & 9 & 0 & 1 & 2 & 2 & 5 & 9 \end{array}\right] \cup$$

$$\left[\begin{array}{ccc|ccccccc|c} 1 & 0 & 2 & 7 & 9 & 8 & 9 & 3 & 1 & 5 & 5 \\ 3 & 4 & 5 & 0 & 1 & 2 & 0 & 3 & 5 & 7 & 7 \\ 6 & 7 & 8 & 6 & 2 & 0 & 0 & 7 & 1 & 1 & 9 \end{array}\right]$$

be two super row bivectors of type B. We see A and B "identical in structure".

Example 3.2.15: Let

$$A = (A_1 \cup A_2) = \left[\begin{array}{ccc|cccccc|c} 3 & 1 & 2 & 3 & 7 & 8 & 1 & 7 & -1 & 1 \\ 5 & 7 & 9 & 1 & 0 & 1 & 5 & 8 & 2 & 2 \\ 0 & 1 & 2 & 7 & 1 & 5 & 6 & 9 & 3 & 3 \\ 1 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 4 & 4 \end{array}\right] \cup$$

$$\left[\begin{array}{cccccc|cccc} 3 & 4 & 1 & 7 & 0 & 1 & 7 & 0 & 8 & 4 \\ 0 & 7 & 0 & 0 & 0 & 1 & 1 & 2 & 5 & 6 \\ 1 & 0 & 0 & 8 & 6 & 2 & 3 & 7 & 0 & 8 \end{array}\right]$$

$$\text{and } B = B_1 \cup B_2 = \left[\begin{array}{ccc|cccccc|c} 1 & 1 & 1 & 1 & 0 & 1 & 3 & 2 & 1 & 3 \\ 2 & 0 & 2 & 0 & 8 & 2 & 6 & 1 & 2 & 7 \\ 3 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 3 & 6 \end{array}\right] \cup$$

$$\left[\begin{array}{ccc|cccc|cc} 2 & 1 & 3 & 7 & 1 & 0 & 4 & 3 & 8 \\ 4 & 8 & 7 & 6 & 2 & 3 & 6 & 7 & 5 \end{array}\right]$$

be two super row bivectors of type B. We see A and B does not have identical structure.

We define now addition of two super row bivectors of type B of identical structure.

DEFINITION 3.2.4: *Let $A = A_1 \cup A_2$*

$$= \left[\begin{array}{ccc|ccc|ccc} a^1_{11} & \cdots & a^1_{1r} & a^1_{1r+1} & \cdots & a^1_{1s} & \cdots & a^1_{1t} & \cdots & a^1_{1m} \\ \vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ a^1_{n1} & \cdots & a^1_{nr} & a^1_{nr+1} & \cdots & a^1_{ns} & \cdots & a^1_{nt} & \cdots & a^1_{nm} \end{array}\right] \cup$$

$$\left[\begin{array}{ccc|cc|cc|cc} b^1_{11} & \cdots & b^1_{1u} & \cdots & b^1_{1r} & \cdots & b^1_{1s} & \cdots & b^1_{1w} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ b^1_{p1} & \cdots & b^1_{pu} & \cdots & b^1_{pr} & \cdots & b^1_{ps} & \cdots & b^1_{pw} \end{array}\right]$$

and if $B = B_1 \cup B_2 =$

$$\left[\begin{array}{ccc|cc|cc|ccc} a^2_{11} & \cdots & a^2_{1r} & a^2_{1r+1} & \cdots & a^2_{1s} & \cdots & a^2_{1t} & \cdots & a^2_{1m} \\ \vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ a^2_{n1} & \cdots & a^2_{nr} & a^2_{nr+1} & \cdots & a^2_{ns} & \cdots & a^2_{nt} & \cdots & a^2_{nm} \end{array}\right] \cup$$

$$\left[\begin{array}{ccc|cc|cc|cc} b^2_{11} & \cdots & b^2_{1u} & \cdots & b^2_{1r} & \cdots & b^2_{1s} & \cdots & b^2_{1w} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ b^2_{p1} & \cdots & b^2_{pu} & \cdots & b^2_{pr} & \cdots & b^2_{ps} & \cdots & b^2_{pw} \end{array}\right]$$

are two super row bivectors of type B. We see A and B are identical in structure. The sum of A and B denoted by $A+B = (A_1 \cup A_2) + (B_1 \cup B_2) = A_1 + B_1 \cup A_2 + B_2$

$$= \left[\begin{array}{ccc|cc|cc|ccc} a^1_{11}+a^2_{11} & \cdots & a^1_{1r}+a^2_{1r} & a^1_{1r+1}+a^2_{1r+1} & \cdots & a^1_{1s}+a^2_{1s} & \cdots & a^1_{1t}+a^2_{1t} & \cdots & a^1_{1t}+a^2_{1t} \\ & & & & & & & & & \\ a^1_{n1}+a^2_{n1} & \cdots & a^1_{nr}+a^2_{nr} & a^1_{nr+1}+a^2_{nr+1} & \cdots & a^1_{ns}+a^2_{ns} & \cdots & a^1_{nt}+a^2_{nt} & \cdots & a^1_{nt}+a^2_{nt} \end{array}\right]$$

$$\cup$$

$$\left[\begin{array}{ccc|cc|cc|cc} b^1_{11}+b^2_{11} & \cdots & b^1_{1u}+b^2_{1u} & \cdots & b^1_{1r+1}+b^2_{1r+1} & \cdots & b^1_{1s}+b^2_{1s} & \cdots & b^1_{1w}+b^2_{1w} \\ & & & & & & & & \\ b^1_{p1}+b^2_{p1} & \cdots & b^1_{pu}+b^2_{pu} & \cdots & b^1_{pr+1}+b^2_{pr+1} & \cdots & b^1_{ps}+b^2_{ns} & \cdots & b^1_{pt}+b^2_{pt} \end{array}\right]$$

is defined to be the sum of the two super row bivectors A and B of type B. Clearly A+B is again a super row bivector of type B with identical structure with A and B.

Now in view of this we have the following interesting theorem.

THEOREM 3.2.1: *Let* $X = \{A = A_1 \cup A_2$ *be the collection of all super row bivector of type B which are identical in structure with A with entries from the field Q or R or C or* Z_p *(p a prime) or from the rings Z or* Z_n*, n not a prime or* $C(Z_p)$ *or* $Z(g_1)$ *or*

$Z(g_2)$ or $Z(g_3)$ (g_1, g_2 and g_3 are new elements such that $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$ and Z can be replaced by Q or R or C or $C(Z_n)$ or Z_n}. Then X is a group under addition.

Example 3.2.16: Let $X = X_1 \cup X_2 =$

$$\left[\begin{array}{ccc|ccccc|cccc} 3 & 2 & 1 & 0 & -1 & 0 & 0 & 1 & 3 & 4 & 0 & 1 \\ 1 & 5 & 2 & 6 & 0 & 1 & 2 & 3 & 1 & 0 & 0 & -1 \\ 0 & 6 & 3 & 0 & 6 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccc|ccccccc} 3 & -1 & 4 & 1 & 4 & -1 & 3 & 0 & 0 & 2 & 1 & 0 & 1 & 1 \\ 6 & 3 & 2 & -1 & 2 & 0 & 1 & 1 & 1 & 0 & 2 & 0 & -1 & 1 \end{array}\right] \text{ and}$$

$$Y = \left[\begin{array}{ccc|ccccc|cccc} -3 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & -4 & 0 & -1 \\ 0 & -1 & 2 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccc|ccccccc} -3 & 0 & 1 & 0 & 3 & 0 & 1 & 0 & 1 & 2 & -1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 1 & 0 & 2 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$= Y_1 \cup Y_2$ be two identical structure with the super row bivector of type B. To find the sum

$$X+Y = (X_1 \cup X_2) + (Y_1 \cup Y_2) = (X_1 + Y_1) \cup (X_2 + Y_2)$$

$$= \left[\begin{array}{ccc|ccccc|cccc} 0 & 3 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 0 & 0 \\ 1 & 4 & 4 & 6 & 1 & 1 & 4 & 3 & 2 & 0 & 0 & 0 \\ 0 & 7 & 6 & 1 & 6 & 2 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccc|ccccccc} 0 & -1 & 5 & 1 & 7 & -1 & 4 & 0 & 1 & 4 & 0 & 0 & 2 & 2 \\ 8 & 4 & 1 & 0 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & -1 & 2 \end{array}\right].$$

We see X+Y is again a super row bivector with identical structure as that of X and Y. The subtraction of X with Y

$X–Y = (X_1 – Y_1) \cup (X_2 – Y_2)$

$$= \left[\begin{array}{ccc|ccccc|cccc} 6 & 1 & 1 & 0 & 2 & 0 & -1 & 0 & 3 & 8 & 0 & 2 \\ 1 & 6 & 0 & 6 & -1 & 1 & 0 & 3 & 0 & 0 & 0 & -2 \\ 0 & 5 & 0 & -1 & 6 & 0 & 0 & -1 & 0 & -2 & -1 & -1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|ccc|ccccccc} 6 & -1 & 3 & 1 & 1 & -1 & 2 & 0 & -1 & 0 & -2 & 0 & 0 & 0 \\ 4 & 2 & 3 & -2 & 2 & -2 & 1 & 0 & 1 & -1 & 2 & -1 & -1 & 0 \end{array}\right].$$

X–Y is again a super row bivector of type B.

Now in a similar way we can define the notion when are two super column bivectors of type B are "identical in structure". Let $A = A_1 \cup A_2$ and $B = B_1 \cup B_2$ be two super column bivectors of type B. We say A and B are super column bivectors of type B are identical in structure if the number of natural rows in A_i is equal to B_i, i=1,2 and the number of columns in A_i equal to B_i and the partition of A_i and B_i are identical. We can as in case of identical structure super row bivectors of type B, we can in case of identical structure of super column bivectors of type B, we can define addition and subtraction of super column bivectors of type B.

Example 3.2.17:

$$\text{Let } A = \left[\begin{array}{cccc} 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ \hline 4 & 0 & 2 & -1 \\ 3 & 2 & 0 & 0 \\ 1 & 0 & -1 & 2 \\ 1 & 2 & 3 & 4 \\ \hline 6 & 7 & 0 & 2 \\ 0 & 2 & 3 & 1 \\ -1 & 0 & 1 & 1 \\ 4 & 2 & 5 & 6 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 8 & 3 \end{array}\right] \cup \left[\begin{array}{ccc} 3 & 2 & 8 \\ 0 & 6 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 6 & 4 & 1 \\ 0 & 0 & 1 \\ \hline 7 & 0 & 2 \\ 0 & 8 & 0 \\ 1 & 0 & 3 \\ 4 & 6 & 2 \\ 6 & 7 & 5 \\ 3 & 2 & 0 \end{array}\right] \text{ and}$$

$$B = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 3 & 6 \\ \hline 1 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ -1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 8 & 0 & 0 & 1 \\ 0 & 1 & 1 & 4 \end{array}\right] \cup \left[\begin{array}{ccc} 0 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 2 \\ 0 & 6 & 3 \\ 1 & 0 & 1 \\ 2 & 1 & 5 \\ 0 & -1 & 0 \\ 0 & 2 & 5 \end{array}\right]$$

be two super column bivector of type B.

The sum of $A+B = (A_1 + B_1) \cup (A_2 + B_2)$

$$\left[\begin{array}{cccc} 3 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ \hline 5 & 0 & 1 & -1 \\ 3 & 3 & 0 & -1 \\ 1 & 1 & -1 & 4 \\ 1 & 2 & 6 & 10 \\ \hline 7 & 8 & 2 & 6 \\ 0 & 2 & 4 & 4 \\ 2 & 0 & 1 & 1 \\ 6 & 2 & 6 & 7 \\ 8 & 1 & 2 & 1 \\ 0 & 2 & 9 & 7 \end{array}\right] \cup \left[\begin{array}{ccc} 3 & 3 & 10 \\ 0 & 8 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \\ \hline 6 & 5 & 2 \\ 1 & 1 & 1 \\ \hline 8 & 0 & 4 \\ 0 & 14 & 3 \\ 2 & 0 & 4 \\ 6 & 7 & 7 \\ 6 & 6 & 5 \\ 3 & 4 & 5 \end{array}\right]$$

is again a super bicolumn bivector of type B.

In view of this we have the following interesting theorem.

THEOREM 3.2.2: *Let $V = \{X = X_1 \cup X_2 /$ collection of all super column bivector of type B with entries from a field or ring which are identical in structure$\}$. Then V is a group under addition.*

However to define multiplication we have to define either minor product or major product.

We proceed onto give the minor product of two super bivectors of type B by numerical illustrations.

Example 3.2.18: Let

$$X = (X_1 \cup X_2) = \left[\begin{array}{ccc|c|cccc} 1 & 5 & 3 & 2 & 5 & 7 & 0 & 1 \\ 0 & 6 & 2 & 1 & 1 & 0 & 1 & 1 \\ 2 & 7 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|cc|ccccc} 1 & 2 & 0 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 5 & 0 & 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 & 3 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\text{and } Y = \left[\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 5 & 3 \\ 0 & 1 & 2 \end{array}\right] \cup \left[\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right] = Y_1 \cup Y_2$$

be two super bivectors.

The minor product of X with Y is given by
$XY = (X_1 \cup X_2)\,(Y_1 \cup Y_2)$
$= X_1 Y_1 \cup X_2 Y_2.$

$$= \left[\begin{array}{ccc|c|cccc} 1 & 5 & 3 & 2 & 5 & 7 & 0 & 1 \\ 0 & 6 & 2 & 1 & 1 & 0 & 1 & 1 \\ 2 & 7 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 5 & 3 \\ 0 & 1 & 2 \end{array}\right] \cup$$

$$\left[\begin{array}{cccc|cc|ccccc} 1 & 2 & 0 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 5 & 0 & 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 & 3 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 5 & 3 \\ 0 & 6 & 2 \\ 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \end{bmatrix} +$$

$$\begin{bmatrix} 5 & 7 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 5 & 3 \\ 0 & 1 & 2 \end{bmatrix} \cup \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 6 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 0 & 5 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 1 & 3 \\ 6 & 0 & 2 \\ 13 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 19 & 15 & 14 \\ 5 & 6 & 6 \\ 2 & 3 & 3 \end{bmatrix} \cup$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 \\ 0 & 5 & 5 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 2 & 1 \\ 0 & 1 & 2 & 5 \\ 2 & 2 & 1 & 5 \\ 1 & 2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 10 & 3 & 7 \\ 0 & 2 & 2 & 1 \\ 1 & 4 & 5 & 1 \\ 1 & 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 16 & 21 \\ 11 & 6 & 10 \\ 15 & 5 & 4 \end{bmatrix} \cup \begin{bmatrix} 7 & 16 & 6 & 8 \\ 2 & 4 & 5 & 8 \\ 3 & 11 & 11 & 6 \\ 5 & 4 & 3 & 5 \end{bmatrix}.$$

It is pertinent to mention that the entries of these super matrices can be replaced by neutrosophic rings, special dual like number neutrosophic rings, special quasi dual number neutrosophic rings and mixed special dual number neutrosophic rings.

Chapter Four

QUASI SUPER MATRICES

A square or a rectangular matrix is called as a quasi super matrix. It cannot be obtained by partition of a matrix. All super matrices are quasi super matrix but quasi matrix in general is not a super matrix. Thus the elements of a quasi matrix are submatrices following some order.

DEFINITION 4.1: *Take a m × n matrix A with entries from real or complex numbers.*

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1r_1} \\ A_{21} & A_{22} & \cdots & A_{2r_2} \\ \vdots & \vdots & & \vdots \\ A_{s_t 1} & A_{s_t 2} & \cdots & A_{s_t r_m} \end{bmatrix}$$

where A_{ij} are submatrices of A with $1 \leq j \leq r_1$, $r_2, \ldots, r_m$ and $1 \leq i \leq s_1, s_2, \ldots, s_t$.

Clearly we do not demand the size of A_{ji} to be same but the only condition to be satisfied is that if A is a m × n matrix then the number of the rows in each column of the matrix adds up to m and the number of the columns in each of rows added up to n. We define A to be a quasi super matrix.

Further we do not demand $r_1 = r_2$ and $r_i = r_j$ in general also. Also $s_i \neq s_j$ in general.

We give examples of one or two quasi super matrix before the proceed on to define some of their basic properties.

Example 4.1: Let A be a 6 × 4 quasi super matrix given by

$$A = \left[\begin{array}{cccc} 3 & 2 & 1 & 2 \\ 0 & 1 & 5 & 0 \\ \hline 9 & 8 & 3 & 2 \\ 1 & 4 & 1 & 4 \\ 2 & 5 & 3 & 0 \\ 3 & 1 & 5 & -1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where

$$A_{11} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 5 \end{bmatrix}, A_{12} = \begin{bmatrix} 2 \\ 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 9 & 8 \\ 1 & 4 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \text{ and } A_{22} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 3 & 0 \\ 5 & -1 \end{bmatrix}.$$

We see the sum of the rows of A_{11} and A_{21} adds up to 4 and sum of the rows of A_{12} and A_{22} adds up to 6 where as some of columns adds up to 4 i.e., the sum of the columns of A_{11} and A_{12} is 4 and that of A_{21} and A_{22} is also 4.

Next we give some more examples.

Example 4.2: Let A be 9 × 8 quasi super matrix given by

$$A = \left[\begin{array}{cccc}
\begin{bmatrix} 2 & 0 & 3 & 1 \\ 0 & 5 & 2 & 1 \end{bmatrix} & & \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} & \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \\
\begin{bmatrix} 3 & 1 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 3 & 3 & 0 & 0 \\ 5 & 1 & 1 & 1 \end{bmatrix} & & \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} \\
\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 1 \end{bmatrix} & \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 4 \end{bmatrix}
\end{array}\right]$$

$$= \begin{bmatrix} A_{11} & & A_{12} & A_{13} \\ A_{21} & & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{34} & A_{33} \end{bmatrix}$$

Clearly the number of columns added up in A_{11} A_{12} and A_{13} to 4 + 2 + 2 = 8.

The sum of the columns added up in A_{21} A_{22} and A_{23} to 4 + 3 + 1 = 8. The sum of the columns of A_{31} A_{32} A_{34} and A_{33} to 3 + 1 + 3 + 1 = 8.

We see basically A is a 8 × 8 matrix but by the division it is made into a quasi super matrix of the given form having 10 number of submatrices. A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{31}, A_{32}, A_{33} and A_{34}.

Now any 8 × 8 matrix can form a division in the following form given in the example 4.3.

Example 4.3: Let A be any 8 × 8 matrix.

$$A = \begin{bmatrix} \begin{bmatrix} 0 & 3 & 1 \\ 1 & 5 & 2 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 5 & 2 & 1 & 0 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 5 & 2 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 9 & 3 & 1 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 2 & 0 & 2 & 5 \\ 3 & 1 & 0 & 2 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ 1 & 1 & 5 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

Here the sum of the rows of the sub matrices A_{11} and A_{21} is 3 + 5 = 8 and sum of the columns of the sub matrices A_{21} and A_{22} is 5 + 3 = 8.

We make the following observation we see all the four sub matrices are square matrices based on these observations we make the following definition.

DEFINITION 4.2: *Let A be the m × m square matrix. If all the sub matrices of A are also square matrices then we call A to a quasi super square matrix.*

The matrix given in example 4.3 is a square quasi super matrix. We give yet another example before we define some more results.

Example 4.4: Let A be a 8×8 matrix with the following representation.

$$A = \left[\begin{array}{cccc}
\multicolumn{3}{c}{\left[\begin{array}{cccccc} 0 & 3 & 1 & 5 & 2 & 1 \\ 1 & 5 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 7 & 9 & 3 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 3 \end{array}\right]} & \begin{array}{c} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} \end{array} \\
\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} & \begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 5 \\ 1 & 5 \end{bmatrix}
\end{array}\right]$$

$$= \left[\begin{array}{cccc}
 & & A_{12} & \\
A_{11} & & A_{22} & \\
 & & A_{23} & \\
A_{21} & A_{31} & A_{24} & A_{25}
\end{array}\right]$$

Sum of the rows A_{11} and $A_{21} = 8$,
Sum of the rows A_{11} and $A_{23} = 8$,
Sum of the rows A_{11} and $A_{24} = 8$,
Sum of the rows A_{12}, A_{22}, A_{23}, A_{25} is 8,
Sum of the columns of A_{11} and A_{12} is 8,
Sum of the columns of A_{11} and A_{22} is 8,
Sum of the columns of A_{11} and A_{23} is 8,
Sum of the columns of A_{21}, A_{31}, A_{24} and A_{25} is 8.

Example 4.5: Let A be a 5×6 matrix which is given in the following.

$$A = \left[\begin{array}{ccc}
\begin{pmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix} & \multicolumn{2}{l}{\begin{pmatrix} 5 & 6 & 1 \\ 3 & 2 & 5 \\ 1 & 0 & 2 \end{pmatrix}} \\
\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
\end{array}\right]$$

$$= \begin{bmatrix} A_{11} & & A_{12} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

where A_{11}, A_{12}, A_{21}, A_{22} and A_{23} are submatrices.

Clearly sum of rows of A_{11} and A_{21} is 5, sum of rows of A_{12} and A_{22} is five and that of A_{12} and A_{23} is five.

Sum of the columns A_{11} and A_{12} is six. Sum of the columns of A_{21} A_{22} and A_{23} is six.

Example 4.6: Let A be a 6 × 6 quasi super matrix with

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A^0 & A^0 \\ A_{21} & A_{22} \end{bmatrix},$$

where A_{11} is a 4 × 4 square matrix and A_{12} is a 2 × 2 square matrix. A_{21} is a 2 × 2 square matrix and A_{22} is a 4 × 4 square matrix.

Now

$$A_{11} = \begin{bmatrix} 0 & 3 & 1 & 2 \\ 1 & 1 & 0 & 5 \\ 5 & 1 & 2 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, A^\circ = A^\circ = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and}$$

$$A_{22} = \begin{bmatrix} 6 & 0 & 1 & 2 \\ 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

$$A = \left[\begin{array}{cccccc} 0 & 3 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 5 & 2 & 1 \\ \hline 5 & 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 6 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ \hline 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \end{array}\right].$$

We see all the six sub matrices are square matrices.

Example 4.7: A be a 4 × 5 matrix which is a quasi super matrix given by

$$A = \begin{bmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \\ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} & \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

Similarly we proceed on to define the notion of quasi super matrix which is rectangular.

DEFINITION 4.3: *Let A be a quasi super matrix.*

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1r_1} \\ A_{21} & A_{22} & \cdots & A_{1r_2} \\ \vdots & \vdots & & \vdots \\ A_{s_1 1} & A_{s_2 2} & \cdots & A_{s_t r_m} \end{bmatrix}$$

A_{ij}'s are rectangular matrices, with $1 < i < r_1, r_2, \ldots, r_m$ and $1 < j < s_1, s_2, \ldots, s_t$, we define A to be a quasi super rectangular matrix.

We give an example of a matrix of this type.

Example 4.8: Let A be a 6 × 8 matrix quasi super matrix.

$$A = \begin{bmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 5 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 5 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 2 & 0 & 2 & 2 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

A is a rectangular quasi super matrix.

Example 4.9: Let A be a 6 × 4 matrix.

$$A = \begin{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 2 & 5 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix} & \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ & A_{21} \end{bmatrix}$$

which is a quasi super matrix. Sum of the columns of A_{11} and A_{12} is four.

Number of columns in A_{21} is four. Sum of rows of A_{11} and A_{21} is six. Sum of rows of A_{12} and A_{21} is six. The quasi super matrix described in example 4.9 is not a rectangular quasi super matrix for it contains a 3 × 3 square matrix in it.

We proceed on to define these type of matrices,.

DEFINITION 4.4: *Let A be a m × n rectangular matrix. A be a quasi super matrix.*

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1r_1} \\ A_{21} & A_{22} & \cdots & A_{1r_2} \\ \vdots & \vdots & & \vdots \\ A_{s_1 1} & A_{s_2 2} & \cdots & A_{s_t r_m} \end{bmatrix}$$

If some of the A_{ij}'s are square submatrices and some of them are rectangular submatrices, then we call A to be mixed quasi super matrix.

Thus we have seen 3 types of quasi super matrices.

We have to give some more examples for we can have a square m × m matrix A: yet A can be a mixed quasi super matrix or a rectangular quasi matrix. Likewise a square m × m matrix can be a mixed quasi super matrix.

Example 4.10: Consider a 6 × 6 quasi super matrix where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where

$$A_{11} = \begin{bmatrix} 2 & 0 & 1 & 4 \\ 5 & 2 & 0 & 6 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A_{12}$$

$$A_{21} = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix} \text{ and } A_{22} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\text{i.e., } A = \left[\begin{array}{cccccc} 2 & 0 & 1 & \multicolumn{1}{c|}{4} & 2 & 1 \\ 5 & 2 & 0 & \multicolumn{1}{c|}{6} & 1 & 2 \\ \hline 3 & 1 & \multicolumn{1}{c|}{2} & 1 & 0 & 1 \\ 1 & 0 & \multicolumn{1}{c|}{1} & 0 & 1 & 0 \\ 0 & 1 & \multicolumn{1}{c|}{0} & 1 & 1 & 1 \\ 2 & 1 & \multicolumn{1}{c|}{0} & 0 & 2 & 2 \end{array}\right].$$

A is a 6 × 6 matrix but A is not a rectangular quasi super matrix or a square quasi super matrix it is only a mixed quasi super matrix.

Thus this example clearly shows a quasi super matrix which is a 6 × 6 square matrix can be a quasi super mixed square matrix.

Next we proceed on to give an example of a square matrix which is only a rectangular quasi super matrix.

Example 4.11: Let A be a 5 × 5 quasi super square matrix.

$$A = \left[\begin{array}{ccc|cc} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ \hline 2 & 3 & 4 & 0 & 1 \\ \cline{4-5} 3 & 4 & 0 & 1 & 2 \\ \hline 4 & 0 & 1 & 2 & 3 \end{array}\right]$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ \multicolumn{2}{c}{A_{51}} \end{bmatrix}$$

where

$$A_{11} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \; A_{12} = \begin{bmatrix} 3 & 4 \\ 4 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 0 \end{bmatrix}, \; A_{22} = [\ 1\ 2\ ] \text{ and}$$

$$A_{31} = [\ 4\ 0\ 1\ 2\ 3\ ].$$

A is a quasi super rectangular matrix.

Example 4.12: Let A be a 6 × 4 quasi super rectangular matrix.

$$A = \left[\begin{array}{cc|cc} 0 & 1 & 2 & 3 \\ 1 & -1 & 0 & 2 \\ \hline 2 & 0 & 1 & 0 \\ 1 & 6 & 7 & 1 \\ 5 & 2 & 0 & 8 \\ 3 & 1 & 9 & 2 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ \multicolumn{2}{c}{A_{21}} \end{bmatrix}.$$

Clearly A is a square quasi super matrix which is not a square matrix. We see all the three submatrices of A are square matrices.

Now having seen three types of quasi super matrices, we now proceed on to define some sort of operation on then. First given any n × m matrix which we are not in a position to give a partition as in case of super matrices.

So we define in case of n × m matrices a new type of relation called quasi division.

A quasi division on the rectangular array of numbers is a cell formation such that the cells are either square, rectangular or row or column or singletons. Clearly or is not used in the mutually exclusive sense.

We just illustrate this by a very simple example.

Example 4.13: Let A be a 7 × 6 quasi super matrix.

$$A = \left[\begin{array}{cc|ccc|c} 0 & 7 & 8 & 9 & 4 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ \hline -1 & 1 & -1 & 1 & 0 & 1 \\ \hline 8 & 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ \hline 2 & 1 & -1 & 4 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 & 1 \end{array}\right].$$

We see the matrix or this 7 × 6 array of numbers which have been divided in 10 cells. Each cell is of a varying size. For we see first cell is a 2 × 2 square matrix.

$$\begin{bmatrix} 0 & 7 \\ 1 & 0 \end{bmatrix} = \text{I cell} = A_{11}$$

The second cell is a rectangular 2 × 3 matrix.

$$\begin{bmatrix} 8 & 9 & 4 \\ 1 & 0 & 1 \end{bmatrix} \text{ is the II cell-} A_{12}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is the III cell - A_{13} is a 2 × 1 column vector or matrix.

The fourth cell is again a column vector given by $\begin{bmatrix} -1 \\ 8 \\ 0 \end{bmatrix} = A_{21}$

The fifth cell is [1], a singleton A_{22}. The sixth cell is [–1 1 0 1] is a row vector A_{23}.

The seventh cell is a column vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix} = A_{31}$. The eighth cell is a 4 × 3 rectangular matrix.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = A_{32}$$

The ninth cell is a 3 × 2 rectangular matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = A_{33}.$$

The 10^{th} cell find its place as A_{44} a row vector [11]. The arrangement of writing or notational order is written in this form

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ & & A_{44} \end{bmatrix}$$

We see the sum of the columns of A_{11}, A_{12} and A_{13} is 6. Sum of the columns of A_{21}, A_{22} and A_{23} is six.

Sum of the columns of A_{31}, A_{32} and A_{33} is six. (1+3+2=6). Also sum of the columns of A_{31}, A_{32} and A_{44} is six.

Now having seen as example we define the notion of cell partition.

DEFINITION 4.5: *A cell partition of m × n rectangular array of numbers A is a division of the m × n array of numbers using cells. The cells can only be singletons or row vectors or column vectors or square matrix and (or) rectangular matrix. i.e., each cell can be called as a sub matrix of A.*

For instance this process is like finding subsets of a set. In that case we have lots of choices and we know given N number of elements in the set; the number of subsets is 2^N. But dividing the n × m array of numbers of the n × m matrix is not identical for the cell division is not arbitrary each cell should have a form and the division can be many for instance we will first illustrate in how many ways a 2 × 2 matrix can be divided using the method of cell division,

Example 4.14: Let $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

We enumerate the number of cell division leading the quasi super matrix.

$$A_1 = \left[\begin{array}{c|c} 0 & 1 \\ \cline{1-1} 2 & 3 \end{array}\right] = \begin{bmatrix} A_{11} & \\ & A_{12} \\ A_{21} & \end{bmatrix},$$

$$A_2 = \left[\begin{array}{c|c} 0 & 1 \\ \cline{2-2} 2 & 3 \end{array}\right] = \left[\begin{array}{cc} A_{11} & A_{12} \\ & A_{22} \end{array}\right],$$

$$A_3 = \left[\begin{array}{cc} 0 & 1 \\ \hline \multicolumn{1}{c|}{2} & 3 \end{array}\right]$$

$$\text{and } A_4 = \left[\begin{array}{cc} \multicolumn{1}{c|}{0} & 1 \\ \hline 2 & 3 \end{array}\right]$$

There are only 4 quasi super matrices constructed using a 2×2 matrix.

However we have several partitions leading to a super matrix.

$$B = \left[\begin{array}{cc} 0 & 1 \\ \hline 2 & 3 \end{array}\right] \text{ is a partition,}$$

B is a super matrix.

$$C = \left[\begin{array}{c|c} 0 & 1 \\ 2 & 3 \end{array}\right] \text{ is a super matrix different from B.}$$

$$D = \left[\begin{array}{c|c} 0 & 1 \\ \hline 2 & 3 \end{array}\right] \text{ is a super matrix different from both B and C.}$$

Thus using 2×2 matrices we have only 3 super matrices. But using cell division leading to quasi super matrix we have 4 quasi super matrices. Thus we have altogether 7 quasi super matrices if we make the rule all super matrices are quasi super matrices. That is the class of super matrices is contained in the class of quasi super matrices.

Example 4.15: Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

be a 3 × 3 matrix.

The number of quasi supermatrices from A are

$$A_1 = \left[\begin{array}{c|cc} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ & A_{22} \end{bmatrix}$$

$$A_2 = \left[\begin{array}{c|cc} 0 & 1 & 2 \\ 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ & A_{22} \end{bmatrix}$$

$$A_3 = \left[\begin{array}{c|cc} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} & A_{12} \\ A_{11} & A_{22} \\ & A_{13} \end{bmatrix}$$

$$A_4 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{3-3} 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & \begin{matrix} A_{13} \\ A_{23} \end{matrix} \end{bmatrix}$$

$$A_5 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ 1 & 0 & 1 \\ \cline{3-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & \begin{matrix} A_{13} \\ A_{23} \end{matrix} \end{bmatrix}$$

$$A_6 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{3-3} 1 & 0 & 1 \\ \cline{3-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & & A_{23} \\ & & A_{33} \end{bmatrix}$$

$$A_7 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ 1 & 0 & 1 \\ \cline{2-2} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & \end{bmatrix}$$

$$A_8 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-2} 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & \end{bmatrix}$$

$$A_9 = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-2} 1 & 0 & 1 \\ \cline{2-2} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & \\ & A_{32} & \end{bmatrix}$$

$$A_{10} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \end{bmatrix}$$

$$A_{11} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-2} 1 & 0 & 1 \\ \cline{3-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \end{bmatrix}$$

$$A_{12} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ \cline{3-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{bmatrix}$$

$$A_{13} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{3-3} 1 & 0 & 1 \\ \cline{2-2} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \end{bmatrix}$$

$$A_{14} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \end{bmatrix}$$

$$A_{15} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{3-3} 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{bmatrix}$$

$$A_{16} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ \cline{2-2} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & A_{32} & \end{bmatrix}$$

$$A_{17} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-2} 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & A_{32} & \end{bmatrix}$$

$$A_{18} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \cline{2-3} 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{bmatrix}$$

$$A_{19} = \left[\begin{array}{c|c|c} 0 & 1 & 2 \\ \hline 1 & 0 & 1 \\ \cline{2-3} 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ & A_{32} & A_{33} \end{bmatrix}$$

and so on.

Thus we see even in case of a 3 × 3 matrix the number of quasi super matrix is very large. If we make the inclusion of super matrix into it. It will still be larger.

Thus at this juncture we propose the following problem.

Problem: Suppose P is any m × m square matrix. i.e., a square array of m × m numbers.

1. Find the number of super matrix constructed using P.
2. Find the number of quasi super matrix constructed using P.

Now we just try to work with a 2 × 4 matrix.

Example 4.16: Let M = $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ be a 2 × 4 matrix.

We just indicate some of the quasi super matrix constructed using M.

$$M_1 = \left[\begin{array}{c|ccc} 0 & 1 & 2 & 3 \\ \cline{2-4} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & \begin{matrix} A_{12} \\ A_{22} \end{matrix} \end{bmatrix}$$

$$M_2 = \left[\begin{array}{c|c|cc} 0 & 1 & 2 & 3 \\ \cline{3-4} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & \begin{matrix} A_{13} \\ A_{23} \end{matrix} \end{bmatrix}$$

$$M_3 = \left[\begin{array}{c|c|cc} 0 & 1 & 2 & \multicolumn{1}{|c}{3} \\ \cline{3-4} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ & & A_{23} & \end{bmatrix}$$

$$M_4 = \left[\begin{array}{c|c|cc} 0 & 1 & 2 & 3 \\ \cline{3-4} 1 & 0 & 1 & \multicolumn{1}{|c}{0} \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & \\ & & A_{23} & A_{24} \end{bmatrix}$$

$$M_4 = \left[\begin{array}{c|c|c|c} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & \\ & & A_{23} & A_{24} \end{bmatrix}$$

$$M_5 = \left[\begin{array}{c|c|c|c} 0 & 1 & 2 & 3 \\ \cline{3-3} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ & & A_{23} & \end{bmatrix}$$

$$M_6 = \left[\begin{array}{cc|cc} 0 & 1 & 2 & 3 \\ \cline{3-4} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ & A_{22} \end{bmatrix}$$

$$M_7 = \left[\begin{array}{cc|c|c} 0 & 1 & 2 & 3 \\ \cline{3-4} 1 & 0 & \multicolumn{2}{c}{1 \quad 0} \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & \end{bmatrix}$$

$$M_8 = \left[\begin{array}{cc|c|c} 0 & 1 & \multicolumn{2}{c}{2 \quad 3} \\ \cline{3-4} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & \\ & A_{22} & A_{23} \end{bmatrix}$$

$$M_9 = \left[\begin{array}{c|c|cc} 0 & 1 & 2 & 3 \\ \cline{1-1} 1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & & \end{bmatrix}$$

and so on. Thus we see even in case of the simple 2 × 4 matrix, we can have several quasi super matrices. We have shown only nine of them.

Now we propose the following problem.

Problem

1. Find the number of quasi super matrices that can be constructed using m × n matrix.

2. Find the number of super matrices that can be constructed using just a m × n rectangular matrix.

Now having defined the notion of cells partition of a matrix which has lead to the definition of quasi super matrices we proceed on the study the further properties of quasi super matrices like matrix addition and multiplication and see how best those concepts can be defined on them.

Example 4.17: Let A be a 9 × 4 rectangular matrix.

$$A = \left[\begin{array}{cc|cc} 0 & 1 & 2 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ \cline{3-4} 1 & 0 & 1 & 0 \\ \cline{1-2} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ \cline{3-4} 1 & 0 & 0 & 1 \\ \hline \multicolumn{1}{c|}{2} & 1 & \multicolumn{1}{c|}{3} & 5 \\ \multicolumn{1}{c|}{1} & 2 & \multicolumn{1}{c|}{3} & 4 \end{array}\right]$$

$$= \begin{bmatrix} A_{11} & A_{12} & \\ A_{21} & A_{22} & \\ & A_{32} & \\ A_{41} & A_{42} & A_{43} \end{bmatrix}.$$

We see A has 8 cells. We can have for the same A we can just have only 4 cells.

$$A = \left[\begin{array}{cc|cc} 0 & 1 & 2 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ \cline{3-4} 1 & 0 & 1 & 0 \\ \cline{1-2} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 5 \\ 1 & 2 & 3 & 4 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

Suppose we have two quasi super matrices how to add? We first proceed on to define addition of a super quasi matrix A with itself.

Example 4.18: Let A be a 5 × 7 quasi matrix with a well defined cell partition defined on it.

$$A = \left[\begin{array}{ccc|cc|cc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ \cline{4-5} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 5 & \multicolumn{1}{c|}{7} & \multicolumn{1}{c|}{8} & 2 & \multicolumn{1}{c}{-1} & 4 & -3 \\ -8 & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{7} & 0 & \multicolumn{1}{c}{2} & 5 & 8 \end{array}\right]$$

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & \\ A_{31} & A_{32} & A_{33} \end{bmatrix}.$$

$$A + A = 2A = \left[\begin{array}{ccc|cc|cc} 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 2 & 2 & 0 & 2 & 2 & 0 & 2 \\ \cline{4-5} 0 & 2 & 2 & 0 & 2 & 2 & 0 \\ \hline 10 & \multicolumn{1}{c|}{14} & \multicolumn{1}{c|}{16} & 4 & \multicolumn{1}{c}{-2} & 8 & -6 \\ -16 & \multicolumn{1}{c|}{2} & \multicolumn{1}{c|}{14} & 0 & \multicolumn{1}{c}{4} & 10 & 16 \end{array}\right]$$

$$= \begin{bmatrix} 2A_{11} & 2A_{12} & 2A_{13} \\ & 2A_{22} & \\ 2A_{31} & 2A_{32} & 2A_{33} \end{bmatrix} = 2A.$$

It is left as an exercise for the reader to verify that for the quasi super matrix A we have A – A = [0]. In this case

$$A - A = \left[\begin{array}{ccc|cc|cc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cline{4-5} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{0} & 0 & \multicolumn{1}{c}{0} & 0 & 0 \\ 0 & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{0} & 0 & \multicolumn{1}{c}{0} & 0 & 0 \end{array}\right].$$

Just a quasi super matrix A under the same cell division can be added any number of times and this will not change the nature of a quasi super matrix cell division.

Thus we can say if A is a quasi super matrix then A + … + A: n times is the same as nA.

If A has A_{11}, A_{12}, …, A_{rt} to be the collection of all its submatrices then nA will have nA_{11}, nA_{12}, …, nA_{rt} to be the collection of all its submatrices.

Note: We can always take any m × n zero matrix and make the cell partition or division in a desired form so that A + (0) = (0) + A = A. For this we first define a simple notion called cell division function.

Suppose we have two m × n matrices A and B. We know the cell division of A how to get the same cell division of B so that the cell divisions of both A and B are the same and both of them are quasi super matrices of same type or similar.

DEFINITION 4.6: *Let A be a m × n quasi super matrix. B any m × n matrix. To make B also a quasi super matrix similar to A; define a cell function F_c from A to B as follows.*

$F_c(A_{11}) = B_{11}$ formed from B by taking the same number of rows and columns from B. So that A_{11} and B_{11} have the same number of rows and columns, not only that the placing of B_{11} in B is identical with that of A_{11} in A.

The same procedure is carried out for every A_{ij} in A.

Thus the cell function F_c converts a usual m × n matrix B into a desired quasi matrix A.

Before we proceed on to define the properties of F_c we show this by some illustrations.

Example 4.19: Let A be a given 3 × 5 quasi super matrix. B be any matrix. To find a similar cell division on B identical with A. Given

$$A = \begin{bmatrix} 0 & 3 & 5 & 2 & 0 \\ 1 & 4 & 4 & 1 & 2 \\ 2 & 5 & 3 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} & & \\ A_{21} & A_{22} & A_{23} & A_{24} \end{bmatrix}$$

is a quasi super matrix with A_{11}, A_{12}, A_{21}, A_{22}, A_{23} and A_{24} as submatrices.

Given

$$B = \begin{bmatrix} 0 & 0.3 & 4 & 0.1 & 2 \\ 1 & 4 & 0 & 2 & 0 \\ 3 & 5 & 1 & 1 & 1 \end{bmatrix}$$

Define the cell function F_c from A to B as follows.

$$F_c(A_{11}) = \begin{bmatrix} 0 & 0.3 \\ 1 & 4 \end{bmatrix} = B_{11},$$

$$F_c(A_{12}) = [4\ 0.1\ 2] = B_{12},$$

$$F_c(A_{21}) = [3\ 5\ 1] = B_{21},$$

$$F_c(A_{22}) = [0] = B_{22},$$

$$F_c(A_{23}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = B_{23}$$

$$\text{and } F_c(A_{24}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = B_{24}.$$

$$B = \left[\begin{array}{cc|ccc} 0 & 0.3 & 4 & 0.1 & 2 \\ \cline{3-5} 1 & 4 & 0 & \multicolumn{1}{|c|}{2} & 0 \\ \cline{1-3} 3 & 5 & 1 & \multicolumn{1}{|c|}{1} & 1 \end{array}\right]$$

$$= \begin{bmatrix} B_{11} & B_{12} & & \\ B_{21} & B_{22} & B_{23} & B_{24} \end{bmatrix}.$$

We enumerate some of the properties of F_c. Let F_c be a cell division function from A to B where A is a quasi super matrix.

$F_c(A) = A$

i.e., A is equivalent to A under F_c; i.e., F_c is reflexive.

If $F_c(A) = B$, i.e.,

A is equivalent to B under F_c then B is equivalent to A under F_c.

i.e., if $F_c(A) = B$ then $F_c(B) = A$.

$(F_c(F_c(A)) = A$ for all A).

If A is any quasi super matrix and B and C any two $m \times n$ matrices. If F_c is the cell division function such that $F_c(A) \rightarrow B$ so that B becomes a quasi super matrix. Suppose $F_c(B) \rightarrow C$ so that C becomes a quasi super matrix, then we see A, B and C and quasi super matrices with

$$A \sim B \text{ and } B \sim C \text{ implies } A \sim C.$$

Thus F_c is an equivalence relation on the class of all $m \times n$ matrices under a special or a specified cell division function.

If we take the collection of all cells partitions of a m × n matrix and denote it by C_{F_c} then C_{F_c} is divided into disjoint classes by this relation i.e., $C_{F_c} = \{F_c \mid F_c;\ A \to B;$ A and B m × n matrices, A is a quasi super matrix and B any m × n matrix}

We illustrate this by an example.

Example 4.20: Let

$$T = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle|\ a, b, c, d \in Z_2 = \{0, 1\} \right\}.$$

To find the partition of T under the cell division function.

First we find the number of elements in C_{F_c} .

$$F_c^0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$F_c^1 = \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right]$$

$$F_c^2 = \left[\begin{array}{c|c} a & b \\ c & d \end{array}\right]$$

$$F_c^3 = \left[\begin{array}{cc} a & b \\ \hline c & d \end{array}\right]$$

$$F_c^4 = \left[\begin{array}{cc} a & b \\ \hline c & \multicolumn{1}{|c}{d} \end{array}\right]$$

$$F_c^5 = \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right]$$

$$F_c^6 = \left[\begin{array}{c|c} a & b \\ \cline{2-2} c & d \end{array}\right]$$

$$F_c^7 = \left[\begin{array}{c|c} a & b \\ \cline{1-1} c & d \end{array}\right]$$

We see $C_{F_c} = \{ F_c^0,\ F_c^1,\ F_c^2,\ F_c^3, F_c^4,\ F_c^5,\ F_c^6$, and F_c^7, i.e., $|C_{F_c}| = 8$.

Now we have to find the number of matrices in T.

T =

$$\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix},\begin{pmatrix}1&1\\1&1\end{pmatrix},\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix},\begin{pmatrix}1&1\\0&0\end{pmatrix},\begin{pmatrix}1&0\\1&0\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}0&1\\0&1\end{pmatrix},\begin{pmatrix}0&0\\1&1\end{pmatrix},\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}1&1\\1&0\end{pmatrix},\begin{pmatrix}1&1\\0&1\end{pmatrix},\begin{pmatrix}0&1\\1&1\end{pmatrix},\begin{pmatrix}1&0\\1&1\end{pmatrix}\right\}.$$

Thus each and every class in C_{F_c} will have all the 16 elements of T with no over lap.

If C_{F_c} is the class then every class contains exactly the same number of elements in the set of matrices on which the quasi super matrices notion is to be defined.

Now as partition yields to a super matrix we see cell division leads to the concept of quasi super matrix.

Now two m × n quasi super matrix can be added if and only if they have the same cell division defined an them, otherwise we say addition is not compatible for the cell division is not compatible.

Example 4.21: Let

$$A = \left[\begin{array}{ccc|cc} 3 & 0 & 1 & 2 & 5 \\ 7 & 1 & 0 & 1 & 1 \\ \hline 6 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ \hline 1 & 2 & 5 & -1 & 1 \\ 1 & 1 & 6 & 2 & 1 \end{array}\right]$$

$$= \begin{bmatrix} A_{11} & A_{12} & \\ A_{21} & A_{22} & A_{13} \\ A_{31} & & \end{bmatrix};$$

where A_{11}, A_{12}, A_{21}, A_{22}, A_{13} and A_{31} are submatrices of A.

$$B = \left[\begin{array}{cc|ccc} 1 & 1 & 0 & 0 & 2 \\ 5 & 0 & 1 & 1 & 0 \\ \hline 6 & 2 & 1 & 0 & 2 \\ \cline{3-5} 1 & 1 & 0 & 1 & 1 \\ \hline 2 & 0 & 1 & 0 & 1 \\ 3 & 0 & 2 & 2 & 0 \end{array}\right] = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & \end{bmatrix}.$$

Clearly the addition of A and B is not compatiable for we see the submatrices B_{11}, B_{12}, B_{21}, B_{22}, and B_{31} are different from A_{11}, A_{12}, A_{21}, A_{22}, A_{13} and A_{31}.

Thus we see addition of A with B is impossible though both A and B are of same order viz., 6×5.

Thus is a quasi super matrix addition is compatiable if and only if (1) A and B are quasi super matrix of same order and

(2) cell division on A and B are the same that is we have a cell division function F_c: A $\rightarrow$ B such that $F_c(A) = B$ true.

Now we illustrate this situation by a simple example.

Example 4.22: Let

$$A = \left[\begin{array}{c|cc|cc} 0 & 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 & 9 \\ \hline 0 & 6 & 2 & 8 & 4 \\ \hline 5 & 1 & 7 & 3 & 9 \\ 1 & 1 & 0 & 1 & 2 \\ 0 & 2 & 1 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} A_{11} & A_{12} & \\ A_{21} & A_{22} & A_{23} \\ A_{31} & & \end{bmatrix}$$

where A_{11}, A_{12}, A_{22}, A_{23} and A_{31} are sub matrix of A.

$$B = \left[\begin{array}{c|cc|cc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} B_{11} & B_{12} & \\ & B_{22} & B_{23} \\ B_{31} & & \end{bmatrix}$$

We see the total number of columns in B_{11} and B_{12} is 5; in B_{22} and B_{23} is 5 in B_{31} and B_{23} is 5.

Now the addition of A and B is compatible.

$$A + B = \left[\begin{array}{c|cccc} 0 & 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 & 9 \\ \cline{2-5} 0 & 6 & 2 & 8 & 4 \\ \hline 5 & 1 & 7 & 3 & 9 \\ 1 & 1 & 0 & 1 & 2 \\ 0 & 2 & 1 & 0 & 1 \end{array}\right] + \left[\begin{array}{c|cccc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ \cline{2-5} 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|cccc} 0+1 & 1+0 & 2+1 & 3+0 & 4+1 \\ 5+0 & 6+1 & 7+0 & 8+1 & 9+0 \\ \cline{2-5} 0+1 & 6+1 & 2+1 & 8+1 & 4+1 \\ \hline 5+1 & 1+1 & 7+0 & 3+0 & 9+1 \\ 1+1 & 1+0 & 0+0 & 1+1 & 2+1 \\ 0+0 & 2+1 & 1+0 & 0+0 & 1+1 \end{array}\right]$$

$$= \left[\begin{array}{c|cccc} 1 & 1 & 3 & 3 & 5 \\ 5 & 7 & 7 & 9 & 9 \\ \cline{2-5} 1 & 7 & 3 & 9 & 5 \\ \hline 6 & 2 & 7 & 3 & 10 \\ 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 1 & 0 & 2 \end{array}\right].$$

It is important and interesting to note that in case of quasi super matrices even addition is not always compatible even if the matrices are of same order.

Now we proceed on to define multiplication in case of quasi super matrices.

What ever be the situation we need to have compatability of order while multiplying.

Example 4.23: Let

$$A = \left[\begin{array}{ccc|cc} 0 & 1 & 2 & 2 & 1 \\ 5 & 1 & 1 & 3 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ \hline 5 & 1 & 0 & 1 & 1 \end{array}\right]_{5\times4}$$

and

$$B = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 \\ \hline 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5x4} .$$

be any two quasi super matrices.

$$A°B = \left[\begin{array}{ccc|cc} 0 & 1 & 2 & 2 & 1 \\ 5 & 1 & 1 & 3 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ \hline 5 & 1 & 0 & 1 & 1 \end{array}\right] \circ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 \\ \hline 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

If the partitions or cell divisions are removed imaginarily and the multiplication takes place we get a 4 × 4 matrix.

$$AB = \left[\begin{array}{ccc|c} 4 & 3 & 3 & 5 \\ 12 & 4 & 7 & 11 \\ 2 & 0 & 2 & 1 \\ \hline 7 & 2 & 6 & 9 \end{array}\right]$$

The quasi super matrix product assumes an approximate cell division which is not unique for it may not be possible to get a cell division exactly as the very order of it is changed.

We call it as the resultant quasi super matrix. What is more important in this situation is that we can find one or more type of cell division on the product. This is not a problem for the very concept of quasi super matrix was found to apply in fuzzy models.

So to over come all these hurdles we make a mention of the following. As in case of super matrices we first apply the product of a quasi super matrix and its transpose we may recall in case of super matrix we had the product of a super matrix with its transpose gave a nice symmetric super matrix.

We observe that in the case of quasi super matrix A, the transpose A^T of A is such that in general the product is not defined. Thus we can define the product only when it is defined.

This is not the case in case of super matrix, for in super matrix we can always define such products but in case of quasi super matrix the product may be defined or not. Only if defined it can be calculated.

This quasi super matrix when its entries are from the fuzzy interval [0,1] we define it to be a fuzzy quasi super matrix or quasi fuzzy super matrix.

All these matrices find their applications when the problem under investigation uses several i.e., one or more fuzzy models simultaneously.

We just illustrate by examples some fuzzy quasi super matrices.

Example 4.24: Let F_s denote the fuzzy quasi super matrix given as

$$F_s = \begin{bmatrix} A_1 & \\ & A_3 \\ A_2 & \end{bmatrix}$$

$$= \left[\begin{array}{cccccc} 0 & 0.1 & 0.4 & 0.5 & 0.3 & 0.7 \\ 1 & 0.1 & 1 & 0 & 0.8 & 0.5 \\ 0.7 & 0.2 & 0.5 & 0.6 & 1 & 0.2 \\ 0.8 & 0.5 & 0.6 & 0.5 & 0.7 & 1 \\ \hline 0.1 & 0.8 & 0.6 & \multicolumn{1}{|c}{1} & 0 & 0.8 \\ 0.8 & 0.5 & 0.1 & \multicolumn{1}{|c}{0.9} & 1 & 0.2 \\ 0.3 & 0.7 & 0.5 & \multicolumn{1}{|c}{0.3} & 0.5 & 1 \end{array}\right],$$

where A_1 is a 4×6 fuzzy rectangular matrix and A_2 and A_3 are 3×3 fuzzy square super matrices.

Example 4.25: Let P_s denote a quasi fuzzy super matrix given by

$$P_s = \left[\begin{array}{cccccccc} 0.1 & 1 & 0.2 & 0.1 & \multicolumn{1}{|c}{0.9} & 0.7 & 0.2 & 0 \\ 0.9 & 0.1 & 1 & 0 & \multicolumn{1}{|c}{0.8} & 0.6 & 0.4 & 0.6 \\ 0.7 & 0.5 & 0.6 & 0.8 & \multicolumn{1}{|c}{0.7} & 0.1 & 0.5 & 0.1 \\ 0 & 1 & 0 & 0.5 & \multicolumn{1}{|c}{0.4} & 0.7 & 0.4 & 0.3 \\ \hline 0.3 & 1 & 0.6 & 0.1 & 0.5 & 0 & \multicolumn{1}{|c}{0} & 0 \\ 0.7 & 0 & 0.2 & 0.4 & 0.6 & 0.7 & \multicolumn{1}{|c}{0} & 0 \\ 0.8 & 1 & 1 & 0 & 1 & 0 & \multicolumn{1}{|c}{1} & -1 \\ \cline{7-8} 0.9 & 0 & 0.7 & 0.8 & 0.9 & 0.6 & \multicolumn{1}{|c}{0.3} & 0.2 \\ 0.5 & 0.2 & 0.6 & 0.4 & 0.5 & 0.1 & \multicolumn{1}{|c}{0.3} & 0.1 \\ 0.6 & 0.3 & 0.4 & 0.3 & 0.2 & 0.7 & \multicolumn{1}{|c}{0.1} & 0 \end{array}\right];$$

where

$$P_s = \begin{bmatrix} A_1 & A_2 \\ & A_4 \\ A_3 & \\ & A_5 \end{bmatrix};$$

where A_1 and A_2 is a 4×4 fuzzy matrix. A_3 is a 6×6 square fuzzy matrix and A_4 and A_5 are 3×2 rectangular fuzzy matrices.

Thus we can have any number of quasi fuzzy super matrices. In case of even quasi fuzzy super matrices we cannot always define the notion of addition or multiplication. Only when special type of compatibility exists we can proceed onto define sum or product or both.

Example 4.26: Let F_s and P_s be two quasi fuzzy super matrices given by

$$F_s = \left[\begin{array}{ccc|cc|cc}
0.3 & 0.1 & 0.2 & 0.7 & \multicolumn{1}{c}{0.2} & 0 & 0.1 \\
1 & 0.2 & 0.5 & 0 & \multicolumn{1}{c}{0} & 0.5 & 1 \\
0 & 0.3 & 1 & 0.1 & \multicolumn{1}{c}{0.1} & 0.2 & 0.3 \\
\cline{1-3}
0 & 0.3 & 0.8 & 0.4 & \multicolumn{1}{c}{1} & 0.1 & 0.7 \\
\cline{4-7}
1 & 0.7 & 0.3 & 1 & 0.1 & 0.7 & 1 \\
0.5 & 0.8 & 0.9 & 0 & 0.3 & 0.2 & 0.5
\end{array}\right]$$

$$= \left[\begin{array}{c|c|c}
A_1 & \multicolumn{2}{c}{A_2} \\
\hline
A_3 & A_4 & A_5
\end{array}\right]$$

and

$$P_s = \left[\begin{array}{ccc|cc|cc}
0 & 0.7 & 1 & 0 & \multicolumn{1}{c}{0.6} & 1 & 0.7 \\
0.3 & 0 & 0.8 & 1 & \multicolumn{1}{c}{0.5} & 0.3 & 1 \\
0.1 & 0.9 & 0 & 0 & \multicolumn{1}{c}{1} & 0.2 & 0 \\
\cline{1-3}
0.9 & 0 & 0.7 & 0.5 & \multicolumn{1}{c}{0.1} & 0.1 & 1 \\
\cline{4-7}
0.2 & 0.3 & 0.5 & 0.3 & 0.5 & 1 & 0.7 \\
0.4 & 0.1 & 0.6 & 1 & 0 & 0 & 0.5
\end{array}\right]$$

$$= \left[\begin{array}{c|c|c}
B_1 & \multicolumn{2}{c}{B_2} \\
\hline
B_3 & B_4 & B_5
\end{array}\right]$$

We see both min $\{P_s, F_s\}$; $P_s + F_s = \min \{P_s, F_s\}$ where min function is well defined.

For if

$$F_s = \begin{bmatrix} (a_{ij}^1) & (a_{ij}^2) & \\ (a_{ij}^3) & (a_{ij}^4) & (a_{ij}^5) \end{bmatrix}$$

where $A_t = (a_{ij}^t)$; t = 1, 2, 3, 4, 5.

and

$$P_s = \begin{bmatrix} (b_{ij}^1) & (b_{ij}^2) & \\ (b_{ij}^3) & (b_{ij}^4) & (b_{ij}^5) \end{bmatrix}$$

where $B_p = (b_{ij}^p)$; p = 1, 2, 3, 4, 5.

$$\min \{F_s + P_s\} = \begin{bmatrix} \min\{a_{ij}^1, b_{ij}^1\} & \min\{a_{ij}^2, b_{ij}^2\} & \\ \min\{a_{ij}^3, b_{ij}^3\} & \min\{a_{ij}^4, b_{ij}^4\} & \max\{a_{ij}^5, b_{ij}^5\} \end{bmatrix}$$

$$= \left[\begin{array}{ccc|cccc} 0 & 0.1 & 0.2 & 0 & 0.2 & 0 & 0.1 \\ 0.3 & 0 & 0.5 & 0 & 0 & 0.3 & 1 \\ 0 & 0.3 & 0 & 0 & 0.1 & 0.2 & 0 \\ \cline{1-3} 0 & 0 & 0.7 & 0.4 & 0.1 & 0.1 & 0.7 \\ \cline{4-7} 0.2 & 0.3 & 0.3 & 0.3 & \multicolumn{1}{c|}{0.1} & 0.7 & 0.7 \\ 0.4 & 0.1 & 0.6 & 0 & \multicolumn{1}{c|}{0} & 0 & 0.5 \end{array}\right]$$

Thus only when compatibility exists we may be in a position to define min function.

Now we proceed onto define max function on F_s and P_s.

$$\max \{F_s + P_s\} = \begin{bmatrix} \max\{a_{ij}^1, b_{ij}^1\} & \max\{a_{ij}^2, b_{ij}^2\} & \\ \max\{a_{ij}^3, b_{ij}^3\} & \max\{a_{ij}^4, b_{ij}^4\} & \max\{a_{ij}^5, b_{ij}^5\} \end{bmatrix}$$

$$= \left[\begin{array}{ccc|cccc}
0.3 & 0.7 & 1 & 0.7 & 0.6 & 1 & 0.7 \\
1 & 0.2 & 0.8 & 1 & 0.5 & 0.5 & 1 \\
0.1 & 0.9 & 1 & 0.1 & 1 & 0.2 & 0.3 \\ \cline{1-3}
0.9 & 0.3 & 0.8 & 0.5 & 1 & 0.1 & 1 \\ \cline{4-7}
1 & 0.7 & 0.5 & 1 & 0.5 & \multicolumn{1}{|c}{1} & 1 \\
0.5 & 0.8 & 0.9 & 1 & 0.3 & \multicolumn{1}{|c}{0.2} & 0.5
\end{array}\right].$$

Now we proceed onto define max min of F_s, P_s.

i.e., max min $\{F_s, P_s\}$

$$= \begin{bmatrix} \max\min\{A_1, B_1\} & \max\min\{A_2, B_2\} & \\ \max\min\{A_3, B_3\} & \max\min\{A_4, B_4\} & \max\min\{A_5, B_5\} \end{bmatrix}$$

$$\max\min \{F_s, P_s\} = \left[\begin{array}{ccc|cccc}
0.1 & 0.3 & 0.3 & 0.2 & 0.6 & 0.7 & 0.7 \\
0.2 & 0.7 & 1 & 0.5 & 0.5 & 0.2 & 1 \\
0.3 & 0.9 & 0.3 & 0.3 & 0.2 & 0.2 & 0.3 \\ \cline{1-3}
0.4 & 0.3 & 0.6 & 1 & 0.5 & 0.4 & 1 \\ \cline{4-7}
0.9 & 0.3 & 0.7 & 0.3 & 0.5 & \multicolumn{1}{|c}{0.7} & 0.7 \\
0.5 & 0.3 & 0.6 & 0.3 & 0 & \multicolumn{1}{|c}{0.2} & 0.5
\end{array}\right]$$

At times we may have only 'addition' to be compatible. For instance consider the example.

Example 4.27: Let F_s and P_s be any two quasi fuzzy super matrices given by

$$F_s = \left[\begin{array}{ccc|cccc}
0.3 & 0.8 & 1 & 0.4 & 1 & 0 & 0.9 \\
0 & 0.4 & 0.5 & 0.8 & 0 & 0.8 & 0.4 \\
0.8 & 1 & 0 & 0.6 & 0.7 & 1 & 0.3 \\
0.4 & 0.3 & 0.8 & 0.1 & 0.5 & 0.6 & 0.1 \\ \cline{4-7}
0.5 & 0.8 & 0.6 & 1 & 0 & 0.8 & 0.1 \\ \cline{1-3}
0.5 & 1 & 0 & 0.5 & 0.6 & 1 & 0 \\
1 & 0 & 0.8 & 1 & 0.4 & 0 & 0.2 \\
0.4 & 0.5 & 1 & 0.3 & 1 & 0.4 & 0.7
\end{array}\right]$$

and

$$P_s = \left[\begin{array}{ccc|cccc}
0.7 & 0.3 & 0.5 & 0.3 & 0.5 & 0.1 & 0 \\
1 & 0.8 & 0.4 & 0.8 & 0 & 0.4 & 0.9 \\
0.7 & 0.6 & 0.3 & 0.1 & 0.8 & 0.1 & 0.3 \\
0.3 & 0.8 & 0.5 & 0.7 & 0.9 & 0.7 & 0.2 \\ \cline{4-7}
0.1 & 0.5 & 0.1 & 0.3 & 0.4 & 0.5 & 0.3 \\ \cline{1-3}
0.3 & 0.2 & 0.4 & 0.7 & 0.5 & 0.1 & 0.8 \\
0.1 & 0.8 & 0.3 & 0.3 & 0.7 & 0.2 & 0.1 \\
0.8 & 0.7 & 0.1 & 0.1 & 0.6 & 0.3 & 0.5
\end{array}\right].$$

Let us denote

$$F_s = \begin{bmatrix} A_1 = \left(a_{ij}^1\right) & A_2 = \left(a_{ij}^2\right) \\ A_3 = \left(a_{ij}^3\right) & A_4 = \left(a_{ij}^4\right) \end{bmatrix}$$

and

$$P_s = \begin{bmatrix} B_1 = \left(b_{ij}^1\right) & B_2 = \left(b_{ij}^2\right) \\ B_3 = \left(b_{ij}^3\right) & B_4 = \left(b_{ij}^4\right) \end{bmatrix}$$

max min $\{F_s, P_s\}$ is not defined as max min $\{A_1, B_1\}$ is not defined min or max is defined by

$$\max\ \{F_s + P_s\} = \begin{bmatrix} \max(a_{ij}^1, b_{ij}^1) & \max(a_{ij}^2, b_{ij}^2) \\ \max(a_{ij}^3, b_{ij}^3) & \max(a_{ij}^4, b_{ij}^4) \end{bmatrix}$$

$$= \left[\begin{array}{ccc|cccc} 0.7 & 0.8 & 1 & 0.4 & 1 & 0.1 & 0.9 \\ 1 & 0.8 & 0.5 & 0.8 & 0 & 0.8 & 0.9 \\ 0.8 & 1 & 0.3 & 0.6 & 0.8 & 1 & 0.3 \\ 0.4 & 0.8 & 0.8 & 0.7 & 0.9 & 0.7 & 0.2 \\ \cline{4-7} 0.5 & 0.8 & 0.6 & 1 & 0.4 & 0.8 & 0.3 \\ \cline{1-3} 0.5 & 1 & 0.4 & 0.7 & 0.6 & 1 & 0.8 \\ 1 & 0.8 & 0.8 & 1 & 0.7 & 0.2 & 0.2 \\ 0.8 & 0.7 & 1 & 0.3 & 1 & 0.4 & 0.7 \end{array}\right].$$

Thus we see at times max and min may be defined but max min may not be defined.

Here also entries of all these quasi super matrices can be replaced by the entries from neutrosophic ring, dual number ring, dual neutrosophic number rings and so on.

These new structures find applications in constructing the fuzzy neutrosophic quasi super models.

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Dr.W.B.Vasantha Kandasamy is an Associate Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 646 research papers. She has guided over 68 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 74th book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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The authors introduce the concept of neutrosophic super matrices and the new notion of quasi super matrices. This new class of quasi super matrices contains the class of super matrices. This larger class contains more partitions of the usual simple matrices. Studies in this direction are interesting and can find more applications in fuzzy models. The authors also suggest in this book some open problems

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