A Self-Gravitational Upper Bound on Localized Energy, Including that of Virtual Particles and Quantum Fields, which Yields a Passable "Dark Energy" Density Estimate

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Abstract

The self-gravitational correction to a localized spherically-symmetric static energy distribution is obtained from energetically self-consistent upgraded Newtonian gravitational theory. The result is a gravitational redshift factor that is everywhere finite and positive, which both rules out gravitational horizons and implies that the self-gravitationally corrected static energy contained in a sphere of radius r is bounded by r times the fourth power of c divided by G. Even in the absence of spherical symmetry this energy bound still applies to within a factor of two, and it cuts off the mass deviation of any quantum virtual particle at about a Planck mass. Because quantum uncertainty makes the minimum possible energy of a quantum field infinite, such a field's self-gravitationally corrected energy attains the value of that field's containing radius r times the fourth power of c divided by G. Roughly estimating any quantum field's containing radius r as c times the age of the universe yields a "dark energy" density of 1.7 joules per cubic kilometer. But if r is put to the Planck length appropriate to the birth of the universe, that energy density becomes the enormous Planck unit value, which could conceivably drive primordial "inflation". The density of "dark energy" decreases as the universe expands, but more slowly than the density of ordinary matter decreases. Such evolution suggests that "dark energy" has inhomogeneities, which may be "dark matter".

Introduction: self-gravitational capping of net localized energy

Einstein's identification of mass as a form of energy and of energy as a source of gravitation has the basic, but regrettably little appreciated, consequence of capping the possible energy content of any localized region at a value which is proportional to (c^4/G) times that region's maximum linear dimension. A very rough heuristic picture of how this comes about can be gleaned from thinking about two equal idealized "point masses" of value $m > 0$ that are separated by a distance $d > 0$. Of course these two "point masses" will gravitationally attract each other with a vector force $-(Gm^2\mathbf{d})/|\mathbf{d}|^3$, where $|\mathbf{d}| = d$, which tends to reduce their separation. We can either impose a constraint to stop that from occurring, or, a bit more elegantly, kick the two particles into a gravitational mutual circular orbit which automatically maintains their desired relative separation d. The kinetic energy of such a circular orbit is half of the negative of its potential energy $-(Gm^2)/d$ from the virial theorem [1], so that with the inclusion of the energy of the two masses the system's total energy comes to $[2mc^2 - ((Gm^2)/(2d))]$. As a function of the value m of those two equal "point masses", this total energy *has a maximum value* which occurs at $m = 2(c^2/G)d$ and equals $2(c^4/G)d$. In view of the fact *that we haven't made an allowance* for the negative gravitational energy *which arises as a consequence of the system's orbital kinetic energy*, this result of $2(c^4/G)d$ *is actually an overestimate* of the system's maximum possible net effective energy.

If we *alternatively* simply *impose* the separation d of the two particles *as a static constraint*, the system's total energy *instead* comes to,

$$E(m; d) = [2mc^2 - ((Gm^2)/d)], \tag{1a}$$

which as a function of m has a maximum value that occurs at $m = (c^2/G)d$ and equals,

$$E_{\max}(d) = (c^4/G)d. \tag{1b}$$

It is therefore apparent *that in any case* the self-gravitation of such a *localized* system indeed *caps its net effective energy* at a value which is proportional to (c^4/G) times its maximum linear dimension d. Now such an energy cap proportional to a system's maximum linear dimension implies that any "point" system must have *zero* energy! So our use of two equal *nonzero* "point masses" to *construct* the systems of the above examples *isn't really physically self-consistent*. The next section will therefore deal with *nonsingularly distributed localized positive energy*, albeit only for such localized positive energy distributions which are static and spherically symmetric; more general nonsingularly distributed static localized positive energy is

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taken up in a later section. The fact that *nonzero point masses do not exist in relativistic gravity theory* has been very strongly emphasized *as well* by Christoph Schiller in connection with his demonstration of the equivalence of his principle of maximum force to the Einstein equation in a context of general covariance [2].

Self-gravitational reduction of spherically-symmetric localized static energy

We denote an initially prescribed *idealized* (i.e., *before* its self-gravitational energy reduction) static spherically-symmetric nonsingular positive-energy density that is localized to a sphere of radius r_s as $T_{G=0}(r)$, where, $T_{G=0}(r) > 0$ for $r < r_s$ and $T_{G=0}(r) = 0$ for $r > r_s$, while we denote the corresponding *physically realistic* self-gravitationally reduced *effective* energy density as $T_G(r)$. These two energy densities *are related to each other through the dimensionless local negative Newtonian gravitational potential* $\phi(r)$ *via* $T_G(r) = T_{G=0}(r)(1 + \phi(r))$. Furthermore, Newtonian gravitational theory *is upgraded to being energetically self-consistent* when $\phi(r)$ follows from the physically realistic self-gravitationally reduced actually *effective* energy density $T_G(r)$ via the Newtonian expression,

$$\phi(r) = \phi(|\mathbf{r}|) = -(G/c^4)\int_{|\mathbf{r}'|\leq r_s} \frac{d^3\mathbf{r}' T_G(|\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} = -((4\pi G)/c^4)\int_0^{r_s} (r')^2 dr' T_G(r')/r_>, \tag{2a}$$

where $r_>$ abbreviates $\max(r, r')$. It is actually *more convenient* to deal *directly* with the dimensionless *local gravitational energy-reduction factor* $\psi(r) \stackrel{\text{def}}{=} (1+\phi(r))$ than with the local Newtonian gravitational *potential* $\phi(r)$. Transcribed in terms of $\psi(r)$ *and the initially prescribed idealized energy density* $T_{G=0}(r)$, Eq. (2a) becomes,

$$\psi(r) = 1 - ((4\pi G)/c^4)\int_0^{r_s} (r')^2 dr' T_{G=0}(r')\psi(r')/r_>, \tag{2b}$$

which transparently manifests the *fed back* character of $\psi(r)$; *elementary* Newtonian gravitational theory which is *not* energetically self-consistent *is recovered* when $\psi(r')$ within the integrand on the right-hand side of Eq. (2b) *is replaced by unity*. We further note that since,

$$T_G(r) = T_{G=0}(r)\psi(r), \tag{2c}$$

Eq. (2b) is readily verified to imply that,

$$\psi(r) = 1 - (r_s/r)((GE_G)/(c^4 r_s)) \text{ when } r \geq r_s, \tag{2d}$$

where E_G is of course the spherically-symmetric static energy distribution's self-gravitationally reduced *effective total energy*.

For a localized photon wave packet the dimensionless local gravitational energy-reduction factor $\psi(r)$ is obviously *also* a local gravitational *frequency reduction factor*. Therefore its dimensionless *inverse* $(1/\psi(r))$ *is the local gravitational redshift or time-dilation factor*. We thus are here *implicitly* dealing with the $g_{00}(r)$ component of a metric tensor [3],

$$g_{00}(r) = (\psi(r))^2. \tag{2e}$$

Since $\psi(r)$ is based on the energetically self-consistent upgraded *Newtonian* gravitational potential $\phi(r)$ of Eq. (2a), our *implicit* metric tensor that is alluded to in Eq. (2e) is expressed in "Newtonian" coordinates.

In order to understand the *behavior* of $\psi(r)$ we now proceed to *repeatedly differentiate* Eq. (2b). To that end we explicitly express $(1/r_>)$ as $\theta(r-r')/r + (1-\theta(r-r'))/r'$, and thus obtain $d(1/r_>)/dr = -\theta(r-r')/r^2$. Applying this result to Eq. (2b) yields,

$$d\psi(r)/dr = ((4\pi G)/(c^4 r^2))\int_0^{\min(r,r_s)} (r')^2 dr' T_{G=0}(r')\psi(r'), \tag{2f}$$

Furthermore, $d^2(1/r_>)/dr^2 = -\delta(r - r')/r^2 - (2/r)d(1/r_>)/dr$, which applied to Eq. (2b) yields,

$$d^2\psi(r)/dr^2 + (2/r)d\psi(r)/dr = \theta(r_s - r)((4\pi G)/c^4)T_{G=0}(r)\psi(r). \tag{2g}$$

Eq. (2g) reveals that $\psi(r)$ satisfies a zero-energy S-wave Schrödinger-equation analogue with a potential barrier produced by $T_{G=0}(r) \geq 0$ in the region $0 \leq r \leq r_s$ and empty space in the region $r > r_s$, where the solution $\psi(r)$ is analytically represented by Eq. (2d). Given the fact of zero energy, the potential barrier resists penetration anywhere that it is positive, so for $0 \leq r \leq r_s$ we would expect $\psi(r)$ *to approximately increase exponentially with increasing* r.

This behavior is *indeed* manifested by the "within the barrier" S-wave WKB approximation,

$$\psi(r) \approx (r_0/r)\sinh\left(\int_0^r dr'[((4\pi G)/c^4)T_{G=0}(r')]^{\frac{1}{2}}\right) \text{ for } 0 \le r \le r_s,$$

which is to be smoothly joined to Eq. (2d) at $r = r_s$, *which would approximately determine both the positive constant* r_0 *and also the self-gravitationally reduced effective total energy* E_G that is needed to make Eq. (2d) definite. By thus utilizing the "within the barrier" WKB approximation together with Eq. (2d) we obtain a cobbled-together picture of the behavior of $\psi(r)$, namely *an increasing positive function of* r *for all* $r \ge 0$ *which smoothly rises from its minimum positive value at* $r = 0$ *toward its asymptotic value of unity as* $r \to \infty$.

By enlisting the help of Eq. (2f) we can *reaffirm* this picture of $\psi(r)$ *without utilizing the WKB approximation.* Eq. (2f) relates the local $r \to 0+$ behaviour of $d\psi(r)/dr$ to $\psi(r = 0)$ as follows,

$$d\psi(r)/dr \sim ((4\pi G)/(3c^4))rT_{G=0}(r = 0)\psi(r = 0), \tag{3a}$$

which implies that as $r \to 0+$,

$$\psi(r) \sim [1 + ((2\pi G)/(3c^4))r^2 T_{G=0}(r = 0)]\psi(r = 0). \tag{3b}$$

Assuming that $T_{G=0}(r = 0) > 0$, we see from Eq. (3b) that if $\psi(r = 0) < 0$, then $\psi(r)$ will initially *decrease from that negative value*, which makes the Eq. (2f) expression for $d\psi(r)/dr$ *negative*, the upshot being a negative $\psi(r)$ *that keeps on decreasing.* If $\psi(r = 0) = 0$ we see from Eq. (3b) that *the first and second derivatives of* $\psi(r)$ *at* $r = 0$ *also vanish.* Therefore, given the linear homogeneous second-order differential Eq. (2g), $\psi(r)$ *vanishes identically all the way to* $r = r_s$, which behaviour *can't be smoothly joined there to the* $\psi(r)$ *form given by* Eq. (2d). Therefore, the $r \to \infty$ *asymptotic value of* $+1$ *for* $\psi(r)$ *that* Eq. (2d) *requires* is compatible *only* with a *positive* value for $\psi(r = 0)$, from which Eq. (2f) tells us $\psi(r)$ *keeps on increasing.* In cases that $T_{G=0}(r = 0)$ *vanishes*, very *similar* arguments regarding the positive and increasing nature of $\psi(r)$ apply for all values of r larger than that r-value beyond which $T_{G=0}(r)$ *first takes on positive values*, with $\psi(r)$ *being a positive constant* for r-values *smaller* than that.

Having thereby shown that,

$$1 \ge \psi(r) > 0 \text{ for all } r \ge 0, \tag{4a}$$

we note that this inequality holds in particular at $r = r_s$, which from Eq. (2d) implies that,

$$0 \le ((GE_G)/(c^4 r_s)) < 1. \tag{4b}$$

Finally, if one *increases* $T_{G=0}(r)$, it is apparent from Eq. (2b) that the positive increasing function $\psi(r)$ responds by *decreasing* in the interval $0 \le r \le r_s$. From Eq. (2b) one sees that the average value of $\psi(r)$ in the interval $0 \le r \le r_s$ times roughly $[1 + ((GE_{G=0})/(c^4 r_s))]$ is equal to unity. Therefore *multiplying* $E_{G=0}$ by the dimensionless factor $N \ge 1$ roughly *divides* the average value of $\psi(r)$ in the interval $0 \le r \le r_s$ by the factor $[1 + N((GE_{G=0})/(c^4 r_s))]/[1 + ((GE_{G=0})/(c^4 r_s))]$.

In any event, taking $E_{G=0} \to \infty$ produces *an impenetrable barrier* in the interval $0 \le r \le r_s$, which implies that,

$$\psi(r) \to 0 \text{ for } 0 \le r \le r_s \text{ when } E_{G=0} \to \infty. \tag{4c}$$

For $r = r_s$, Eq. (4c) together with Eq. (2d) imply that,

$$((GE_G)/(c^4 r_s)) \to 1 \text{ when } E_{G=0} \to \infty. \tag{4d}$$

The inequality $\psi(r) > 0$ of Eq. (4a) *rules out gravitational horizons*—which occur where the gravitational redshift factor $(1/\psi(r))$ is locally infinite—*regardless of the strength of the nonnegative* $T_{G=0}(r)$. However, Eq. (4c) tells us that the positive *finite* gravitational local redshift factor $(1/\psi(r))$ can certainly be made *arbitrarily large*. These *dual aspects* of the behavior of the gravitational local redshift factor are related to the fact that Eq. (2b) *feeds back that factor's inverse* $\psi(r)$.

The issue of whether gravitational horizons can *ever* be physically realized in General Relativity *was fundamentally settled in the negative* by Christoph Schiller *in the course of his demonstration* that his principle of the *unattainable* least upper bound of $(c^4/(4G))$ on force magnitudes is, against a backdrop of general covariance, *equivalent* to the Einstein equation [2].

With regard to Schiller's principle vis-à-vis the inequalities given by our Eqs. (4a) and (4b), we note that the *Newtonian gravitational force magnitude* between *two identical* localized spherically-symmetric static systems, each with radius r_s and self-gravitationally reduced effective total energy E_G is equal to, *when they just touch*, $G(E_G)^2/[c^4(2r_s)^2]$. Requiring this to be *less* than Schiller's unattainable least upper bound $(c^4/(4G))$ *also* yields our key Eq. (4b) inequality $((GE_G)/(c^4r_s)) < 1$ *that reflects the nonexistence of a gravitational horizon at* $r = r_s$.

Although Schiller's principle *rules out physical realization of gravitational horizons*, the fact that physical systems can come *arbitrarily close* to attaining horizons (which is explicitly noted above in the paragraph below Eqs. (4c) and (4d)) *actually plays a crucial role in the demonstration that Schiller presents* [2].

The key inequality $((GE_G)/(c^4r_s)) < 1$ of Eq. (4b) *also* implies that if a localized static, spherically symmetric energy distribution is shrunk to a *point*, i.e., if its radius r_s is taken to zero, then *that sphere's effective energy* E_G *is as well forced to zero*. Therefore nonzero effective static *point energies* indeed *don't exist*, as noted in the introductory section of this paper—Schiller stresses that appreciation of that fact is crucial to proper understanding of General Relativity [2].

From Eq. (2d) we see that in the *empty-space* Schwarzschild region $r > r_s$ our "Newtonian" $g_{00}(r)$ of Eq. (2e) has the value $(1-((GE_G)/(c^4r)))^2$. Since the well-known "isotropic" Schwarzschild metric tensor [4] has $g_{00}(\rho) = (1 - ((GE_G)/(2c^4\rho)))^2/(1 + ((GE_G)/(2c^4\rho)))^2$, it can be readily verified that the "isotropic" Schwarzschild metric tensor is mapped into our "Newtonian" Schwarzschild metric tensor by the simple coordinate transformation $\rho(r) = r - (GE_G)/(2c^4)$. Applying that transformation to the *full* "isotropic" Schwarzschild metric tensor explicitly yields our *full* "Newtonian" Schwarzschild metric tensor,

$$\begin{aligned} ds^2 = (1 - ((GE_G)/(c^4r)))^2(cdt)^2 - (1 - ((GE_G)/(2c^4r)))^{-4}dr^2 - \\ (1 - ((GE_G)/(2c^4r)))^{-2}((rd\theta)^2 + (r\sin\theta d\phi)^2), \end{aligned} \tag{5}$$

which has the empty-space region of validity $r > r_s$. Since $((GE_G)/(c^4r_s)) < 1$ from Eq. (4b), it follows that in this "Newtonian" Schwarzschild metric tensor's empty-space *region of validity* $r > r_s$, $((GE_G)/(c^4r)) < 1$. That inequality makes it manifest that within its region of validity the Eq. (5) "Newtonian" Schwarzschild metric tensor *has no horizon nor any other unphysical anomaly*.

Of course *the reason that no horizon occurs* is that the self-gravitational *capping* of localized energy which is *inherent* to Eq. (4b) *always* locates the *putative* horizon of this "Newtonian" Schwarzschild metric tensor in a region that *is not in fact in empty space*; i.e., that is *outside* of the *empty-space region of validity* of the "Newtonian" Schwarzschild metric tensor.

That the putative horizon *always* "occurs" in a non-empty-space region where the "Newtonian" Schwarzschild metric tensor *is inherently invalid* is *indeed fortunate*, because *at its* $r = (GE_G)/c^4$ *putative horizon* the Eq. (5) "Newtonian" Schwarzschild metric tensor has bizarre (3 + 0) dimensions, *which violates the metric-tensor signature theorem* [5]!

Interacting quantum particles that have forbidden energy or mass

Because of its wave character, a quantum particle which interacts with a potential can penetrate a short distance into a region where the potential's value exceeds the particle's energy—such penetration is forbidden to classical particles. In such a region the quantum particle's kinetic energy and momentum squared effectively assume *negative* values, and its penetration length λ into that energetically forbidden region can be roughly described as,

$$\lambda \approx \hbar/(-p_{\text{eff}}^2)^{\frac{1}{2}}. \tag{6a}$$

Since for a nonrelativistic interacting particle,

$$E = p^2/(2m) + V, \tag{6b}$$

we can rewrite Eq. (6a) as,

$$\lambda \approx \hbar/(2m(V - E))^{\frac{1}{2}}, \tag{6c}$$

and the energetically forbidden region corresponds to $V > E$. If V should vary significantly from the edge of the forbidden region to the depth λ, Eq. (6c) will need to be regarded as an approximate *implicit relationship* which actually needs to be *solved* for λ. That implicit relationship is set up by reexpressing V *as a function of the distance from the edge of the forbidden region*, and the resulting new independent variable is then identified as λ.

A highly energetic interacting quantum particle can similarly deviate from the natural rest mass which it has when it is free, and thus enter a region of forbidden rest mass. Just as there is an effective length limit for quantum particle penetration into a region of forbidden energy, so there is an effective proper time limit, i.e., lifetime, for quantum particle penetration into a region of forbidden rest mass, namely,

$$\tau \approx \hbar/(\Delta_m c^2), \tag{7a}$$

where Δ_m is the rest-mass deviation experienced by an energetic interacting quantum particle and τ is the lifetime of that deviant-mass state. Given its limited lifetime, such a deviant-mass *virtual particle* is as well *limited in space* to a spherical region of radius $R \approx c\tau$,

$$R \approx c\tau \approx \hbar/(\Delta_m c). \tag{7b}$$

Given that radius R of Eq. (7b), Eq. (4b) then yields a self-gravitational approximate upper bound on the total energy E of this deviant-mass particle,

$$E \mathrel{\widetilde{<}} (c^4/G)R \approx c^3\hbar/(G\Delta_m). \tag{7c}$$

Since $\Delta_m c^2$ cannot exceed the deviant-mass particle's total energy E, it follows from Eq. (7c) that,

$$\Delta_m c^2 \mathrel{\widetilde{<}} c^3\hbar/(G\Delta_m). \tag{7d}$$

Therefore,

$$(\Delta_m)^2 \mathrel{\widetilde{<}} \hbar c/G, \tag{7e}$$

which implies that,

$$\Delta_m \mathrel{\widetilde{<}} (\hbar c/G)^{\frac{1}{2}}, \tag{7f}$$

namely that Δ_m is approximately bounded by the *Planck mass* $(\hbar c/G)^{\frac{1}{2}}$.

This approximate Planck-mass upper bound on the mass deviation of any interacting quantum virtual particle universally cuts off the *ultraviolet divergences* which bedevil quantum particle scattering amplitude calculations.

Self-gravitational reduction of the infinite energies of quantum fields

A noninteracting field (e.g., the source-free electromagnetic field) always decomposes into an infinite number of independent simple harmonic oscillators whose frequency spectrum has no upper bound. Upon quantization, each such oscillator has a *minimum positive energy* (i.e., quantum ground state energy) which is equal to its frequency times $\hbar/2$: that minimum energy is quantum theoretically *inviolable*, being *completely mandated by the quantum uncertainty principle*. The fact that there are an *infinite number* of such simple harmonic oscillators with an *unbounded frequency spectrum* implies that the corresponding quantum field *always has infinite energy*. If the field is set up in a bounded region, then its quantum counterpart has *not only* infinite energy, but necessarily *infinite average energy density* as well.

Therefore for quantum fields the uncertainty principle baldly confronts us with an unphysical nightmare, yet *without* this *selfsame* uncertainty principle *likewise* mandating a definite minimum energy, Rutherford's nuclear atom can't be sustained.

The last apparent hope for the beleaguered theorist in this harrowing circumstance lies with *the self-gravitational reduction of an initially infinite energy*, $E_{G=0} \to \infty$, which is contained in a spherical region of radius r_s, namely the situation described by Eq. (4d). A grace note is that the self-gravitationally reduced energy *result* for that situation *is both finite and simple*,

$$E_G = (c^4/G)r_s. \tag{8a}$$

Of course it makes no difference whatsoever *how many* or *what types* of quantum fields are contained in that spherical region of radius r_s; *any infinite* initial energy contained in that region produces the result of Eq. (8a).

The *total* self-gravitationally reduced energy of quantum fields, which is what Eq. (8a) is supposed to describe, isn't *directly* measurable. However, by making use of the fields' containing radius r_s, we can obtain from Eq. (8a) their self-gravitationally reduced *averaged energy density* $\bar{\rho}$,

$$\bar{\rho} = (3c^4/(4\pi G r_s^2)). \tag{8b}$$

It now remains to puzzle out what conceivable physics could produce the quantum fields' containing radius r_s. It is, of course, apparent that *no material substance* can serve to contain *the arbitrarily high frequencies* which such fields are able to muster. The universe' *cosmological redshift*, however, in principle ought to defang *any* frequency, and indeed appears to serve as the "containment" for what we can possibly hope to survey. Therefore a not altogether implausible crude estimate of the quantum fields' "containing radius" r_s ought to be given by the age of the universe [6] times the speed of light, which comes to about 1.3×10^{26} meters. Putting that value of r_s into Eq. (8b) yields about 1.7×10^{-9} joules per cubic meter (i.e., 1.7×10^{-8} ergs per cubic centimeter or 1.7 joules per cubic kilometer) as a crude estimate of the universe' average "dark energy" density. This is in fact of the same order of magnitude as what is yielded by observations [7, 8].

In addition to the ability of Eq. (8b) to yield a passable crude estimate of the current universe' average "dark energy" density, its systematics also seem fascinating. If we project it all the way back to the universe' birth, when r_s was presumbably of the order of magnitude of the Planck length $(G\hbar/c^3)^{\frac{1}{2}}$, then $\bar{\rho}$ approaches of order unity in Planck units of energy density, which is roughly 120 orders of magnitude greater than its value for the current universe.

Theorists *who did not attempt to actually model the physics which produces self-gravitational energy correction* have favored this particular *enormous* value of "dark energy" density because of their adoption of a physically-blinkered "universal fix" for infinite results, namely the replacement of any such infinity by one Planck unit of the appropriate dimensions [8]. Neither physical modeling of self-gravitational energy correction nor observations have much overlap with such undiscriminating replacement of the quantum energy-density infinity by its Planck-unit value, but it is still fascinating to consider that enormous Planck unit of "dark energy" density as being relevant to the *early* universe, as that would apparently provide an *automatic* mechanism for the heretofore puzzling "inflation" of that early universe.

Finally, Eq. (8b) suggests that the average "dark energy" density ought to decrease toward zero as the universe continues its expansion. This brings to mind the not infrequently expressed theorist preference for *exactly vanishing* "dark energy" density over its *observed* value [8], which while *immensely smaller* than the Planck unit of energy density, nonetheless *absolutely fails to vanish*. In fact, completely to the *contrary*, it *dominates* the *net* average energy density of our universe [7]. We see that Eq. (8b) apparently caters for *all* tastes in average "dark energy" density, whether those tastes gravitate toward the enormous Planck unit of energy density, zero energy density, or anything in between, including a passable rendition of the observed average "dark energy" density which actually obtains at the current stage of evolution of our universe. It is to be cautioned, however, that while Eq. (8b) indeed has average "dark energy" density decreasing toward zero as the universe continues to expand, the average density of *normal* matter would be expected to decrease *at a faster rate*, so that "dark energy" relative *dominance* would continue to grow.

Must self-gravitation be quantized to correct quantum energy infinities?

Gravity, like electromagnetism, is a *gauge theory*, and the issues surrounding its quantization formally parallel those issues in electromagnetism. In both cases there are dynamical, nondynamical and redundant fields present, and the *dynamical* fields in *both* cases are *two* in number and describe *transverse radiation*. *Only these two dynamical radiation fields are subject to quantization.*

What remains after the two transverse dynamical radiation fields are accounted for splits evenly into nondynamical and redundant fields, *neither* of which, of course, are subject to quantization. The four-potential of electromagnetism yields one redundant field and one nondynamical field of Coulombic character. The symmetric metric tensor of gravity yeilds four redundant fields and four nondynamical ones. One of the nondynamical fields very roughly corresponds to Newtonian gravity with roughly an energy-density source, while the other three merely round out a relativistic four-vector representation, and therefore have roughly a momentum-flux source.

If we look back at the previous parts of this article, it is clear that the self-gravitational corrections which are of overarching importance can all be profitably pondered in a quasi-static or outright static framework. The basic ingredients for self-gravitational corrections tend to be *Newtonian*, albeit *an energetically self-consistent form* of gravitational Newtonianism.

Gravitational radiation *doesn't* physically enter into self-gravitational correction, so gravity quantization *cannot* be an issue in such correction, any more than electromagnetic quantization can be an issue in electrostatics. Gravitostatics is merely more subtle than electrostatics because of its energetic self-consistency.

Self-gravitational reduction of arbitrary smooth localized static energy

In the second section of this article we showed that any spherically-symmetric static nonsingular positive energy distribution of radius r_s has, after self-gravitational reduction, a cap on its total effective energy E_G that is given by Eq. (4b), namely $E_G < (c^4/G)r_s$.

We now *extend* our study of the self-gravitational energy-reduction process begun in the second section to *any* specified *static energy density* $T_{G=0}(\mathbf{r})$ which is nonnegative, smooth and globally integrable, i.e.,

$$T_{G=0}(\mathbf{r}) \geq 0, \tag{9a}$$

$$\nabla_{\mathbf{r}}\left(T_{G=0}(\mathbf{r})\right) \text{ is continuous,} \tag{9b}$$

and,

$$\int T_{G=0}(\mathbf{r}')d^3\mathbf{r}' < \infty. \tag{9c}$$

Here we *specifically refrain* from making the assumption of the second section of this article that $T_{G=0}(\mathbf{r})$ *possesses spherical symmetry*, nor do we assume that it possesses *any other particular symmetry*.

Now if it were the case that we *actually had in hand the self-gravitational correction* $T_G(\mathbf{r})$ of the specified static energy density $T_{G=0}(\mathbf{r})$, we could calculate the negative Newtonian gravitational work done to bring the *infinitesimal original static energy* $T_{G=0}(\mathbf{r})d^3\mathbf{r}$ from infinity to its position at $\mathbf{r}$ while subject to the static gravitational field that is provided by $T_G(\mathbf{r})$, which yields the result $-(G/c^4)\int d^3\mathbf{r}' T_G(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^{-1}T_{G=0}(\mathbf{r})d^3\mathbf{r}$. However, because the static gravitational interaction inherently occurs between *pairs* of infinitesimal energies, we must take care to *avoid double-counting*, so we assign only *half* of this *negative gravitational work correction* to the infinitesimal static energy located at $\mathbf{r}$, and thus obtain,

$$T_G(\mathbf{r})d^3\mathbf{r} = \left[1 - \tfrac{1}{2}(G/c^4)\int d^3\mathbf{r}' T_G(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^{-1}\right] T_{G=0}(\mathbf{r})d^3\mathbf{r}. \tag{10a}$$

From Eq. (10a) we see that the dimensionless *local gravitational energy-reduction factor* $\mathcal{F}_G(\mathbf{r})$ that satisfies,

$$T_G(\mathbf{r})d^3\mathbf{r} = \mathcal{F}_G(\mathbf{r})T_{G=0}(\mathbf{r})d^3\mathbf{r}, \tag{10b}$$

is given by,

$$\mathcal{F}_G(\mathbf{r}) \stackrel{\text{def}}{=} 1 - \tfrac{1}{2}(G/c^4)\int |\mathbf{r}-\mathbf{r}'|^{-1}T_G(\mathbf{r}')d^3\mathbf{r}'. \tag{10c}$$

This static local gravitational energy-reduction factor $\mathcal{F}_G(\mathbf{r})$ is obviously the *inverse* of the corresponding gravitational time-dilation factor, and thus is equal to $(g_{00}(\mathbf{r}))^{\frac{1}{2}}$ [3]. If we insert the instruction implicit in Eq. (10b) into the right-hand side of Eq. (10c), we obtain the following inhomogeneous linear integral equation for $\mathcal{F}_G(\mathbf{r})$,

$$\mathcal{F}_G(\mathbf{r}) = 1 - \tfrac{1}{2}(G/c^4)\int |\mathbf{r}-\mathbf{r}'|^{-1}T_{G=0}(\mathbf{r}')\mathcal{F}_G(\mathbf{r}')d^3\mathbf{r}'. \tag{10d}$$

In light of Eq. (9c), we can deduce from Eq. (10d) that,

$$\lim_{|\mathbf{r}|\to\infty} \mathcal{F}_G(\mathbf{r}) = 1. \tag{11a}$$

Furthermore, since the integral transform *kernel* $-1/(4\pi|\mathbf{r}-\mathbf{r}'|)$ is the Green's function of the Laplacian operator $\nabla^2_{\mathbf{r}}$, we in addition deduce from Eq. (10d) that,

$$\nabla^2_{\mathbf{r}}\mathcal{F}_G(\mathbf{r}) = (2\pi G/c^4)T_{G=0}(\mathbf{r})\mathcal{F}_G(\mathbf{r}),$$

which is readily reexpressed as a zero-energy stationary-state nonrelativistic *Schrödinger equation* for the dimensionless wave function $\mathcal{F}_G(\mathbf{r})$, namely,

$$\left(-\hbar^2\nabla^2_{\mathbf{r}}/(2m) + V(\mathbf{r})\right)\mathcal{F}_G(\mathbf{r}) = 0, \tag{11b}$$

whose repulsive potential $V(\mathbf{r})$ is defined as,

$$V(\mathbf{r}) \stackrel{\text{def}}{=} \left[\pi\hbar^2 G/(mc^4)\right] T_{G=0}(\mathbf{r}). \tag{11c}$$

Because of Eq. (9c) it is clear from Eq. (11c) that,

$$\lim_{|\mathbf{r}|\to\infty} V(\mathbf{r}) = 0, \tag{11d}$$

which, in turn, implies that the large-$|\mathbf{r}|$ limit of $\mathcal{F}_G(\mathbf{r})$ that is given by Eq. (11a) is *consistent* with the Eq. (11b) zero-energy Schrödinger equation.

Having established the connection of the Eq. (10d) *gravitational* integral equation to the Eq. (11b) *zero-energy* stationary-state Schrödinger equation, we now furthermore note that for stationary states of *positive energy* $E > 0$ this Schrödinger equation becomes,

$$\left(-\hbar^2\nabla_{\mathbf{r}}^2/(2m) + V(\mathbf{r})\right)\langle\mathbf{r}|\psi_E\rangle = E\langle\mathbf{r}|\psi_E\rangle. \tag{12a}$$

From Eq. (11d) we see that as $|\mathbf{r}| \to \infty$, Eq. (12a) becomes simply,

$$\left(-\hbar^2\nabla_{\mathbf{r}}^2/(2m)\right)\langle\mathbf{r}|\psi_E\rangle = E\langle\mathbf{r}|\psi_E\rangle, \tag{12b}$$

whose solutions include *all* the dimensionless *plane waves* $e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$ for which $\mathbf{p}$ satisfies $|\mathbf{p}|^2 = 2mE$, as well, of course, as the *linear combinations* of these which comprise the the full set of angularly-modulated outgoing and ingoing free *spherical waves* which have *this same scalar wave number* $k = (2mE)^{\frac{1}{2}}/\hbar$ [9]. Thus we see that Eq. (12a) has a massive *inherent solution degeneracy*. A useful *resolution* of this solution degeneracy can be achieved by *reexpressing* Eq. (12a) in an *inhomogeneous* linear form that *can only be satisfied* by the *particular* permissible $|\mathbf{r}| \to \infty$ asymptotic behavior which *properly accords* with the *design* of a specified experiment.

That idea underlies the Lippmann-Schwinger *inhomogeneous modification of* Eq. (12a), which *forces* its wave function to behave as a specifically chosen *single* permitted plane wave $e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$ plus *only outgoing* angularly-modulated spherical waves in the asymptotic region $|\mathbf{r}| \to \infty$ where Eq. (12a) is adequately described by Eq. (12b). If we denote as $\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle$ the solution of Eq. (12a) which *satisfies* this *particular* permitted $|\mathbf{r}| \to \infty$ asymptotic behavior, then the *inhomogeneous* Lippmann-Schwinger equation that *uniquely* describes $\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle$ is [10],

$$\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar} - \langle\mathbf{r}|(-\hbar^2\widehat{\nabla^2}/(2m) - |\mathbf{p}|^2/(2m) - i\epsilon)^{-1}\widehat{V}|\psi_{\mathbf{p}}^+\rangle. \tag{13a}$$

From a *static gravitational* standpoint the *relevant feature* of the Eq. (13a) inhomogeneous Lippmann-Schwinger equation and its wave function $\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle$ is that,

$$\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle\big|_{\mathbf{p}=\mathbf{0}} = \mathcal{F}_G(\mathbf{r}), \tag{13b}$$

as is seen from comparison of the $\mathbf{p} = \mathbf{0}$ case of Eq. (13a) with Eq. (10d)—to make this comparison one must use Eq. (11c) to express $V(\mathbf{r})$ as the appropriate constants times $T_{G=0}(\mathbf{r})$, and one must also use the fact that the integral transform kernel $-1/\left(4\pi|\mathbf{r} - \mathbf{r}'|\right)$ is the coordinate-representation inverse (i.e., Green's function) of the Hilbert-space "Laplacian" operator $\widehat{\nabla^2} = -|\widehat{\mathbf{p}}|^2/\hbar^2$, namely that,

$$\langle\mathbf{r}|(\widehat{\nabla^2})^{-1}|\mathbf{r}'\rangle = -1/\left(4\pi|\mathbf{r} - \mathbf{r}'|\right).$$

Note that the *negative* imaginary infinitesimal $-i\epsilon$ which appears in Eq. (13a) is *unnecessary* when $\mathbf{p} = \mathbf{0}$, which represents a *purely static* state of affairs that has *no* distinguishable outgoing versus ingoing spherical waves. Indeed the massive solution degeneracy of the stationary-state Schrödinger equation given by Eq. (12a) *collapses* when $E = 0$.

Given the Eq. (13b) close relationship of the local gravitational energy-reduction factor $\mathcal{F}_G(\mathbf{r})$ to the Lippmann-Schwinger wave function $\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle$, it would seem logical to apply well-known general solution methods for Lippmann-Schwinger equations to our gravitational Eq. (10d). Unfortunately, however, the only widely-applied fully general solution method for Lippmann-Schwinger equations is *perturbational* in character, and therefore is *inherently* subject to *failure*.

The struggle to transcend the perturbational Born trap

If we bring the second term on the right-hand side of the Eq. (13a) Lippmann-Schwinger equation to its left-hand side, we obtain,

$$\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle + \langle\mathbf{r}|(\widehat{K} - E_{\mathbf{p}} - i\epsilon)^{-1}\widehat{V}|\psi_{\mathbf{p}}^+\rangle = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}, \tag{14a}$$

where $\widehat{K} \stackrel{\text{def}}{=} (-\hbar^2\widehat{\nabla^2}/(2m))$ is the kinetic energy *operator* and $E_{\mathbf{p}} \stackrel{\text{def}}{=} (|\mathbf{p}|^2/(2m))$ is the kinetic energy *c-number scalar* that corresponds to the c-number momentum vector $\mathbf{p}$. Taking $\langle\mathbf{r}|\mathbf{p}\rangle \stackrel{\text{def}}{=} e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$, the formal solution of Eq. (14a) is,

$$\langle\mathbf{r}|\psi_{\mathbf{p}}^+\rangle = \langle\mathbf{r}|[1 + (\widehat{K} - E_{\mathbf{p}} - i\epsilon)^{-1}\widehat{V}]^{-1}|\mathbf{p}\rangle. \tag{14b}$$

The only way forward at this point would seem to be expansion of the inverse of the operator in square brackets in the well-known Born geometric perturbation series, which involves successive *powers* with alternating signs of the particular operator,

$$\widehat{X} \stackrel{\text{def}}{=} (\widehat{K} - E_{\mathbf{p}} - i\epsilon)^{-1}\widehat{V},$$

acting on the momentum eigenstate $|\mathbf{p}\rangle$ [11]. If the operator $\widehat{X}$ dominates the identity on the momentum eigenstate $|\mathbf{p}\rangle$, it is not unlikely that the Born geometric series *diverges*. One might suppose that in such instances one could simply *recast* the Born geometric expansion to be in powers of the *inverse* of $\widehat{X}$, since one is formally free to choose *either* the expansion $[1+\widehat{X}]^{-1} = 1-\widehat{X}+\widehat{X}^2-\cdots$ *or* the expansion $[1+\widehat{X}]^{-1} = \widehat{X}^{-1}[1+\widehat{X}^{-1}]^{-1} = \widehat{X}^{-1}-(\widehat{X}^{-1})^2+(\widehat{X}^{-1})^3-\cdots$. Most unfortunately, however, since $\widehat{X}^{-1} = \widehat{V}^{-1}\left(\widehat{K} - E_{\mathbf{p}}\right)$, the operator $\widehat{X}^{-1}$ *vanishes altogether when applied to the momentum eigenstate* $|\mathbf{p}\rangle$. This unanticipated abrupt setback is a stark warning that hidden snares beset Born-style geometric perturbation expansions for the Lippmann-Schwinger equation.

For the $\mathbf{p} = \mathbf{0}$ case of the Lippmann-Schwinger equation that applies to *static gravitation*, we are, of course, *particularly* interested in *arbitrarily large* energy densities $T_{G=0}(\mathbf{r})$, and therefore in *arbitrarily strong* operators $\widehat{V}$ and $\widehat{X}$. Thus it is clear that we need to *entirely abjure* Born-style geometric perturbation expansion, but how could that *conceivably* be accomplished *in practice*? The *only* possibility is through exploration of *nonstandard manipulations* of the Lippmann-Schwinger equation.

Returning to Eq. (14a) we now *deliberately shun* the *elegant and natural* factorization of the *operator* $[1+\widehat{X}]$ on its left-hand side, which can only lead us down the primrose path to the Born geometric perturbation series, and *instead* opt to forcibly factor that side into two *mere functions of the vector coordinate* $\mathbf{r}$, the *first* of which is *still* $\langle\mathbf{r}|\psi^+_{\mathbf{p}}\rangle$ because we aim for a result which is at least *akin* to Eq. (14b), but the *second* of which is *repulsively inelegant*, being *merely the product of* $(\langle\mathbf{r}|\psi^+_{\mathbf{p}}\rangle)^{-1}$ with the *original left-hand side of* Eq. (14a). There *is* in fact "method" in that gross ugliness, however, because we can now *actually arithmetically divide* the plane wave $e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$ on the *right-hand side* of Eq. (14a) by that gauche *second* factor *without resorting to any kind of perturbation expansion*. A very heavy price has been paid in the coin of gross inelegance, but the goal of *no perturbation expansion whatsoever* has been achieved. To be sure, the almost childish manipulations just described *haven't* extracted any final *result* from Eq. (14a), what they have produced is *only* a basis for *refinement through iteration*. It is readily seen, however, that the iteration process is *devoid of perturbational characteristics*; it *instead* resembles a *continued fraction*.

The iteration formula we have just extracted from Eq. (14a) is explicitly,

$$\langle\mathbf{r}|\psi^{(n+1)+}_{\mathbf{p}}\rangle = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}/[1 + (\langle\mathbf{r}|\psi^{(n)+}_{\mathbf{p}}\rangle)^{-1}\langle\mathbf{r}|(\widehat{K} - E_{\mathbf{p}} - i\epsilon)^{-1}\widehat{V}|\psi^{(n)+}_{\mathbf{p}}\rangle], \tag{14c}$$

for $n = 0, 1, 2, \ldots$, where, of course, $\langle\mathbf{r}|\psi^{(0)+}_{\mathbf{p}}\rangle = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$. Therefore $(\langle\mathbf{r}|\psi^{(0)+}_{\mathbf{p}}\rangle)^{-1}$ is obviously well-defined, and the form of Eq. (14c) makes it apparent that for $n = 1, 2, \ldots$, $(\langle\mathbf{r}|\psi^{(n)+}_{\mathbf{p}}\rangle)^{-1}$ is well-defined as well. That the iteration formula of Eq. (14c) does not have perturbational characteristics, but rather those of a continued fraction is also manifest.

Finally, on inserting $\mathbf{p} = \mathbf{0}$ into Eq. (14c) we obtain the iteration formula for the gravitational energy-reduction factor $\mathcal{F}_G(\mathbf{r})$, which is,

$$\mathcal{F}^{(n+1)}_G(\mathbf{r}) = 1/[1 + \tfrac{1}{2}(G/c^4)(\mathcal{F}^{(n)}_G(\mathbf{r}))^{-1}\textstyle\int |\mathbf{r} - \mathbf{r}'|^{-1}T_{G=0}(\mathbf{r}')\mathcal{F}^{(n)}_G(\mathbf{r}')d^3\mathbf{r}'], \tag{15a}$$

for $n = 0, 1, 2, \ldots$, where, of course, $\mathcal{F}^{(0)}_G(\mathbf{r}) = 1$.

Since $T_{G=0}(\mathbf{r}) \geq 0$ from Eq. (9a), $T_{G=0}(\mathbf{r})$ is smooth from Eq. (9b), and $\int T_{G=0}(\mathbf{r}')d^3\mathbf{r}' < \infty$ from Eq. (9c), it is clear from Eq. (15a) that,

$$\text{if } 1 \geq \mathcal{F}^{(n)}_G(\mathbf{r}) > 0, \text{ then } 1 \geq \mathcal{F}^{(n+1)}_G(\mathbf{r}) > 0. \tag{15b}$$

Therefore we can conclude that,

$$1 \geq \mathcal{F}_G(\mathbf{r}) > 0. \tag{15c}$$

From Eqs. (9a), (10b) and (15c) we can deduce that,

$$0 \leq T_G(\mathbf{r}) \leq T_{G=0}(\mathbf{r}). \tag{15d}$$

From Eq. (10c) and the fact that the gravitational energy-reduction factor $\mathcal{F}_G(\mathbf{r})$ satisfies $\mathcal{F}_G(\mathbf{r}) > 0$ we can deduce that,

$$2(c^4/G) > \int |\mathbf{r} - \mathbf{r}'|^{-1}T_G(\mathbf{r}')d^3\mathbf{r}' = \int |\mathbf{r}''|^{-1}T_G(\mathbf{r} + \mathbf{r}'')d^3\mathbf{r}'',$$

where $\mathbf{r}'' \stackrel{\text{def}}{=} (\mathbf{r}' - \mathbf{r})$. Furthermore, we have that,

$$\int |\mathbf{r}''|^{-1} T_G(\mathbf{r} + \mathbf{r}'') d^3\mathbf{r}'' \geq \int_{|\mathbf{r}''| \leq R} |\mathbf{r}''|^{-1} T_G(\mathbf{r} + \mathbf{r}'') d^3\mathbf{r}'' \geq R^{-1} \int_{|\mathbf{r}''| \leq R} T_G(\mathbf{r} + \mathbf{r}'') d^3\mathbf{r}''.$$

Therefore from the two foregoing lines of displayed integral inequalities we can conclude that,

$$(c^4/G)(2R) > \int_{|\mathbf{r}''| \leq R} T_G(\mathbf{r} + \mathbf{r}'') d^3\mathbf{r}'', \tag{15e}$$

namely that the self-gravitationally corrected static energy contained in any sphere cannot exceed the diameter of that sphere times (c^4/G). Note that this result is *completely independent of any assumption concerning symmetry properties of the energy distribution*, and that it is in agreement with the extremely nonspherical result for two discrete particles which is given by Eq. (1b). Therefore it is likely to be overly conservative in practice. A more practical energy upper-bound estimate can be obtained by simply averaging the maximum possible and minimum possible values of $|\mathbf{r}''|$ that occur when integrating over the sphere of radius R, which yields $R/2$, and therefore the "rough" bound,

$$(c^4/G)R \mathrel{\tilde{>}} \int_{|\mathbf{r}''| \leq R} T_G(\mathbf{r} + \mathbf{r}'') d^3\mathbf{r}'',$$

which is in line with the result of Eq. (4b), for which spherical symmetry was assumed.

No doubt the most interesting aspect of what has just been presented here is the unfastening of the shackles of the perturbational Born-expansion paradigm for a class of equation systems that incorporate linear operators. A superior iteration method has been developed by deliberately shunning an attractive natural relationship that involves those linear operators in favor of concocting a clumsy artificial relationship that involves *only function values*. The point of proceeding in this way is that purely *arithmetic* operations with *function values* require *no approximations*, whereas even quite elementary-looking operations involving *operators* may *not* be practically feasible without making use of potentially disastrous perturbation expansions. Indeed function manipulations can, on the contrary, be *directed* toward the goal of achieving iteration schemes that have *continued fraction* rather than perturbational character.

Conclusion

We have constructed a spherically-symmetric Newtonian gravitostatic model with energy feedback that yields a simple and apparently very useful upper bound on the amount of self-gravitationally corrected energy which can be contained in a spherical region. That bound is (c^4/G) times the radius r_s of the sphere, a relationship which, inter alia, implies that the Schwarzschild radius *never* lies in free space, making the Schwarzschild singularity physically unrealizable. We have, moreover, been able to extend this Newtownian gravitostatic model to *nonsymmetric* energy distributions by means of an interesting nonperturbational continued-fraction iteration-solution method. That extension turns out to yield *qualitatively the same upper bound* (i.e., to within a factor of two) on the amount of self-gravitationally corrected static energy which can be contained in a spherical region.

Such a (c^4/G) times radius upper bound on a sphere's contained energy cuts off the mass deviation of an interacting quantum virtual particle at approximately the Planck mass, which in principle does away with the ultraviolet divergences that bedevil quantum particle scattering amplitude calculations.

The (c^4/G) times radius bound on a sphere's contained energy ought to be *attained* for contained quantum fields, which have *infinite energy* before self-gravitational correction, due to the combination of their unbounded frequency spectra and the quantum uncertainty principle. But *only* the universe *itself*, with its cosmological redshift, is actually capable of "containing" the arbitrarily high frequencies of a quantum field. Roughly estimating the radius of the universe as its age times the speed of light, and then dividing the (c^4/G) times this radius by the corresponding spherical volume that has this radius, yields a rough averaged "dark energy" density estimate of about 1.7 joules per cubic kilometer, which is of the same order of magnitude as observational data. The *same* formula suggests that the *early* universe might have had an immensely greater "dark energy" density, perhaps as much as a Planck unit of energy density, which would be roughly 120 orders of magnitude times its present value. It is interesting that this seems to provide an *automatic* mechanism for the inflation of the early universe. The formula also suggests that the "dark energy" density will be decreasing toward zero as the universe expands, but that it won't decrease as rapidly as the density of ordinary matter will, which will increase the relative dominance of dark energy.

Finally, there seems to be *no reason* why the *presently evolved* "dark energy" density *should not share the small inhomogeneities which are so typical of the rest of the universe*, such as the small peaks in the cosmic microwave background, and the galaxies, groups, filaments and voids in the distribution of luminous matter. If "dark energy" indeed has inhomogeneities, then might not those inhomogeneities *themselves* be the thing we call "dark matter"? In spite of all the gravitational evidence for "dark matter", there is apparently no observationally-known non-gravitational signal whatsoever for it. It would be a relief if something so elusive ultimately turned out to *not* have an independent existence.

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