Pedagogical use of relativistic mass at better visualization of special relativity

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Abstract. Relativistic mass is not incorrect. The main argument against it is that it does not tell us anything more than the relativistic energy tells us. In this paper it is shown that this is not true, because new aspects of special relativity (SR) can be presented. One reason for this definition is to show a relation between time dilation and relativistic mass. This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means even a simpler presentation than is shown with Einstein's general covariance. Therefore, this opposes that SR is only a theory of space-time geometry, but it needs also rest mass. Phenomenon of increasing of relativistic mass with speed can be used for a gradual transition from Newtonian mechanics to SR. It also shows how relativistic energy can have properties of matter. The postulates, which are used for the definition of SR, are therefore still clearer and the whole derivation of the Lorentz transformation is clearer. Such derivation also gives a more realistic example for the confirmation of Duff's claims.

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1. Introduction

Fundamental physics include quantum field theory, quantum mechanics, general relativity and special relativity (SR). Understanding of the elementary physical theories and of their various aspects is important both for students and for researchers of still undiscovered theories, such as quantum gravity. Although nowadays it is made many efforts to present fundamental physics more clearly, this is not yet achieved. Some calculations of it agree very precisely with measurements, but visualizations of their calculations are not perfect. Probably today the most unexplained physical theory is quantum physics. But there exist some explanations, which make it more understandable [1, 2, 3, 4]. General relativity is also tried to be presented more clearly [5, 6]. SR is the most simple of them, but it is not clear enough and it not agree with our feeling of Newtonian physics, to which we are accustomed to. In the paper it will be shown that one problem of incomprehensibility of SR is also in Einstein's advice not to use relativistic mass (m_r).

m_r is used here for a different interpretation of the theory of special relativity (SR). The shortest way to define m_r is:

$$m_\mathrm{r} = W/c^2\,, \tag{1}$$

where W is the total (or relativistic) energy and c is the speed of light. One argument against using m_r is that it confuses students. The main argument against using m_r is that there is no sense to do so [7, 8, 9, 10], as it does not tell us anything more than W tells us; although it is not incorrect. But, here the reasons will be shown that the definition of m_r is useful:

- A new relation between time dilation and m_r is presented.
- This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means a simpler presentation of background independence than it is shown with Einstein's *general covariance* [5, page 138].
- Such derivation gives a more realistic example for the confirmation of Duff's claims [11, 12] that the units and the dimensionful constants are physically nonexistent.
- The modified postulates of SR additionally clarify SR and the derivation of the Lorentz transformation.
- The next sense is to show, how increasing of m_r with speed can be used for a pedagogical gradual transition from Newtonian mechanics to SR. It is similarly with other aspect of derivations. Connection with Newtonian physics is important, because it is more imaginable and understandable to us.
- Even laymen are very familiar with the equation $E = mc^2$, where (1) is only generalization of this equation, therefore the SR derivation can become more familiar to laymen.
- m_r gives us a different aspect as W.

- After this derivation, the author better understands SR and he supposes that other will also.
- Other reasons for $m_{\rm r}$ can be found in [8, 13] and in references therein.

In section 2 it is shown somewhat a different derivation of SR, where $m_{\rm r}$ is used. It is evident also, how insight in SR is clearer. In section 3 is alternatively shown how $m_{\rm r}$ is increased at acceleration. This is shown in semi-SR approach and passes over to a full SR-approach. This additionally visualizes SR. In section 4 it is shown how space-time and matter are connected. It is shown with the common interpretation of SR and with this new interpretation with $m_{\rm r}$ that space-time does not exist without matter. This connection is also shown from other aspects. In section 5 it is shown how use of $m_{\rm r}$ confirms Duff's claims that units and the dimensionful physical constants are physically nonexistent.

2. Derivation with use of $m_{\rm r}$

Let us imagine a trolley inside of a moving rocket that moves perpendicularly to the direction of the rocket. When the rocket increases velocity, the trolley moves a little bit slower than before (we observe transversal component according to a rest observer). The common explanation is that a cause is time dilation. But, an alternative explanation can be that the transversal momentum is constant with the rocket velocity, hence increasing of $m_{\rm r}$ means smaller velocity of such a trolley. Such explanation shows a relation between $m_{\rm r}$ and time dilation. Such a relation also gives hint that space-time does not exist without matter.

An essential difference of such interpretation of SR with the common interpretation of SR is that here the following transformations are used:

$$m = m_{\rm r}/\gamma\,, \tag{2}$$

$$t' = t''/\gamma\,, \tag{3}$$

where m is mass of the trolley inside of the rocket, t' is transformed time inside of the rocket which is obtained with the common Lorentz transformation,‡ and γ is defined as

$$\gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \tag{4}$$

with v the velocity of the rocket. Of course, it is understandable that physics is the same if these transformations are used.

But, let us show a detailed derivation in order how things will be clearer. We will also see how input postulates can be simplified. Einstein's postulates of the SR are:

(i) The laws of physics are the same for all observers in an inertial state of motion.

(ii) All inertial observers always measure the speed of light as being the same.

‡ Time for a rest observer is commonly assigned as t. See also equations (10) and (11), which are the Lorentz equations, respecting equation (3).

Let us add to these two postulates still two known, acceptable postulates

(iii) Space is isotropic for all observers.

(iv) A maximal speed inside of every inertial system is a speed of light c'' (which is not necessarily equal to c).

But, let us omit postulate (ii). The reason for the omission of this postulate is that postulate (ii) will be derived in the following paragraphs.

Let us synchronize clocks in another inertial system so that we see them to move with the same rate as our clocks. (Of course, this does not mean synchronization in the opposite direction, thus an observer in another inertial system does not see both clocks synchronized.) Therefore, a speed of light in the transversal direction regarding an observer in a rocket (with respect of the our synchronization) can be simply calculated. An observer from the stationary system sees the speed of light equal to c, of course. He can use Pythagoras' theorem, and therefore he calculates that the observer in the rocket sees a transversal speed of light $c''_{\rm trans}$ equal to

$$c''_{\rm trans} = c/\gamma \,. \tag{5}$$

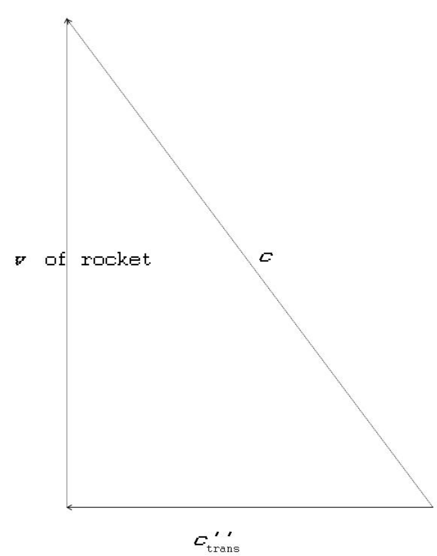

Figure 1. If clocks in a rocket are synchronized with the same rate as our clocks then an observer from the rocket measures smaller transversal speed of light, $c''_{\rm trans}$.

Postulate (iii) gives also that the longitudinal speed of light equals to the transversal speed of light, this means

$$c''_{\rm long} = c/\gamma \,. \tag{6}$$

Because of equal status of both inertial systems, an observer in a moving system must calculate γ in (4) with the same value, but this gives that he sees smaller velocity of the inertial system (v'') than it is seen by the first observer:

$$v'' = v/\gamma\,, \tag{7}$$

therefore, from his point of view all velocities are proportionally smaller.§ It can be said for another observer that he sees the same γ, but not the same c. This is one step closer to calculation with dimensionless numbers [11, 12], which are physically more fundamental.

The derivation of the *Lorentz transformation* can be started in the similar way as in the common derivation of SR; this means that we begin with the two initial equations of the Lorentz transformation:

$$x'' = \eta(x - vt)\,, \tag{8}$$

$$x = \eta(x'' + v''t'')\,, \tag{9}$$

where x and t are space and time coordinates of a stationary system and x'' and t'' are space and time coordinates of a moving system (for instance, a rocket), which moves with velocity v in the direction x.‖ η means a factor of Lorentz contraction and it still needs to be calculated by applying (6) and (7).¶ Hence, calculation gives that η equals γ. These equations also give how time is transformed:

$$t'' = \gamma^2(t - vx/c^2)\,, \tag{10}$$

$$t = t'' + v''x''/c''^2\,. \tag{11}$$

Now let us respect that everything in the moving system is moving slower, hence also its clocks and also processes in brains. Therefore, we can use the transformation (3). Equation (3) together with (6) to (11) gives back the common equations of the Lorentz transformation and hence gives the equal speed of light in all inertial systems. So, this is a transition from the above three postulates (i), (iii) and (iv) to the common two postulates (i) and (ii). But, further analyses will be done with t'', as it is calculated in (3).

The smaller speed of light in (5) and (6) (and proportionally smaller all velocities, for instance (7)), can be compensated by larger mass, so the momentum in the transversal direction is preserved.

Admittedly, the derivation of Lorentz transformation above is too long and it does not take advantage of γ calculated with figure 1, but it calculates it once again. The essence of equation (8) is that moving observer sees η times (γ times) larger distances between the points in x''-axis, than it is seen by the rest observer. But, this conclusion can be already obtained with the equation (6). Distance between two points on x''-axis

§ If this was not true, the velocity of the first system v'' would exceed c'', what is against postulate (iv).

‖ If we are more precise, then the minus sign can stand before $v''t''$ in (9) because, in truth, v'' means opposite velocity. In this case those four equations (8) to (11) become still more symmetric.

¶ The procedure with η (γ) is also used in the common derivation of the Lorentz transformation.

can be measured with a ray with speed $c''_{\rm long} = c''$ and this so that time of ray-travel is measured as first, and afterwards this time is used at definition of length. Thus

$$t''_R = c''/R\,, \tag{12}$$

where R means a momentary distance between two points, as seen by the rest observer. The rest observer measures time of travelling of a ray as

$$t_R = c/R\,. \tag{13}$$

Because of formula (6), it is valid

$$t''_R = \gamma t_R\,. \tag{14}$$

Now, let us assume that time of travelling of a ray is also a measure for distance, thus the relation between distances R and R'' is

$$R'' = \gamma R\,, \tag{15}$$

thus a moving observer sees larger longitudinal distances than a rest observer.

Equation (15) is not yet the end of the story. We need still to replace the moving observer with the rest observer (and vice versa), as it is given by the principle of relativity. This gives us still other Lorentz equations, as (9) and (11). Admittedly, relativity of speeds c and c'' is not perfect, so we need this replacement.

This derivation ignores simultaneity and unsimultaneity which are otherwise important at presentation of Lorentz contraction. Despite of this, the result is correct. Of course, simultaneity and unsimultaneity are consequences of the Lorentz contraction. It is not necessary to solve system of equations (8) and (9) to imagine factor γ of Lorentz contraction and time dilatation. So, the Lorentz contraction and time dilatation are explained in fewer steps.

Equations (10) and (11) are less symmetric than in the common SR derivation, because factors before (8) to (11) are γ, γ, γ^2, and 1, but in the common derivation all of them are equal to γ. However, we can also notice some simplifications:

- simplifications of the postulates (i) and (ii),
- simpler and clearer calculation of γ,
- dimensionless numbers are more frequently used,
- it is easier to get mental picture of derivation of the Lorentz contraction.
- presentation with momentum is clearer as also will be seen further,
- we can use the minus sign for velocity v''.

3. Some aspects of relativistic mass at acceleration

Now it is seen, how it is with conservation of the momentum in the transversal direction. Let us see still more clearly, how it is enlargement of momentum in the longitudinal direction, and let us look at, how it is with increasing of W with v. This can also be additionally clarified with use of m_r, and therefore this is a further visualization of these equations.

For the beginning, let us naively assume that space is Euclidean, and that acceleration increases W and hence also increases m_r. Then the equation for increasing of W of an accelerating body is:

$$\mathrm{d}W = c^2\mathrm{d}m_\mathrm{r} = m_\mathrm{r}a\mathrm{d}x = m_\mathrm{r}v\mathrm{d}v\,, \tag{16}$$

where a means acceleration, x means distance and v means velocity. A solution of (16) is

$$2\ln(m_\mathrm{r}/m_\mathrm{r0}) = (v/c)^2\,, \tag{17}$$

where m_r0 is mass at velocity zero, and ln is the logarithm with base e. The result is incorrect, because the real relation is

$$m_\mathrm{r} = \gamma m_\mathrm{r0}\,. \tag{18}$$

If formula (16) is corrected to

$$\mathrm{d}W = c^2\mathrm{d}m_\mathrm{r} = \gamma^2 m_\mathrm{r}a\mathrm{d}x = \gamma^2 m_\mathrm{r}v\mathrm{d}v\,, \tag{19}$$

the final result is (18), what is correct.

The equation equivalent to (19) is known from the common calculations of SR:

$$\mathrm{d}W/\mathrm{d}t = \gamma^3 mva\,. \tag{20}$$

Equation (20) is a consequence of a fact that

$$\mathrm{d}\gamma/\mathrm{d}(v/c) = \gamma^3(v/c)\,, \tag{21}$$

where v/c also means more natural unit for velocity. Equation (20) is interpreted that longitudinal relativistic mass (m_rl) equals

$$m_\mathrm{rl} = \gamma^3 m_\mathrm{r0}\,. \tag{22}$$

This anisotropy of m_r is disturbing, thus it is useful to find some evident analogy for factor γ^2, which was inserted in (19). One option is that factors γ are attributed to the speed of light, thus that enlargement of energy is connected with c'' and not with c. Thus, (19) should be modified into

$$\mathrm{d}W'' = c''^2\mathrm{d}m_\mathrm{r} = \mathrm{d}W/\gamma^2 = c^2\mathrm{d}m_\mathrm{r}/\gamma^2 = m_\mathrm{r}a\mathrm{d}x = m_\mathrm{r}v\mathrm{d}v\,, \tag{23}$$

and the result is (18), what is correct. Otherwise, c'' has an obvious meaning, but admittedly, W'' does not have obvious analogy.

Thus, in the present example the part γ^2 is attributed to the reduction of the speed of light and not to m_rl, what is clearer.

Explanation with c'' is equivalent to explanation with the Lorentz contraction, as it is explained in the section 2. That means that longitudinal distances x'' in the rocket seen from the rocket are larger than the same distances x seen from the rest system. This can also be comprehended from (8) to (11):

$$\mathrm{d}x'' = \gamma \mathrm{d}x \,. \tag{24}$$

If this is corrected in (16), the new equation is:

$$c^2 \mathrm{d}m_\mathrm{r} = m_\mathrm{r} a'' \mathrm{d}x'' = m_\mathrm{r} v'' \mathrm{d}v'' \,, \tag{25}$$

or

$$c^2 \mathrm{d}m_\mathrm{r} = m_\mathrm{r} \gamma a (\gamma \mathrm{d}x) = m_\mathrm{r} (\gamma v) \gamma \mathrm{d}v \,. \tag{26}$$

and the result is (18), what is correct.

Equations (25) and (26) show just oppositely than the equation (7). The reason is, because v'' in (7) presents velocity as it seen by an observer in the moving inertial system, whereas v'' in (25) means the velocity seen by a rest observer, but by considering length of rocket as seen by a moving observer. The same is valid for a''.

In principle, (23), (25) and (26) are only visualization of (21). Here we have only sides of the rectangular triangle and a number of possible options for analogies is small, thus possibility of mistaken analogy is small.

- Thus, properties of the mass (m_r), such as inertia or resting, are also important, not only properties of its energy counterpart (W).
- Therefore it is obtained with the use of analysis of acceleration, how W expressed with m_r increases with increasing of v. It is also presented, how addition of $\mathrm{d}m_\mathrm{r}$ is different as a first imagination.
- This derivation also tells us, how to include Lorentz contraction.
- At the same time, this is also a pedagogical gradual transformation from Newtonian mechanics to SR.

4. A connection between matter and space-time

A complaint is possible that rest observers see a larger mass inside the rocket, but observers in the rocket see larger masses in rest system. Therefore, it seems that larger mass is not realistic. However, this is exactly the same problem as in the common Lorentz equations, where relations for time show the same paradox. Yet, SR is a correct theory, and both the common interpretation of SR and the interpretation with m_r are correct. It is not necessarily to look at the same time from two inertial systems; it is enough to look at once from one inertial system and at another time from another inertial system. In short, it is not incorrect that mass-time relationship is only one-way.

This slower speed of time with increasing of m_r can also be generalized out of SR to big and small elementary particles. If a human body was made from the same particles, but 1000 times lighter ones, the speed of time would seem to us much smaller than now. Hence one second would seem very long. (This example is not relative, because it gives the same results from both observers.)

This can be generalized still further. A fly feels a longer second than an elephant, because of smaller mass of the fly brain the brain processes are faster. Although particles are not smaller, this can also be an analogy for enlargement or reduction of the "mass". Another example from biology is either a cold lizard or a warm one. For the first one a second seems shorter. Therefore examples of various time speeds have a very similar key foundation, this is the momentum or movement. The common derivation of SR avoids discussing this effect of increasing of mass at acceleration and thus avoids discussing its consequences.

We know from the common interpretation of SR that rest matter cannot be accelerated to $v = c$. It can only be approached to this speed. But, anywhere close to c this matter is moving, always we can find an inertial system, where this matter is at rest. The speed of a photon equals c. We cannot find an inertial system where it is at rest. Time flows where rest matter exists, but time does not flow for a photon. Therefore rest matter defines that time flows; hence time is dependent of rest matter. Thus, this can be found also from the common interpretation of SR.

Interpretation of SR with m_{r} tells us still more. It tells us that speed of time depends on largeness of mass. Therefore this is another clarification that space-time does not exist without rest matter. Hence, this is a simplified explanation of background independence, according to what is shown with general covariance [5, page 138]. Introduction of the Lorentz contraction in the above derivation also partially shows on the background independence. This alternative derivation of Lorentz contraction shows this more clearly than the common derivation, because it shows how distances become relative.

Hence formally, one time is really attributed to every point of space, but truly time flows only if rest matter is present, or differently saying, that matter is a reference for this space-time. Therefore space-time without rest matter does not exist. This fact means also consequences for quantum gravity theory.

But two details should still be clarified. Seemingly, time flows for a photon

- because it has some frequency,
- if it is calculated for rest matter that time does not flow at $v = c$, this does not mean automatically that time does not flow for a photon. Precisely said, it means only that time does not flow for rest matter if it is accelerated to $v = c$. And, of course, it never reaches this speed.

Frequency of a photon is dependent from rest matter, or from inertial system, where this matter is at rest; and, a privileged inertial system does not exist. Therefore rest matter cannot be ignored where the existence of photons is mentioned. Thus, photons exist because of rest matter. It is similarly in general relativity, where it is claimed that gravitational waves exist independently of matter. But indirectly they are connected with matter.

Hence energy shows a property of matter, this is inertia, what is expressed with the momentum. Energy also shows another property of matter that can be at rest. Of course, energy of photons cannot be at rest, but regarding all above, space-time does not exist without rest matter, therefore energy of photons also does not exist without rest matter.

5. Influence of m_{r} on Duff's claims

Duff presents an example, where the elementary dimensionless constants vary and he shows that variations of dimensionless "constants" is only physically significant, whereas the dimensionful "constants"[+] (Gravitational constant G, Planck's constant $\hbar$, c, masses of various elementary particles m_i) are dependent from the system of units, which is defined [11, 12]. He shows that the dimensionful constants are always redundant according the dimensionless constants, thus variations of the dimensionful constants do not show any physical background, but only show agreement about system of units. Therefore elementary units, kilogram, meter and second do not exist physically. He says that G, $\hbar$ and c are only conversion factors between different units [11, 12]. For instance, one system of units, which he uses for explanation, are Schrödinger units:

$$c''' = c/\alpha \,. \tag{27}$$

The above derivation with m_{r} gives an example with a similar conclusion as given by Duff. Although μ_is in this case are constant, it is evident that it is not necessary that c is constant in all inertial systems. ($\mu_i^2 = m_i^2 G/(\hbar c)$.) (Constancy of c is almost a holy thing in SR, but here it is shown that this is not necessary.) Thus, a conclusion is similar as at the Duff's example, whereas the example with m_{r} is more realistic and more concrete, thus less abstract. The m_{r} case also does not need changed system of units, it only needs a changed definition what an observer sees. Additionally, in [12, page 8] Duff also gives an example for the Lorentz transformation, where c is ignored so that he uses $c = 1$. But this is still ever an example, where c is constant, whereas the author's example breaks off the taboo that c should be constant.

Many times it is said that m_{r} does not give anything new - likewise it can be asked, what is sense of Duff's $c''' = c/\alpha$. The answer is that c''' gives new information, and, similarly, m_{r} also gives new one.

Besides, mathematics itself is based on dimensionless quantities, when physics is practically mathematics. Although the physics is not dimensionful, we imagine it as classical (or Newtonian) physics, which is dimensionful. It is dimensionful, because we do not directly imagine maximal speed c, we do not directly imagine the principle of uncertainty (thus we imagine as $\hbar = 0$), even curved space-time is not imagined enough. Thus, imagination of those "dimensionless theories"[*] helps to imagine physics correctly. Thus we can expect that SR should be explained more intuitively imaginable. The same is true also for quantum mechanics and general relativity. This dimensionless nature of

[+] If a quantity varies, it is no more a constant, therefore the quotation marks are used.
[*] SR, quantum mechanics, and general relativity

physics is also an excuse and confirmation for those queer theories and it is hoped that their queerness will be explained one day.

Oas comments that acceleration gives rise to energy whereas number of particles is not enlarged by it [10]. (So, by him, m_{r} does not tell us anything new according to relativistic energy (W).) But, this also tells us that m_{r} is increased inside of elementary particles, therefore elementary particles are these essential things.♯ So, dimensionless numbers μ_i are creators of all space-time.

Duff uses also a provocative question, why three elementary units exist, why not seven or any other number of them. His answer is that the number of basic elementary units is zero. A next answer was given by Okun that three basic units are consequence of "cube of theories", respectively because of three theories: QM, SR and general relativity, which describe physics separately and combined [11, page 25]. However, the author has a different theory. He claims that three units for length, time and mass (kilogram, meter and second), are important, because they form momentum. Momentum is important because of the law of conservation of momentum, and also because of importance of momentum at time feeling in the above derivation with m_{r}. Of course it is understandable, that length, time and mass can use the same units, for instance seconds, or they are even without units. Only their three dimensional structure is important, as described above.

The next example for visualization of SR is the Minkowski space. Let us define that time-axis is real and space-axes are imaginary. Thus SR metric can also be described with the formula:

$$\mathrm{d}s^2 = -\mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 + c^2\mathrm{d}t^2 \,. \tag{28}$$

s means distance between two events, x, y, and z are space axes, and t is time-axis. In this case imaginary distance in space-time means that information between two events cannot be transmitted if speed is not superluminal. (Distances between events are spacelike.)

Otherwise, despite of m_{r} the author agrees that the term space-time is correctly used in physics. But as a detail of (28), it is useful to mention that time is physically distinct from distance, what can be deciphered in (28) by a positive sign before t. This is not mentioned in Duff's article and anywhere, but it should be.

6. Conclusion

The common derivation and interpretation of SR is short enough and mathematically symmetric, but it is not imaginable enough and comparable with the common Newtonian physics. Derivation with m_{r} is, otherwise, dissuaded, because it should not tell us anything new. But, it tells a lot of new things, because derivation with m_{r} better shows on differences with Newtonian physics, on which we are intuitively more accustomed. It

♯ Rest matter is built up from elementary particles, but it can be built up also from black holes. Maybe those two things are the same. The answer is hidden in a quantum gravity theory.

more precisely shows what the most intuitively strange differences with the Newtonian physics are. Time dilation is connected with increasing of m_{r}. It is also clearer what is a role of Pythagoras' theorem in SR. (Another options of it is [14].) Time runs only in potentially rest objects. Thus space-time without rest matter does not exist. The Lorentz contraction can also be a consequence of distinct speeds of light in different inertial systems. It also gives that space-time without rest matter does not exist. A similar conclusion is also given by general covariance in the general relativity, but this conclusion in SR is simpler. Postulates are essence of physics. Different postulates can be found which tell more and are shorted. This is partially succeeded here.

SR is also one of the theories whose simplify physics. One of its simplifications is that speed of light is a natural conversion factor between length and time. Duff's argues that elementary units c, $\hbar$ and G do not exist. The above derivation simplifies his arguments. The next simplification is that space-time without matter does not exists.

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