

On the Planck Scale Potential Associated with Particles

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Abstract

As the particles originating from point-like entities are associated with infinite self energies, a postulate, that the scalar-potential associated with particles are bounded by the Planck scale potential is presented. By defining the self-energy of a particle in terms of its scalar-potential, equivalences between charge-energy and mass-energy are obtained. The electromagnetic energy-momentum equation and de-Broglie's electromagnetic wave-length and frequency associated with a charge particle in motion are derived. Relativistically covariant electromagnetic energy and momentum expressions are obtained, resolving the "4/3" discrepancy. The non-covariance nature of the present classical electrodynamics is discussed and how the proposed postulate makes it a fully covariant theorem with the rest of the classical electrodynamics is presented. How the electromagnetic energy-momentum equation could potentially resolve the stability-problem of a charge particle is discussed and thereby a theoretical explanation to electron's spin is presented.

Keywords: Planck scale potential, mass-energy equivalence, charge-energy equivalence, energy-momentum relation, classical electron theory, electromagnetic momentum, de-Broglie's matter wave frequency, de-Broglie's wave length.

1 Introduction

In classical electrostatics, the self-energy of a charge particle, whose charge is assumed to be uniformly distributed over its surface, constructed by bringing in infinitesimal amounts of charge quantities from infinity, diverges or goes to infinity, when the radius approaches to zero. However, to avoid the infinite energy problem, if the charge particle is assumed to be associated with a finite radius, then it gives rise to a problem of explaining how the like-charge distribution of the particle is held together, against the repulsive nature of the like-charges, known as the "stability-problem". Historically, a few different approaches have been proposed in order to account for the self-energy problem arising from the point-like charge particles and the stability-problem arising from the finite radius models. Stokes showed in 1844 [1] that the inertia

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of a body moving in an incompressible perfect fluid is increased. Noticing that the electromagnetic momentum and energy, and thereby the mass of charged bodies depend on their speed, Thomson in 1881 [2] showed that it is harder to set in motion, a charged sphere, moving in a space filled with a medium of a specific inductive capacity than an uncharged body. Due to this self-induction effect on charged bodies, the electromagnetic energy was considered to behave as having some sort of a momentum and apparent electromagnetic mass, which increases the ordinary mechanical mass of the charged bodies, or in modern terms, the increase should arise from their speed dependent electromagnetic self-energy. Using this model, Searle in 1897 derived the relativistic electromagnetic energy of a moving charge spheroid shell [3]. Then in 1904, Lorentz [4] computed the electromagnetic momentum associated with a moving charge spheroid shell. However, the electromagnetic energy and the momentum expressions they obtained were neither relativistically covariant nor transformed as an energy-momentum four-vector and lead to the “4/3” discrepancy problem. In order to remove this discrepancy, Poincare postulated that a non-electromagnetic force was necessary, to hold the like-charge distribution, and by doing so, he was able to solve both the stability and the “4/3” discrepancy problems [5]. However, the non-covariant ad-hoc forces associated with Poincare’s postulate are assumed to compensate for the non-covariance nature of the electromagnetic force, so that the entire electron system becomes covariant. Thereafter, many others attempted to resolve the “4/3” discrepancy problem and noteworthy among the efforts are those of Fermi [6], Mandel [7], Wilson [8], Dirac [9], Pryce [10], Kwal [11] and Rohrlich [12]. By showing that the Trouton-Noble experiment’s null result [13] can be explained if the energy density of an electromagnetic field can be expressed as the difference in electric and magnetic energy in vacuum, rather than their addition, Butler derived [14] a new energy density expression. He then discussed the non-covariance nature of the present classical electrodynamics and derived a relativistically covariant energy-momentum four-vector and resolved the “4/3” discrepancy problem. Further, he showed that the source of the non-variance of the energy and momentum expressions arise from the procedure used to derive the Poynting’s theorem, which is covariant only in the absence of charges in the moving frames [15]. Stratton also pointed out that the classical interpretation of Poynting’s theorem appears to rest to a considerable degree on hypothesis [16], while Pauli stated that “*Maxwell-Lorentz electrodynamics is quite incompatible with the existence of charges, unless it is supplemented by extraneous theoretical concepts*” [17]. A similar analysis on hidden momentum and electromagnetic mass of a charge body has been carried out by Hnizdo [18].

In QED (Quantum Electro-Dynamics), with renormalization techniques, the energy associated with an electron is separated into two parts: the energy associated by its interactions with other charge particles and the self-energy associated by its interactions with itself. In renormalization, the part that interacts with itself is removed or taken out from the theory and therefore, the electron’s charge doesn’t fly-off or repel itself. With this treatment, the infinities which arise, when the radius of the spherical electron goes to zero, is removed.

In the present paper, a new postulate that the scalar-potential associated with particles are bounded by the Planck scale potential is introduced. Using this postulate to define the self-energy of a charge particle, equivalence between charge and energy is obtained. Using a similar procedure, the conventional mass-energy equivalence is re-affirmed. Deriving the relativistic energy-momentum relation for a charge particle in motion, both the relativistically covariant energy and momentum expressions are obtained, resolving the “4/3” discrepancy. Extending de-Broglie’s concept, both the matter wave-length and matter-frequency in terms of electromagnetic energy and momentum expressions associated with a charge particle in motion are presented. With the proposed postulate, by deriving an energy-momentum four-vector, a covariant form of the classical electrodynamics is obtained. Discussing how the proposed postulate could potentially resolve the stability problem of a charge particle, a theoretical explanation to the electron’s spin is presented.

2 Self-energy of a charge particle

The energy of a charge particle of charge q , assumed to be uniformly distributed over the surface of the particle’s body of radius r , is given by:

$$U = \left(\frac{q^2}{8\pi\epsilon_0 r} \right) \quad (1)$$

Similarly, if the charge q is assumed to be distributed over the particle's volume with a constant charge density, the energy expression reads:

$$U = \frac{3}{5} \left(\frac{q^2}{4\pi\epsilon_0 r} \right) \quad (2)$$

The energy of a charge particle due to surface charge density or volume charge density diverges or said to give rise to a singularity, when the radius approaches to zero. In other words, the corresponding energies become infinite for point-like charge particles. Further, they only take into account the amount of potential energy that get stored in constructing the corresponding configurations in bringing infinitesimal amounts of charge quantities from infinity, and assume that the subsequent infinitesimal charge amounts do not possess self-energies associated with them, i.e. the conventional energy expressions given in equations 1 and 2 are based on the assumption that charge is continuous and indefinitely sub-divisible, and that, the subdivided infinitesimal charge quantities alone do not possess self-energies associated with them. Thus, the expressions given in equations 1 and 2 can be regarded as the potential energies of a system comprising infinite number of infinitesimal charge quantities, whose individual self-energies are assumed to be zero.

Proposing the equivalence between mass and energy in 1905 [19], Einstein concluded that the mass of a body is a measure of its energy content. That is, if the energy changes by E , the mass changes in the same sense give by:

$$E = mc^2 \quad (3)$$

Further, the relativistic energy of a particle with rest mass m and velocity u is obtained from the energy-momentum relation:

$$E^2 = (\gamma m u c)^2 + (m c^2)^2 \quad (4)$$

Nevertheless, in the context of relativity, mass is not considered as an additive quantity, in the sense that a collection of rest masses of particles in a system adds up to give the total rest mass of the system. Instead, the energy-momentum equation quantifies the amount of total invariant mass M of the system:

$$(M c^2)^2 = \left(\sum_{i=1}^n E_i \right)^2 - \left| \left(\sum_{i=1}^n \gamma_i m_i \mathbf{u}_i c \right) \right|^2 \quad (5)$$

where the velocities \mathbf{u}_i of each particle m_i are obtained with relative to the center-of-momentum of the mass body M . Therefore, a system composed of a collection of particles obeys the equation 5, whereas the equations 3 and 4 represent individual particles at rest and in constant velocity motion respectively. In contrast to the charge particle energy models, the mass-energy equivalence given in equation 3 is not constructed by bringing in an infinitesimal amount of mass quantities, each associated with zero self-energy content at infinity. That is, even at infinity, a corresponding infinitesimal amount of mass quantity δm , is assumed to be associated with a self-energy given by:

$$E = \delta m c^2 \quad (6)$$

In view of the self-energy problem arising from point-like charge particle models, the zero self-energies associated with infinitesimal amounts of charge quantities at infinity arising from finite radius charge particle models and the "4/3" problem associated with the classical electron theory, a postulate that the scalar-potential associated with particles are bounded by the Planck scale potential is put forth to construct a physical theory consistent with the theory of relativity. The self-energy associated with a charge quantity q , interacting with its own field ϕ_E is given by:

$$E = q\phi_E = q \left(\frac{q}{4\pi\epsilon_0 r} \right) \quad (7)$$

As the radius approaches to zero, the scalar-potential ϕ_E becomes infinite and thereby the corresponding self-energy of the charge particle becomes infinite as well. However, the proposed postulate states that the scalar-potential associated with particles are bounded by the Planck scale potential or the Planck-voltage V_{planck} :

$$\phi_E = \left(\frac{q}{4\pi\epsilon_0 r} \right)_{planck} = V_{planck} \quad (8)$$

Therefore, the self-energy of a charge particle is obtained as:

$$E = qV_{planck} \quad (9)$$

3 Mass-energy equivalence

The self-energy associated with a spherical body with mass quantity m , interacting with its own field ϕ_G is given by:

$$E = m\phi_G = m \left(\frac{Gm}{r} \right) \quad (10)$$

where G is the universal gravitational constant. As the radius approaches to zero, the scalar-potential ϕ_G becomes infinite and thereby the corresponding self-energy of the mass particle becomes infinite as well. However, from the proposed postulate, the scalar-potential associated with particles are bounded by the Planck scale potential, such that:

$$\phi_G = \left(\frac{Gm}{r} \right)_{planck} = (velocity)_{planck}^2 = c^2 \quad (11)$$

Thus, the self-energy of a mass particle is obtained as:

$$E = mc^2 \quad (12)$$

Historically, in 1900 Poincare arrived at equation 12 based on the concept of the radiation pressure, associated with electromagnetic radiation energy with a fictitious-fluid having mass and momentum. Einstein first entered the discussion in 1905 by relating the radiation energy to the change in mass. He then affirmed this relation in many different presentations, revisiting the discussion with different experiments, devised to affirm this equivalence. However, Hecht asserts that Einstein was not able to provide a conclusive general proof of this seminal hypothesis from the first principles [20] [21].

4 Relativistic energy-momentum relation associated with a charge particle in motion

Starting from the classical interpretation, a generalized relation between the change in energy dE and momentum $d\mathbf{p}$ can be derived, for a particle in motion from its velocity \mathbf{u} .

$$\frac{dE}{d\mathbf{p}} = \mathbf{u} \quad (13)$$

The electromagnetic vector-potential \mathbf{A} , located on the surface of a charge particle with charge q and radius r , moving at a velocity \mathbf{u} defined from Helmholtz theorem is given below:

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{vol} \left(\frac{\mathbf{J}}{r} \right) dV = \frac{1}{(c^2 4\pi\epsilon_0)} \int_{vol} \left(\frac{\rho\mathbf{u}}{r} \right) dV = \frac{\mathbf{u}}{c^2} \int_{vol} \left(\frac{\rho dV}{4\pi\epsilon_0 r} \right) = \frac{\mathbf{u}}{c^2} \int_{vol} \left(\frac{dq}{4\pi\epsilon_0 r} \right) = \frac{\mathbf{u}}{c^2} \phi_E \quad (14)$$

$$\mathbf{A} = \frac{\mathbf{u}}{c^2} \phi_E \quad (15)$$

where \mathbf{J} is the current density, $\mu_0 \epsilon_0 = \frac{1}{c^2}$, ρ is the charge density and $\rho dV = dq$ and $\phi_E = \frac{q}{4\pi \epsilon_0 r}$ is the electrical scalar-potential. The electromagnetic momentum \mathbf{p}_q of a particle with charge q and velocity \mathbf{u} is defined as:

$$\mathbf{p}_q = q\mathbf{A} = q \left(\frac{\mathbf{u}}{c^2} \phi_E \right) \quad (16)$$

As the radius of the charge particle approaches to zero, the corresponding scalar-potential ϕ_E and the vector-potential \mathbf{A} become infinite and give rise to unbounded quantities. However, by using the postulate presented in this paper, the-scalar-potential and the vector-potential become bounded:

$$\phi_E = V_{planck} \quad (17)$$

$$\mathbf{A} = \left(\frac{\mathbf{u}}{c^2} V_{planck} \right) \quad (18)$$

and the electromagnetic momentum expression in equation 16 reads:

$$\mathbf{p}_q = q \left(\frac{\mathbf{u}}{c^2} V_{planck} \right) \quad (19)$$

Combining equations 13 and 19, equation 13 reads:

$$\frac{dE}{d\mathbf{p}} = \mathbf{u} = \frac{\mathbf{p}_q c^2}{qV_{planck}} \quad (20)$$

The momentum and energy equations, leading to energy-momentum equations for both mass and charge particles in motion are tabulated below. The constants of integration were obtained by introducing the rest-frame energies, $m_0 c^2$ and $q_0 V_{planck}$ associated with mass and charge particles at rest, respectively.

<i>mass – particle</i>	<i>charge – particle</i>
$\mathbf{p} = m\mathbf{u}$ $\mathbf{p} = \left(\frac{m\mathbf{u}}{c^2} \right) \phi_G$ $\frac{dE}{d\mathbf{p}} = \mathbf{u} = \frac{\mathbf{p}c^2}{m\phi_G}$ $(mc^2) dE = (pc^2) dp; (\phi_G = c^2)$ $E dE = (pc^2) dp$ $E^2 = (pc)^2 + (m_0 c^2)^2$ $(mc^2)^2 = (muc)^2 + (m_0 c^2)^2$ $(mc^2)^2 \left(1 - \frac{u^2}{c^2} \right) = (m_0 c^2)^2$ $mc^2 = \gamma m_0 c^2$	$\mathbf{p}_q = q\mathbf{A}$ $\mathbf{p}_q = \left(\frac{q\mathbf{u}}{c^2} \right) \phi_E$ $\frac{dE}{d\mathbf{p}_q} = \mathbf{u} = \frac{\mathbf{p}_q c^2}{q\phi_E}$ $(qV_{planck}) dE = (p_q c^2) dp_q; (\phi_E = V_{planck})$ $E dE = (p_q c^2) dp_q$ $E^2 = (p_q c)^2 + (q_0 V_{planck})^2$ $(qV_{planck})^2 = \left(qu \frac{V_{planck}}{c} \right)^2 + (q_0 V_{planck})^2$ $(qV_{planck})^2 \left(1 - \frac{u^2}{c^2} \right) = (q_0 V_{planck})^2$ $qV_{planck} = \gamma q_0 V_{planck}; \left(\gamma = \left(1 - \frac{u^2}{c^2} \right)^{-\frac{1}{2}} \right)$

Thus, the energy-momentum equation for a charge particle in motion is obtained:

$$E^2 = (qu)^2 \left(\frac{V_{planck}}{c} \right)^2 + (q_0 V_{planck})^2 \quad (21)$$

Further, both the mass-energy and charge-energy equivalences can be shown as relativistically covariant expressions.

$$mc^2 = \gamma m_0 c^2 \iff E = \gamma E_0 \quad (22)$$

$$qV_{planck} = \gamma q_0 V_{planck} \iff E = \gamma E_0 \quad (23)$$

In like manner, the relativistic-momentum expressions associated with both mass and charge particles in motion can be obtained.

$$m\mathbf{u} = \gamma m_0 \mathbf{u} \iff q\mathbf{u} \left(\frac{V_{planck}}{c^2} \right) = \gamma q_0 \mathbf{u} \left(\frac{V_{planck}}{c^2} \right) \quad (24)$$

Since the relativistic-mass γm_0 is derived from the relativistic energy $\gamma m_0 c^2$ or relativistic momentum $\gamma m_0 \mathbf{u}$ expressions of the system, the relativistic-mass is not a good concept. Einstein wrote *"It is not good to introduce the concept of the mass $M = \gamma m_0$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the rest-mass m_0 . Instead of introducing M , it is better to mention the expressions for the momentum and energy of a body in motion"* [22]. The same set of arguments holds true for the relativistic charge γq_0 associated with relativistic charge-energy ($\gamma q_0 V_{planck}$) and relativistic charge-momentum $\gamma q_0 \mathbf{u} \left(\frac{V_{planck}}{c^2} \right)$ expressions obtained.

5 Electromagnetic energy and momentum associated with de-Broglie's matter-wave hypothesis

The distinction between waves propagating according to a set of laws known as Maxwell's equations and particles considered to consist of localized entities was challenged when Einstein in 1905 [23] introduced the concept that light behaved as a collection of localized energy packets or energy quanta, which was later termed 'photons'. de-Broglie expanded Einstein's hypothesis to all matter particles, arguing that, just as light exhibits a wave-particle duality, all particles must also be associated with a wave into which they are incorporated [24] and showed that for every particle of matter with mass m and velocity u , a real-wave with a wave-length known as de-Broglie's wave-length, associated with its momentum exists. That is, the wave-length associated with a particle in motion is inversely proportional to the momentum of the particle:

$$\lambda = \frac{h}{p} = \frac{h}{(\gamma m u)} \quad (25)$$

Also the frequency of the matter-wave associated with a particle, as deduced by de-Broglie, is directly proportional to the particle's total energy.

$$f = \frac{E}{h} = \frac{\gamma m c^2}{h} \quad (26)$$

Extending the concept of de-Broglie to charge particles, where they are associated with a matter-wave in to which they are incorporated, the corresponding de-Broglie's wave-length in terms of electromagnetic momentum p_q can be obtained as:

$$\lambda_q = \frac{h}{p_q} = \frac{h}{\left(\gamma q u \frac{V_{planck}}{c^2} \right)} \quad (27)$$

and the frequency of the matter-wave associated as:

$$f_q = \frac{\gamma q V_{planck}}{h} \quad (28)$$

In 1927, Davisson and Germer confirmed that electrons diffraction, thought to have arisen from the electromagnetic mass of electrons [25]. However, the matter-wave relations presented in equations 27 and 28 for a charge particle in motion were derived from a relativistically covariant electromagnetic energy-momentum relation, and therefore they could potentially account for the observed scattering of the electrons and give rise to a diffracted wave-like behavior in a relativistically covariant manner.

6 Relativistically covariant energy-momentum four-vector

Abraham [26] and Lorentz [27], based on Maxwell's theory of electricity and magnetism developed the first set of theories for the classical electron. From classical electrostatics, the rest energy U_0 of a spherical charge body with radius r , associated with total charge e , uniformly distributed over its surface can be obtained from equation 1. The relativistic electromagnetic energy U of the moving charge e , can be derived, similar to those derivation given by Panofsky [28]:

$$U = \frac{1}{2} \int_{all-space} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} H^2 \right) d^3r = \gamma \frac{e^2}{8\pi\epsilon_0 r} \left(1 + \frac{1}{3} \beta^2 \right) \quad (29)$$

and its relativistic electromagnetic momentum \mathbf{p} as:

$$\mathbf{p} = \epsilon_0 \int_{all-space} (\mathbf{E} \times \mathbf{B}) d^3r = \frac{4}{3} \gamma \left(\frac{e^2}{8\pi\epsilon_0 r c^2} \right) \mathbf{u} \quad (30)$$

However, according to mass-energy equivalence and the theory of relativity, the equivalent electromagnetic invariant mass m_e of an electron with charge e is given by:

$$m_e = \frac{U_0}{c^2} = \frac{e^2}{8\pi\epsilon_0 r c^2} \quad (31)$$

Therefore, the relativistic electromagnetic energy and momentum expressions given in equations 29 and 30 can be written in terms of the electromagnetic invariant mass m_e as:

$$U = \gamma m_e c^2 \left(1 + \frac{1}{3} \beta^2 \right) \quad (32)$$

$$\mathbf{p} = \frac{4}{3} \gamma m_e \mathbf{u} \quad (33)$$

where $\beta = \frac{u}{c}$. From equations 32 and 33, it is immediately obvious that the terms U and \mathbf{p} are neither relativistically covariant nor transform properly as an energy-momentum four-vector.

However, by using the charge-energy equivalence obtained in equation 9, the electromagnetic invariant mass m_e of an electron with charge e can be obtained as:

$$m_e = \frac{U_0}{c^2} = e \frac{V_{planck}}{c^2} \quad (34)$$

from which the expressions for the relativistic electromagnetic energy and momentum given in equations 23 and 24 can be presented, in terms of the electromagnetic invariant mass m_e as shown below.

$$U = \gamma m_e c^2 \quad (35)$$

$$\mathbf{p} = \gamma m_e \mathbf{u} \quad (36)$$

The relativistic electromagnetic energy and momentum expressions obtained in equations 35 and 36 are relativistically covariant and they form a relativistically covariant energy-momentum four-vector in Minkowskian space-time:

$$P^\mu = \left(\frac{E}{c}, \mathbf{p} \right) = \left(\gamma e \frac{V_{planck}}{c}, \gamma e \frac{V_{planck}}{c^2} \mathbf{u} \right) \quad (37)$$

which gives rise to the energy-momentum relation as shown below.

$$P^\mu P_\mu = \left(\gamma e \frac{V_{planck}}{c}, \gamma e \frac{V_{planck}}{c^2} \mathbf{u} \right) \left(\gamma e \frac{V_{planck}}{c}, -\gamma e \frac{V_{planck}}{c^2} \mathbf{u} \right) \quad (38)$$

$$P^\mu P_\mu = \left(\gamma e \frac{V_{planck}}{c} \right)^2 \left(1 - \frac{u^2}{c^2} \right) = \left(e \frac{V_{planck}}{c} \right)^2 \quad (39)$$

$$(\gamma e V_{planck})^2 = \left(\gamma e u \frac{V_{planck}}{c} \right)^2 + (e V_{planck})^2 \quad (40)$$

$$U^2 = (\mathbf{p}c)^2 + (U_0)^2 \quad (41)$$

7 Stability problem associated with finite radius charge particle models

Charge particle models with finite radii cannot explain how the repulsive like-charge distributions are held together. In order to explain the stability problem, Poincare, in 1905 introduced a postulate, that a non-electromagnetic force was required, to hold the like-charge distribution together.

However, in view of the energy-momentum expression for a collection of mass particles given in equation 5, the electromagnetic energy-momentum relation alone would quantify the total energy of a system comprising of many charge particles, without having to incorporate the potential energies arising between its constituent particles. That is, the composite energy of the system associated with the energy-momentum equation, includes both the kinetic energy and the potential energy of the system. The electromagnetic energy-momentum relation obtained for a charge particle in motion, can be extended to a collection of many charge particles, and obtain the total invariant charge Q as given below.

$$(QV_{planck})^2 = \left(\sum_{i=1}^n E_i \right)^2 - \left| \left(\sum_{i=1}^n \gamma_i q_i \mathbf{u}_i \frac{V_{planck}}{c} \right) \right|^2 \quad (42)$$

Further, if the total energy of a collection of infinitesimal charge quantities can be quantified by the expression given in equation 42, then the momentum energy components associated with constituent charge quantities can be shown to give rise to a set of attractive magnetic forces between themselves, if they are associated with a relative motion in the same sense of direction, which could potentially resolve the stability problem associated with finite radius models. Therefore, the model presented in this paper for charge particles such as electrons with finite radius models demands that the corresponding charge distributions to be associated with an intrinsic spin and thereby a theoretical explanation to the electron's spin is obtained.

8 Conclusion

To treat the electric and gravitational scalar potentials associated with a particle, a Planck-scale cut-off potential was introduced, so that they become finite and bounded. This led to the derivation of both the charge-energy and mass-energy equivalences. Extending this postulate to a charge particle in motion, its corresponding energy-momentum equation was derived and the obtained total electromagnetic energy and the electromagnetic momentum were relativistically covariant. de-Broglie's electromagnetic wave-length and frequency in terms of electromagnetic momentum and energy were derived. The stability problem associated with a repulsive like-charge distribution was potentially resolved by demanding that electron's are associated with an intrinsic spin, consistent with the quantum mechanical description of the elementary particles. The present paper is a call for a revision of the classical electrodynamics to make it a fully covariant system with the rest of the classical physics.

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