

# **The light velocity in the special relativity theory in the Earth's gravity field**

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## **ABSTRACT**

The theory is the light velocity's theory in the special relativity theory in the gravity field. You can consider that this gravity field treats the Earth gravity field. In this time,

if Earth mass is  $M$  and Earth radius is  $R_0$ , the light's velocity is  $\frac{c}{\alpha_0} = c\sqrt{1 - \frac{2GM}{R_0c^2}}$

instead of  $c$ . Hence, In the Earth, every special relativity's experiment is more accurate

by using the light speed  $c(M, R_0) = \frac{c}{\alpha_0} = c\sqrt{1 - \frac{2GM}{R_0c^2}}$  is measured in the Earth.

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## I. Introduction

The article treats that the light velocity theory is in the special relativity theory in the gravity field. You can consider that this gravity field treats the Earth gravity field.

In the general relativity theory, the light speed is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] = 0 \quad (1)$$

In the Earth, if the Earth radius is  $R_0$  and the Earth mass is  $M$ , the light speed  $V$  is

$$r = R_0, dr = 0, V = \frac{ds}{dt} = \sqrt{R_0^2 \left(\frac{d\theta}{dt}\right)^2 + R_0^2 \sin^2 \theta \left(\frac{d\phi}{dt}\right)^2} = c \sqrt{1 - \frac{2GM}{R_0 c^2}} = \frac{c}{\alpha_0}$$

$$\alpha_0 = 1 / \sqrt{1 - \frac{2GM}{R_0 c^2}} \quad (2)$$

Hence, the light's velocity of this theory is  $\frac{c}{\alpha_0}$  in the Earth. In this theory, be able to consider that the

light has the velocity  $\frac{c}{\alpha_0}$ . Therefore, in the free time-space, the speed of Electro-magnetic wave  $c$  is

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (3)$$

$\epsilon_0$  is the permittivity constant in the free time-space,

$\mu_0$  is the permeability constant in the free time-space

$$\frac{c}{\alpha_0} = c \sqrt{1 - \frac{2GM}{R_0 c^2}} = \frac{1}{\sqrt{\epsilon_0(M, R_0) \mu_0(M, R_0)}} \quad (4)$$

$\epsilon_0(M, R_0)$  is the permittivity constant in the Earth's gravity field,

$\mu_0(M, R_0)$  is the permeability constant in the Earth's gravity field.

In the Earth, every special relativity's experiment is more accurate by using the light speed

$$c(M, R_0) = \frac{c}{\alpha_0} = c \sqrt{1 - \frac{2GM}{R_0 c^2}} \text{ is measured in the Earth.}$$

## II. Additional chapter

Therefore, in the Earth, in the special relativity theory, if it uses the light speed  $\frac{c}{\alpha_0} = c \sqrt{1 - \frac{2GM}{R_0 c^2}}$

instead of  $c$ .

$$t = \frac{\tau}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \quad (5) \quad \alpha_0 = 1/\sqrt{1 - \frac{2GM}{R_0 c^2}}.$$

In this theory,

$$\begin{aligned} d\tau^2 &= dt^2 \left(1 - \alpha_0^2 \frac{u^2}{c^2}\right) = dt^2 - \alpha_0^2 \frac{1}{c^2} (dx^2 + dy^2 + dz^2) \\ &= dt^2 \left(1 - \alpha_0^2 \frac{u^2}{c^2}\right) = dt^2 - \frac{1}{c^2} \alpha_0^2 (dx^2 + dy^2 + dz^2) \\ &= dt'^2 \left(1 - \alpha_0^2 \frac{u'^2}{c^2}\right) = dt'^2 - \frac{1}{c^2} \alpha_0^2 (dx'^2 + dy'^2 + dz'^2) \quad (6) \end{aligned}$$

$$\begin{aligned} x &= \frac{x' + v_0 t'}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}}, t = \frac{t' + \alpha_0^2 \frac{v_0}{c^2} x'}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}}, x' = \frac{x - v_0 t}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}}, t' = \frac{t - \alpha_0^2 \frac{v_0}{c^2} x}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}} \quad (7) \\ y &= y', z = z' \end{aligned}$$

$$V = \frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v_0}{1 + \alpha_0^2 \frac{v_0}{c^2} \frac{dx'}{dt'}} = \frac{u + v_0}{1 + \alpha_0^2 \frac{v_0}{c^2} u}, \quad u = \frac{dx'}{dt'} \quad (8)$$

In the example, the light is

$$\begin{aligned} d\tau^2 &= dt^2 \left(1 - \alpha_0^2 \frac{u^2}{c^2}\right) = dt^2 - \frac{1}{c^2} \alpha_0^2 (dx^2 + dy^2 + dz^2) = 0 \\ cdt &= \alpha_0 ds, \quad ds = \sqrt{dx^2 + dy^2 + dz^2}, \quad \frac{ds}{dt} = \frac{c}{\alpha_0} \\ d\tau^2 &= dt'^2 - \frac{1}{c^2} \alpha_0^2 (dx'^2 + dy'^2 + dz'^2) = 0 \\ cdt' &= \alpha_0 ds', \quad ds' = \sqrt{dx'^2 + dy'^2 + dz'^2}, \quad \frac{ds'}{dt'} = \frac{c}{\alpha_0} \quad (9) \end{aligned}$$

The light's velocity of the this theory is  $\frac{c}{\alpha_0}$

In this time, the mass  $m_0$  is

$$m = \frac{m_0}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \quad (10)$$

$$m = \frac{E}{c^2} \alpha_0^2$$

In this theory, the particle's the force  $F$  and the kinetic energy  $KE$ , the power  $P$ , the momentum  $p$ , the total energy  $E$  are

$$p^\alpha = m_0 \frac{dx^\alpha}{d\tau}$$

$$F = m_0 a = \frac{d}{dt} \left( \frac{m_0 u}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \right) = \frac{dp}{dt}$$

$$KE = \int_0^u u d \left( \frac{m_0 u}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \right) = \frac{m_0 c^2 / \alpha_0^2}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} - m_0 c^2 / \alpha_0^2 = E - m_0 c^2 / \alpha_0^2$$

$$P = \frac{d(KE)}{dt} = \frac{d}{dt} \left( \frac{m_0 c^2 / \alpha_0^2}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} - m_0 c^2 / \alpha_0^2 \right) = F \cdot u = \frac{d}{dt} \left( \frac{m_0 u}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \right) \cdot u \quad (11)$$

And

$$E^2 = \frac{m_0^2 c^4 / \alpha_0^4}{1 - \alpha_0^2 \frac{u^2}{c^2}} = m_0^2 c^4 / \alpha_0^4 + p^2 c^2 / \alpha_0^2 = m_0^2 c^4 / \alpha_0^4 + \frac{m_0^2 u^2 c^2}{1 - \alpha_0^2 \frac{u^2}{c^2}}$$

$$= \frac{m_0^2 c^4 (1 - \alpha_0^2 \frac{u^2}{c^2}) \frac{1}{\alpha_0^4} + m_0^2 u^2 c^2 / \alpha_0^2}{1 - \alpha_0^2 \frac{u^2}{c^2}} = \frac{m_0^2 c^4 / \alpha_0^4}{1 - \alpha_0^2 \frac{u^2}{c^2}}$$

$$E' = \frac{m_0 c^2 / \alpha_0^2}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}}, \quad p' = \frac{m_0 u}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}},$$

$$V = \frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v_0}{1 + \alpha_0^2 \frac{v_0}{c^2} \frac{dx'}{dt'}} = \frac{u + v_0}{1 + \alpha_0^2 \frac{v_0}{c^2} u}, \quad u = \frac{dx'}{dt'}$$

$$\begin{aligned}
E &= \frac{m_0 c^2 / \alpha_0^2}{\sqrt{1 - \alpha_0^2 \frac{V^2}{c^2}}} = \frac{m_0 c^2 (1 + \alpha_0^2 \frac{v_0}{c^2} u) \frac{1}{\alpha_0^2}}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}} \sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}} = \frac{E' + v_0 p'}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}} \\
p &= \frac{m_0 V}{\sqrt{1 - \alpha_0^2 \frac{V^2}{c^2}}} = \frac{m_0 (u + v_0)}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}} \sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}} = \frac{p' + \frac{v_0}{c^2} \alpha_0^2 E'}{\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}}} \quad (12)
\end{aligned}$$

If  $a = a_0$ ,

$$\begin{aligned}
a &= a_0 = \frac{d}{dt} \left( \frac{u}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \right), \quad u = \frac{dx}{dt} = \frac{a_0 t}{\sqrt{1 + \frac{\alpha_0^2 a_0^2 t^2}{c^2}}} \\
x &= \frac{c^2}{a_0 \alpha_0^2} \left( \sqrt{1 + \frac{\alpha_0^2 a_0^2 t^2}{c^2}} - 1 \right) \quad (13)
\end{aligned}$$

In this theory, the Maxwell-equation is

$$\begin{aligned}
& \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) = \vec{\nabla} \cdot \vec{E} = 4\pi\rho \\
& \left[ \left( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) i - \left( \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right) j + \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) k \right] = \vec{\nabla} \times \vec{B} \\
& = \frac{1}{c/\alpha_0} \left[ \left( \frac{\partial E_x}{\partial t} + 4\pi j_x \right) i + \left( \frac{\partial E_y}{\partial t} + 4\pi j_y \right) j + \left( \frac{\partial E_z}{\partial t} + 4\pi j_z \right) k \right] = \frac{1}{c/\alpha_0} \left( \frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j} \right) \\
& \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) = \vec{\nabla} \cdot \vec{B} = 0 \\
& \left[ \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) i - \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) j + \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) k \right] = \vec{\nabla} \times \vec{E} \\
& = -\frac{1}{c/\alpha_0} \left[ \frac{\partial B_x}{\partial t} i + \frac{\partial B_y}{\partial t} j + \frac{\partial B_z}{\partial t} k \right] = -\frac{1}{c/\alpha_0} \frac{\partial \vec{B}}{\partial t} \quad (14)
\end{aligned}$$

Therefore,

$$\vec{B} = \vec{\nabla} \times \vec{A}, \quad \vec{E} = -\vec{\nabla} \phi - \frac{1}{c/\alpha_0} \frac{\partial \vec{A}}{\partial t} \quad (15)$$

In this time, uses Lorentz gauge.

$$\frac{1}{c/\alpha_0} \frac{\partial \phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0 \quad (\text{Lorentz gauge}) \quad (16)$$

Therefore,

$$\left(\frac{1}{c^2/\alpha_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 4\pi\rho, \left(\frac{1}{c^2/\alpha_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{A} = \frac{4\pi}{c/\alpha_0} \vec{j} \quad (17)$$

The transformation of 4-vector operator  $\left(\frac{1}{c/\alpha_0} \frac{\partial}{\partial t}, \vec{\nabla}\right)$  is

$$\frac{1}{c/\alpha_0} \frac{\partial}{\partial t} = \gamma \left( \frac{1}{c/\alpha_0} \frac{\partial}{\partial t'} - \frac{v_0}{c/\alpha_0} \frac{\partial}{\partial x'} \right), \frac{\partial}{\partial x} = \gamma \left( \frac{\partial}{\partial x'} - \alpha_0 \frac{v_0}{c} \frac{1}{c/\alpha_0} \frac{\partial}{\partial t'} \right)$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}, \quad \gamma = 1/\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}} \quad (18)$$

The transformation of the Electro-magnetic 4-vector potential  $(\phi, \vec{A})$  is

$$\phi = \gamma \left( \phi' + \alpha_0 \frac{v_0}{c} A_{x'} \right), \quad A_x = \gamma \left( A_{x'} + \alpha_0 \frac{v_0}{c} \phi' \right)$$

$$A_y = A_{y'}, \quad A_z = A_{z'}, \quad \gamma = 1/\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}} \quad (19)$$

Therefore, the transformation of Electro-magnetic field  $\vec{E}, \vec{B}$  is

$$E_x = E_{x'}, \quad E_y = \gamma E_{y'} + \gamma \alpha_0 \frac{v_0}{c} B_{z'}, \quad E_z = \gamma E_{z'} - \gamma \alpha_0 \frac{v_0}{c} B_{y'}$$

$$B_x = B_{x'}, \quad B_y = \gamma B_{y'} - \gamma \alpha_0 \frac{v_0}{c} E_{z'}, \quad B_z = \gamma B_{z'} + \gamma \alpha_0 \frac{v_0}{c} E_{y'}$$

$$\gamma = 1/\sqrt{1 - \alpha_0^2 \frac{v_0^2}{c^2}} \quad (20)$$

In the quantum theory,

$$E = \frac{m_0 c^2 / \alpha_0^2}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} = h\nu \quad (21)$$

The Compton effects is

$$\lambda' - \lambda = \frac{h}{m_0 c / \alpha_0} (1 - \cos \phi) \quad (22)$$

De Broglie wavelength  $\lambda$  is

$$\lambda = \frac{h}{p} = \frac{h}{mu} , \quad m = \frac{m_0}{\sqrt{1 - \alpha_0^2 \frac{u^2}{c^2}}} \quad (23)$$

### III. Conclusion

If  $M = 0, \alpha_0 = 1$ , this theory does the special relativity theory in the free time-space.

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