The effective mass tensor in the General Relativity

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In this paper we define a four-dimensional effective mass tensor that can be suitable for the describe of the weak gravitational interactions in GR. In the metric of the weak gravitational field the components of the effective mass tensor depend on the components of the metric tensor, what can suggest that this tensor can in GR exist.

general theory of gravity, Einstein's field equation, effective mass tensor

PACS: 04.20.-q, 04.20.Cv

Introduction

In Newton's theory of gravity the mass (inertial or gravitational) is a scalar. The concept of mass in General Relativity (GR) is more complex than the concept of mass in special relativity. In fact, GR does not offer a single definition for the term mass, but offers several different definitions which are applicable under different circumstances [1, 4].

The effective mass in the solid-state physics

The effective mass is well-known in the solid-state physics. When an electron is moving inside a solid material, the force between other atoms will affect its movement and it will not be described by Newton's law. So we introduce the concept of effective mass to describe the movement of electron in Newton's law. The effective mass can be negative or different due to circumstances. Generally, in the absence of an electric or magnetic field, the concept of effective mass does not apply. In the solid-state physics a three-dimensional effective mass tensor (EMT) is define:

$$\left[\frac{1}{m^*}\right]_{ij} = \frac{d^2E}{dp_i dp_j} \tag{1}$$

where: E is the energy, p is the momentum of the electron, i and j are Cartesian coordinates. For the classical kinetic energy $E = p^2/2m$ and in the isotropic medium the effective mass $m^* = m$, where m is the scalar [5].

The EMT in the gravitation

Similarly to (1) we are define a four-dimensional EMT for the gravitational field in the form:

$$\left[\frac{1}{m^*}\right]_{\mu\nu} \overset{\text{def}}{=} \frac{d^2E}{dp_\mu dp_\nu} \tag{2}$$

where: E is the total energy, p is the three-dimensional momentum of the system, m_{00} is the time dependent effective mass component, m_{0i} or m_{i0} is the space-time dependent effective mass component, and m_{ii} is the space dependent (three-dimensional) effective mass component.

Now we calculate the EMT in the metric of the weak gravitational field

$$ds^2 = \left(1+\frac{2\varphi}{c^2 r}\right)c^2 dt^2 - \left(1-\frac{2\varphi}{c^2 r}\right)\left(dx^2+dy^2+dz^2\right) \tag{3}$$

The total energy E of the system in metric (3) we can describe as

$$m_0^2 c^2 = p_\mu p^\mu = p^\alpha p^\beta g_{\alpha\beta} = \left(1+\frac{2\varphi}{c^2 r}\right)\left(p^0\right)^2 - \left(1-\frac{2\varphi}{c^2 r}\right)p^2 \tag{4}$$

where: p is the total momentum of the system, φ – Newtonian potential. After calculation, and assuming that $\frac{2GM}{c^2 r} << 1$, we have a formula for the E in the weak gravitational field

$$E \approx m_0 c^2 + \frac{p^2}{2m_0}\left(1+\frac{2\varphi}{c^2}\right) - \frac{GMm_0}{r} \tag{5}$$

According to (2) and (5) we can calculate the EMT and then we have

$$\begin{aligned} m_{11}^* &\approx m_{22}^* \approx m_{33}^* \approx m_0\left(1+\frac{2\varphi}{c^2}\right)^{-1} \\ m_{00}^* &\approx m_0 \end{aligned} \tag{6}$$

or in the matrix form

$$\left(\frac{1}{m^*}\right)_{\mu\nu} \approx \begin{pmatrix} \frac{1}{m_0} & 0 & 0 & 0 \\ 0 & \frac{1}{m_0}\left(1+\frac{2\varphi}{c^2}\right) & 0 & 0 \\ 0 & 0 & \frac{1}{m_0}\left(1+\frac{2\varphi}{c^2}\right) & 0 \\ 0 & 0 & 0 & \frac{1}{m_0}\left(1+\frac{2\varphi}{c^2}\right) \end{pmatrix} \tag{7}$$

Let's notice, that in the metric (3) **the EMT components depend on the metric tensor components**. It is a very interesting result, so what can suggest that EMT can exist in GR. Now we will try to find the EMT in GR.

The EMT in GR

The Einstein field equation in GR has form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} \tag{8}$$

where: $R_{\mu\nu}$ is the Ricci curvature tensor, R the Ricci scalar, $g_{\mu\nu}$ the metric tensor, G is Newton's gravitational constant, c speed of light in the vacuum, and $T_{\mu\nu}$ the stress-energy tensor.

Let's multiply both sides of the equation (8) by the factor c^2/G and then we get

$$\rho^{*}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\rho = -\frac{8\pi}{c^2} T_{\mu\nu} \tag{9}$$

where: $\rho^{*}_{\mu\nu} = (c^2/G)R_{\mu\nu}$ we will call *the effective mass density tensor*, $\rho = (c^2/G)R$ *is the mass density*.

The simple two-sided multiplication changed the physical sense of the equation (8): in (8) the distribution of matter and energy in the space-time determines his **curvature** directly and unambiguously but in (9) the distribution of matter and energy in the space-time determines his **effective mass density tensor** directly and unambiguously.

Solving the equation (9) we can finds all components of the EMT.

Summary

The concept of effective mass is a very attractive because effective mass in the equations of the motion includes full information about all fields (gravitational, electromagnetic etc.) surrounding the body without their exact analysis (9). Effective mass can be isotropic or anisotropic, positive or negative. For the free body his effective mass is equal normal mass.

In the metric (3) the EMT components depend on the metric tensor components. It is a very interesting result, what can suggest that this EMT in GR exist (9).

We would like to postulate a new version of the Mach's Principle in the sound: **the effective mass** is a result of interaction between the body and the bodies in the Universe[1] or the distribution of the matter in the Universe determines the effective mass density tensor.

Bibliography

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[1] The old version of the Mach's Principle sounded: **the mass** is a result of interaction between the body and the bodies in the Universe[1].