Matter as gravitational waves. (On the nature of electron) .

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Abstract

Background: At present in physics there are 2 large and seemingly incompatible theories: the theory of General Relativity and Quantum Mechanics. A model to derive Quantum Mechanics from General Relativity is presented here.

Results: A six dimensional model with the following characteristics is proposed:

6D
- 1 temporal dimension
- 5 spatial dimensions
  - 3 extended spatial dimensions
  - Plane of compacted dimensions; where elementary particles (partons, electrons) move at the speed of light in elliptical paths with a perimeter equal to a half Compton wavelength.

The charge / mass ratio and the intrinsic magnetic momentum of the electron solely from its mass are estimated.

The gravitational wave equations are solved for the particular case of a flat three-dimensional space. The boundary conditions are set assuming that the waves are guided by the curvature of compacted dimensions. In particular, exact solutions are obtained for the case of a motionless particle-pulsation, uniform linear motion particle-pulsation and the relativistic hydrogen atom. These solutions justify the postulates of quantum mechanics and provide numerical solutions compatible with the experimental data. Finally a possible origin of inertia is proposed.

Conclusions: We should review the dual wave-particle concept in favour of a solely gravitational wave nature. It is remarkable to note that the same conclusions can be drawn with other configurations of compacted dimensions (whether it be in number, size or topology)

keywords: Quantum Mechanics, General Relativity, extra dimensions , Kaluza-Klein, Hydrogen Atom , inertia.

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Assumptions:

Hypothesis is limited to circumstances in which 3-dimensional space can be considered plane, therefore the equations of general relativity decrease of complexity and may be written in a similar way to Maxwell's laws, this is known as gravitomagnetism. We also neglected the influence of the expanding universe. The assumptions on which it is based are:

1) The equation that relates relativistic energy of a body with its speed reflects a movement at the speed of light in an hypothetical direction perpendicular to the three known spatial dimensions.

2) This extra dimension is the same that it was postulated by Kaluza in 1919.

3) The topology of the Kaluza dimension is closed and very small. (Klein 1926).

4) It is established that in order to conserve space isotropy in the extended dimensions the existence of at least another compacted dimension is necessary.

5) The movement of particles at high speeds in the plane of these compacted dimensions results in a vibration that is reflected in the law that governs the emission of a black body . Because of this movement there is an independent temperature term $E_r = \frac{h\upsilon}{2}$

6) The rest energy of elementary particles (electrons, partons) is due to this movement. This fact implies that the 4th spatial dimension of the assumption No. 4 (which we will call ξ) can be identified with the inverse of the mass of elementary particles. $\xi_0 = \frac{\bar{h}}{2 m_0 c}$

7) The 5th spatial dimension (η) is closely related to the imaginary coordinate of Minkowski spacetime.

8) The constants G, μ, ε, etc. are due to the formulation in three flat spatial dimensions of a five dimensional space (with two of its dimension extremely curved and compacted) and therefore disappear or are greatly simplified when we done calculations in 6 dimensions (5 spatial+ time).

9) The plane of the extra dimensions has elliptical topology.

Table of physical constants.

Electron mass $m_e = 9{,}10938291 \times 10^{-31}\, kg$

Bohr's magneton $\mu_B = 9{,}27400915 \times 10^{-24}\, J \cdot T^{-1}$

Intrinsic magnetic moment of electron $\mu_e = -1{,}001159652 \times \mu_B$

Elemental electric charge $e = 1{,}602176 \times 10^{-19}\, C$

Reduced Plank's constant $\bar{h} = 1{,}054571628 \times 10^{-34}\, J \cdot s$

Light velocity $c = 299792458\, m/s$

Gravitational constant $G = 6{,}67384 \times 10^{-11}\, N \cdot m^2/kg^2$

Six dimensional gravitational constant $\hat{G}$

Vacuum magnetic permeability $\mu_B = 4\pi \times 10^{-7}\, T \cdot m \cdot A^{-1}$

Symbols

$\hat{} \equiv if$ over $a\, constant,\, 6\, dimensions\, constant$

$\vec{B} \equiv Magnetic\ Field$

$\vec{B}_g \equiv Gravitomagnetic\ Field$

$\beta r \equiv Propagation\ constant\ of\ a\ wave$

$\oint \vec{E}\, dl \equiv \vec{E}\ circulation$

$dl \equiv length\ diferential$

$dS \equiv surface\ differential$

$E \equiv Energy$

$\vec{E} \equiv electric\ field$

$\vec{E}_g \equiv gravitational\ field$

$E_r \equiv residual\ energy$

$E_0 \equiv Rest\ energy\ of\ a\ particle$

$E_c \equiv kinetic\ energy$

$E_m \equiv mechanical\ energy$

$\oint \vec{E}\, dS \equiv Flux\ of\ \vec{E}$

$\vec{H} \equiv H\ magnetic\ field$

$\vec{H}_g \equiv H\ gravitomagnetic\ field$

$i \equiv current\ intensity$

$i_g \equiv mass\ current\ intensity$

$j \equiv current\ density$

$j_m \equiv mass\ current\ density$

$K \equiv Boltzman's\ constant$

$k \equiv circular\ wavenumber$

$k_c \equiv cutoff\ circular\ wavenumber$

$L \equiv Length$

$\lambda \equiv Wavelength$

$\lambda_{apa} \equiv apparent\ wavelength$

$\lambda_0 \equiv rest\ particle's\ wavelength$

$\vec{\nabla}^2 \equiv laplacian$

$\vec{\nabla}^2_{6D} \equiv 6\ dimensions\ laplacian$

$\vec{\nabla}^2_{3D} \equiv 3\ dimensions\ laplacian$

$m \equiv mass$

$m_0 \equiv rest\ mass$

$\mu \equiv magnetic\ moment$

$\mu_g \equiv gravitomagnetic\ moment$

$\vec{n} \equiv normal\ vector$

$\nu \equiv\ frecuency$

$p \equiv lineal\ momentum$

$p_e \equiv perimeter$

$Q\ , q \equiv electric\ charge$

$q_e\ , e \equiv electron\ charge$

$R \equiv scalar\ curvature$

$r \equiv radius$

$T \equiv stress - energy\ tensor$

$v \equiv velocity$

$v_g \equiv group\ velocity$

$v_p \equiv\ propagation\ velocity$

$\omega \equiv angular\ \ frecuency$

$\xi \equiv Compacted\ radial\ dimension$

$\xi_0 \equiv Compacted\ radial\ dimension\ of\ \ a\ particle$

1.Introduction. Kaluza-Klein theory.

Kaluza-Klein theory aims to unify the two fundamental forces of gravity and electromagnetism through the introduction of a fourth spatial dimension. It was first enunciated by the Polish mathematician Kaluza, who extended general relativity to a space-time of 5 dimensions. The resulting equations can be divided into several groups of equations, one of which corresponds to the Einstein field equations (gravity), another with Maxwell's equations (electromagnetism) and finally an scalar field with an obscure.physical meaning

Einstein field equations

$$\begin{array}{|cccc|c|} g_{11} & g_{12} & g_{13} & g_{14} & \text{M} \\ g_{21} & g_{22} & g_{23} & g_{24} & \text{a x w} \\ g_{31} & g_{32} & g_{33} & g_{34} & \text{e l l} \\ g_{41} & g_{42} & g_{43} & g_{44} & \\ \hline \text{M a x w e l l} & & & & \Phi \end{array} \quad \text{Scalar field}$$

That is, the mere fact that each particle is free to move through an additional dimension allows the unification of gravity with electromagnetism. Despite this spectacular result theory suffered from a serious problem, and that is, where is this 4th dimension? .If we lived in a universe with four spatial dimensions, gravity would diminish with the cube of the distance, a circumstance which contradicts everyday experience since it decreases with the square of the distance.
In order to try to explain why the extra dimension does not affect the physical laws Oscar Klein in 1926 he proposed that the 4th spatial dimension is curved on itself in a circle of extremely small radius (below 10^{-18} m) A particle moving a small distance in the direction of this dimension should return to the starting point. Distance a particle must travel before returning to its starting point is defined as the size of the dimension and this extra dimension is said to be compacted

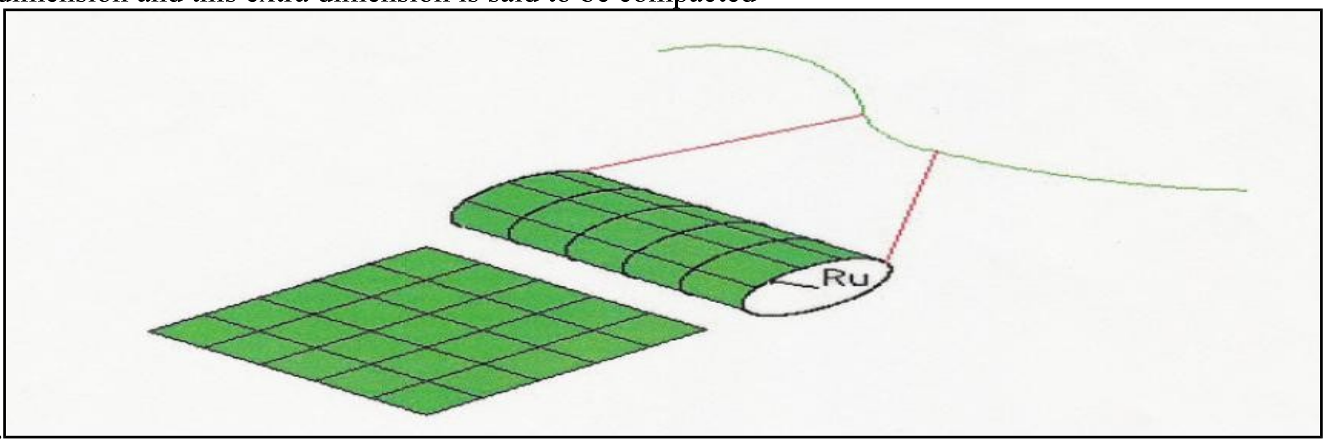


Fig 1. Compaction process of a dimension. If compacted radius is sufficiently small a three-dimensional cylinder appears to be a one-dimensional wire.

Therefore we should represent space-time as if there were a small circle at each point in which particles can move freely.
In the Kaluza-Klein theory pure geometry of an empty space-time (massless) of five dimensions leads to

a four-dimensional space-time with mass. Unfortunately, the application of this theory to the study of the charge-mass rate provides a prediction that differs from the experimental about 20 orders of magnitude, reason why it was largely abandoned for several decades.

2.Considerations about Kaluza-Klein theory.

2.1 On the circular topology of dimensions.

The curvature of a dimension requires the existence of another dimension on which it can be curved, as we can see by simply drawing a circle. If we focus on the hypothetical 4th spatial dimension of circular topology from Kaluza-Klein theory we have 2 options:

1. The 4th spatial dimension is curved over one of the known spatial dimensions, resulting in space would not be isotropic.

2. The 4th spatial dimension curves over another additional spatial dimension also compacted, for example in the case of a toroid. It is easily seen that, regardless of their number, we can separate dimensions in two groups, extended and compacted.

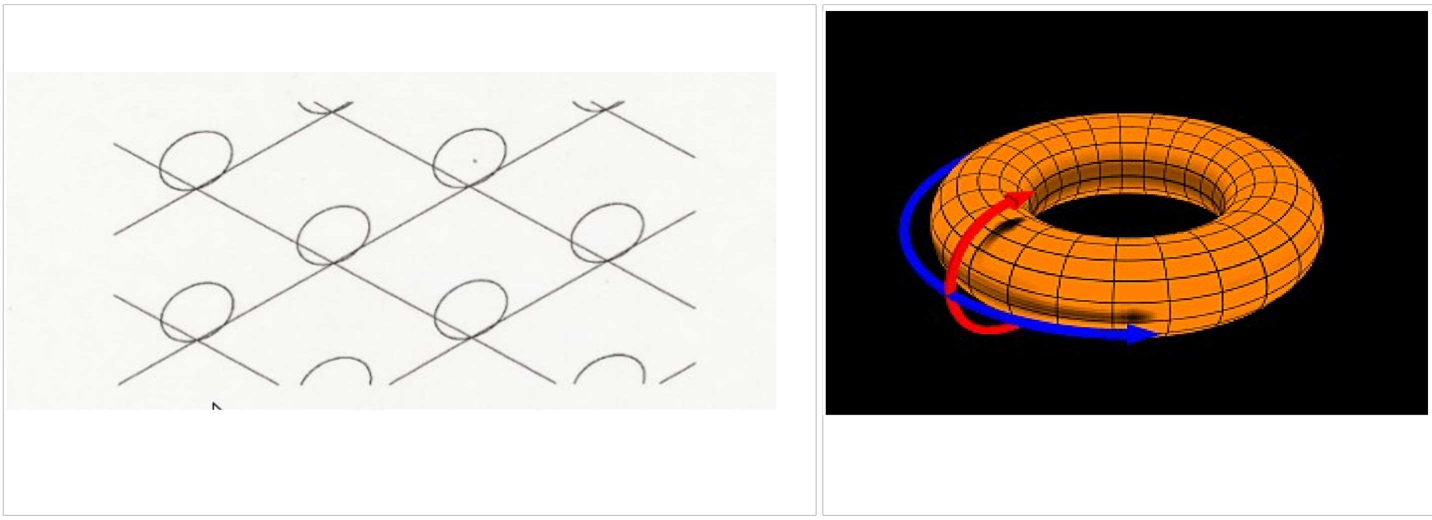
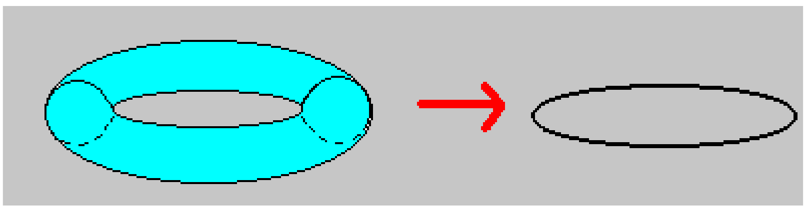


Figura 2. The hypothetical fourth spatial dimension wound on an extended or compacted dimension.



3.Physical meaning of the 2 additional spatial dimensions

3.1 The relativistic energy formula.

The relativistic energy of a moving body is:

$E^2=\left(m_0 c^2\right)^2+\left(p\,c\right)^2$ where:

- E =Energy of a moving body

- m_o= rest mass
- c= light velocity
- p= lineal momentum of a body.

If we write energy as a function of velocity components Vx, Vy and Vz then we have:

$$\boxed{E^2=(m_0\cdot c\cdot c)^2+(m_0\cdot c\cdot V_x)^2+(m_0\cdot c\cdot V_y)^2+(m_0\cdot c\cdot V_z)^2}$$

This equation suggests that all bodies move at the speed of light in a direction perpendicular to x, y and z. Within the theory of relativity the term ($m_o c^2$) is interpreted as the result of displacement of bodies through time at the speed of light. Another possibility is to identify this term with the energy due to a movement in the direction of the fourth spatial dimension postulated in the Kaluza-Klein theory.

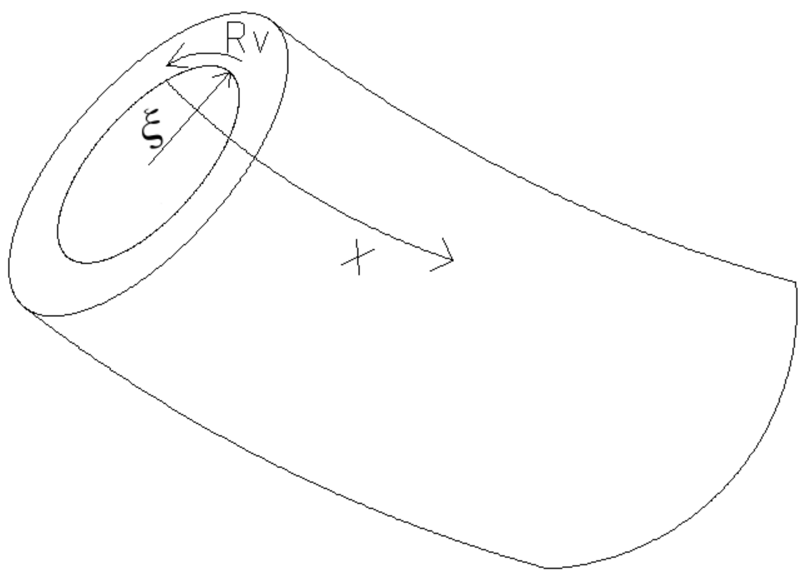


Fig. 3Coordinate system for the compacted dimensions.

This movement at speed of light of elementary particles would be in the R_v

This movement, seen perpendicularly from the expanded dimensions, should be perceived as a vibration due to the circular topology of the extra dimensions.

If we analyze the Albert Einstein and Otto Stern modification to the formula derived in 1900 by Max Plank for an isolated radiator of energy we have:

$$E=\frac{h\cdot \upsilon}{e^{\frac{h\cdot\upsilon}{K\cdot T}}-1}+\frac{h\cdot\upsilon}{2}$$

where:

h= Plank's constant , K=Boltzmann’s constant , υ= frequency T= Absolute temperature

it can be seen that even at the absolute zero temperature any particle has a residual vibration energy equal to:

$$E_r=\frac{h\cdot\upsilon}{2}$$

This equation has been interpreted on numerous occasions as a consequence of quantum vacuum energy. However that vibration could be interpreted as the projection in our 3-dimensional of the particle's motion at the speed of light in the 4th spatial dimension R_v. In this case the energies should be equal

$$E = m_0 c^2$$

$$m_0 c^2 = \frac{h \cdot \upsilon}{2} \Longrightarrow \upsilon = \frac{2 \cdot m_0 c^2}{h}$$

$$E_r = \frac{h \cdot \upsilon}{2}$$

frequency of particles according to their mass.

If the trajectory was circular and assuming that all particles travel at the speed of light it can be deduced the radius of this circular motion:

$$\upsilon = \frac{c}{2\pi \xi_0}$$

$$\xi_0 = \frac{h}{4\pi m_0 c} = \frac{\bar{h}}{2 m_0 c}$$ In the electron case we have:

$$\upsilon = \frac{2 m_0 c^2}{h}$$

$$\xi_e = \frac{\bar{h}}{2 m_e c} = 1{,}93079616 \cdot 10^{-13}\, m$$

where h bar is reduced Plank's constant and ξ is the radius of the circular movement trough the two compacted dimensions.

Perimeter would be:

$$\boxed{p_e = \frac{h}{2 m_0 c}}$$

which represents half D'Broglie wavelength of a particle moving at light speed.

3.2 Interpretation of mass as the inverse of a length.

The previous relationship provides a physical interpretation of the rest mass as the inverse of a length.

$$\xi_0 = \frac{\bar{h}}{2 m_0 c} = \frac{1{,}7588 \cdot 10^{-43}}{m_0}$$ In S.I. Units.

This interpretation allows a new point of view of phenomena already known, for example if we analyze dimensionally the energy:

Energy = Force * displacement = $[M LT^{-2} * L] = [L^{-1} * L^2 T^{-2}] = [LT^{-2}]$

ie provides acceleration units for energy, which match with gravitational phenomenon.

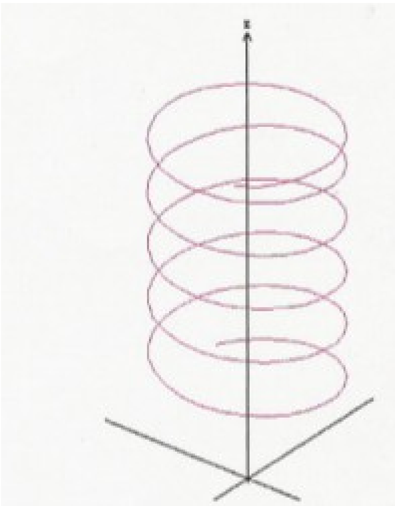
if we analyze dimensionally the density:

Density =Mass/Volume= [L $^{-4}$] i.e. , curvature units, coinciding with the general relativity theory, that directly linked matter-energy density with spacetime curvature.

On the other hand if we consider the equation relating scalar curvature R with the matter-momentum tensor T we have: $-R=\frac{8\pi G}{c^4}\cdot T$ where $\frac{8\pi G}{c^4}=2{,}0766\cdot 10^{-43}$ is of the same order of magnitude as the factor $\frac{\hbar}{2c}=1{,}7588\cdot 10^{-43}$.

3.4 D'Broglie wavelength.

The sum of the circular motion in the plane of the compacted dimensions with a movement in the extended dimensions would form helical paths.



The velocities triangle can be plotted:

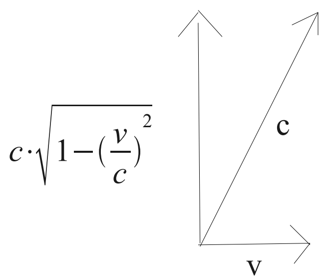


Fig. 4 Helical paths

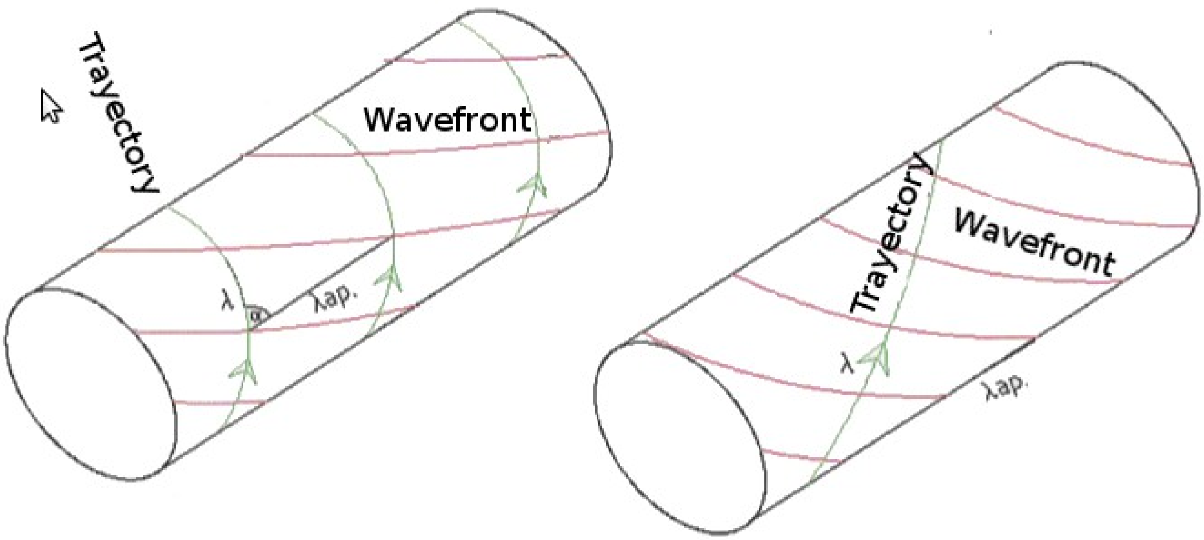
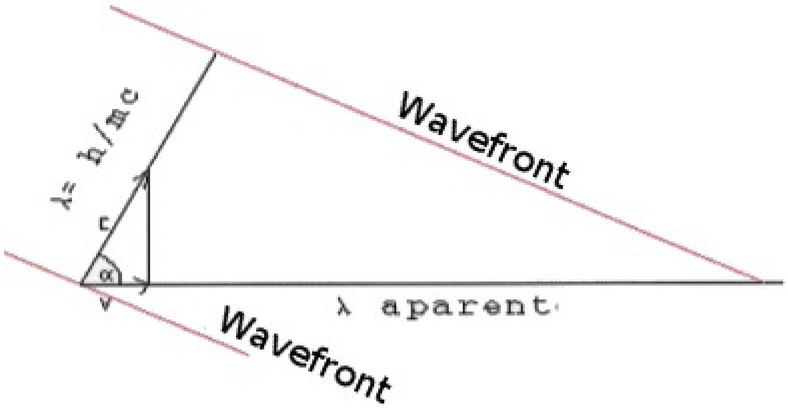


Fig 5. Real particles paths.

Transverse wave associated with a physical particle moving at the speed of light would have a wavelength equal to:

$$\lambda_0 = \frac{h}{m_0 \cdot c}$$

However, for a four-dimensional observer this phenomenon seems to him as if material particle has an apparent associated wave λ equal to the projection on extended dimensions.



Thus:

Fig 6. Velocity triangle. (Alcerro)

$$\cos\alpha = \frac{v}{c}$$

$$\frac{v}{c} = \frac{\dfrac{h}{m_0 \cdot c}}{\lambda\, apa} \quad \Rightarrow \quad \lambda\, apa = \frac{h}{m_0 \cdot v}$$

$$\cos\alpha = \frac{\lambda_0}{\lambda\, apa} = \frac{\dfrac{h}{m_0 \cdot c}}{\lambda\, apa}$$

As apparent wavelength is a dimension in the direction of movement it appears contracted by the relativistic effect $\lambda\, apa = \frac{h}{m_0 \cdot v} \cdot \sqrt{1 - (\frac{v}{c})^2}$, coinciding with D'Broglie wavelength of material particles.

3.4 Interpretation of the uncertainty principle.

The uncertainty principle for position and momentum states that $\boxed{\Delta x \cdot \Delta p \geqslant \frac{\hbar}{2}}$ thus momentum uncertainty must satisfy $\boxed{\Delta p \geqslant \frac{\hbar}{2 \Delta x}}$, if we use the relativistic equation that links energy with momentum $p = \gamma\, m_0 v$ when momentum uncertainty is greater than $m_0 c$ then energy uncertainty would be greater than $m_0 c^2$, enough to create another particle of the same type. Therefore it must be a fundamental limitation in position uncertainty

$$\boxed{\Delta x \geqslant \frac{1}{2} \cdot \left(\frac{\hbar}{m_0 c} \right)} \quad \text{thus} \quad \boxed{\Delta x \geqslant \xi_0} \; .$$

It follows therefore that uncertainty principle arises from the fact that we are studying five dimensional phenomena as if it were three dimensional phenomena.. It is not surprising therefore that the compton wavelength represents the limit between the behavior as a particle or as a wave.

3.5 Qualitative influence of the curvature of space in phenomena occurring at large scales.

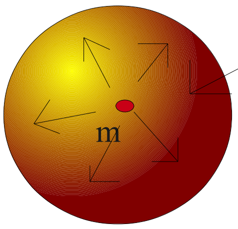
In the traditional analysis of Kaluza-Klein theories physical constants, ($\mu_0, G, \varepsilon_0 ....$) should vary according to the number of dimensions used. This is based on considerations similar to those discussed below.

If we take the equivalent of Gauss's law for the gravitational field in its integral form we have:

$$\oint_s E_g \cdot dS = 4\pi G \cdot m$$

"The gravitational flux through any closed surface is proportional to the enclosed mass"

$$4\pi r^2 \cdot E_{g5D} = 4\pi G \cdot m_0$$



Note that the resulting equation is a four-dimensional equation , as $m_0 = \frac{\bar{h}}{2\xi_0 c}$, and therefore $E = E(L_x, L_y, L_z, \xi_0)$

Performing an exercise of imagination we can assume a 5D universe in which one of the extended spatial dimensions linearly compactase to an extent "a" as shown in Figure 7. Let's see what happen when physicists of this universe analyze Gauss' law in 4 D.

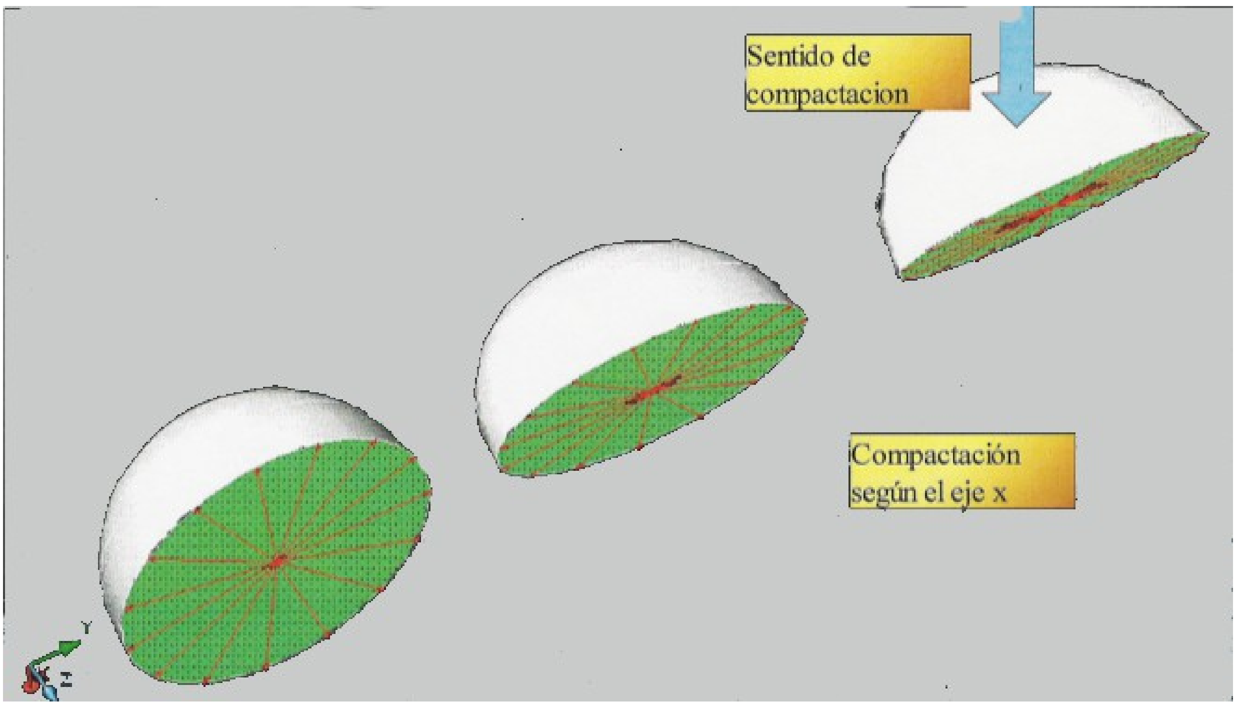
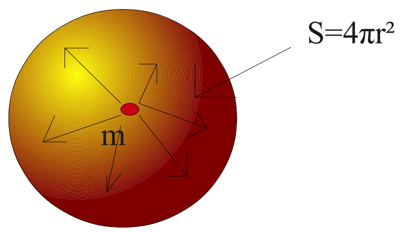


Fig 7. Effect on Gauss's law when a spatial dimension is compacted linearly.

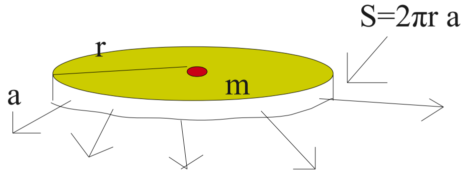
$4\pi r^2 \cdot E_{g5D} = 4\pi m_0$



Universe uncompacted **5D**

5D compacted

$2\pi ra \cdot E_{g5D} = 4\pi \cdot m_0$



Universe compacted
r>>>a

Scientists from this 4D plane universe would measure an Eg field and try to apply Gauss's law obtaining the following results:

$2\pi r \cdot E_{g4D} = 4\pi \cdot m_0$

4D interpretation

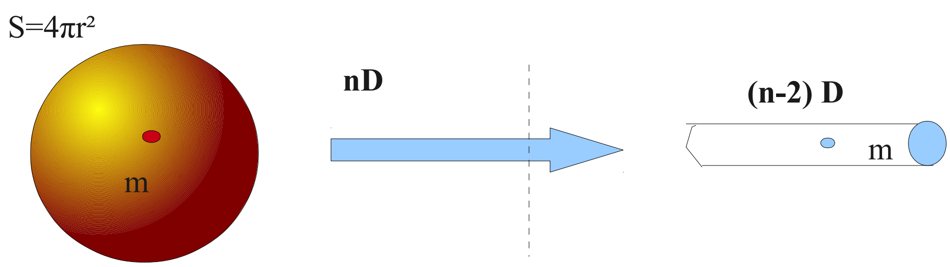
By equating the two equations we have:

$2\pi r \cdot E_{g4D} = 2\pi ra \cdot E_{g5D}$ then $E_{g4D} = a \cdot E_{g5D}$

As E_{g4D} has to be equal to E_{g5D} we should add a constant to the formula in 4D in order to obtain a correct result:

$2\pi r \cdot E_{g4D} = 4\pi\, a \cdot m_0$, i.e. appears a constant of gravitation G = a.

As "a" is very small the field measured when compacting a dimension is much smaller.
In the case two dimensions that are compacted into a circle of radius "a" we would have:



2D Surface of a 3D sphere

$4\pi a^2 \cdot E_{g3D} = m_0$

$E_{g1D} = 2\pi a^2 \cdot E_{g2D}$

$2\, E_{g1D} = m_0$

0D surface of a 1D sphere

If we compact circularly two dimensions field would be altered by a factor equal to $2\pi a^2$, this would allow us to estimate the radius of compacted dimensions:

$$G \approx 2\pi \cdot R_u^2 \quad \rightarrow \quad R_u \approx \sqrt{\frac{G}{2\pi}} \simeq 3 \cdot 10^{-6}\, m$$

Clearly, since we have used the flat space approximation for the gravitational field, estimation of the compacted dimensions radius can not be very accurate, but it gives us the order of magnitude of the compacted dimensions.

On the other hand, at lower scales force fields diminish according to inverse cube law, so appear to have higher intensity. In short, the curvature act as a converging lens, decreasing the intensity of distant phenomena and increasing the apparent intensity at very small scales. This fact could justify qualitatively the difference in scale between the 4 fundamental forces of nature.

If we consider that according to the postulates of this paper the dimensions of the mass are the inverse of a length it we would
$[G]=L^3 M^{-1} T^{-2}=L^4 T^{-2}$ and interpreting time as a length: $[G]=L^2$.

Consequently curvature of six-dimensional spacetime justifies the relationship between inertial mass and gravitational mass when we talk about phenomena that occur at large scales. Therefore most of the constants should disappear when we perform 6D calculations. ($\hat{\mu}_{0g}=1, \hat{G}=1,....$).

4.Origin of the electric field.

4.1 About gravitomagnetism.

If we write gravitomagnetism equations comparing them with Maxwell's equations.

GRAVITOMAGNETISM	ELECTROMAGNETISM
$\vec{\nabla}\vec{E}_g = -4\pi G\rho$	$\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$
$\vec{\nabla}\vec{B}_g = 0$	$\nabla B = 0$
$\vec{\nabla}\times\vec{E}_g = \frac{-1}{c}\frac{\partial\vec{B}g}{\partial t}$	$\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$
$\nabla\times B_g = \frac{-4\pi G}{c}\cdot\vec{j}_m + \frac{1}{c}\frac{\partial\vec{E}_g}{\partial t}$	$\nabla\times B = \mu_0\cdot\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

Despite obvious similarities gravitomagnetism equations differs in two signs of Maxwell's equations, the first sign indicates that can only exist attractive forces between the masses, the second sign indicates that **two mass streams flowing in the same direction repel,** contrary to what happens in electromagnetism in which they attract.

4.2 Gravitomagnetic field produced by elementary particles.

Since elementary particles are moved at the speed of light in circular paths with a very small radius then they produce a strong induction field Bg. This causes a force field which is qualitatively similar to the electric field, as shown in Figure 8.

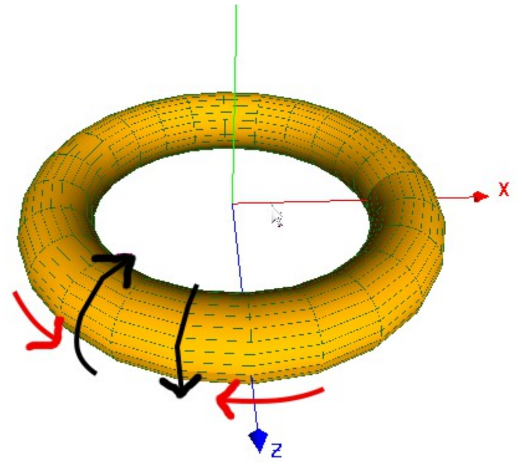
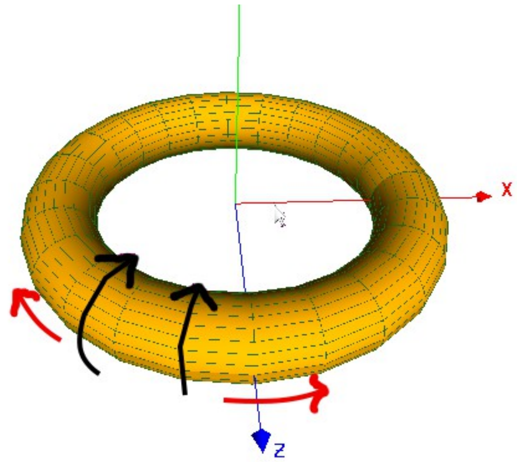
	
Different direction of rotation → Attraction	Same direction of rotation → Repulsion

Fig 8. Example 3 dimensional. A circular mass movement may appear to be an electric charge.

Gravitomagnetism laws expressed in six dimensions should not need any constant, then Ĝ=1.
To calculate induction field generated by elementary particles it can be assimilated to the field generated by a circular loop.

.

$$B=\frac{\mu_0 i}{2\mathrm{R}}$$

ELECTROMAGNETISM 5D $B=B(L_x, L_y, L_z, M, T)$ *

$$B_g=\frac{-4\pi\hat{G}}{c}\cdot\frac{i_g}{2R^2}$$

GRAVITOMAGNETISM 6D $B=B(L_x, L_y, L_z, \xi, \eta, T)$

* Note: If we consider mass as the inverse of a length any equation containing the mass dimension (or electric charge, since the charge / mass ratio is constant for each type of elementary particles) should be considered as a 5 dimension equation , 4 spatial dimensions plus one time dimension.

If electric field is the five dimensions expression of gravitomagnetic field in six dimensions then B = Bg

$$\frac{B}{B_g}=\frac{\mu_0 2\mathrm{cR}^2}{-4\pi\hat{G}\cdot 2\mathrm{R}}\cdot\frac{i}{i_g}=1$$

the ratio of electric current to mass current is the same as the ratio of charge to mass of an elementary particle. So you can write:

$$\frac{\mu_0 cR}{-4\pi \hat{G}} \cdot \frac{q}{m_0} = 1$$

Given that we have postulated that Ĝ=1 and $R = \xi_e = \frac{\bar{h}}{2 m_e c} = 1{,}93079616 \cdot 10^{-13}$

$$\frac{\mu_0 c \xi_e}{-4\pi} \cdot \frac{q_e}{m_e} = 1$$

$\frac{q_e}{m_e} = 1{,}72759870 \cdot 10^{11}$ S.I units.

Compared with the experimental ratio of charge to mass of electron:

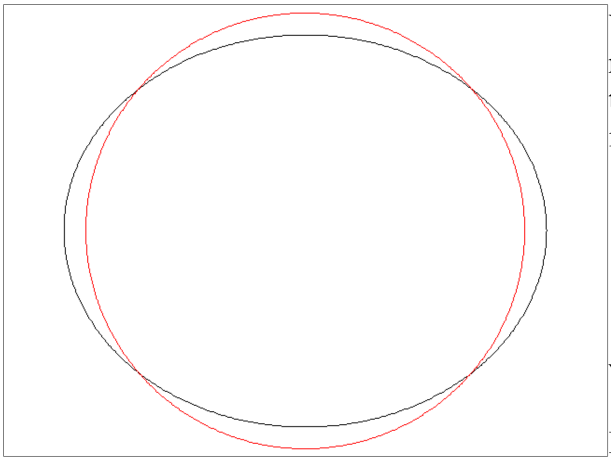
$$\frac{q}{m_0} = \frac{e}{m_{e0}} = \frac{1{,}602176 \cdot 10^{-19}}{9{,}10938291 \cdot 10^{-31}} = 1{,}75881946 \cdot 10^{11}$$

which it differs by 1.8% of the estimated value.

Therefore, when considering the mass as the inverse of a length it is possible to save the main difficulty presented by the Kaluza-Klein theory.

Note that just considering a value of Ĝ = 1,01807176 we can obtain a correct value of the ratio of charge to mass.

4.3 Elliptical topology of the compacted dimensions.



We need to increase the magnetic induction, while retaining the perimeter. The easiest way is to transform the circular path into elliptical path. The magnetic induction in the center of an elliptica current loop is:

$$\boxed{B_z = \mu_0 I \frac{l}{4S}}$$

where l = perimeter, S = surface, I= electric current.

In order to estimate perimeter it has been used the following approximate formula:

$$L \approx \pi \left(3(a+b) - \sqrt{(3a+b)(a+3b)} \right)$$

We can chose a circular path with radius equal to one and we can deform it maintaining constant the perimeter. Simply choose a loop of semiaxes a = 1.10576 and b = 0.8883 in order to increase the perimeter-surface relationship and therefore the magnetic induction field B by a factor of 1.018068, which would provide the correct value of ratio of mass to charge for the electron.

The perimeter of the previous ellipse has been calculated using the formula:

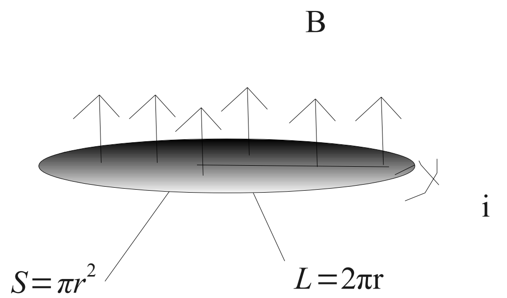
$$L=4a\int_0^{\pi/2}\sqrt{1-e^2\sin^2\theta}\,d\theta$$

where e is the ellipse eccentricity. The mistake made when using the approximate formula is 3,42 10^{-6} by one.

Despite trajectories are elliptical it is possible to continue using the circular assumption using the constant Ĝ = 1,01807176, which is now considered a shape factor.

4.4 Application example. Intrinsic magnetic moment of the electron.

Let us see how we can convert electromagnetic 5D formulas to gravitomagnetic 6D.



$$\boxed{\mu=i\cdot\pi r^2}$$

Gravitomagnetic moment is the product of mass current by surface embraced by the loop. However as distances involved are lower than compacted dimensions we must to use 6D.
To convert the formula to 6D we start from the definition of magnetic moment.

$$\mu=\frac{1}{2}\int r\,x\,idl$$

The steps to follow are the following:

1. Remove shape factor.
   $$\mu=\int r\,x\,idl$$
2. Divide by r to move from 5D to 6D
   $$\mu=\frac{1}{r}\int r\,x\,idl=i\int dl=i\cdot 2\pi R$$
3. Use mass current instead electric current.

$\mu = i_m \cdot 2\pi R$

4. Replace electromagnetic constant by gravitomagnetic constant. This is accomplished by multiplying by the factor $\dfrac{-4\pi \hat{G}/c}{\mu_0}$.

$$\boxed{\mu_g = \frac{-4\pi \hat{G}/c}{\mu_0} \cdot i_m \cdot 2\pi\mathrm{R}}$$

The mass current will be $i_m = m_0 \cdot v$ where $v = \dfrac{n^o\, vueltas}{segundo} = \dfrac{c}{2\pi\mathrm{R}}$ since we had postulated that electrons travel at light speed in compacted dimensions.

Therefore

$$\boxed{\mu_g = \frac{-4\pi \hat{G}}{\mu_0 c} \cdot \frac{m_0 c}{2\pi\mathrm{R}} \cdot 2\pi\mathrm{R} = \frac{-4\pi \hat{G}\, m_0}{\mu_0}}$$

in electron's case and using Ĝ = 1,01807176 we have

$$\boxed{\mu_g = \frac{-4\pi \cdot 9{,}109\, 10^{-31} \cdot 1{,}01807176}{4\pi\, 10^{-7}} = -9.2740055\, 10^{-24}}$$ in SI units.

This value is very similar to Bohr's magneton, 9.274 00915 10 $^{-24}$ en in S.I. Units.

Note: The above formula can also easily be obtained from the expression of the magnetic moment from espin $\mu = \dfrac{q}{2\mathrm{m}} = \dfrac{q}{2\mathrm{m}} S = \dfrac{q}{2\mathrm{m}} \dfrac{\hbar}{2}$ and by substituting $\dfrac{q}{m} = \dfrac{-4\pi \hat{G}}{\mu_0 c} \cdot \dfrac{2mc}{\hbar}$

5. Electron as gravitomagnetic pulsation.

5.1 Gravitomagnetic wave equation.

Gravitomagnetic equations are:

$$\vec{\nabla}\vec{E}_g=-4\pi G\rho$$

$$\vec{\nabla}\vec{B}_g=0$$

$$\vec{\nabla}\times\vec{E}_g=\frac{-1}{c}\frac{\partial\vec{B}g}{\partial t}$$

$$\nabla\times B_g=\frac{-4\pi G}{c}\cdot\vec{j}_m+\frac{1}{c}\frac{\partial\vec{E}_g}{\partial t}$$

We propose gravitomagnetism equations in a massles space. As we have postulated that $\hat{\mu}_{0g}$ must be equal to the unit then B should be equal to H.

$$\vec{\nabla}\vec{E}_g=0(a)$$

$$\vec{\nabla}\vec{H}_g=0(b)$$

$$\vec{\nabla}\times\vec{E}_g=\frac{-1}{c}\frac{\partial\vec{H}g}{\partial t}(c)$$

$$\nabla\times H_g=\frac{1}{c}\frac{\partial\vec{E}_g}{\partial t}(d)$$

Operating in (c) we have $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_g)=\vec{\nabla}\times\left(\frac{-1}{c}\frac{\partial\vec{H}g}{\partial t}\right)$

therefore $\vec{\nabla}\times\vec{\nabla}\times\vec{E}_g=\frac{-1}{c}\frac{\partial\left(\nabla\vec{H}g\times Hg\right)}{\partial t}$

and by substituting $\nabla\times H_g=\frac{1}{c}\frac{\partial\vec{E}_g}{\partial t}$ y $\vec{\nabla}\times\vec{\nabla}\times\vec{E}_g=\vec{\nabla}^2\vec{E}_g$

and finally $\boxed{\vec{\nabla}^2\vec{E}_g=\frac{-1}{c^2}\frac{\partial\nabla^2\vec{E}g}{\partial t^2}}$

Phase velocity would be $v_p = \frac{w}{k}$ so $v_p = \frac{1}{\sqrt{\frac{1}{c^2}}} = c$ and thus:

$$\boxed{\vec{\nabla}^2 \vec{E}_g + \frac{1}{v_p^2}\frac{\partial \vec{E}g}{\partial t^2} = 0}$$

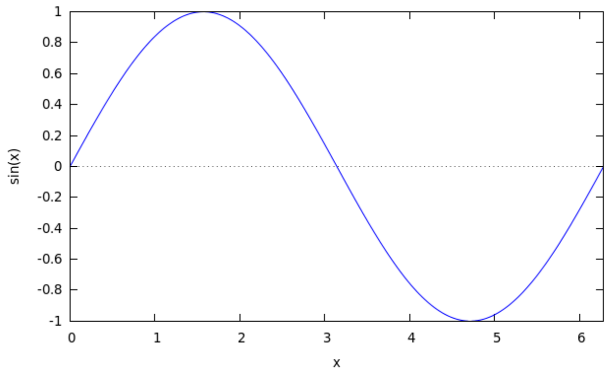
Similarly we can obtain::

$$\boxed{\vec{\nabla}^2 \vec{H}_g + \frac{1}{v_p^2}\frac{\partial \vec{H}g}{\partial t^2} = 0}$$

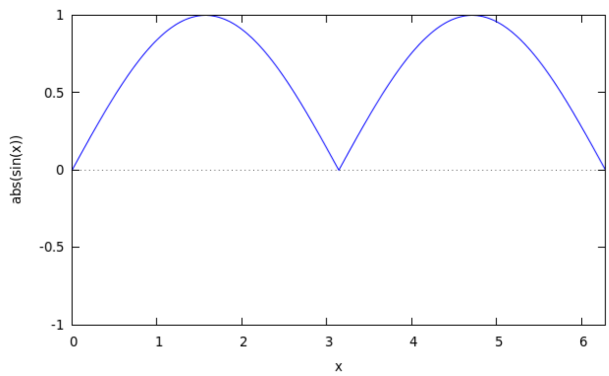
If we assume that field has harmonic time dependence:

$\boxed{\vec{\Psi} = \left|\vec{\Psi}_0 e^{-wt}\right|}$ then: $\boxed{\vec{\nabla}^2 \vec{E}_g + \frac{w^2}{v_p^2}\vec{E}g = 0}$ and naming wavenumber to the ratio $\frac{w}{v_p}$:

$\boxed{\vec{\nabla}^2 \vec{E}_g + k^2 \vec{E}g = 0}$ completely analogous to Helmholtz equations, so solutions would be:



However magnetic waves are different, since gravitational field can not be negative.



In the figure we can observe the equivalent gravitomagnetic wave.

For the same frequency the gravitomagnetic wave has a wavelength that is half of the equivalent electromagnetic, therefore wavenumber k should be defined as:

$$\boxed{k = \frac{\pi}{\lambda}}$$

Fig 9. Gravitomagnetic wave vs electromagnetic wave

5.2 Scalar gravitomagnetic wave equation in 6D. Free particle-pulsation solution.

Because of space topology gravitomagnetic waves can not travel freely, but must conform to a strict boundary conditions. The most similar physical phenomenon is found in the transmission of electromagnetic waves by circular or elliptical waveguides, but in this case confinement is due to the curvature of space and not to a metal walls.

We are going to use a cylindrical- elliptic coordinate system in five spatial dimensions:

Expanded spatial dimensions: Cartesian coordinates x,y,z.
Compacted spatial dimensions : Elliptic coordinates, curves with ξ =constant are confocal ellipses, and curves with η=constant are hyperboles perpendicular to previous ellipses. In the limit case $f \rightarrow 0$, this coordinates transform to circular coordinates, where $radius(\xi), angle(\eta)$

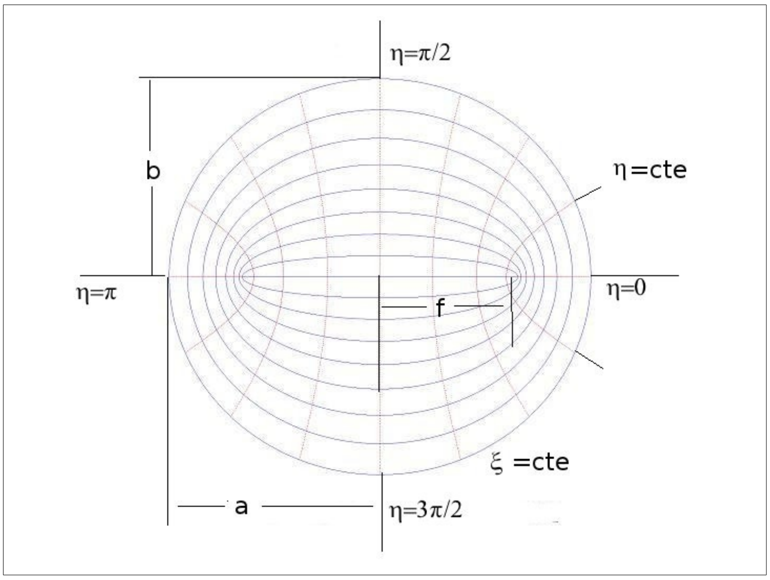


Fig 10. Elliptic coordinates.

We can transform elliptic coordinates into Cartesian coordinates:

$$x = f\cosh\xi\cos\eta \quad , \quad y = f\, senh\xi\, sen\,\eta$$

Six-dimensional wave equation would be:

$$(\nabla_{6D}^2 + k^2)\cdot H = 0$$

We will solve the equation for two cases, first assuming a circular topology and then generalizing it to an elliptical topology:

5.2.1 Circular topology of compacted dimensions.

Using Laplacian in cylindrical coordinates:

$$\boxed{\left(\frac{\partial^2}{\partial^2\xi}+\frac{1}{\xi}\cdot\frac{\partial}{\partial\xi}+\frac{1}{\xi^2}\cdot\frac{\partial}{\partial\eta^2}+\frac{\partial^2}{\partial^2 x}+\frac{\partial^2}{\partial^2 y}+\frac{\partial^2}{\partial^2 z}+k^2\right)\cdot H=0}$$

We can solve it by separation of variables

$$H(\xi,\eta,x,y,z)=G(\xi)\cdot N(\eta)\cdot F(x,y,z)$$

and replacing:

$$G''NF+\frac{1}{\xi}\cdot G'NF+\frac{1}{\xi^2}GN''F+GN\cdot\nabla_{3D}^2 F+k^2GNF=0$$

Divide by GNF

$$\boxed{\frac{G''}{G}+\frac{1}{\xi}\cdot\frac{G'}{G}+\frac{1}{\xi^2}\cdot\frac{N''}{N}+\frac{\nabla_{3D}^2 F}{F}+k^2=0}$$

And as is usual in waveguide calculations we can separate wave number on 2 addends:

$k^2=\beta^2+k_c^2$ where β is the “propagation constant” and k_c is the “cut-off wave-number” and it represents the minimum frequency for a wave in order to can be propagated through the guide.

So we can obtain 2 equations:

$$\left.\begin{array}{l}\boxed{\frac{G''}{G}+\frac{1}{\xi}\cdot\frac{G'}{G}+\frac{1}{\xi^2}\cdot\frac{N''}{N}+k_c^2=0}\quad (1)\\[2ex] \boxed{\frac{\nabla_{3D}^2 F}{F}+\beta^2=0}\quad (2)\end{array}\right\}$$

If we multiply (1) by ξ^2:

$$\boxed{\frac{\xi^2\cdot G''}{G}+\xi\cdot\frac{G'}{G}+\frac{N''}{N}+\xi^2\cdot k_c^2=0}\quad (3)$$

So we can obtain 2 equations that depend on just one variable. As the terms of the equation add a constant then each term must be constant:

$$\frac{N''}{N}=-m_s^2 \;\rightarrow\; \boxed{N''+m_s^2 N=0}\quad (3a)$$

Solution is: $N(\eta)=A_0\cdot sen(m_s\cdot\eta)+B_0\cdot\cos(m_s\cdot\eta)$

we substitute $\frac{N''}{N}=-m_s^2$ in (3) and we obtain:

$$\boxed{\xi^2\cdot\frac{G''}{G}+\xi\cdot\frac{G'}{G}-m_s^2+\xi^2\cdot k_c^2=0} \rightarrow$$

$$\boxed{\xi^2\cdot G''+\xi\cdot G'+(\xi^2\cdot k_c^2-m_s^2)\cdot G=0}$$ (4) it is known as Bessel's differential equation.

Let's study m_s :

logically $H(\xi,\eta,x,y,z)=H(\xi,\eta+\pi l,x,y,z)\, where\, l=integer$

So m_s *must be a half-integer.*

Remember that:

$N(\eta)=A_0\cdot sen(m_s\cdot\eta)+B_0\cdot\cos(m_s\cdot\eta)$ it can be written as:

$$\boxed{N(\eta)=C_0\cdot sen[m_s\cdot(\eta-\eta_0)]}$$

We can assume that electrons are gravitational waves, for the case of an electron at rest we have that the propagation constant β=0, in this case:

$k^2=0^2+k_c^2=k_c^2$

We can associate this cutoff frequency to the vibration that present all electrons:

$k=\frac{\pi}{\lambda}$ where wavelength would be equal to electron's circular path perimeter, so: $\lambda=\frac{h}{2\mathrm{m}_0 c}$ and:

$k=k_c=\frac{\pi}{\lambda}=\frac{2\pi m_0 c}{h}=\frac{m_0 c}{\hbar}$ and thus circular wave-number must be:

$\boxed{k_c=\frac{m_0 c}{\hbar}}$ so we can write (4) in this way $\boxed{\xi^2\cdot G''+\xi\cdot G'+\left(\xi^2\cdot\left(\frac{m_0 c}{\hbar}\right)^2-m_s^2\right)\cdot G=0}$ (5)

Solutions to Bessel's differential equation are:

- J α → Bessel function of the first kind.

– Y_α → Bessel function of the second kind.

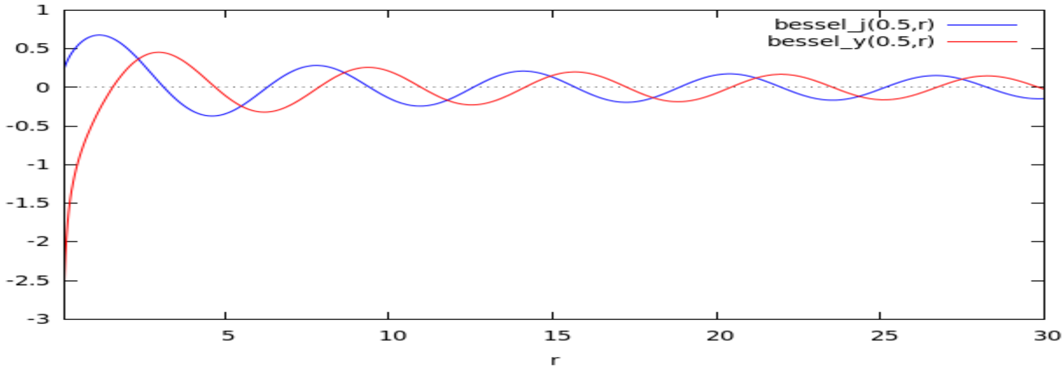


Fig 11. Bessel functions.

Both are periodic functions, therefore not valid, because otherwise the waves would leave the universe. Hankel functions are not valid because they are linear combinations of J_α and Y_α

If we postulate that k_c is an imaginary number then::

$$\boxed{k_c=\frac{m_0 c}{\hbar}i} \rightarrow \boxed{\xi^2 G''+\xi G'-\left(\xi^2\left(\frac{m_0 c}{\hbar^2}\right)^2+m_s^2\right)G=0} \quad (6) \text{ or modified Bessel's equation}$$

the general solution is:

$$G=C_2 I_{m_s}\left(\frac{2 m_0 c}{\hbar}\cdot\xi\right)+C_3 K_{m_s}\left(\frac{2 m_0 c}{\hbar}\cdot\xi\right) \quad (7)$$

where I_{m_s} and K_{m_s} are modified Bessel functions of first and second kind and order m_s. Notice scale factor 2, which corrects the different definition of wave number. We can plot both functions:

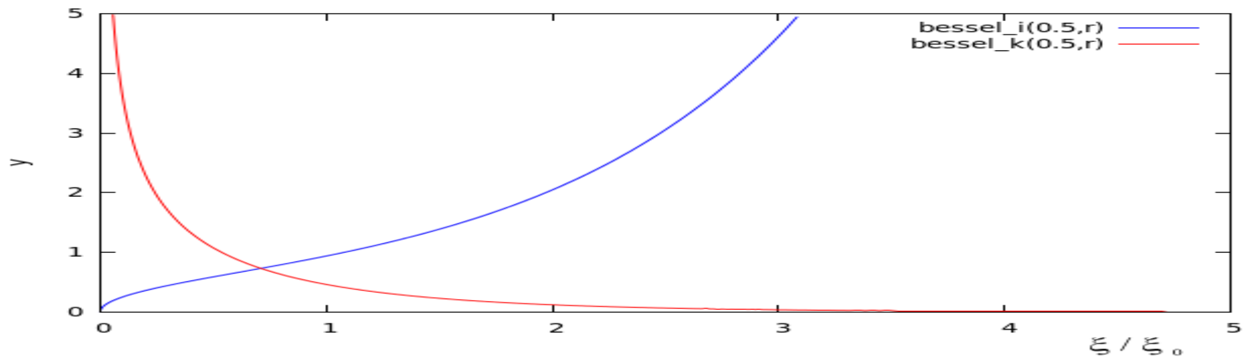


Fig 12 Bessel modified functions.

That is, none of the separate functions can fulfill the boundary conditions, however, if we realize that actually r_u represents an event horizon it is easy to see that the solution would be:

$$\left.\begin{aligned} &Si\,\xi \leqslant \xi_0 \rightarrow G = C_2\, I_{m_s}\left(\frac{2\,m_0\,c}{\hbar}\cdot\xi\right) \\ \\ &Si\,\xi \geqslant \xi_0 \rightarrow G = C_3\, K_{m_s}\left(\frac{2\,m_0 c}{\hbar}\cdot\xi\right) \end{aligned}\right\}$$

Boundary condition is that both solutions would be equal at $\boxed{\xi_0=\frac{\hbar}{2\mathrm{m}_0\,c}}$ and therefore $\boxed{\frac{2\mathrm{m}_0\,c}{\hbar}\,\xi_0=\frac{2\mathrm{m}_0 c}{\hbar}\cdot\frac{\hbar}{2\mathrm{m}_0\,c}=1}$

If we assume that electron is the simplest wave $m_s=\pm 1/2$ then

$C_2\cdot I_{\frac{1}{2}}(1)=C_3\cdot K_{\frac{1}{2}}(1)$, so: $C_2=0{,}4916\,C_3$

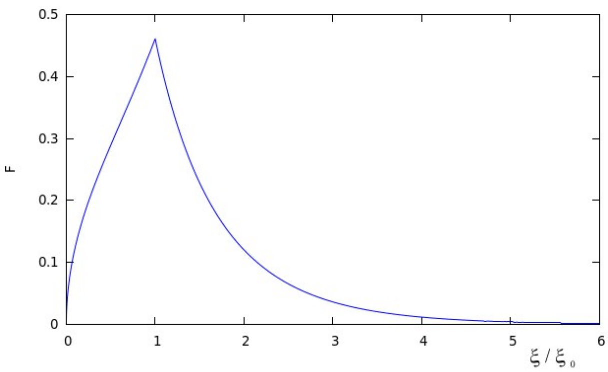
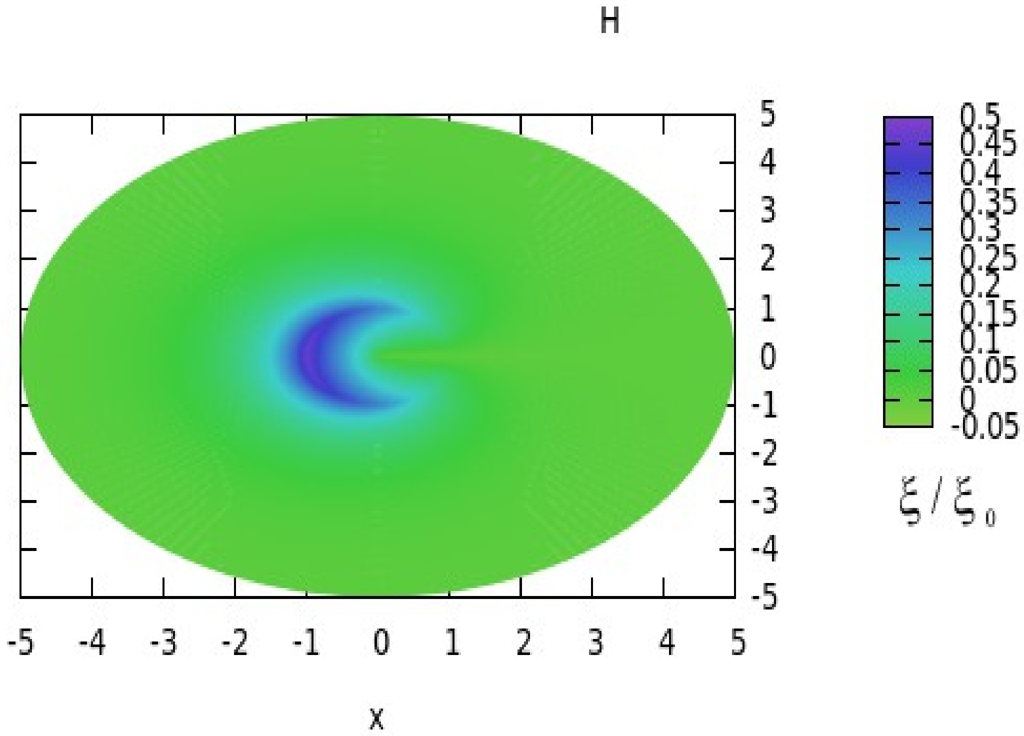


Fig 13 Solution versus ξ/ξ $_0$



We can plot then G(ξ)N(η) in the compacted dimensions plane:

Fig 14. Compacted dimensions plane solution.

This is an stationary wave. Energy flux of a wave would be expressed by quadratic wave-fuction $<\Psi>^{2,}$ Ψ can be E_g or H_g.

If we plot $<E_g>^2$ versus ξ/ξ_0 then:

$$Si\,\xi \leqslant \xi_0 \rightarrow G = 0{,}4916\, I_{\frac{1}{2}}\left(\frac{2m_0 c}{\hbar}\cdot\xi\right)$$

$$Si\,\xi > \xi_0 \rightarrow G = K_{\frac{1}{2}}\left(\frac{2m_0 c}{\hbar}\cdot\xi\right)$$

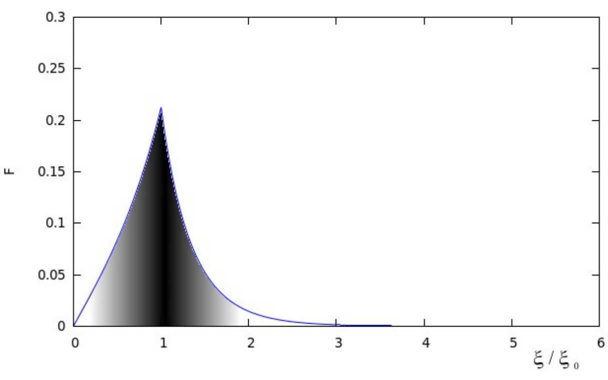


Fig 15 ***Energy flux versus*** **ξ/ξ_0**

We can plot $(GN)^2$ function in the compacted dimensions plane:

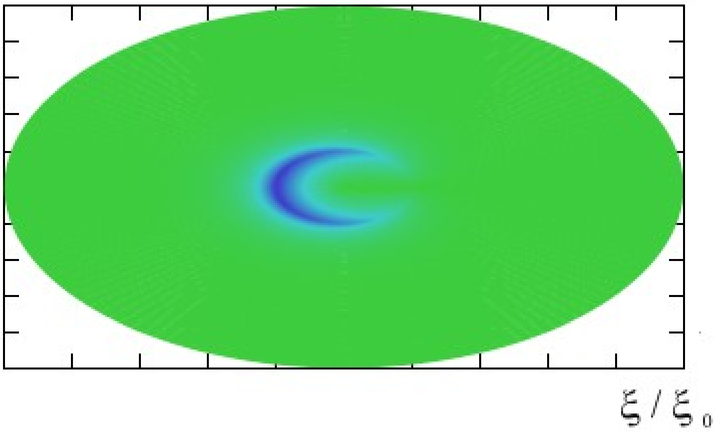


Fig 16. $G(\xi)^2 N(\eta)^2$ Energy flux in the compacted dimensions plane.

We can calculate the center of mass of the function $< E_g >^2$

$$\xi_{cdm} = \frac{\int \xi E^2 d\xi}{\int E^2 d\xi}$$ and:

$$\int E^2 d\xi = \int_0^1 \left(0{,}4916\, I_{0,5}(\xi)\right)^2 d\xi + \int_1^{\infty} \left(K_{0,5}(\xi)\right)^2 d\xi$$

$$\int \xi E^2 d\xi = \int_0^1 \left(0{,}4916\, \xi I_{0,5}(\xi)\right)^2 d\xi + \int_1^{\infty} \left(\xi K_{0,5}(\xi)\right)^2 d\xi$$

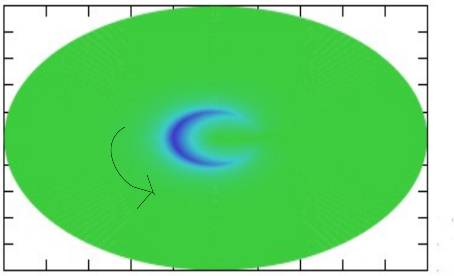
Integrals are solved numerically by Romberg method. The integration limits are [0,1] U [1,20]

$\xi_{cdm} = \frac{\int \xi E^2 d\xi}{\int E^2 d\xi} = 1{,}006495$, therefore, circular topology is not valid because of energy flux is not in the coordinate $\xi = \frac{\hbar}{2 m_0 c}$.

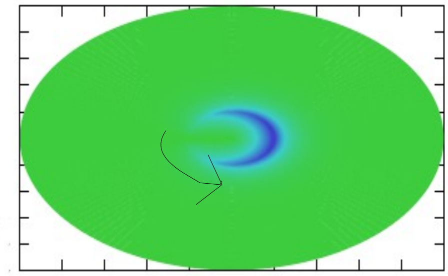
Circular topology is a good approximation if we want to estimate angular momentum of the movement in the compacted dimensions:

$$L = m \times r \cdot v = m_e \xi c = m_e \frac{\hbar}{2 m_e c} \cdot c = \frac{\hbar}{2}$$

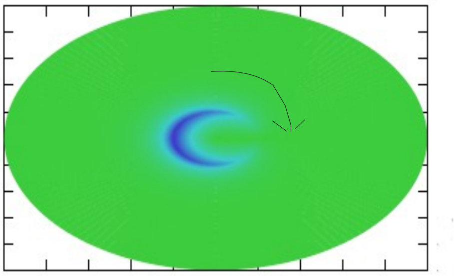
Based on the above result we can assign quantum property of spin to the standing wave of electrons in the compacted dimensions. We can identifie it with the constant m_s, the sign of this constant represents the phase difference and the different direction of rotation explain the difference between electrons and positrons.



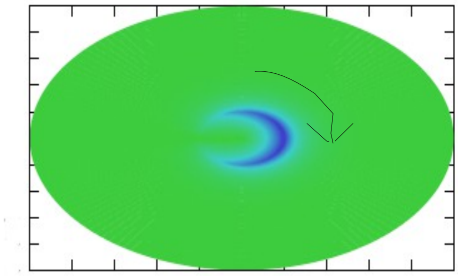
Electron spin +1/2



Electron spin -1/2



Positron spin +1/2



Positron spin -1/2

Fig 17. Intuitive representation of electron spin.

It is easy to see that the results can be extrapolated to estimate the angular momentum of spin particles with different spin, resulting in:

$$\boxed{L_s = m_s \cdot \hbar}$$

5.2.2 Elliptic topology of the compacted dimensions.

Laplacian is also separable, so:

$$H(\xi,\eta,x,y,z) = D(\xi,\eta) \cdot F(x,y,z)$$

$$\left.\begin{array}{l} \boxed{\dfrac{\nabla^2_{\xi,\eta} D(\xi,\eta)}{D(\xi,\eta)} + k_c^2 = 0} \quad (3) \\[2ex] \boxed{\dfrac{\nabla^2_{3D} F(x,y,z)}{F(x,y,z)} + \beta^2 = 0} \end{array}\right\}$$

In the elliptic coordinate system equation (3) can be written as:

$$\frac{2}{f^2(\cosh(2\xi) - \cos(2\eta))}\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) D + k_c^2 \cdot D = 0$$

and assuming that $D(\xi,\eta)=G(\xi)\cdot N(\eta)$ then:

$$\frac{1}{G}\frac{\partial^2 G}{\partial\xi^2}+\frac{k_c^2}{2}f^2\cosh(2\xi)=\frac{-1}{N}\frac{\partial^2 N}{\partial\eta^2}+\frac{k_c^2}{2}f^2\cos(2\eta)$$

this equation can be separated by a constant "a" (Please do not confuse with the semi-major axis of the ellipse)

$$\left.\begin{array}{l} G''-(a-2\mathrm{q}\cosh 2\xi)G=0 \\ N''-(a-2\mathrm{q}\cos 2\eta)N=0 \end{array}\right\}$$

where:

$q=\dfrac{k_c^2 f^2}{4}$ and separation constant "a" just depend on q parameter, a= a(q).

In the limit case q=0 solutions are the Bessel functions.

The second equation is the "angular" dependence and is known as Mathieu equation, first equation is the "radial" dependence and is known as modified Mathieu equation.

As we have postulated that $\boxed{k_c=\dfrac{m_0 c}{\hbar}i}$ then q <0 because kc is imaginary.
In electron case parameter q will be:

$$k_c=\frac{m_0 c}{\hbar}i=\frac{9{,}10938291\,10^{-31}\cdot 299792458}{1{,}054571628\,10^{-34}}i=2{,}58960532\cdot 10^{12}\,i$$

as compacted dimensions radius have been estimated in:

$$r_u=\sqrt{\frac{G}{2\pi}}=3{,}259\cdot 10^{-6}\,m$$

and assuming a elliptic topology of parameters

$$a=1{,}10576\cdot r_u$$

$$b=0{,}8883\cdot r_u$$

and the focus of the ellipse will be:

$$f=\sqrt{a^2-b^2}=r_u\sqrt{1{,}10576^2-0{,}8883^2}=2{,}146\cdot 10^{-6}\,m$$

therefore we can estimate q parameter:

$$q = \frac{k_c^2 f^2}{4} = \frac{(2{,}146 \cdot 10^{-6})^2 (2{,}58960532 \cdot 10^{12})^2}{4} = -7{,}7208 \cdot 10^{12}$$

Taking into account the same considerations as in section 5.2.1 solutions that can be identified with the electron will be:

$N(\eta) = \left| se_{\frac{1}{2}}(\eta, -7.7208 \cdot 10^{12}) \right|$ or absolute value of odd angular Mathieu function of ½ order(or elliptic sine).

Regarding the modified Mathieu equation none of the radial functions can be solution by itself, so as in the circular case radial solution is given by:

If $0 < \xi < \xi_0$ $\quad G(\xi) = I_{o1/2}(2 k_c \xi, -7.7208 \cdot 10^{12}) = I_{o1/2}\left(\frac{\xi}{\xi_0}, -7.7208 \cdot 10^{12}\right)$

or evanescent radial Mathieu function of first kind and order ½.

If $\xi > \xi_0$ $\quad G(\xi) = K_{o1/2}(2 k_c \xi, -7.7208 \cdot 10^{12}) = K_{o1/2}\left(\frac{\xi}{\xi_0}, -7.7208 \cdot 10^{12}\right)$

or evanescent radial Mathieu function of second kind and order ½.

Since q is very large, value of separation constant can be approximated by the following relationship

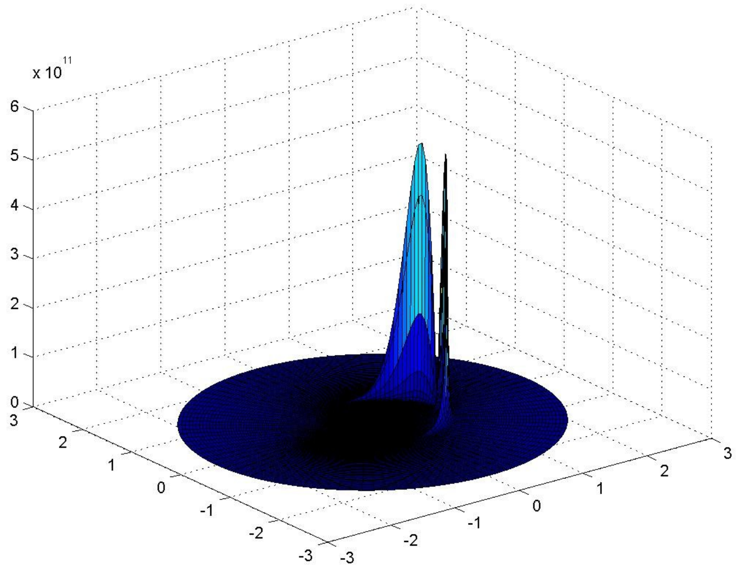
$$a_r = -2q + (2 + 4r)\sqrt{q} - \frac{1}{4} - \frac{1}{2r} - \frac{1}{2r^2} + \left(\frac{-1}{32} - \frac{3r}{32} - \frac{3r^2}{32} - \frac{r^3}{10}\right)\frac{1}{\sqrt{q}} + \ldots.$$

(from Algebraic methods to compute Mathieu functions) where r is the function order.

Solutions to Io and Ko functions of half integer order are not published, but when q → ∞ then a ½ → a 1 and therefore:

$$I_{O\frac{1}{2}} \simeq I_{O1} \qquad K_{O\frac{1}{2}} \simeq K_{O1}$$

As q is too large it is difficult to solve numerically the equations. So we plot the solution for q = -300 in order to have an intuitive vision of possible solutions:



5.2.3 Solution for the extended dimensions.

Now we solve equation (2)

$$\boxed{\frac{\nabla^2_{3D} F}{F} + \beta^2 = 0}$$ we will solve two cases:

CASE A. PARTICLE-PULSATION AT REST. β=0

We have then:

$$\boxed{\frac{\nabla^2_{3D} F}{F} = 0}$$ (8) , and the solutions are:

$$\boxed{F = constant = C_1} \quad (9)$$

$$\boxed{F = \frac{C_2}{x^2 + y^2 + z^2}} \quad (10)$$

The solution (10) is completely analogous to gravitational and electric potential.

However the solution is not valid for x = y = z = 0, as it provides infinite values. The following solution is therefore proposed:

$$Si\, r \leqslant \lambda_c ----------\rightarrow F = C_1$$

$$Si\, r > \lambda_c ---\rightarrow F = \frac{C_2}{x^2+y^2+z^2} + C_3$$

where λc is Compton wavelength of the electron.

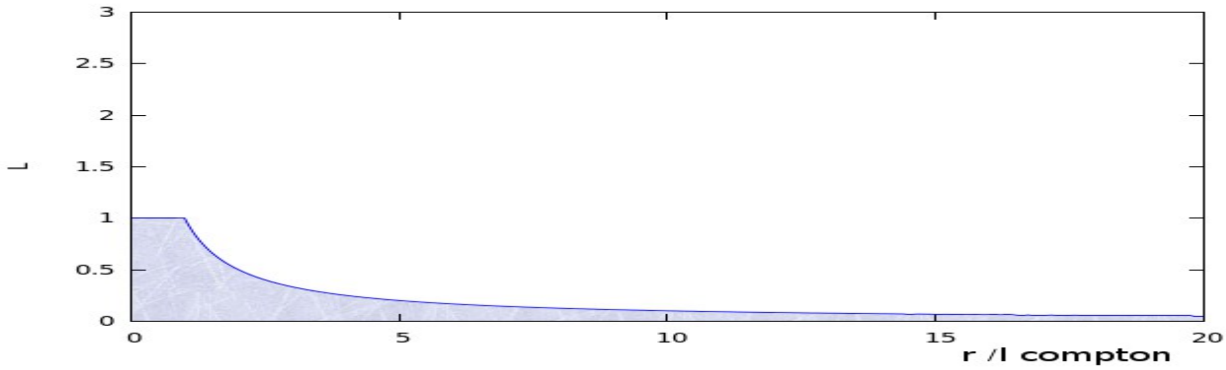


Fig 14. Solution vs radius/ Comptonwavelength .

Gravitomagnetics pulsations in six dimensions are interpreted as sources of gravitational and electric field in a four-dimensional space. This is due to the restrictions imposed by the topology of space

CASE B. PARTICLE-PULSATION IN UNIFORM MOTION.

$$\frac{\nabla^2_{3D} F}{F} + \beta^2 = 0$$

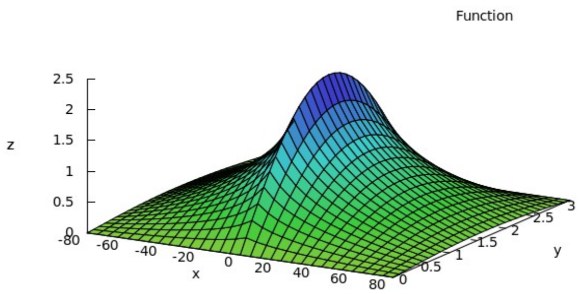
If the particle is in uniform motion along the axis Z the following solution is proposed:

$$If\ \sqrt{x^2+y^2} \leqslant \xi_0 ----------\rightarrow F = C_4 Sen(\beta z)$$

$$If\ \sqrt{x^2+y^2} > \xi_0 ----------\rightarrow F = C_5 Sen(\beta z) \cdot \log C_6 \sqrt{x^2+y^2}$$

That is, the product of a plane wave by a two-dimensional potential in the perpendicular plane to the movement.

We can plot it .
Z represents the amplitude:



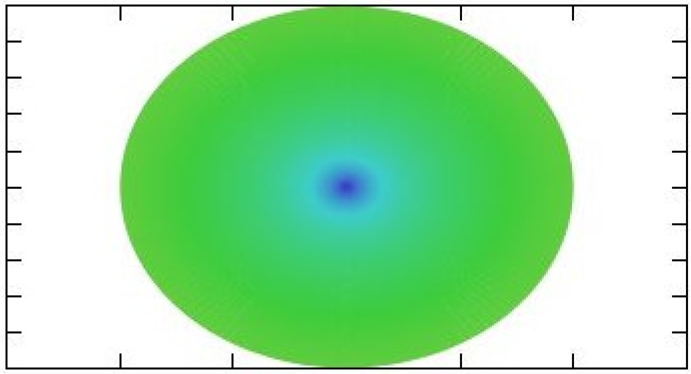


Fig 15 Frontal solution for a free particle-pulsation with uniform motion..

In front view electron appears to be a source of electric and gravitatory field. But transversely view it appears to be a wave.

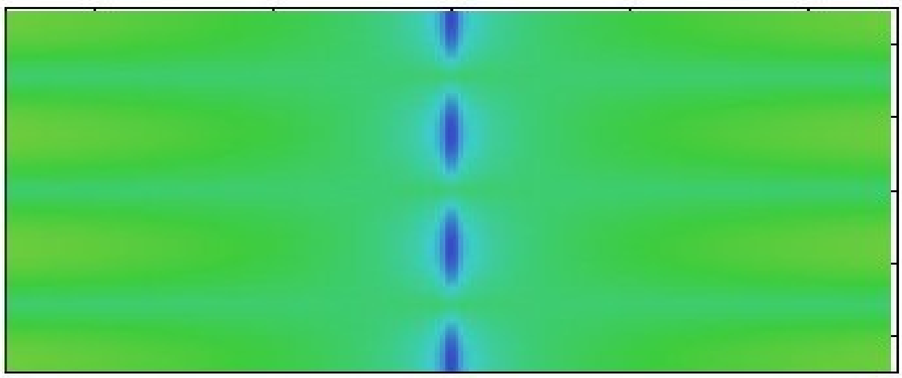
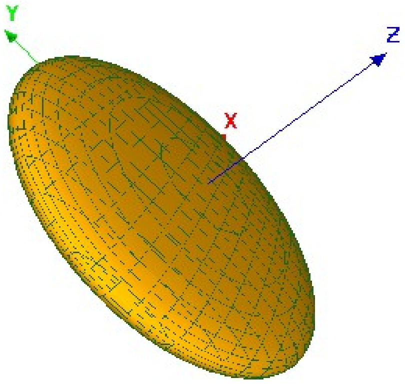


Fig 16. Transversal solution for a free particle-pulsation with uniform motion.



Electron's shape would be then:
Fig 17. Isosurface.
This is an orbital that represents a free electron with an uniform motion along z axis. We just plot an isosurface. The electron extends to infinity in the XY plane, but is compressed by the effect of movement in the Z axis

6.Discussion. Physical meaning of quantum mechanics

6.1 Particle concept. Origin of inertia.

The solution of the gravitomagnetic wave equation for a free pulsation seems to be particle, because it is a source of gravitational and electric field, but seen transversely fully justifies its wavelike behavior. (D'Broglie hypothesis).

Thus, considering the electrons as gravitomagnetics pulsations can we explain:

- The double-slit experiment, in which each electron interferes with itself effectively.
- Aharonov-Bohm effect, in which an electron is affected by a solenoid, despite being confined to a region in which magnetic field is zero. This is explained by considering that part of the particle-pulsation in fact goes through the solenoid.
- The fact that the electron looks like a dimensionless object (point object) without any internal structure.

Furthermore, this leads to the absence of action at a distance. The electric or gravitational field of electrons are perceived because we are inside electrons.

For determining origin of inertia is very interesting to observe the propagation of electromagnetic waves in a waveguide as those used to transmit electromagnetic signals. Waves whose frequency is less than a minimum frequency, called the cut-off frequency, are not transmitted. Among the other waves, the higher the frequency, the higher velocity.
The propagation mode of a wave with a frequency ω in a waveguide with a cut-off frequency ω_0 is:

$$\beta=\frac{1}{c}\sqrt{\omega^2-\omega_0^2}$$

Transmission rate of information and energy within a waveguide is represented by the group velocity, which is defined as the derivative of the frequency respect to propagation mode dω/dk.

Taking derivatives with respect to ω :

$$\frac{d\beta}{d\omega}=\frac{\omega}{c\sqrt{\omega^2-\omega_0^2}}$$

so group velocity of a wave with a frequency ω in a waveguide with a cut-off frequency ω_0 is:

$$v_g=\frac{d\omega}{d\beta}=\frac{c\sqrt{\omega^2-\omega_0^2}}{\omega}=c\sqrt{1-\left(\frac{\omega_0}{\omega}\right)^2}$$

and reordering:

$$\left(\frac{v_g}{c}\right)^2=1-\left(\frac{\omega_0}{\omega}\right)^2 \quad\rightarrow\quad \frac{\omega_0}{\omega}=\sqrt{1-\left(\frac{v_g}{c}\right)^2} \quad\rightarrow\quad \frac{\omega}{\omega_0}=\frac{1}{\sqrt{1-\left(\frac{v_g}{c}\right)^2}}$$ and thus:

$$\omega=\frac{\omega_0}{\sqrt{1-\left(\frac{v_g}{c}\right)^2}}$$

We can multiply by Plank's constant h:

$$\omega h=\frac{\omega_0 h}{\sqrt{1-\left(\frac{v_g}{c}\right)^2}}$$ and if we remember that $E=h\omega$

then: $$E=\frac{E_0}{\sqrt{1-\left(\frac{v_g}{c}\right)^2}}$$

So an electromagnetic wave acquires the same properties as a material particle when is guided by a metal frame or other boundary conditions, such as glass fiber.
Onwards we assume that there are a kind of gravitomagnetic waves which can interact between them by interchanging energy. These waves should be the elemental particles, such electrons. Because of this assumption, integration constant shouldn't be normalized to 1, but should be normalized to adjust energy flux of elementary particles.

6.2 Klein-Gordon Equation. D'Broglie wavelength.

Gravitomagnetic wave equation in 6D is:

$$(\nabla_{6D}^2+k^2)\cdot H=0$$

As $k^2=\left(\frac{m_0 c}{\hbar}i\right)^2+\beta^2$ then: $\rightarrow$ $\boxed{\left[\nabla_{6D}^2+\left(\frac{m_0 c}{\hbar}i\right)^2+\beta^2\right]\cdot H=0}$ (a)

Group velocity will be: $v_g = \frac{c^2 \beta}{2\pi f_0} \rightarrow \beta = \frac{v_g \cdot 2\pi f_0}{c^2}$

and $\omega = 2\pi f_0$ so we can write:

$$\beta = \frac{v_g \cdot \omega}{c^2} = \left(\frac{v_g}{c}\right) \cdot \left(\frac{\omega}{c}\right)$$

As $k = \frac{\omega}{c}$ and substituting in (a)

$$\left(\frac{\omega}{c}\right)^2 = \left(\frac{m_0 c}{\hbar} i\right)^2 + \left(\frac{v_g}{c}\right)^2 \cdot \left(\frac{\omega}{c}\right)^2$$

$$\left(\frac{\omega}{c}\right)^2 \left[1 - \left(\frac{v_g}{c}\right)^2\right] = \left(\frac{m_0 c}{\hbar} i\right)^2 \text{ , as } \frac{v_g}{c} = \varepsilon \text{ then } \left(\frac{\omega}{c}\right)^2 \left[1 - \varepsilon^2\right] = \left(\frac{m_0 c}{\hbar} i\right)^2$$

as we postulated that k is imaginary:

$$k = \frac{m_0 c}{\hbar} \cdot \frac{1}{\sqrt{1-\varepsilon^2}} \cdot i \text{ , and } m = \frac{m_0}{\sqrt{1-\varepsilon^2}}$$

finally substituting in the wave equation:

$$\boxed{\left(\nabla^2_{6D} + \left(\frac{m c}{\hbar} i\right)^2\right) \cdot H = 0}$$ similar to independent time Klein-Gordon equation.

This equation should be solved for six dimensions, not for four dimension. Because of this fact, this equation failed when applied to hydrogen atom.

If we multiply and divide by c: $\boxed{k = \frac{1}{\hbar c} \cdot \frac{m_0 c^2}{\sqrt{1-\varepsilon^2}} \cdot i = \frac{Energy_{wave}}{\hbar c} \cdot i}$

This equation allows to solve the wave equation of electron in a force field.

On the other hand:

$$k = \frac{m_0 c}{\hbar} \cdot \frac{1}{\sqrt{1-\varepsilon^2}} \cdot i \text{ and considering that } k^2 = \left(\frac{m_0 c}{\hbar} i\right)^2 + \beta^2$$

We can write $\left(\frac{m_0 c}{\hbar} i\right)^2 \cdot \frac{1}{1-\varepsilon^2} = \left(\frac{m_0 c}{\hbar} i\right)^2 + \beta^2$

$$\left(\frac{m_0 c}{\hbar} i\right)^2 \cdot \left[\left(\frac{1}{1-\varepsilon^2}\right) - 1\right] = \beta^2 \quad \text{Thus} \quad \beta^2 = \left(\frac{m_0 c}{\hbar} i\right)^2 \cdot \left[\frac{1-1+\varepsilon^2}{1-\varepsilon^2}\right] = \left(\frac{m_0 c}{\hbar} i\right)^2 \cdot \left[\frac{\varepsilon^2}{1-\varepsilon^2}\right]$$

then:

$$\beta = \frac{m_0 c}{\hbar} \cdot \left[\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\right] i \quad \text{and as} \quad \frac{v_g}{c} = \varepsilon \quad \text{we finally obtain:}$$

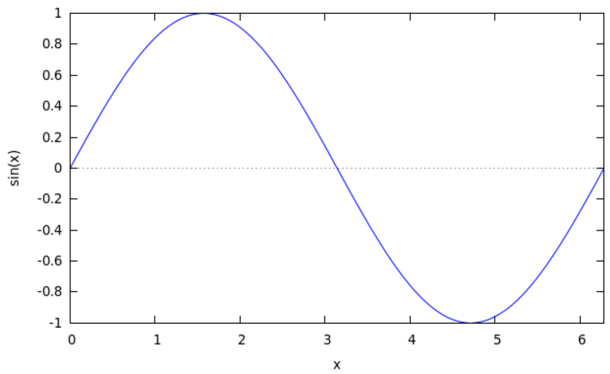
$$\beta = \frac{m_0 v_g}{\hbar} \cdot \left[\frac{1}{\sqrt{1-\varepsilon^2}}\right] i$$

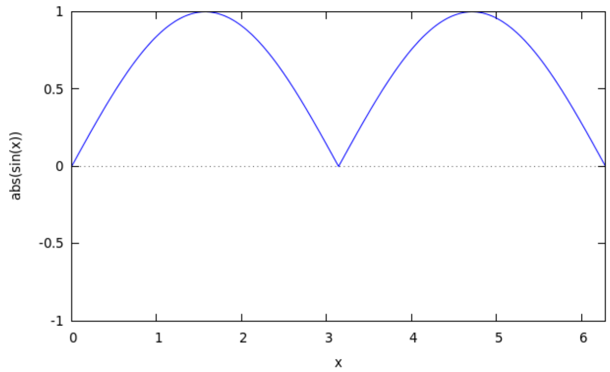
Wavelength associated to the propagation mode will be:

$$\beta = \frac{\pi}{\lambda} \rightarrow \rightarrow \lambda = \frac{\pi}{\beta} = \frac{\pi \hbar \cdot \sqrt{1-\varepsilon^2}}{m_0 v_g} i = \frac{\pi h \cdot \sqrt{1-\varepsilon^2}}{2 \pi m_0 v_g} i$$

and finally:

$$\boxed{\lambda = \frac{h \cdot \sqrt{1-\varepsilon^2}}{2 m_0 v_g} i} \quad \text{that is, a semi D'Broglie wavelength.}$$





7. Application of gravitomagnetic wave equation to the hydrogen atom.

7.1 Wave equation of the hydrogen atom.

If we use a spherical coordinate system for the extended dimensions and a circular coordinate system for the compacted dimensions the wave equation would be:

$$(\nabla_{6D}^2 + k^2)\cdot H = 0 \quad \text{where} \quad k = \frac{E_{onda}}{\hbar c} i$$

Total energy of the pulsation was formed by:

- Rest energy: $E_0 = mc^2$
- Kinetic energy: E_c

As kinetic energy is unknown we can obtain it as mechanical energy minus electric potential energy:

- Mechanical energy: E_m
- Electric potential energy. If we assume that because of its large mass relative to the electron proton remains motionless then

$$E_{ELEC} = \frac{-1}{4\pi\varepsilon_0}\frac{Qq}{r} = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

thus:

$$k = \frac{E_{onda}}{\hbar c} i = \frac{mc^2 + E_m - \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}}{\hbar c} i = \left(\frac{mc^2 + E_m}{\hbar c} - \frac{e^2}{\hbar c\, 4\pi\varepsilon_0}\frac{1}{r}\right) i \quad \text{and if we define} \quad \alpha = \frac{e^2}{\hbar c\, 4\pi\varepsilon_0}$$

we can write:

$$\nabla_{6D}^2 H - \left(\frac{mc^2 + E_m}{\hbar c} - \frac{\alpha}{r}\right)^2 H = 0$$

By operating:

$$\nabla^2 H - \left(\frac{m^2c^4}{\hbar^2 c^2} + \frac{E_m^2}{\hbar^2 c^2} + \frac{2E_m mc^2}{\hbar^2 c^2} + \frac{\alpha^2}{r^2} - \frac{2\mathrm{mc}^2\alpha}{\hbar c r} - \frac{2E_m\alpha}{\hbar c r}\right) H = 0$$

and:

$$\nabla^2 H - \left(\frac{mc}{\hbar}\right)^2 H - \left(\frac{E_m}{\hbar c}\right)^2 H - \frac{2\text{mc}^2}{\hbar c}\left(\frac{E_c}{\hbar c} - \frac{\alpha}{r}\right) H - \left(\frac{\alpha^2}{r^2} - \frac{2 E_m \alpha}{\hbar c r}\right) H = 0$$

It can be solved by variables separation

$$H(\xi, \eta, x, y, z) = \Phi(\xi, \eta) \cdot \Psi(r, \theta, \phi)$$

Laplacians are separable:

$$\nabla^2_{\xi,\eta} \Phi - \left(\frac{mc}{\hbar}\right)^2 \Phi = 0 \quad \text{Equal to free particle case} \quad \text{(I)}$$

$$\nabla^2_{r,\theta,\phi} \Psi - \left[\left(\frac{E_m}{\hbar c}\right)^2 - \frac{2 E_m \alpha}{\hbar c r}\right] \Psi - \frac{2\text{mc}^2}{\hbar c}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right) \Psi - \left(\frac{\alpha^2}{r^2}\right) \Psi = 0 \qquad \text{(II)}$$

Taking out common factor in (II)

$$\boxed{\nabla^2 \Psi - \frac{E_m}{\hbar c}\left[\frac{E_m}{\hbar c} - \frac{2\alpha}{r}\right] \Psi - \frac{2\text{mc}^2}{\hbar c}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right) \Psi - \left(\frac{\alpha^2}{r^2}\right) \Psi = 0}$$

In non relativistic case $mc^2 >>> E_m$ and term $\frac{E_m}{\hbar c}\left[\frac{E_m}{\hbar c} - \frac{2\alpha}{r}\right] \Psi$ is negligible compared to

$\frac{2\text{mc}^2}{\hbar c}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right) \Psi$ and therefore:

$$\boxed{\nabla^2 \Psi - \frac{2\text{m}_0 c}{\hbar}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right) \Psi - \left(\frac{\alpha^2}{r^2}\right) \Psi = 0} \quad \text{(III)}$$

7.2 Schrödinger's equation.

We can write Schrödinger's equation:

$$\frac{i}{c}\cdot\frac{\partial}{\partial t}\Psi = \frac{-1}{2}\frac{\hbar}{m_0 c}\nabla^2\Psi - \frac{\alpha}{r}\Psi$$

As $\frac{i}{c}\cdot\frac{\partial}{\partial t}\Psi = \frac{E}{\hbar c}\Psi$ then:

$\left(\frac{E}{\hbar c} - \frac{\alpha}{r}\right)\Psi = -\left(\frac{1}{2}\frac{\hbar}{m_0 c}\right)\nabla^2\Psi$ and operating:

$\boxed{\nabla^2\Psi + \frac{2\mathrm{m}_0 c}{\hbar}\left(\frac{E}{\hbar c} - \frac{\alpha}{r}\right)\Psi = 0}$ this is very similar to (III)

7.3 Solving the wave equation for the extended dimensions. Non relativistic case.

We can apply laplacian in spherical coordinates to (III):

$$\boxed{\nabla^2\Psi - \frac{2\mathrm{m}_0 c}{\hbar}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right)\Psi - \left(\frac{\alpha^2}{r^2}\right)\Psi = 0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\,Sen\theta}\frac{\partial}{\partial\theta}\left(Sen\,\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\,Sen^2\,\theta}\frac{\partial^2\Psi}{\partial\phi} - \frac{2\mathrm{m}_0 c}{\hbar}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right)\Psi - \left(\frac{\alpha^2}{r^2}\right)\Psi = 0$$

If we assume that $\Psi(r,\theta,\phi) = R(r)P(\theta)T(\phi)$ then:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 R' PT\right) + \frac{1}{r^2\,Sen\,\theta}\frac{\partial}{\partial\theta}\left(Sen\,\theta\, RP' T\right) + \frac{1}{r^2\,Sen^2\,\theta}RPT'' - \frac{2\mathrm{m}_0 c}{\hbar}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right)RPT - \left(\frac{\alpha^2}{r^2}\right)RPT = 0$$

we can multiply by $\frac{r^2\,Sen^2\,\theta}{RPT}$

$$\frac{Sen^2\theta}{R}\frac{d}{dr}\left(r^2 R'\right) + \frac{Sen\,\theta}{P}\frac{d}{d\,\theta}\left(Sen\,\theta\, P'\right) + \frac{T''}{T} - r^2\,Sen^2\,\theta\frac{2\mathrm{m}_0 c}{\hbar}\left(\frac{E_m}{\hbar c} - \frac{\alpha}{r}\right) - r^2\,Sen^2\,\theta\left(\frac{\alpha^2}{r^2}\right) = 0$$

As we have a term that only depends on φ and the addition must be constant then:

$$\frac{T''}{T}=cte=-m_l^2$$

and the solution is: $T(\phi)=C_4 e^{-i m_l \phi}$ with m_l half-integer.

Substituting and dividing by $Sen^2\theta$

$$\frac{1}{R}\frac{d}{dr}\left(r^2 R'\right)+\frac{1}{P\,Sen\theta}\frac{d}{d\theta}\left(Sen\theta\,P'\right)-\frac{m_l^2}{Sen^2\theta}-r^2\frac{2m_0 c}{\hbar}\left(\frac{E_m}{\hbar c}-\frac{\alpha}{r}\right)-\alpha^2=0$$

Now we have separated the variables.

$$\frac{1}{R}\frac{d}{dr}\left(r^2 R'\right)-r^2\frac{2m_0 c}{\hbar}\left(\frac{E_m}{\hbar c}-\frac{\alpha}{r}\right)-\alpha^2=l(l+1)$$

$$\frac{1}{P\,Sen\theta}\frac{d}{d\theta}\left(Sen\theta\,P'\right)-\frac{m_l^2}{Sen^2\theta}=-l(l+1)$$

As $\alpha^2 \ll l(l+1)$ we can write:

$$\frac{1}{R}\frac{d}{dr}\left(r^2 R'\right)-r^2\frac{2m_0 c}{\hbar}\left(\frac{E_m}{\hbar c}-\frac{\alpha}{r}\right)=l(l+1) \quad \text{(a)}$$

$$\frac{1}{P\,Sen\theta}\frac{d}{d\theta}\left(Sen\theta\,P'\right)-\frac{m_l^2}{Sen^2\theta}=-l(l+1) \quad \text{(b)}$$

$$\frac{T''}{T}=-m_l^2 \quad \text{(c)}$$

Equation (b) is the associated Legendre function, and with equation (c) allow to obtain spherical harmonics solutions. Boundary conditions are l=0,1,2,.... and $0 \leq |\, m_l \,| \leq 1$

Mathematically m_l can be a half-integer, but Legendre polynomial of half-integer order have a singularity in θ=1, so they can not have physical meaning.

We are going to work in equation (a) in order to obtain energy levels:

If we apply the rule chain

$$2rR'+r^2R''-r^2\left(\frac{2m_0 c^2 E_m}{(\hbar c)^2}-\frac{2m_0 c^2\alpha}{\hbar c\, r}\right)R=l(l+1)R$$

we define the function

$$u(r)=rR$$

$$u'(r)=rR'+R$$

$$u''(r) = R' + rR'' + R' = 2\mathrm{R}' + rR''$$

so we can write the above equation in a simplified way:

$$2\mathrm{rR}' + r^2 R'' = r(2\mathrm{R}' + rR'') = ru''$$

and thus:

$$u'' - \left(\frac{2\mathrm{m}_0 c^2 E_m}{(\hbar c)^2} - \frac{2\mathrm{m}_0 c^2 \alpha}{\hbar c r} + \frac{l(l+1)}{r^2} \right) u = 0$$

If $r \to \infty$ we can write:

$$u'' - \left(\frac{2\mathrm{m}_0 c^2 E_m}{(\hbar c)^2} \right) u = 0$$

we can define

$$\beta^2 = \frac{2\, mc^2 E_c}{(\hbar c)^2}$$

then:

$$u'' - \left(\beta^2 - \frac{2\mathrm{m}_0 c\, \alpha}{\hbar r} + \frac{l(l+1)}{r^2} \right) u = 0$$

Dividing by β^2:

$$\frac{u''}{\beta^2} - \left(1 - \frac{2\mathrm{m}_0 c \alpha}{\beta^2 \hbar r} + \frac{l(l+1)}{\beta^2 r^2} \right) u = 0$$

As r always multiply to β we can make next variable change $\rho = \beta r$ and define the function $U(\rho)$:

$$U'' - \left(1 - \frac{2\mathrm{m}_0 c \alpha}{\beta \hbar \rho} + \frac{l(l+1)}{\rho^2} \right) U = 0$$

we define $\rho_0 = \dfrac{2\mathrm{m}_0 c\, \alpha}{\beta \hbar}$

then

$$U'' - \left(1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2} \right) U = 0 \quad \text{(d)}$$

Equation (d) is found in a very similar way when radial Schrödinger's equation is solved for the hydrogen atom. It can be solved by asymptotic study and series expansions. The condition need is that for at least any value of j the next equation would be true:

$\boxed{2(j+l+1)=\rho_0}$ where j is an integer.

And defining $n=j+l+1$:

$\boxed{2n=\rho_0}$

If we remember:

$$\beta^2=\frac{2mc^2E_m}{(\hbar c)^2} \quad \text{y} \quad \rho_0=\frac{2m_0 c\alpha}{\beta\hbar}$$

then:

$$\frac{2mc\alpha}{\hbar\frac{\sqrt{2mE_m}}{\hbar}}=2n \quad \rightarrow \quad \frac{mc\alpha}{\sqrt{2mE_m}}=n$$

and squaring:

$\frac{mc^2\alpha^2}{2E_m}=n^2$ we can select the negative solution → $\boxed{E_m=\frac{-mc^2\alpha^2}{2n^2}}$

therefore it provides the same energy levels that Schrodinger's equation.

In fact equation (a) is a variation of associated Laguerre function with the same solutions. Final solution would be :

$H(\xi,\eta,x,y,z,t)=\Phi(\xi,\eta)\cdot\Psi(r,\theta,\varphi)\cdot e^{-\omega t i}$, and we can obtain angular momentums:

- Orbital angular momentum $L=\sqrt{l(l+1)}\cdot\hbar$
- Z component of orbital angular momentum $L_z=m_l\hbar$
- Spin angular momentum $L_s=m_s\hbar=\frac{\pm 1}{2}\hbar$

7.4 Solving the wave equation for the extended dimensions. Relativistic case.

The wave equation for the extended dimensions is:

$$\boxed{\nabla^2\Psi-\frac{E_m}{\hbar c}\left[\frac{E_m}{\hbar c}-\frac{2\alpha}{r}\right]\Psi-\frac{2mc^2}{\hbar c}\left(\frac{E_m}{\hbar c}-\frac{\alpha}{r}\right)\Psi-\left(\frac{\alpha^2}{r^2}\right)\Psi=0}$$

Taking out common factor and operating:

$$\boxed{\nabla^2\Psi-\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]\Psi-\left(\frac{\alpha^2}{r^2}\right)\Psi=0}$$

we can apply laplacian in spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{1}{r^2\,Sen\theta}\frac{\partial}{\partial\theta}\left(Sen\,\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{r^2\,Sen^2\theta}\frac{\partial^2\Psi}{\partial\phi}-\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]\Psi-\left(\frac{\alpha^2}{r^2}\right)\Psi=0$$

and defining $\Psi(r,\theta,\varphi)=R(r)P(\theta)T(\varphi)$ then:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2R'PT\right)+\frac{1}{r^2\,Sen\theta}\frac{\partial}{\partial\theta}\left(Sen\,\theta\,RP'T\right)+\frac{1}{r^2\,Sen^2\theta}RPT''-\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]RPT-\left(\frac{\alpha^2}{r^2}\right)RPT=0$$

If we multiply by $\dfrac{r^2\,Sen^2\theta}{RPT}$

$$\frac{Sen^2\theta}{R}\frac{d}{dr}\left(r^2R'\right)+\frac{Sen\,\theta}{P}\frac{d}{d\theta}\left(Sen\,\theta\,P'\right)+\frac{T''}{T}-r^2\,Sen^2\theta\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]-r^2\,Sen^2\theta\left(\frac{\alpha^2}{r^2}\right)=0$$

As we have a term that only depends on φ and the addition must be constant then:

$$\frac{T''}{T}=cte=-m_l^2$$

and the solution is: $T(\varphi)=C_4\mathrm{e}^{-im_l\varphi}$ with m_l half-integer.

Substituting and dividing by $Sen^2\theta$

$$\frac{1}{R}\frac{d}{dr}\left(r^2R'\right)+\frac{1}{P\,Sen\theta}\frac{d}{d\theta}\left(Sen\theta\,P'\right)-\frac{m_l^2}{Sen^2\theta}-r^2\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]-\alpha^2=0$$

Now we have separated the variables.

$$\frac{1}{R}\frac{d}{dr}\left(r^2R'\right)-r^2\left[\frac{E_m^2+2mc^2E_m}{(\hbar c)^2}-\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\hbar c r}\right]=l'(l'+1)$$

$$\frac{1}{P\,Sen\,\theta}\frac{d}{d\theta}\left(Sen\,\theta\,P'\right)-\frac{m_l^2}{Sen^2\theta}-\alpha^2=-l'(l'+1)$$

If we define $\alpha^2 - l'(l'+1) = -l(l+1)$:

$$\frac{1}{R}\frac{d}{dr}\left(r^2 R'\right) - r^2\left[\frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2} - \frac{\left(2E_m + 2\mathrm{mc}^2\right)\alpha}{\hbar c r}\right] = l'(l'+1) \quad \text{(a')}$$

$$\frac{1}{P\, Sen\theta}\frac{d}{d\theta}\left(Sen\theta\, P'\right) - \frac{m_l^2}{Sen^2\theta} = -l(l+1) \quad \text{(b')}$$

$$\frac{T''}{T} = cte = -m_l^2 \quad \text{(c')}$$

Second equations only have solution if l is a positive integer, so we can obtain l' values according l values.

$$\alpha^2 - l' - l'^2 = -l(l+1) \quad \rightarrow \quad l'^2 + l' - \alpha^2 - l(l+1) = 0$$

This second grade equation was solved for some l values:

l	**l'**
0	$-5{,}3254190509\text{x}10^{-5}$
	-0,9999467485
1	0,9999822494
	-1,9999822494
2	1,9999893497
	-2,9999893497
3	2,9999923927
	-3,99.....

The solution with physical meaning should be the first, thus:

l	l'	δ=l-l'
0	-5,3254190509x10^{-5}	5,325419051x10^{-5}
1	0,9999822494	1,775055653x10^{-5}
2	1,9999893497	1,065029359x10^{-5}
3	2,9999923927	7,607344624x10^{-6}
4	3,9999940832	5,916821056x10^{-6}
5	4,999995159	4,841x10^{-6}
6	5,9999959037	4,096259329x10^{-6}
7	6,9999964499	3,55009114x10^{-6}

Now we can solve the equation (a'):

$$\frac{1}{R}\frac{d}{dr}\left(r^2 R'\right)-r^2\left[\frac{E_m^2+2\,mc^2 E_m}{\left(\hbar c\right)^2}-\frac{\left(2\,E_m+2\mathrm{mc}^2\right)\alpha}{\hbar c\,r}\right]=l'\left(l'+1\right)$$

we can apply the rule chain and multiply by R:

$$2\mathrm{rR}'+r^2 R''-r^2\left[\frac{E_m^2+2\,mc^2 E_m}{\left(\hbar c\right)^2}-\frac{\left(2\,E_m+2\mathrm{mc}^2\right)\alpha}{\hbar c\,r}\right]R=l'\left(l'+1\right)R$$

We can define:

$$u(r)=rR$$

$$u'(r)=rR'+R$$

$$u''(r)=R'+rR''+R'=2\mathrm{R}'+rR''$$

so we can write it in a simplified way:

$$2\mathrm{rR}'+r^2 R''=r\left(2\mathrm{R}'+rR''\right)=ru''$$

and thus:

$$r u'' - r^2 \left[\frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2} - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\hbar c r} \right] \frac{u}{r} = l'(l'+1)\frac{u}{r}$$

operating:

$$u'' - r \left[\frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2} - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\hbar c r} \right] u = l'(l'+1)\frac{u}{r}$$

Dividing by r:

$$u'' - \left[\frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2} - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\hbar c r} + \frac{l'(l'+1)}{r^2} \right] u = 0$$

When r → ∞ we can write:

$$u'' - \left[\frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2} \right] u = 0$$

and defining

$$\beta^2 = \frac{E_m^2 + 2mc^2 E_m}{(\hbar c)^2}$$

we can write:

$$u'' - \left[\beta^2 - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\hbar c r} + \frac{l'(l'+1)}{r^2} \right] u = 0$$

Dividing by β^2 :

$$\frac{u''}{\beta^2} - \left[1 - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\beta^2 \hbar c r} + \frac{l'(l'+1)}{\beta^2 r^2} \right] u = 0$$

As r always multiply to β we can make next variable change ρ = βr and define the function U(ρ):

$$U'' - \left[1 - \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\beta \hbar c \rho} + \frac{l'(l'+1)}{\rho^2} \right] U = 0$$

If we define $\rho_0 = \frac{(2E_m + 2\mathrm{mc}^2)\alpha}{\beta \hbar c}$ we can write:

$$\boxed{U'' - \left[1 - \frac{\rho_0}{\rho} + \frac{l'(l'+1)}{\rho^2} \right] U = 0} \quad \text{(d')}$$

Equation (d') is found in a very similar way when radial Schrödinger's equation is solved for the hydrogen atom. It can be solved by asymptotic study and series expansions. The condition need is that for at least any value of j the next equation would be true

$\boxed{2(j+l'+1)=\rho_0}$ where j is an integer. If we write l' depending of l:

$2(j+l-\delta(l)+1)=\rho_0$ and defining $n=j+l+1$:

$2(n-\delta(l))=\rho_0$ and $n'(l)=n-\delta(l)$ the condition must be :

$$\boxed{2n'(l)=\rho_0}$$

If we remember

$$\beta^2=\frac{E_m^2+2mc^2E_m}{(\hbar c)^2} \quad \text{and} \quad \rho_0=\frac{(2E_m+2\mathrm{mc}^2)\alpha}{\beta\hbar c}$$

we can obtain the energy levels of electrons in an Hydrogen atom. We have to consider that mechanical energy is negative an therefore the square root is imaginary:

$$n'(l)=\frac{(E_m+mc^2)\alpha}{i\sqrt{E_m^2+2mc^2E_m}}$$

and squaring:

$$n'^2=\frac{-(E_m+mc^2)^2\alpha^2}{E_m^2+2mc^2E_m}$$

operating

$$n'^2E_m^2+2n'^2mc^2E_m=-(E_m\alpha+\alpha mc^2)^2$$

so

$$n'^2E_m^2+2n'^2mc^2E_m+E_m^2\alpha^2+\alpha^2(mc^2)^2+2E_m\alpha^2mc^2=0$$

and finally:

$$(n'^2+\alpha^2)E_m^2+2mc^2(n'^2+\alpha^2)E_m+\alpha^2(mc^2)^2=0$$

A second grade equation in E_m of the kind $ax^2+bx+c=0$

where

$$a=n'^2+\alpha^2$$

$$b=2mc^2(n'^2+\alpha^2)$$

$$c=\alpha^2(mc^2)^2$$

And the solution will be:

$$\boxed{E_m = -mc^2\left[1 \pm \sqrt{\frac{\alpha^2}{n'^2 + \alpha^2}}\right]}$$

Second solution agrees numerically with the first order relativistic correction to Schrödinger equation.

$$E = -\frac{\alpha^2}{n^2}\left(\frac{3}{4\mathrm{n}} - \frac{1}{l + \frac{1}{2}}\right)$$

Numerical solutions are shown in the next page:

Above solutions do not agree with the fine structure in a quantitative way, neither agree with hyperfine structure in a qualitative way because we did not include the magnetics terms, but this approximation is enough to show that both formulations are equivalent.

l	l'	l'-l	µ	c	α	n	l	n'(l)	En eV	E (n') eV	E(n,l) eV	Dif. diezmilesimas
0	-5,3282E-005	-5,3254E-005	9,10E-031	299792458	0,0072973526	1	0	0,9999467458	-13,5982875367	-13,599192797	-13,5991926957	-0,001012656
1	0,9999822401	-1,7751E-005										
2	1,9999893441	-1,0650E-005			0,0072973526	2	0	1,9999467458	-3,3995718842	-3,3997189861	-3,3997189725	-0,0001362814
3	2,9999923887	-7,6073E-006			0,0072973526	2	1	1,9999822494	-3,3995718842	-3,399598285	-3,3995982846	-3,21175530615E-006
4	3,9999940801	-5,9168E-006										
5	4,9999951564	-0,000004841			0,0072973526	3	0	2,9999467458	-1,5109208374	-1,5109677754	-1,5109677716	-3,86328924407E-005
6	5,9999959016	-4,0963E-006			0,0072973526	3	1	2,9999822494	-1,5109208374	-1,5109320123	-1,5109320122	-1,28389521237E-006
7	6,999996448	-3,5501E-006			0,0072973526	3	2	2,9999893497	-1,5109208374	-1,5109248604	-1,5109248603	-2,78053136071E-007
					0,0072973526	4	0	3,9999467458	-0,849892971	-0,8499134801	-0,8499134786	-1,57074475649E-005
					0,0072973526	4	1	3,9999822494	-0,849892971	-0,8498983926	-0,8498983926	-4,91273688397E-007
					0,0072973526	4	2	3,9999893497	-0,849892971	-0,8498953754	-0,8498953754	5,70987701565E-008
					0,0072973526	4	3	3,9999923927	-0,849892971	-0,8498940823	-0,8498940823	-3,18922666054E-008
					0,0072973526	5	0	4,9999467458	-0,5439315015	-0,5439422193	-0,5439422186	-7,64989516178E-006
					0,0072973526	5	1	4,9999822494	-0,5439315015	-0,5439344945	-0,5439344945	-1,67669211848E-007
					0,0072973526	5	2	4,9999893497	-0,5439315015	-0,5439329497	-0,5439329497	8,13449307913E-008
					0,0072973526	5	3	4,9999923927	-0,5439315015	-0,5439322877	-0,5439322877	-5,49404965966E-008
					0,0072973526	5	4	4,9999940832	-0,5439315015	-0,5439319198	-0,5439319199	5,83488812822E-008

me	mp
9,11E-031	1,67E-027

$$E_n = -\frac{mc^2 \alpha^2}{2}$$

$$E_{n'} = -mc^2 \left[1 \pm \sqrt{\frac{\alpha^2}{n'^2 + \alpha^2}} \right]$$

$$E(n, l) = -\frac{mc^2 \alpha^2}{2} - \frac{\alpha^2}{n^2} \left(\frac{3}{4\mathrm{n}} - \frac{1}{l + \frac{1}{2}} \right)$$

Conclusions

Despite the great successes of physics in the last 80 years, the truth is that we have two great theories to describe reality that are incompatible with each other (General Relativity Theory and Quantum Mechanics). These incompatibilities have increased with the course of time and and they have led to the development of theories and hypotheses that either are based on the accumulation of free parameters without theoretical foundation (mass, charge, spin, .. standard model) or postulates facts impossible to verify (String Theories,. ..). All these attempts to reconcile the smallest with the biggest share inability to make basic predictions. It is therefore imperative to carry out a deep revision of the principles on which we base physics today.

Seeking alternative interpretations to the special relativity and specifically to the equation that links the energy of a body in motion with its speed it was found that perhaps this theory implicitly involves the displacement at the speed of light in at least one additional dimension. (This assumption was developed independently by Alcerro and his work is previous to this paper)

From this point of view geometrization of time postulated by Minkonswki would be a misinterpretation and it could be replaced with advantage by assuminig the displacement at the speed of light of all the particles in a new Kaluza-like dimension. The subsequent adoption of the postulates of Klein about the size and topology of the Kaluza dimension along with the experimental isotropy of the universe would force to postulate the existence of at least one extra dimension, which would lead to the existence of a plane of compacted dimensions in which the particles in apparent rest would move in closed paths with the speed of light.

Every particle at rest is possessed of a certain energy, and the two theories give us different formulations. On the one hand the GR tells us that it takes the value of E = mc², while the expression of the residual vibrational energy of a quantum oscillator is $E_r = \frac{h \cdot \upsilon}{2}$. Since both theories have had great success in their respective field of application, why not assume that both are correct? If we consider that both energies should be equal and assuming circular motions it is possible to estimate the radius of the trajectories in the plane of the extra dimensions. This allows us to interpret the mass of elementary particles as the inverse of the radial dimension of the compacted dimensions $\xi_0 = \frac{\bar{h}}{2 m_0 c}$, which in the case of the electron would be 1.93079616 10 $^{-13}$ m.

The combination of this circular motion in the compacted dimensions with any movement in the extended dimensions produce helical paths. The time speed should be regarded as the velocity of the particles in the plane of the compacted dimensions.

A more detailed study of these helical movements provides a coherent explanation to the D'Broglie wavelength and allows us to infer that the uncertainty principle arises from the fact of trying to analyze phenomena occurring in five spatial dimensions as if they had only 3 spatial dimensions.

Einstein's equations in the weak-field approximation can be linearized, allowing to write them in a similar manner to electromagnetism.This formulation is known as gravitomagnetism. Since in the gravitomagnetism two parallel mass currents circulating in the same direction repel each other is possible to find an intuitive explanation to the electrical charge as forces between parallel currents mass.

If we analyze qualitatively the effect that the curvature of space has on physical laws we can see that this curvature acts in a similar way to how a converging lens acts on an image, that is, increases the effects at short distances, while decreases the effects at long distance.

Therefore many physical constants ($\mu_0, G, \varepsilon_0 \ldots$) should approach to unit when we enunciate the laws of nature in the six postulated dimensions. That is, the constants are the consequence of trying to analyze phenomena occurring in five spatial dimensions as if they had only three spatial dimensions.

On the other hand, this analysis allows to interpret the gravitational constant G as the compacted dimensions surface, and we can estimate the dimensions compacted radius $\xi_u \approx \sqrt{\frac{G}{2\pi}} \simeq 3\cdot 10^{-6} m$.

By following these guidelines we can interpret the electrical induction vector as the formulation in 5 dimensions of gravitomagnetic induction vector in 6 dimensions, this allows us to obtain the charge to mass ratio of the electron

$$\frac{q}{m_0^2} = \frac{-8\pi \hat{G}}{\mu_0 \bar{h}} = 1{,}89650465\cdot 10^{41} \quad .$$

The experimental value differs slightly $\frac{e}{m_e^2} = 1{,}93077784\cdot 10^{41}$.

In order to obtain a correct value we just have to take a value of Ĝ = 1,01807176 . We can assume an elliptic form instead circular form of the compacted dimensions. The intrinsic magnetic moment of the electron would be

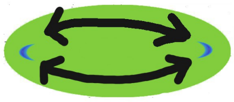
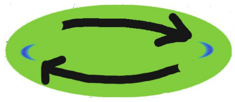
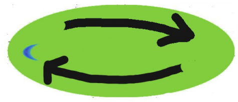
$\mu_g = \frac{-4\pi \hat{G}\, m_0}{\mu_0}$, the value of this expression agrees with Bohr magneton. So we can estimate charge and mass of electron just from its mass.

Having explained the origin of the electric field, magnetism can be obtained from electrical potential across the postulates of relativistic electrodynamics.
If we apply Einstein's equations in the weak field approach to an space as the postulate in this work we obtain solutions in the form of waves. These waves travel in helical paths due to confinement produced by the curvature of the two compacted dimensions. Because of the use of a cylindrical elliptical coordinate system the laplacian is separable. So we can choose a solution composed of the multiplication of two functions, one dependent on the compacted dimensions and another dependent on extended dimensions.

$$H(\xi, \eta, x, y, z) = \Phi(\xi, \eta)\cdot \Psi(r, \theta, \phi) \quad .$$

The solution for the compacted dimensions is an stationary wave in the form of Mathieu functions of half-integer order and negative q parameter. The order of the Mathieu function is interpreted as the quantum property spin.

These standing waves are asymmetrical in its direction of rotation, that is, they are not composed of two equal waves that rotate in opposite directions. This asymmetry causes the appearance of the electromagnetic field due to the forces between parallel mass flows. The direction of rotation marks the sign of the charge (and allows differentation between particles and antiparticles).
Odd spin particles-pulsation also have geometric asymmetry (fermions), while particles with even spin are symmetrical geometrically (bosons with mass). The association of two electrons with opposite spins (Cooper pair) and their similar behavior to bosons demonstrates this idea.

		
Pair particle -antiparticle before their annihilation	Boson with mass	Fermion

If we apply initial assumptions to this waves then cut-off circular wave number would be $k_c = m_0 c / \hbar i$. It has been shown that this relation allows to relate the group velocity of the wave with its frequency, so that the relativistic energy equation of a body is obtained. That is, the confinement of the gravitational wave is the origin of inertia.

It was shown that we should use six dimension Klein-Gordon equation to model particle-pulsation. We have study some solutions in the extended dimensions:

- Rest particle: Wave is a point-like gravitatory and electric field source.

- Uniform movement particle: In front view electron appears to be a source of electric and gravitatory field. But transversely view it appears to be a plane wave with a wavelength equivalent to D'Broglie wavelength. This solution justify dual conception of matter.(wave-particle)

- Hydrogen atom. We can obtain same solutions that Schrödinger equation,both the relativistic case, as the nonrelativistic.

All this leads to postulate that electrons (and possibly the rest of elementary particles) are constituted by gravitational pulses (solitons) guided by the curvature of the compacted dimensions. Therefore they can not be considered as point particles and we should interpret the square of the wave function as the energy flow of the gravitational wave, rejecting the Copenhagen interpretation of quantum mechanics. This solves most of the paradoxical experiments, such as the double slit, for example.
On the other hand, such as standing waves, they modify the propagation medium, so they may interact between them, while the phase differences between the waves introduce the random component. Indeed, the latter provides a simple explanation to tunnel effect

We should review the dual conception wave-particle in favor of just gravitational wave conception with $\hat{G} \simeq 1$, because in this way we find a path to the unification of all forces and provides a unique basis of the two great theories of present physics. It is remarkable to note that the same conclusions can be drawn with other configurations of the compacted dimensions (either in number, size or topology), although the hypothesis developed here is the simplest that the author has been able to find.

In the year of our Lord of 2016

References

[1] Digital Library of Digital Functions . National Institute of Standard and Technology.

[2] The hydrogen atom . Michael Fowler November 2016.

[3] Sobre las dimensiones extras espaciales. César Mora y O. Pedraza 2007

[4] Equivalent photon model of matter. Alcerro Mena

[5] Gravitomagnetism and the speed of gravity. Kopeikin 2008

[6] Gravitomagnetic effects. Schäfer. 2004

[7] Gravitomagnetism and the clock effect. Bahram Mashhoon , Frank Gronwald , and Herbert I.M. Lichtenegger. 1999

[8] The Many Faces of Gravitoelectromagnetism obert T. Jantzen ,Paolo Carini and Donato Bini 2001.

[9] Theory and applications of Mathieu functions . MacLachlan 1947

[10] Mathieu functions, a visual approach ,J. C. Gutierrez-Vegaa) , R. M. Rodrıguez-Dagnino and M. A. Meneses-Nava