F - Theory in a Nutshell

Arka Prabha Banik

arkapb@imsc.res.in

The Institute of Mathematical Sciences , CIT Campus , Chennai , TN , INDIA , 600 113

1 *Introduction with M theory :*

M-Theory is the name for a unified version of string theory, proposed in 1995 by the physicist Edward Witten. At the time of the proposal, there were 5 (three superstrings and two heterotic strings) variations of string theory, but Witten believed that each was a manifestation of a single underlying theory . He and others identified several forms of duality between the theories which, together with certain assumptions about the nature of the universe, could allow for them to all be one single theory: M-Theory. One of the major components of M-Theory is that it required adding yet another dimension on top of the already-numerous extra dimensions of string theory, so that the relationships between the theories could be worked out.

Three confluent developments brought about the so-called M - theory: it turned out that all five SuperString theories were related, and also with the mentioned maximal 11D supergravity; there were also higher dimension extended objects (p-Branes, p > 1, with p = 0 for particles, p = 1 for strings, etc.), and several nonperturbative results were obtained; besides, as the theory had no adjustable parameters, it seemed to be potentially selfsufficient. The hopes for a unique theory were higher than ever; many previously skeptical physicists were converted to the superstring Credo.

In 1995, ten years later, three confluent developments brought about the so-called Mtheory: it turned out that all five SuperString theories were related, and also with the mentioned maximal 11D supergravity(SUGRA); there were also higher dimension extended objects (p-Branes, p > 1, with p = 0 for particles, p = 1 for strings, etc.), and several nonperturbative results were obtained; besides, as the theory had no adjustable parameters, it seemed to be potentially selfsufficient. The hopes for a unique theory were higher than ever; many previously skeptical physicists (including this humble reviewer) were converted to the superstring Credo.

The full set of relations between the different types of Superstring Theory together with 11D Supergravity are schematized in the famous hexagon .

1.1 *Salient features of M - theory :*

Though physicists have still not uncovered the secrets of M-Theory, they have identified several properties that the theory would have:

- *11 dimensions of spacetime*
- *contains strings and branes (originally called membranes)*

- *methods of using compactification to explain how the extra dimensions reduce to the four spacetime dimensions we observe*
- *dualities and identifications within the theory that allow it to reduce to special cases of the string theories known, and ultimately into the physics we observe in our universe*

1.2 *Difficulties with M-theory.*

Inspite of these separate advances , the theory languished , both for lack of new stimulus, as for being unable to complete its deficiencies . Among the unsatisfactory features of M-theory as first established we can quote:

- *There is no clear origin for the IIB theory: indeed, the relation within the hexagon is by means of T-duality, which means going to nine (or less) dimensions; one should hope to obtain the proper IIB theory in 10D in some limit of the (future) M-theory.*
- *Even the Heterotic Exceptional corner of the hexagon is a bit far fetched if related to the 11D Sugra: where do the two E8 groups come from? As we said, Witten and Hoˇrawa were careful enough to state "IF the 11D/Segment reproduces a string theory, THEN it has to be the H-E corner". Of course, the two gauge groups E8 at boundaries are forced by anomaly cancelation, but one does not "see" them directly in the M-theory. This argument is really more powerful: it is only the IIA corner which fits reasonably well with the 11D theory; indeed, neither the H-O theory nor the Type I come directly from M-theory, one has to recur to T-duality; also, even as regards the IIA theory, see point 5) below.*
- *There is lack of a dynamical principle; in this sense string theory, at least, is more conventional than M-theory: excited strings can be treated, at least perturbatively, as a Quantum Theory; however, membranes and higher (p > 2) Branes have no known quantization scheme. This is related to the dimensionless character of the 2-dim quantum fields, which allows very general background couplings, and this is simply not true for membranes etc. A related argument is this: for particles and strings, the geodesic problem (minimal volume) in the "Polyakov form" is equivalent to gravitation in one or two dimensions (Weyl invariance is needed in the string case). However, this is no longer true from membranes onward: although there is no really graviton degrees of freedom in 3D, gravitation is "conic", and presents definite phenomena.*
- *The maximal natural gauge group in M-theory, in its 11D version, seems to be the* $\binom{8}{2} = 28$*-dim orthogonal* $O(8)$ *group, generated by the 28 massless gauge fields down to 4D . But this group is too small, and unable to accomodate*

the minimal GUT group of the standard model, O(10) (SU(5) is insufficient to account for massive neutrinos); there are possible way outs, like composite fields, SU(8) as gauge group, etc., but none really very convincing [43].

- *There is the so-called massive IIA theory which again does not fit well with M-theory. It is a new version of the IIa nonchiral supergravity in ten dimensions, in which the two-form B "eats" the one-form A (the vector field) and grows massive: it is exactly the Higgs mechanism one degree further; but then the analogy with the reduced SUGRA in 11D no longer subsists, and hence it corresponds to no clear corner in the M-theory hexagon.*

2 *F theory :*

Cumrum Vafa [1]seems to have been the first, back in February 1996 , to publish an "F" Theory, in 12 dimensions with (−2,+10) signature, after the first introduction of the M-theory by Witten in March, 1995 (although the name, M-theory, was given a bit later), and as an extension of the same; for another early hint on 12D space .

The argument of Vafa was related to the IIB superstring theory ; as it lives in 10D, it cannot come directly from M-theory in 11 dimensions: the two possible one-dimensional compactifications from 11D were on a circle S1, giving the IIA theory, and in a segment D1, giving the Het-Excep theory; besides, there were no questions of any strong coupling limit: as we said, the IIB string was a case in which selfduality under $g_s \to 1/g_s$ *was proposed, because the two scalars (dilaton and axion) make up a complex field z, which transform homographically under the discrete residue of SL(2,R), a well known invariance group (although noncompact!) of IIb Sugra; the two B fields (fundamental and RR) also transform naturally under this SL(2,R) group; Townsend was the first to propose then that a discrete SL(2, Z) subgroup remained, and it was then obvious that the duality included inversion of the string coupling,* $g_s \to 1/g_s$*, where* g_s *is the exponential of the vev of the dilaton .*

The point of Vafa was that the group SL(2, Z), the so-called modular group, was the moduli group for a torus: the inequivalent conformal structures in the torus are labeled by the modular group. And Vafa, of course, interpreted this torus, with metric (−1,+1), as a compactifying space from a (−2, 10) signature space in twelve dimensions; the name "F-theory" was proposed by Vafa himself, meaning probably "father" or "fundamental" ; the argument for increasing one time direction is subtle and we shall show it more clearly later below. So the idea is that the IIB theory on space M_{10} *comes from an elliptic fibration with fiber a 2-torus, in a certain 12D space: symbolically*

$$T^2 \to V_{12} \to M_{10} \tag{2.1}$$

Then, F-theory on V_{12} *is equivalent to IIB on* M_{10}*.*

2.1 *Some arguments for F-theory*

There are several other arguments, mainly aesthetic or of completion, in favour of this Ftheory. We express all of them rather succintly, as none are really thoroughly *convincing. However, there are so many that we believe taken together they give some force to the idea of a 12-dimensional space with two times; for the particle content in this space see a proposal later.*

- *IIB theory really comes from 12 dimensions, with toroidal compactification; this was the original argument of C. Vafa . The space V12 admits an elliptic fibration, and the quotient is the frame for the IIB theory. Most of compactifications from strings (10D) or M-theory (in 11D) can be carried out from 12D F-theory .*

- *There is a Chern-Simons (CS) term in the 11d Sugra lagrangian, together with the conventional kinetic terms for the graviton, gravitino and 3-form C, and another "Pauli type" coupling*

$$L = ... + C \wedge dC \wedge dC + ... \tag{2.2}$$

  Now a CS term can be understood as a boundary term, hence claiming for an extra dimension interpretation (dC)^3 . This favours the interpretation of the group E_8 as a gauge group in M-theory .

- *There is a (2, 2) Brane extant in the Brane Scan, once one allows for some relax in supersymmetry dimension counting. In fact, in the brane scan of Townsend, if one insists on (1,D − 1) signature, one finds the four series of extended objects (and their duals), ending up with the 11D p = 2 membrane . By relaxing the signature, but still insisting in supersymmetry (in the sense of bose-fermi matching), one encounters a few new corners ; Susy algebras in this most general context were already considered in . Indeed, there is a (2, 2) membrane living in (2, 10) space. The membrane itself was studied carefully in ; by doubly dimension reduction, this (2, 2) membrane supposes to give rise to the string in IIB-theory .*

- *4 The algebra of 32 supercharges of 11D Sugra still operates in 12 = (2, 10) dimensions, with the (anti-)conmutation relations*

$$\{Q, Q\} = 2{-}form + 6^{\pm}{-}form \tag{2.3}$$

  so dim Q = $2^{12/2\ -1}$ and 528 = $\begin{pmatrix} 12 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ 6 \end{pmatrix}/2$ *. Now, in 11D, dim Q = 2(11−1)/2 = 32 (type +1, real), and the superalgebra is*

$$\{Q, Q\} = P_\mu + Z^{(2)} + Z^{(5)} \qquad 32 \cdot 33/2 = 11 + 55 + 462 \tag{2.4}$$

  namely, 11-dim translations plus a two-form and a 5-form, understood as central charges. But the 11D superalgebra clearly comes from the simpler 12-dim algebra of above: the 2-form gives 1-form and 2-form, and the selfdual 6-form gives rise to the five-form. The signature must be (2, 10), which gives 0 mod 8 for the type of Q; in case of (1, 11) signature is 2 mod 8: the charges would be *complex, or 2 · 32 = 64 real: twelve dimensional two-times space is maximal for 32 real supercharges. On the other hand, the direct reduction from 12D space to 10D via a (1, 1) torus would yield the IIB string from the (2, 2) membrane and the self-four form $D^{\pm}$ from the selfdual 6-form .*

- *We have remarkable relations in dimensions 8D, 9D, 10D which is the effective dimensions of a (1, 9) string theory, M-theory in (1, 10) and F-theory in (2, 10) as regards supersymmetry . The fundamental supersymmetry extant in 8 effective dimensions in string theory is 8v−8s between the vector (v) and the spinor (s), and squaring*

$$(8_v - 8_s) \cdot (8_v - 8_{s'}) = h - \psi + C = 44 - 128 + 84 \tag{2.5}$$

That is, the square fits in the content of 11d Sugra with 9 effective dimensions . But another square

$$(h - \psi + C)^2 = 2^{15} - 2^{15} \tag{2.6}$$

gives a 27-plet in $D_{\mathrm{eff}} = 10$ (or (2, 10) = 12D) as we shall see later, because this is related to our proprosal for the particle content of F-theory. The fact that the square of the fundamental irreps of Susy in 8 effective dimensions fits nicely in irreps of 9D, and the square again fits in irreps of a effective 10D theory, is most notorious and unique, it is certaintly related to octonion algebra, and it was first noticed by I. Bars before the M-theory revolution.

- *The content of 11D Sugra is related to the symmetric space (Moufang projective plane over the octonions)*

$$OP2 = F_4/Spin(9) \tag{2.7}$$

where O(9) is the massless little group in 11D; this was shown by Kostant [56]. There is a natural extension by complexification to the space

$$OP^2C = E_6/Spin^c(10) \tag{2.8}$$

related naturally to the 12 = (2, 10) space as

$$Spin^c = Spin(10) \times_{/2} U(1) \tag{2.9}$$

, where O(10)×O(2) is the maximal compact group of the O(2, 10) tangent space group in 12D. We shall explain this in detail in section four . It will be enough to remark here that now the candidate GUT group is O(10) , and there is no problem with fitting the SM group (U(3, 2, 1)) within it. 7) Compactification from 11 dimensions to 4 is preferable through a manifold of G_2 holonomy in order to preserve just N = 1 Susy in 4D; G_2 is a case of 7D exceptional holonomy, the only other being Spin(7), acting in 8 dimensions, very suitable for our 12 → 4 descent ; again, the 11D case generalizes naturally and uniquely to 12 ; and we have the nice split 12 = 4+2 · 4. Indeed, it seems that an argument like 4) can be also made here. Trouble is, we really need 8-dim manifolds with (1, 7) signature and exceptional holonomy , which are not yet fully studied .

For the moment. As for the selfdual 6-form , it is hoped it will be related to the matter content, in the same sense as the central charges in 11D Supergravity are related to membranes. Is it possible to relate this selfdual 6-form to the extant (2, 2)-Brane ? How do we incorporate two times in a theory of physics, in which the arrow of time is so characteristic . At face value, there are two ways out: either one of the times compactifies, so possible violations of causality are of the order of Planck´s length, or there is a gauge freedom to dispose of one of the times; I. Bars favours the second, but there are also consistent schemes with the first alternative .

2.2 *M theory vs F theory :*

Here I have tried to make a list of differences between M and F theory ,

though there is a link with T dualty .:

M theory	
Herein lies an intersecting "6-brane" models	*In*
This is the 7-dimensional gauge theory on a R $_3$-manifold X3	*F theory GU*
The Charged matter localized at points in X_3	*T*
Yukawas exponentially suppressed but no candidate model of flavor	*Here*
It is difficult to study	

3 *Some Progresses (Aspects) in F theory*

3.1 *Abelian gauge symmetries in F-theory*

U(1) gauge symmetries are ubiquitous in string compactifications. They are well known to play an important role in model building as selection rules in the effective action. As such they have also been heavily invoked in recent approaches to F-theory model building. However, in this framework the geometric origin of abelian gauge fields is relatively poorly understood. Thus the quest for U(1) symmetries in F-theory is a topic both of phenomenological relevance and of interest in its own right. We will discuss the appearance of U(1) symmetries in F-theory from a geometric and field theoretic perspective and compare the situation to the well understood case of perturbative Type IIB orientifolds. We will discuss two different mechanisms that can make a perturbatively present U(1) symmetry disappear in F-theory: A geometric Stuckelberg mechanism and Higgsing. Both have very different geometric origins and also bear different consequences for the phenomenology of the 4-dimensional field theory. As an application we will identify the restrictions on the elliptic fibration that guarantee a U(1) symmetry in F-theory compactifications with a Grand Unified SU(5) gauge symmetry.

3.1.1 *Little Mathematics behind it :*

In studying Gaugegroups in F-theory Many people studied U(1) symmetries that lead to selection rules on allowed Yukawa couplings direct phenomenological relevance.U(1) symmetries easily included in Type IIB models with D7-branes . Massless U(1) gauge fields arise in an expansion of 3-form-field C_3 into harmonics

$$C_3 = A^j \wedge \omega^i + \dots \qquad \omega_i \epsilon H^{(1,1)}(Y) \tag{3.1}$$

. Cartan U(1)'s of SU(N) realized via forms at singularities . The Idea behind that was to make Diagonal U(1) from fibering collapsing Torus cycle over a chain in the base with matching couplings of IIB theory .F-Theory is nonperturbative "completion" of IIB . It is the Only the nature of non-abelian ADE groups in F-theory well-understood . Now to prepare a theory on Nonabeliangauge groups in F-theory , elliptically fibered 4-fold Y_4has been considered . Singularity of fibration along $S \subset B_3$has been proposed along with ADE group G along S.

To study the geometry the singularities has been considered and found nonsingularity in 4-fold Y_4 . The S has been replaced by blow-up divisors , D_i= 1(1)rk(G) .The Intersections of Γ_i,Γ_0 has been studied in detail and extended Dynkin diagram of G has been constructed .it is found that the extended nodesare not homologically independent

3.1.2 *Massive U(1)´s in F-theory (A proposal)*

Geometrically massive gauge boson A^{0A} has been constructed at KK-scale . This stuffs can be consistently omitted in low energy theory .But in thesecases perturbative selection rules remain as an accidental symmetries . For instance , it has been observed that the same pattern of allowed Yukawa couplingsis there in F-theory upon uplifting IIB models .Now , in F-theory models without a IIB limit are also present , e.g. in the cases of models with E8 symmetry, different selection rules are possible . In addition, selection rules may be broken non-perturbativelyor by Higgsingof U(1)´s which has been studied by Grimm and Wignard .

In Summary I can highlight , the Upliftment of massless diagonal U(1) that generally described in F-theory by resolution divisor over self-intersection curve . Geometrically massive U(1)'s can be described by non-harmonic forms and live at the KK-scale . The M-theory reduction involves these non-harmonic forms precisely reproduces the structure of the axion gaugings and D-terms found in Type IIB . In addition , The corresponding perturbative selection rules survive in the low energy theory in the form of accidental symmetries

I think ,there is a lot of future work in this field - the precise geometric understanding of non-harmonic forms and the uplift of cohomology groups from $X_3$to Y_4which is desirable . The study of turning on fluxes along massive U(1)'s, in particular reproducing the flux-induced Stückelberg gauging . There's a lot of scope for study of the effect of massive U(1)'s on D3/M5 instantons and their selection rules . It is Important to understand how/if known mechanisms of IIB uplift to F-theory In particular: instanton potentials important for moduli stabilization and how they are affected by massless and massive U(1) gauge symmetries .

3.2 *TOP-DOWN 26d STRING THEORY*

It seems that Vafa's 1T 12d theory may be extended by Bars' 2T physics to get a 1T 12d Universe along with a 1T 14d Multiverse. The mathematical details of the relationships between M-theory, F-theory, S-theory and Multiverse theory are outside the scope of this paper. As an aside, it is interesting that Vafa named his superstring theory F-theory where the F stands for Father, under the consideration that the M in Witten's superstring theory stands for Mother. We presume that *the S in Bars' 13d theory then stands for Son. And so a Bars-type 14d superstring theory would be called D-theory where the D stands for Daughter. Also it is a convenient oversimplification to picture compactification as resulting in a grid of wires connecting junctions.*

Consider the following simple model:

The original superstring theories were in 10 dimensions (10d) which is the most basic form of string theory. In such theories, 6d of the 10d are COMPACTIFIED. That is, 6d dimensions shrink down to the Planck scale or less, as 3D space dimensions INFLATE in the Big Bang and one dimension remains time T'. The inflated dimensions are referred to as (3+1) spacetime. In 11d M-theory, the extra dimension just allows all 10d string theories plus 11d point-particle quantum gravity to be duals. (Duality just means it's the same physics in a different approximation, like the wave-particle duality.)

But in F-THEORY, two dimensions are compactified into a torus before or perhaps coincident with the 6d compactification. That leaves 14d for the Multiverse. A conjecture is that all of the dimensions of the Multiverse are also compactified except for three Space and one Time Dimension. If so, then 10d dimensions form a fine mesh screen or matrix in the Multiverse. But if the creation of the Multiverse is like that of a universe; that is, 6d compactify as 3D inflate; then 4d may have compactified beforehand into a fine-mesh grid at or below the Planck scale.

Occam's razor suggests that the 4d grid is Cartesian containing three space coordinates and one time-like coordinate into which the 6d compactification provides for 3 expanded Mspace Dimensions. The Multiverse Compact Manifold is presumed to exist at only a single time T which is the same time as in each universe, i.e., T=T'. Such a Multiverse grid and spacetime are likely infinite in the Time Dimension.

To summarize, reality may consist of 24 space-like dimensions of which all but 6 Space Dimensions have compactified into essentially two zero-volume matrices of wires and their junctions at or below the Planck scale. A similar 6d matrix called the Compact Manifold CM in string physics exists in both Mspace and Uspace. We shall refer to each individual junction as a compact manifold, the triple intersection points of seven-branes in 6d geometry.

3.3 *The large N limit of Jordan Matrix Models and F theory*

The large N limit of the E7 $\times$ SU(N) Matrix Model and 28-dim Bosonic F Theory .To construct the E7 $\times$ SU(N) Matrix model, we must firstly begin with the results of Gunaydin who have shown that there are no quadratic $E_{7(7)}$ invariants in the 56-dim representation of E7 but instead a real quartic invariant I_4 can be built by means of the Freudenthal ternary product among the elements X, Y,Z... of a Freudenthal algebra

$$Fr[O] = J_3[O] \oplus J_3[O] \oplus R^2 \tag{3.2}$$

of 56 real-dimensions 27 + 27 + 2 = 56 which is compatible with the 3-grading decomposition of the 56-dim representation of $E_{7(7)}$

under the $E_{6(6)}\times$Dilations : $1\oplus 54\oplus 1$.

Hence, the ternary product $X\times Y\times Z\to W$ and a skew-symmetric bilinear form $<X, Y>$ yields a quartic E_7 invariant of the form

$$I_4 = \frac{1}{48} < (X,X,X), X >= X^{ij}X_{jk}X^{kl}X_{li} - 1/4.X^{ij}X_{ij}X^{kl}X_{kl} + .$$

$$1/96.\epsilon^{ijklmnpq}X_{ij}X_{kl}X_{mn}X_{pq} + 1/96.\epsilon_{ijklmnpq}X^{ij}X^{kl}X^{mn}X^{pq}. \quad (3.3)$$

where the symplectic invariant of two 56-dim representations, like the area element in phase space $\int dp\wedge dq$, is given by :

$$< X, Y >= X^{ij}Y_{ij} - X_{ij}Y^{ij}$$

the fundamental 56 dimensional representation of $E_{7(7)}$ is spanned by the antisymmetric real tensors (bi-vectors) X^{ij} ,X_{ij} built from the SL(8,R) group indices $1\leq i, j\leq 8$ and such that the net number of degrees of freedom is 56 = 28 + 28 because an SL(8,R) bi-vector has 28 independent components. There are 28 coordinates X^{ij} and 28 momenta coordinates $X_{ij} = P_{ij}$.

The next step is to construct $E_{7(7)}\times SU(N)$ invariants in the large N limit. This is straightforward once we follow the same steps in the previous section and after defining the matrix-valued coordinates $M^AT_A = X^{ijA}T_A$ which take values in the Lie algebra $e_{7(7)}\times su(N)$. The quartic $E_7\times SU(N)$ invariant which we propose is defined by

$$I_4 = \frac{1}{2}f^E_{AB}f_{CDE}[X^{ijA}X^B_{jk}X^{klC}X^D_{li} - 1/4.X^{ijA}X^B_{ij}X^{klC}X^D_{kl} + .$$

$$\frac{1}{96}\epsilon^{ijklmnpq}X^A_{ij}X^B_{kl}X^C_{mn}X^D_{pq} + \frac{1}{96}\epsilon_{ijklmnpq}X^{ijA}X^{klB}X^{mnC}X^{pqD}. \quad (3.4)$$

where the four-index SU(N) invariant tensor is defined in terms of the structure constants $[T_A, T_B] = f^C_{AB}T_C$ as $\rho_{ABCD} = f^E_{AB}f_{CDE}$. It is not difficult to verify that one can re-write the terms of the trace of SU(N) commutators, since

$$Trace(T^AT^B) = 1/2\delta^{AB} \quad as \quad (3.5)$$

$$Trace([X^{ijA}T_A, X^B_{jk}T_B][X^{klC}T_C, X^{Dli}T_D]) = \frac{1}{2}f^E_{AB}f_{CDE}X^{ijA}X^B_{jk}X^{klC}X^D_{li}. \quad (3.6)$$

and similar equalities hold . In the large N limit of one has the correspondence$X^{ijA}T_A \rightarrow X^{ij}(\sigma^a)$; the SU(N) commutaors [ ] $\rightarrow$ { }$_{PB}$ and the Trace operation $\rightarrow \int d^n\sigma$. Therefore, the large N limit of the expression I_4 , after rewriting each single term in terms of the trace of SU(N) commutators , is given by a generalized nonlinear sigma model action associated with the maps from a 4-dimensional base manifold onto the bivector-valued target space coordinates $X^{ij}(\sigma^a) = -X_{ji}(\sigma^a)$ and momenta $P_{ij}(\sigma^a) = X_{ij}(\sigma^a) = -X_{ji}(\sigma^a)$, respectively, that parametrize a 28 complex dimensional space C^{28} associated with a 56 real-dimensional phase space realization of the Freudenthal algebra Fr[O] . To sum up, the large N limit of the quartic invariant furnishes the generalized nonlinear sigma model action on a 56-real dimensional phase space

$$S = \int [d^4\sigma][\{X^{ij}, P_{jk}\}\{X^{kl}, P_{li}\} - 14\{X^{ij}, P_{ij}\}\{X^{kl}, P_{kl}\} + .$$

$$1/96.\epsilon^{ijklmnpq}\{P_{ij}, P_{kl}\}\{P_{mn}, P_{pq}\} + 1/96.\epsilon_{ijklmnpq}\{X^{ij}, X^{kl}\}\{X^{mn}, X^{pq}\}]. \quad (3.7)$$

the Poisson brackets are defined with respect to the 4 coordinates $\sigma^a = \sigma^1, \sigma^2, \sigma^3, \sigma^4$ associated with the 4 dimensional base manifold by

$$\{X^{ij}, X_{jk}\} = \Omega^{ab}\frac{\partial X^{ij}}{\partial\sigma^a}\frac{\partial X_{jk}}{\partial\sigma^b}. \quad (3.8)$$

Ω^{ab} is the Poisson symplectic two-form .

A further analysis reveals that the last two terms are zero due to the antisymmetry $\{P_{kl}, P_{ij}\} = -\{P_{ij}, P_{kl}\}$and the condition $\epsilon^{klijmnpq} = \epsilon^{ijklmnpq}$ (an even permutation of indices). Thus, we are left only with

$$S = \int [d^4\sigma][\{X^{ij}, P_{jk}\}\{X^{kl}, P_{li}\} - 14\{X^{ij}, P_{ij}\}\{X^{kl}, P_{kl}\}. \quad (3.9)$$

It remains to be studied whether or not the 4-dim base manifold can be identified with the 4-dim world volume of a 3-brane. The action bears a resemblance with the action of previous equation corresponding to the 3 + 1 dimensional world volume of a 3-brane. Namely, if one can interpret the elements X, Y,Z... of the Freudenthal Fr[O] algebra in terms of the 28 complex coordinates corresponding to the embeddings of *a complexified 4d world-volume (associated with a 3-brane) onto a complexified 12-dim spacetime, and which result from the foliations of the 28 complex-dim spacetime into 16 complex-dimensional leaves (like the projective plane (C ×O)P^2) along the 12 complex-dimensionaspacetime M_{12}. Another picture is naturally to study the compactifications of a 28 complex-dim spacetime on 16 complex-dimensional internal spaces, like the projective plane (C × O)P^2 whose isometry group is E_6, yielding Einstein- Yang-Mills actions in 12 complex-dimensions. We conjecture that this novel E_7 ×SU(N) matrix model would be the appropriate arena for a bosonic formulation of F theory, in the same vein as the formulation of the heterotic string is based upon compactifications of the 26-dimensional bosonic string on 16-dim lattices.*

Concluding, the generalized Nonlinear sigma model action should describe the global dynamics of a complexified 3-brane embedded in 28-complex dim (56 real dimensional phase space) corresponding to a complexified bosonic formulation of F theory. Identical results can be attained when the phase space coordinates X_{ij} , P_{ij} belong to the complexification of the Freudenthal algebra Fr[C × O] algebra of 4 × 28 = 112 real-dimensions. In this case one would have the quaternionic version of the bosonic formulation of F theory in 28 quaternionic-dimensional spaces. The connection between F theory and Jordan algebras of degree four J4[H] have been described by Smith .

To finalize we shall present a modification of the Dirac-Nambu-Goto membrane action in terms of a 3 × 3 × 3 cubic matrix Habc. A generalization of a determinant for matrix elements of non-associative Jordan algebra has been provided by Freudenthal det X = 1/ 3 (X,X,X) in terms of the cubic-form . Despite that the non-associativity of octonions precludes the ordinary definition of a determinant, another interesting possibility to explore is to write the cubic matrix X^{ABC} of 3 × 3 × 3 = 27 entries that matches precisely the number of 27 independent components of the Jordan 3 × 3 hermitian matrices belonging to J_3[O] algebra, and whose hyper-determinant is :

$$DetX \sim \epsilon^{A_1A_2A_3}\epsilon^{B_1B_2B_3}\epsilon^{C_1C_2C_3}X_{A_1B_1C_1}X_{A_2B_2C_2}X_{A_3B_3C_3}. \tag{3.10}$$

one could then construct a generalization of the Dirac-Nambu-Goto membrane action : S= $\int d^3\sigma[|DetH|]^{1/3}$. (2.7) where the hyper-metric H represented by the 3 × 3 × 3 hyper-matrix (cubic matrix) Habc is defined as the pullback of

$$H_{\mu_1\mu_2\mu_3} which\ is\ of\ the\ form \qquad H_{abc} = H_{\mu_1\mu_2\mu_3}\partial_a X^{\mu_1}\partial_b X^{\mu_1}\partial_c X^{\mu_1}. \tag{3.11}$$

and the Finslerian-like space-time interval is of the form :

$$(ds)^2 = [H_{\mu_1\mu_2\mu_3}dx^{\mu_1}dx^{\mu_2}dx^{\mu_3}]^{2/3}. \tag{3.12}$$

Finslerian-like geometries are related to W_N geometries . The Exceptional (magical) Jordan algebras J_3[R,C,H,O] were instrumental in deciphering important algebraic structures in W_3,W_N gravity . For these reasons, theinterplay among W_N algebras, Jordan algebras, Finsler geometry and modified Dirac-Nambu-Goto membrane actions warrants further investigation .

3.4 *F-Theory Compactification :*

String Theory that is defined by 2D CFT on worldsheet is an lternative description of dynamics of massless states . The Kaluza-Klein reduction on M10 = R(1,3) x M6 is an Effective theory in 4D . Now , at low energy the prescription is that to carry out the Wilsonian effective action: integrate out massive states which are heavier than a certain energy scale .The effective action of Type IIB String Theory involves axio-dilaton field . Defining 'axion' scalar field $C_{(0)} = A_z$ *, where Az is as follows : The B-field is locally given by the curl of a vector potential and in 3 dimensions this would be* $B_r = \partial_\theta A_z$ *. The axion is only well-defined up to a shift by one.*

$$Q_{mag} = \oint_{S^1} \overrightarrow{B}.\hat{r} d\theta = 2\pi = \oint_{S^1} \partial_\theta C_{(0)} \tag{3.13}$$

So that we can conclude $C_{(0)}$ *has to be replaced by* $C_{(0)} + 1$ *as* $\theta \to \theta + 2\pi$.

Henceforth , we must inflict a monodromy upon the axion .Now , denfining the complex scalar $\tau = C_{(0)} + ie^{\phi}$*and theLagrangian of IIB SUGRA is given by*

$$L \sim R - \frac{\partial_z \tau \partial_{\overline{z}} \overline{\tau}}{2(\Upsilon\tau)^2} \tag{3.14}$$

By conjecture, exact symmetry of IIBwill be as :

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad for \qquad a; b; c; d\epsilon\mathbb{Z} \qquad and \quad ad - cd = 1. \tag{3.15}$$

F-theory compactications take into account the backreaction of the 7-branes on the geometry. Everything is neatly encoded in the geometry of the elliptically bered space.

To study the geometry of F-theory an elliptically fibered Calabi-Yau fourfold Y4 is taken which is complexified g_s *encoded in* T_2 *fibration over the base* B_3 *Gauge Symmetry , where fiber degenerates a co-dim 1 singularity signified a location (p,q) 7-branes in the base B3 . Consider the case of the* Y_4 *as* T_2 *over* B_3*Instanton the geometry is over Euclidean D3 brane (ED3) wrapping divisor in* B_3 *.*

It is little contrasting to string/M-theory in the case of no. i.e., 12-dimensional F-theory effective action which is also a fundamental formulation and is poorly understood .F-theory physics is often studied using limits and dualities for example in the case of weak coupling limit with D7-branes and O7-planes . Again for F-theory / heterotic duality many local geometries are involved .The only known way to extract generic features of F-theory effective actions is via its formulation as a limit of M-theory. It is remarkworthy that if objects like G - flux and M_5*-branes are used in the context of F- theory this limit is always understood with full surity .4D chiral* *index for F-theory compactifications using M-theory G -flux in the F-theory limit has been derived . So also the corrections to the F-theory gauge coupling function due to flux using M-theory warping are derived precisely . F-theory effective action can be reliably studied with bulk + 7-brane physics in a unified N=1 framework but the M-theory origin of various F-theory effects can be unexpected .*

Type IIB has non-perturbative symmetry rotating . The interpretation as complex structure of a two-torus (2 auxiliary dimensions) which is the minimally supersymmetric F-theory compactifications . Moreover , F-theory on torus fibered Calabi-Yau 4-fold i.e. , 4 dim . Now , for N=1 supergravity theory the base is a Kähler manifold . The singularities of the fibration are crucial to encode 7-brane physics . There the pinching torus indicates presence of 7-branes magnetic charged under the fibration .Morrison and Vafa had showedthe brane and bulk physics encoded by complex geometry.

In 6D F-theory compactifications on Calabi - Yau manifold is the effective theory of strong constraints from anomalies (gauge+gravitational) . F-theory geometry: topological properties of resolved elliptic fibrations can be matched with the anomaly constraints relating terms in the effective action and spectrum . Green-Schwarz had shown this complications from a general 4D theory by distinguishing the moduli fields (all chiral multiplets in 4D), as in 5D . The scalar potential e.g. due to fluxes . focus on light fields . corrections and additional axion-(curvature) , e.g. dilaton-axion at weak string coupling , additional het. axion . To analyze 4D F-theory it is done in D=3, N=2 supergravity on Coulomb branch F-theory on X_4 x S_1 = M-theory on X_4 .

Matching of two effective theories possible only at 1-loop in F-theory (by integrating out massive matter) = classical supergravity terms in F-theory compactications preserve the minimal amount of supersymmetry . E.g. compactifying F-theory on a Calabi-Yau fourfold (eight real dimensions) yields and N = 1; d = 4 theory MSSM-like models .

I think there is many open questions like

- *Why M5-branes instantons behave non-trivially in the M-theory to F-theory limit*
- *How to extend constraint analysis in 4D, including fluxes and potentials and the method of constraining continuos parameters*

To summarize , F-theory provides an ideal setup for:

- *Unifying 7-brane and bulk physics and also for N_{brane} & N_{geom} in complex geometries .*
- *Evolving in the promising phenomenological scenarios (GUTs, moduli stabilization) . For the cases of Geometric features of particle physics with those of intersecting branes & exceptional gauge symmetries that are common in the heterotic string – at finite string coupling g_s*

3.5 *F theory GUTs Models :*

F theory GUT [4] is a phenomenologically viable models from string theory . It is an imprint of UV completion upon low energy theorywhich will build complete string models . The main challenge is the generic statements that it should valid for a large class of models .

The method is the Bottom-up approach which is systematically build models starting with effective theory on branes. It incorporate constraints from embeddability into compact model . There is Three-step strategy , they are described as follows [5]:

Local *Models : It is an Effective field theory on 7-branes described on The SU(5) GUT . Low energy gauge decoupling gravity dof is given by :* $\frac{M_{GUT}}{M_{Pl}}$ 10^{-3}. *The SU(5) SUSY GUT theory 'has been described with* A_4 *singularity and 3×10M . There is also an SO(10) enhancement . There is also Yukawa couplings from triple intersection of matter curves* G_p *on* $SU(5)\times U(1)_1\times U(1)_2$ *. Now defining :*

1. $$10_M = \begin{pmatrix} Q & \sim & (3,2)_{+1/6} \\ U^C & \sim & (\overline{3},1)_{-2/3} \\ E^C & \sim & (1,1)_{+1} \end{pmatrix}$$

2. $$\overline{5}_M = \begin{pmatrix} D^C & \sim & (\overline{3},1)_{1/3} \\ L & \sim & (1,2)_{-1/2} \end{pmatrix}$$

3. $$5_H = \begin{pmatrix} H_U & \sim & (1,2)_{+1/2} \\ H_U^{(3)} & \sim & (3,1)_{-1/3} \\ & & \end{pmatrix}$$

4. $$\overline{5}_H = \begin{pmatrix} H_d & \sim & (1,2)_{-1/2} \\ H_d^{(3)} & \sim & (\overline{3},1)_{+1/3} \\ & & \end{pmatrix}$$

The superpotential term is going to be

$$W_u \sim \lambda_u 5_H \times 10_M \times 10_M + \lambda_d 5_H \times \overline{5}_M \times 10_M \tag{3.16}$$

Geometrically, a local model corresponds to a system where gauge theory localizes on a (4 + k)-dimensional subspace of the ten-dimensional string theory. Starting from the ten-dimensional Einstein- Hilbert action, and the (4+k)-dimensional Yang-Mills action, the resulting gauge coupling constant and four-dimensional Newton's constant respectively depend on the internal volumes as

$$(g_{YM}^{4d})^2 \propto Vol(M_k)^{-1} \qquad , \qquad G_N^{4d} \propto Vol(M_6)^{-1}. \tag{3.17}$$

In this theory Gravity has been decoupled and the method of decoupling gravity corresponds to a limit where the ratio of the characteristic radii becomes parametrically small.This theory has been Geometrically consistent with SL(2 , Z) .

Vafa etc has constructed the Favourable flavour structure . GUT structures naturally explain some aspects of flavor. For example, the mass of the b quark andτ *lepton unify at the GUT scale , which fits with embedding their Yukawas in the interaction term* $\overline{5}H \times \overline{5}M \times 10M$. *Geometrically, order one coefficients for both the* $\overline{5}H \times \overline{5}M \times 10M$ *and* $5H \times 10M \times 10M$ *seem the easiest to arrange, and this is the case we focus on here. Since the top and bottom quark have different masses, this is most compatible with large* $tan\beta$ *scenarios.*

Application : Phenomenologically viable SU(5) SUSY GUTs realized in local F-theory 7-brane intersections and including the promising flavour and SUSY-breaking phenomenology .

Semi-local *Model : In this model general conditions imposed for embedding into local* CY_4 *In Global* CY_4 *, we use to do elliptic fibration over B* $y^2 = x^3 + fx + g$ *and*$E \to X_4 \to B \supset S_{GUT}$. *But in Local CY4 and in ALE-fibration over* S_{GUT} *and the scheme is like* $ALE \to X4 \to S_{GUT}$ *.Embeddability implies strong phenomenological restrictions . Local geometry around F-theory 7-branes is a deformed* E_8 *singularity* $y^2 = x^3 + b_5xy + b_4x^2z + b_3yz^2 + b_2xz^3 + b_0z^5$. E_8 *singularitytells that* $b_{2,3,4,5} = 0$ *. The E8 gauge theory has broken to SU(5) by adjoint VEVs .All matter arises from E8 and i give masses to SU(5)GUT multiplets .GUT-fields carry 4 independent U(1) charges:* $(\lambda_1, \cdots, \lambda_5)$ *Superpotential couplings dictated by 4 independent U(1)'s . In this model , U(1)'s get identified by monodromies highly constrains embeddable models .Geometry of E8 singularity given by* b_n *where*

$$b_n(\lambda_i) = b_0 P_n(\lambda_i) \tag{3.18}$$

$$b_5 \sim b_0\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5 \qquad , \qquad b_4 \sim b_0 \sum_{i<j<k<l} \lambda_i\lambda_j\lambda_k\lambda_l$$

$$b_3 \sim b_0 \sum_{i<j<k} \lambda_i\lambda_j\lambda_k \qquad , \qquad b_2 \sim b_0 \sum_{i<j} \lambda_i\lambda_j$$

- *In this , Inversion $\lambda_i(b_n)$ generically has branch-cuts . Now , the monodromy group $G \subset S_5$=Weyl group of SU(5)$_\perp$ acts on λ_i. Independent gauged U(1)s are encoded inthe orbits of monodromy group*
- *So , in short ,F-theory leads to GUT embedded into E8 singularity . In this theory ,the relaxed constraints are Non-GUT exotics from 10 and 5 .*
- *Now , the required Embedding into Semi-local Model is the E_8 gauge theory and highly constraining which is non-minimal SU(5) GUT with non-GUT exotics and non-minimal gauge mediation models*

Global *Model : In this case , the explicit realization in compact CY4 . Now, as we know , X_4 = elliptically fibered CY4 with three-fold base B . So that , $E_\tau \to X_4 \to B \supset S_{GUT}$ .Te basic procedure is the construction of elliptically fibered CY_4 realizing semi-local models Global model . The constraint upon B_6 is that in the $X4$ Calabi-Yau Manifold , B is almost Fano i.e. $K^{-1}{}_{B3}$ is semi-ample . For MSSM-running: With*

$$\beta_1 = 3, \beta_2 = -1, \beta_3 = -33/5 \qquad and \quad \alpha_i^{-1}(M_{GUT}) = \alpha_i^{-1}(m_z) - \frac{\beta_i}{2\pi} ln(M_{KK}/m_z) \tag{3.19}$$

Winjholt has shown that the KK-thresholds for 8d theory 7-brane worldvolume theory must has divergence logΛ. But the external contribution (from bulk) to cancel log divergence and the divergence is capped off at winding scale

$$M_{Winding} = Windingscale > M_{KK} \tag{3.20}$$

This can be written in a 4d looking way

$$\alpha_i^{-1} \to \alpha_i^{-1} - \frac{\beta^{KK}}{2\pi} ln(\frac{M_{winding}}{M_{KK}}) \tag{3.21}$$

There is also Non GUT exotic contribution in this model :

$$\alpha_i^{-1} \to \alpha_i^{-1} - \frac{\beta^{Exotic}}{2\pi} ln(\frac{M_{KK}}{M_{Exotic}}) \tag{3.22}$$

so that the whole formulae leads to

$$\alpha_i^{-1}(M_{GUT}) = \alpha_i^{-1}(m_z) - \frac{\beta_i}{2\pi} ln(M_{KK}/m_z) - \frac{\beta^{KK}}{2\pi} ln(\frac{M_{winding}}{M_{KK}}) - \frac{\beta^{Exotic}}{2\pi} ln(\frac{M_{KK}}{M_{Exotic}})$$

Again the splittingsof $\alpha_i^{-1}(m_Z) - \alpha_j^{-1}(m_z)$ *is independent of GUT but sensititive to threshold contributions . Again , MSSM alone agrees with experiment to within 0.5% and with correction , on threshold of acceptibility . Most rosy estimates for can lead to 0.5% but most are somewhat larger .*

Condition for consistency with gauge couplings at m_z *is :*

$$\frac{\beta_i}{2\pi} ln(M_{KK}/m_z) + \frac{\beta^{KK}}{2\pi} ln(\frac{M_{winding}}{M_{KK}}) + \frac{\beta^{Exotic}}{2\pi} ln(\frac{M_{KK}}{M_{Exotic}}) \sim 0 \qquad (3.23)$$

Lift of semi-local models to global models is the fate of the U(1) symmetries .

3.6 *F theory and Phenomenology :*

Grand Unified Theories (GUTs) is reviewed in F-theory . This leads to a specific class of geometric ingredients which are necessary in order to realize the matter content, and interaction terms of the Minimal Supersymmetric Standard Model[3]. Imposing the condition that gravity can in principle decouple endows the models with surprising predictive power. We study gauge mediated supersymmetry breaking in F-theory GUTs, and find that the soft scalar masses contain an intrinsically stringy ingredient which at lower energies leads to observable consequences . The flavor physics of quarks and leptons are in accord with experiment, and lead to predictions in the neutrino sector. It is found that the model non-trivially satisfies many cosmological constraints. Cosmology also provides a window into the stringy deformation of the supersymmetry breaking sector .Recently , the central extension of the SUSY algebra in the BLG theory for M2-branes. The central charges (related to other M-brane configurations) transform as the 2-form, 28, and the self-dual four-form, 35+, of the transverse SO(8) symmetry and they are included in the Lagrangian.the singularity repels neutral fields and this fact actually leads to new exponential hierarchies where the exponent is related to the ratio of the Planck and GUT scales or its powers (multiplied by a gauge coupling).

Such a construction may create natural Dirac neutrino masses [6]around 50 meV (plus minus half an order of magnitude). Another scenario shows that the heavy Majorana neutrino masses are around $10^{\{12\}}$ *GeV (plus minus 1 or 2 orders of magnitude).The string analogue of the GUT scale* $3 \cdot 10^{17}$ *GeV does not exactly coincide with the phenomenologically favorite value* $MGUT \approx 2 \cdot 10^{16}$ *GeV, but the discrepancy is small enough to be explainable (in principle) in terms of the perturbative string threshold corrections. There are three ways how GUTs may arise from string theory: heterotic strings, non-perturbative type IIA (M-theory on G2 singularities), non-perturbative type IIB (F-theory). They study the latter because it involves holomorphic geometry that they know really well . F-theory includes the singularities which are technically 7-branes of 9+1-dimensional type IIB theory, localized* *in 2 transverse spatial dimensions. To get a realistic model, they must become spacetime-filling branes in the real world, i.e. 3-branes. It means that the 7-branes must be compactified on a 4-manifold. The holomorphic structures must exist on this 4-manifold so it must actually be 2-complex-dimensional. After some analyses, they decided that it must be a del Pezzo (complex) surface. And dP_8 is the most general one - others can be obtained from dP_8 by shrinking its cycles. But I still haven't said what kind of a singularity - what kind of a 7-brane - is wrapping the del Pezzo (complex) surface. It turns out that the rank of the gauge group on the brane is between 4 and 6. The minimal case, 4 with the SU(5) group, is already viable and a unique mechanism of symmetry breaking (via the flux) exists in this case; the choice of the U(1) group is unique as well.I simplified a bit . The 7-brane wrapped on the del Pezzo 4-cycle of the type IIB manifold carries the GUT gauge group but it is not the only one . There are other 7-branes . Their intersections with the GUT del Pezzo 7-brane has real codimension of 2. That's where additional fields live.*

Now, one should remember that the F-theory GUTs are better than an ordinary GUTs as it preserves the beauties of GUTs but removes their warts. For example, the fermion representations are beautiful but the Higgses suck. So the F-theory GUTs keeps the complete representations for the fermions - they live on Riemann surfaces with a vanishing flux through them - while the Higgs multiplets break into pieces because they live on Riemann surfaces with a nonzero flux . The proton decay is suppressed by "sequestering", if you wish , the up-Higgses and down-Higgses live on different, separated Riemann surfaces inside the del Pezzo manifold: the mu-term is then zero . The symmetry-breaking flux also influences the masses of lighter generations: the similarity of the tau and bottom masses is preserved while the corresponding similarities for lighter generations are destroyed by changes of the wavefunction induced by the flux . So the picture is that all fields of the GUT sector live somewhere on the (real) 4-dimensional del Pezzo (complex) surface but some fields such as fermions and Higgses live on (real) 2-manifolds i.e. Riemann surfaces inside the del Pezzo manifold. That's a rather simple geometry. You draw a well-known 4D geometry and 2-dimensional submanifolds in it carry particles. The only difference from the octopi is that it actually generates the right type of low-energy effective field theory. If this difference doesn't hurt, the octopus researchers should like the F-theory framework . They say that the construction can't have a heterotic dual because of a cohomology argument and Green-Schwarz-generated string-scale masses that are avoided in the F-theory picture . Attempts to realize GUTs from String Theory has been done by Weakly coupled E8 $\times$ E8 Heterotic String (heterotic orbifolds) . The needed things are the large threshold corrections at MX . The GUT breaking via discrete Wilson lines and F-theory / Type IIB compactifications with (p, q)$-$ 7-branes . It Solves the 10 10 5_H Yukawa problem of orientifolds . The GUT brane wraps a shrinkable 4-cycle and GUT breaking via U(1)Y flux .

The grand unified group is Higgsed by a characteristic F-theoretical player, a flux, *to a flipped GUT model or the MSSM itself. They say that such a possibility doesn't exist in the heterotic model building which is true but, in my opinion, misleading because the heterotic models have other tools that play a similar role (the Wilson line symmetry breaking). The symmetry breaking by the flux automatically solves the doublet-triplet splitting problem (the Higgses may come in incomplete multiplets, naturally without any triplets), explains the longevity of the proton, and even reproduces some qualitative features of the GUT light mass relations.*

The Program is to embed the local ideas into a global framework such as , F-theory on elliptically fibered four-folds with shrinkable 4-cycles . The derivation of the global consistency conditions is to lift and generalise Type IIB orientifold consistency conditions to genuine F-theory models . then , we do the study of consequences of U(1)Y flux which means the gauge coupling unification . So that , the moduli stabilization is done via flux and the instantons has to generate superpotentials .

Generally, a string model has four spacetime dimensions, N = 1 supersymmetry and a large gauge symmetry $G = \prod_a G_a$ *including the SU(3) × SU(2) × U(1) of the Standard Model as well as additional, 'hidden' factors. At the tree level, all the gauge couplings ga are controlled by the expectation value of the dilaton field S,*

$$4\pi/g_a^2 \equiv 1/\alpha_a = k_a < ReS > \tag{3.24}$$

k_a *being fixed integer or rational coefficients. The universality of this relation naturally leads to the desired GUT-like pattern of the Standard Model's gauge couplings. The perturbative string theory suffers from an exact degeneracy which leaves <S> completely undetermined ; likewise, the vacuum expectation values of several other moduli fields (collectively denoted T) are also indeterminate to all orders of the string perturbation theory .*

The working hypothesis is that decoupling of GUT scale from Planck scale and the localisation of GUT physics on del-Pezzo surfaces . The universality of this relation naturally leads to the desired GUT-like pattern of the Standard Model's gauge couplings. The perturbative string theory suffers from an exact degeneracy which leaves hSi completely undetermined; likewise, the vacuum expectation values of several other moduli fields (collectively denoted T) are also indeterminate to all orders of the string perturbation theory.

It is an One-loop running of the three Standard Model gauge couplings with MSSM matter spectrum above the TeV scale,

In the simplest scenario, a confining hidden sector generates a dynamical superpotential W Λ_{hid}^3 *where*

$$\Lambda_{hid}\ e^{-2\pi kS/b} M_{Pl} \tag{3.25}$$

is the confinement scale and b the appropriate - function coefficient.

Taking several such hidden sectors together and allowing for moduli-dependent pre-exponential factors, one generally has

$$W_{eff}(S,T) = M_{Pl}^3 \sum_a C_a(T) e^{-6\pi k_a S/b_a} \tag{3.26}$$

Where , a runs over the confining hidden sectors , which leads to an effective scalar potential

$$V(S,T) = e^K(|DW|^2 - 3|W|^2)$$

Phenomenologically, this effective potential should have a stable minimum with spontaneously broken supersymmetry and zero cosmological constant. Furthermore, the observable sector (i.e., the Standard Model) should feel the breakdown of supersymmetry at the electroweak scale MW; this requires

$$W^{eff} = O(M_W M_{Pl}^2) \tag{3.27}$$

or equivalently confinement scales hid in the 1013 GeV to 1014 GeV range.

Likewise, extrapolating the Supersymmetric Standard Model all the way up to the GUT scale and using eq. (1), one needs hReSi ≈ −1 GUT ≈ 23.[6

According to Dine and Seiberg, for any string model with unbroken supersymmetry at the tree level, the effective potential exponentially asymptotes to zero in the weak coupling regime Re S ¨ ‡ and hence, the stable minima of the potential, if any, must lie at strong coupling. õ For example, the superpotential (3) with a generic KNahler function K(S, T) and no special tuning of the coefficients Ca(T) and ka/ba, leads to stable vacua only when some of the exponential factors e.6fÎkaS/ba are not too small (O(1))

and hence hRe Si <~ O(1/6) max a (ba/ka)

However, from the heterotic stringfs point of view, this scenario . or any other scenario which needs very large or complicated hidden sectors . conflicts with the universal central charge constraint, which limits the rank of the entire (perturbative) four-dimensional gauge group: rank(G) . 22 ; (5) this leaves rather limited room for the hidden sectors. Consequently, the perturbative heterotic string theory with only field-theoretical non-perturbative corrections has extreme difficulty combining a stable vacuum with a large dilaton expectation value and a large hierarchy. The inherently stringy non-perturbative effects are now gradually becoming understood in terms of duality relations between various string theories, M.theory and F.theory. In particular, the N = 1, d = 4 compactifications of the heterotic string are dual to F–theory compactifications on elliptically fibered Calabi-Yau fourfolds

The precise behavior of the resulting scalar potential can only be analyzed on the model-by-model basis, but a crude order-of-magnitude analysis suggests that its stable minima (if any) should have 6 hid(hTi) = O(bhid). (7) Again, we see that large hidden sectors naturally lead to small hid .

The non-perturbative string theory or F–theory allow for essentially unilimited hidden sectors. This makes it relatively easy to arrange for a stable vacuum state where supersymmetry is broken at a hierarchically low scale. On the other hand, the GUT-like unification of the Standard Model's gauge couplings is no longer automatic but instead has to be imposed as a phenomenological constraint. We do not propose any specific models but merely outline a general scenario for obtaining viable phenomenology from the F–theory. Indeed, it is hard to be specific without a better understanding of the moduli dependence of the gauge couplings in F–theory or even general rules for obtaining the spectra of the charged matter fields. However, we believe our scenario is a useful starting point for future work.

Shortcomings

- *Missing stringy global consistency conditions: landscape vs. swampland*

.Physics of abelian gauge symmetries: Green-Schwarz mechanism, Freed-Witten anomalies, (Grimm, Weigand)

- *Need local mechanism for Susy breaking ! gauge mediated susy breaking*

- *closed string Moduli stabilisation, need to explain why susy breaking is subleading to gauge mediation*

3.7 Conclusion:

Recently , Vafa etc has shown that E-type Yukawa points are required in order to generate a top quark mass in F-theory GUTs. D3-brane probes of such E-points are then a very well-motivated extension of the Standard Model. In this paper we have studied the effects of the Standard Model on such a probe sector, and conversely, the effects of the probe on the Standard Model. They have also presented the evidence for the existence of a strongly interacting conformal fixed point for this system, and moreover, have shown that various properties of this system, such as the infrared R-symmetry, the scaling dimensions of operators, and the effects of the probe on the running of the gauge coupling constants are all computable. 14.

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