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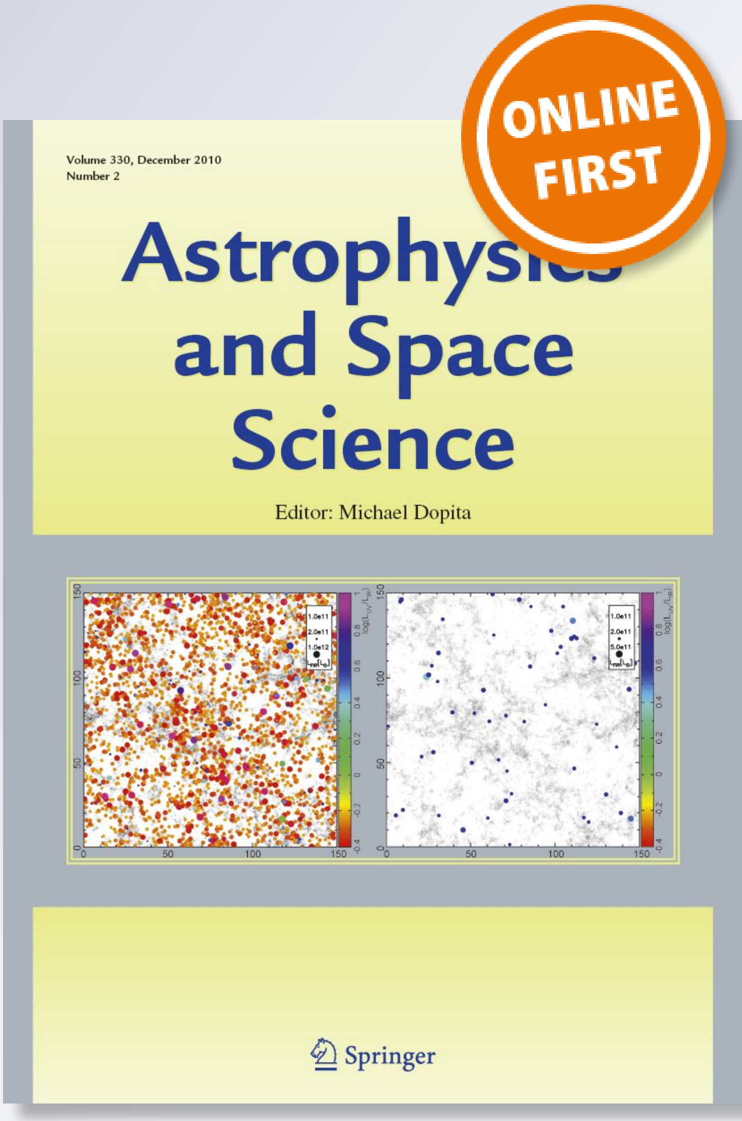

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ORIGINAL ARTICLE

Radar signal delay in the Dvali-Gabadadze-Porrati gravity in the vicinity of the Sun

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Abstract In this paper we examine the recently introduced Dvali-Gabadadze-Porrati (DGP) gravity model. We use a space-time metric in which the local gravitation source dominates the metric over the contributions from the cosmological flow. Anticipating ideal possible solar system effects, we derive expressions for the signal time delays in the vicinity of the Sun. and for various ranges of the angle θ of the signal approach, The time contribution due to DGP correction to the metric is found to be proportional to $b^{3/2}/c^2 r_0$. For r_0 equal to 5 Mpc and θ in the range $[-\pi/3, \pi/3]$, Δt is equal to 0.0001233 ps. This delay is extremely small to be measured by today's technology but it could be probably measurable by future experiments.

Keywords Dvali-Gabadadze-Porrati gravity · Radar signal delays · De Sitter background · Friedman-Lemaitre-Robertson-Walker phase · Accelerating phase

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1 Introduction

There is recent attention for the so-called Dvali-Gabadadze-Porrati (DGP) model. This is a five dimensional gravity model that explains the observed acceleration of the expansion of the Universe. Furthermore, it predicts minor post-Einstein effects, testable at local scales resulting to information on the Universe's global properties in relation to the ongoing cosmological expansion (Iorio 2005a, 2005b, 2005c). So far, two-body scenarios have been investigated in which the time rates of change for the longitude of pericenter and the mean anomaly of the secondary have been carried out (Lue and Starkman 2003; Iorio 2005b), with the effects being functions of eccentricity. Following Iorio (2005b, 2005c), one might say that the "ideal test-bed for such tests is the inner planets of the solar system". Measurements of such precessions lie in the limit of precision of today's planetary data. For a more detailed and complete overview on the DGP gravity, see Lue (2006).

The DGP model is based on an extra flat dimension w, and a free crossover parameter r_0 which defines a radius beyond which the four-dimensional gravitational theory transitions into a five-dimensional regime. The last parameter is defined by $r_0 = k^2/2\mu$. The constants μ^2 and k^2 define the energy scales of the theories of gravity: the first one is Newton's constant, $\mu^2 = 8\pi G$, while the second represents the energy scale of the bulk gravity (Sawicki et al. 2007). The crossover parameter is also fixed from observations of IA type supernova to a value approximately equal to 5 Gpc (Lue and Starkman 2003). For distances greater than 5 Gpc Newtonian gravity needs to be modified, which can lead to different explanations for the dark matter when somebody tries to interpret the accelerations observed in the Universe.

In this short contribution we consider the case when the Newtonian-Einstein gravity is modified according to the

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Dvali-Gabadadze-Porrati braneworld model, We will calculate the resulting radar signal time delay in the vicinity of the Sun We proceed with in a way similar to that used in Haranas and Ragos (2011) and Haranas et al. (2011). We base our analysis on the angle θ defined by the distance of the closest approach, and any other point along the path of a traveling radar signal. We apply our results for various angular ranges.

2 The Dvali-Gabadadze-Porrati metric

Following Lue and Starkman (2003), the metric of a spherical source in a cosmological de Sitter background can be described by the following line element:

$$ds^2 = c^2 N^2(r,w)dt^2 - A^2(r,w)dr^2 - B^2(r,w)\left(d\theta^2 + \sin^2 d\phi^2\right) - dw^2. \tag{1}$$

In the region where the local gravitation dominates the metric over the contributions from the cosmological flow, the de Sitter solution of the five-dimensional field equations can be expressed by Lue and Starkman (2003):

$$N(r,z) = 1 + n(r,z) = 1 - \frac{GM}{rc^2} \pm \sqrt{\frac{GMr}{r_0^2}}, \tag{2}$$

$$A(r,z) = 1 + a(r,z) = 1 + \frac{GM}{rc^2} \mp \sqrt{\frac{GMr}{r_0^2}}, \tag{3}$$

$$B(r,z) = r\left(1 + b(r,z)\right), \tag{4}$$

where M is the gravitating mass, G is the constant of universal gravitation, w is the fourth special coordinate, and $b(r,z), n(r,z), a(r,z)$ are functions of the coordinates, that can be calculated using the derived field equations. For distances scales r much smaller than r_0, Newton-Einstein gravity is obtained with few exceptions that include minor corrections. With reference to Lue and Starkman (2003) we can write the DGP line element in the following way:

$$\begin{aligned} ds^2 &= c^2\left(1 - \frac{GM}{rc^2} \pm \sqrt{\frac{GMr}{r_0^2}}\right)^2 dt^2 \\ &\quad - \left(1 + \frac{GM}{rc^2} \mp \sqrt{\frac{GMr}{r_0^2}}\right)^2 dr^2 \\ &\quad - r^2\left(1 + b(r,z)\right)^2 d\Omega^2 - dw^2. \end{aligned} \tag{5}$$

To deal with the DGP effect on the propagation of electromagnetic signals in the vicinity of the Sun, we incorporate an additional DGP correction term in the Schwarzschild space-time metric coefficients of the line element. Next we modify the Schwarzschild metric used in the solar system when general relativistic effects are taken into account. Therefore, if r, θ, ϕ are the polar coordinates of any point along the signal's path, and Ω is the corresponding solid angle, the photon transmission time can be written as follows:

$$dt' = \frac{ds}{c}, \tag{6}$$

and the line element takes the form:

$$\begin{aligned} dt &= \left(1 + \frac{GM}{rc^2} \mp \sqrt{\frac{GMr}{r_0^2}}\right)^2 dt' \\ &= \left(1 + \frac{GM}{rc^2} \mp \sqrt{\frac{GMr}{r_0^2}}\right)^2 \frac{ds}{c}, \end{aligned} \tag{7}$$

where the plus sign is related to the Friedman-Lemaitre-Robertson-Walker phase of the universe, while the minus sign is related to a self accelerating phase (Iorio 2005a). In this paper, we first consider the plus sign or Friedman-Lemaitre-Robertson-Walker phase and, then, the minus sign representing the accelerating phase.

Since $r = b/\cos\theta$, where b is the distance of the closest signal approach, we have that:

$$dr = -\frac{b\sin\theta}{\cos^2\theta}d\theta. \tag{8}$$

Also, $ds = \sqrt{dr^2 + r^2 d\theta^2}$ and after substitution and simplification, Eq. (7) becomes:

$$dt = \frac{b}{c}\left(\begin{array}{l} 1 + \dfrac{2GM}{bc^2}\cos\theta + \dfrac{G^2M^2}{b^2c^4}\cos^2\theta \\ + \dfrac{GMb}{c^2r_0^2}\sec\theta + 2\sqrt{\dfrac{GMb}{c^2r_0^2}\sec\theta} \\ + \dfrac{2GM}{bc^2}\sqrt{\dfrac{GMb\sec\theta}{c^2r_0^2}} \end{array}\right)\sec^2\theta d\theta. \tag{9}$$

Equation (9) contains classical, general relativistic, DGP time delays. Since we are interested in the DGP delay only, we may neglect the term $1 + \frac{2GM}{bc^2}\cos\theta + \frac{G^2M^2}{b^2c^4}\cos^2\theta$. Then, we, then, have to integrate the following expression:

$$\begin{aligned} dt &= \frac{b}{c}\left(\frac{GMb}{c^2r_0^2}\sec\theta + 2\sqrt{\frac{GMb}{c^2r_0^2}\sec\theta}\right. \\ &\quad \left. + \frac{2GM}{bc^2}\sqrt{\frac{GMb\sec\theta}{c^2r_0^2}}\right)\sec^2\theta d\theta. \end{aligned} \tag{10}$$

Omitting order $O(c^{-3})$ and $O(c^{-4})$ terms for being too small, we integrate over various angular subintervals of the range $(-\pi/2, \pi/2)$ to avoid the singularities at $\theta = \pm\pi/2$. For any such interval $[\alpha, \beta]$ the corresponding radar signal time delay will be:

$$\Delta t = \frac{b}{c}\int_\alpha^\beta \left(\frac{GMb}{c^2r_0^2}\sec\theta + \frac{2}{cr_0}\sqrt{GMb\sec\theta}\right)\sec^2\theta d\theta. \tag{11}$$

We start at $[-\pi/6, \pi/6]$. The integration over that range results that:

$$\Delta t = \frac{2GMb^2}{3c^3r_0^2} + \frac{GMb^2\ln(27)}{6c^3r_0^2} + \frac{8b\sqrt{2bGM}}{3^{7/4}c^2r_0} + \frac{8b\sqrt{bGM}}{3c^2r_0}F\left(\frac{\pi}{12}, 2\right), \tag{12}$$

where, F is the elliptic integral function of the first kind. Similarly, integrating over the range $[-\pi/4, \pi/4]$, we obtain that:

$$\Delta t = \frac{GMb^2\sqrt{2}}{c^3r_0^2} + \frac{8\times 2^{1/4}b\sqrt{GMb}}{3c^2r_0} + \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{8}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left[\frac{\cos(\pi/8)+\sin(\pi/8)}{\cos(\pi/8)-\sin(\pi/8)}\right]. \tag{13}$$

Next, for the range $[-\pi/3, \pi/3]$ we have that:

$$\Delta t = \frac{b^2\sqrt{2}GM}{c^3r^2} + \frac{8b}{c^2r_0}\sqrt{\frac{2GMb}{3}} + \frac{8b\sqrt{GMb}}{3c^2r}F\left(\frac{\pi}{6}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right), \tag{14}$$

Finally, for the range $[-4\pi/10, 4\pi/10]$, we obtain the following expression

$$\Delta t = \frac{GMb^2\sqrt{50+22\sqrt{5}}}{c^3r_0^2} + \frac{8b\sqrt{bGM}\sqrt{15+7\sqrt{5}}}{3c^2r_0} + \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{5}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left(\frac{1+\sqrt{5}+\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}-\sqrt{10-2\sqrt{5}}}\right). \tag{15}$$

Next, in the same way, we proceed to calculate the signal delays when the DGP correction to the metric appears with a negative sign. For any interval $[\alpha, \beta] \subset (-\pi/2, \pi/2)$ the corresponding radar signal time delay will be:

$$\Delta t = \frac{b}{c}\int_\alpha^\beta \left(\frac{GMb}{c^2r_0^2}\sec\theta - \frac{2}{cr_0^2}\sqrt{GMb\sec\theta}\right)\sec^2\theta d\theta. \tag{16}$$

Over $[-\pi/6, \pi/6]$, we obtain:

$$\Delta t = \frac{2GMb^2}{3c^3r_0^2} - \frac{8b\sqrt{2GMb}}{3^{7/4}c^2r_0} - \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{12}, 2\right) + \frac{GMb^2}{6c^3r_0^2}\ln(27). \tag{17}$$

Similarly, and for $[-\pi/4, \pi/4]$, we get that:

$$\Delta t = \frac{\sqrt{2}GMb^2}{c^3r_0^2} - \frac{8\times 2^{1/4}b\sqrt{GMb}}{3c^2r_0} - \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{8}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left(\frac{\cos(\pi/8)+\sin(\pi/8)}{\cos(\pi/8)-\sin(\pi/8)}\right), \tag{18}$$

for $[-\pi/3, \pi/3]$, the integration results:

$$\Delta t = \frac{GMb^2}{c^3r_0^2} - \frac{8b}{c^2r_0}\sqrt{\frac{2GMb}{3}} - \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{6}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left[\frac{1+\sqrt{3}}{-1+\sqrt{3}}\right] \tag{19}$$

and, finally, for $[-4\pi/10, 4\pi/10]$, we obtain that:

$$\Delta t = \frac{\sqrt{50+22\sqrt{5}}GMb^2}{c^3r_0^2} - \frac{8b\sqrt{15+7\sqrt{5}}\sqrt{GMb}}{3c^2r_0} - \frac{8b\sqrt{GMb}}{3c^2r_0}F\left(\frac{\pi}{5}, 2\right) + \frac{GMb^2}{c^3r_0^2}\ln\left[\frac{1+\sqrt{5}+\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}-\sqrt{10-2\sqrt{5}}}\right]. \tag{20}$$

In order to calculate the values of the elliptic function F used above, we have utilized an up to 4$^{\text{th}}$ order series expansion of this function. This expansion is given below:

$$\begin{aligned} F(\theta, k) \approx{}& \theta + \left(\frac{\sin^{-1}(\sin\theta) - \sin\theta\sin^2\theta}{4}\right)k \\ &+ \left(\frac{9(\sin^{-1}(\sin\theta) - \cos^2\theta\sin\theta)}{64} - \frac{3\cos^2\theta\sin^3\theta}{32}\right)k^2 \\ &+ \left(\frac{25(\sin^{-1}(\sin\theta) - \sin\theta\sin^2\theta)}{256} - \frac{25\cos^2\theta\sin^3\theta}{384} - \frac{5\cos^2\theta\sin^5\theta}{96}\right)k^3 \\ &+ \left(\frac{1225(\sin^{-1}(\sin\theta) - \cos^2\theta\sin\theta)}{16384} - \frac{1225\cos^2\theta\sin^3\theta}{24576} - \frac{245\cos^2\theta\sin^5\theta}{6144} - \frac{35}{1024}\cos^2\theta\sin^7\theta\right)k^4 \\ &+ O(k^5). \end{aligned} \tag{21}$$

Then:

$$F\left(\frac{\pi}{12}, 2\right) \approx -\frac{585}{512} + \frac{3339\sqrt{3}}{16384} + \frac{1379\pi}{4096} \approx 0.268087, \tag{22}$$

$$F\left(\frac{\pi}{8}, 2\right) \approx \frac{413}{1024} - \frac{285}{128\sqrt{2}} + \frac{4137\pi}{8192} \approx 0.415422, \tag{23}$$

$$F\left(\frac{\pi}{6}, 2\right) \approx -\frac{14491\sqrt{3}}{16384} + \frac{1379\pi}{2048} \approx 0.583429, \tag{24}$$

$$\left(\frac{\pi}{5}, 2\right) \approx -\frac{\sqrt{3309806810 + 65996498\sqrt{5}}}{32768} + \frac{4137\pi}{5120} \approx 0.744014. \tag{25}$$

3 Numerical results

To apply the above analysis in the case of the Sun, we have used that the mass of the Sun is $M = 1.99 \times 10^{30}$ kg. For the Friedman-Lemaitre-Robertson-Walker phase, the signal time delay in the vicinity of the Sun for $-\pi/6 \leq \theta \leq \pi/6$, we have:

$$\Delta t = 5.977 \times 10^{-6}\left(\frac{b^2}{r_0^2}\right) + 3.033 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right) \approx 3.033 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right). \tag{26}$$

Similarly, for $-\pi/4 \leq \theta \leq \pi/4$ we obtain:

$$\Delta t = 1.128 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) + 5.477 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right) \approx 5.477 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right). \tag{27}$$

Next, for $-\pi/3 \leq \theta \leq \pi/3$ we get that:

$$\Delta t = 2.350 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) + 1.035 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right) \approx 1.035 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right). \tag{28}$$

Finally, for $-4\pi/10 \leq \theta \leq 4\pi/10$:

$$\Delta t = 5.802 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) + 2.144 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right) \approx 2.144 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right). \tag{29}$$

After applying that $b \approx R_S = 6.96 \times 10^8$ m and $r_0 \approx 5$ Gpc $= 1.542 \times 10^{23}$ m (Lue and Starkman 2003), the delays in picoseconds [ps] for the above mentioned ranges of θ are tabulated in Table 1.

Similarly, for the accelerating phase of the universe we obtain that:

Table 1 Radar signal time delays due to the DGP gravity in the vicinity of the Sun. Results related to the Friedman-Lemaitre-Robertson-Walker phase of the universe

Angular range of closest approach (rad)	Signal time delays (ps)
$-\pi/6 \leq \theta \leq \pi/6$	0.0000361
$-\pi/4 \leq \theta \leq \pi/4$	0.0000652
$-\pi/3 \leq \theta \leq \pi/3$	0.0001233
$-4\pi/10 \leq \theta \leq 4\pi/10$	0.0002553

Table 2 Radar signal time delays due to the DGP gravity in the vicinity of the Sun. Results related to the accelerating phase of the universe

Angular range of closest approach (rad)	Signal time delays (ps)
$-\pi/6 \leq \theta \leq \pi/6$	−0.0000361
$-\pi/4 \leq \theta \leq \pi/4$	−0.0000652
$-\pi/3 \leq \theta \leq \pi/3$	−0.0001233
$-4\pi/10 \leq \theta \leq 4\pi/10$	−0.0002553

$$\Delta t = 5.977 \times 10^{-6}\left(\frac{b^2}{r_0^2}\right) - 3.033 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right) \approx -3.033 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right), \tag{30}$$

$$\Delta t = 1.128 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) - 5.477 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right) \approx -5.477 \times 10^{-7}\left(\frac{b^{3/2}}{r_0}\right), \tag{31}$$

$$\Delta t = 2.350 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) - 1.035 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right) \approx -1.035 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right), \tag{32}$$

$$\Delta t = 5.802 \times 10^{-5}\left(\frac{b^2}{r_0^2}\right) - 2.144 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right) \approx -2.144 \times 10^{-6}\left(\frac{b^{3/2}}{r_0}\right). \tag{33}$$

Then, the signal delays are given in Table 2. Next, Fig. 1 gives a 3D plot of the signal time delay in the vicinity of the Sun, related to the Friedman-Lemaitre-Robertson-Walker phase of the universe, as a function of the DGP parameter r_0 and the distance of closest approach b, for the range $[-4\pi/10, 4\pi/10]$. Similarly, Fig. 2 represents a 3D plot of the time signal delay in the vicinity of the Sun, related to the accelerating phase of the universe, as a function of the same parameters and for the same angular range.

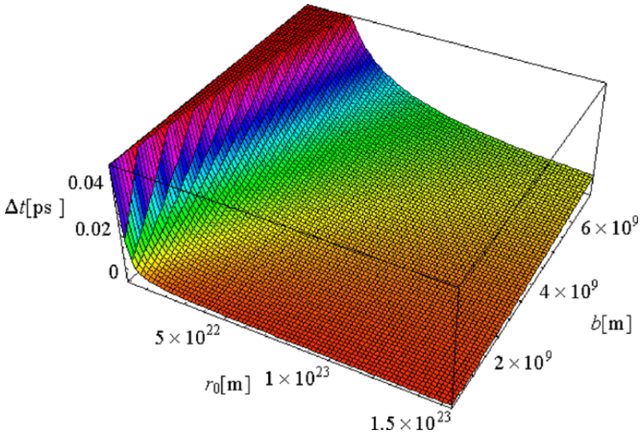

Fig. 1 Signal time delay in the vicinity of the Sun, related to the Friedman-Lemaitre-Robertson-Walker phase of the universe, is plotted as a function of the DGP parameter r_0 and the distance of closest approach b, for the range $[-4\pi/10, 4\pi/10]$

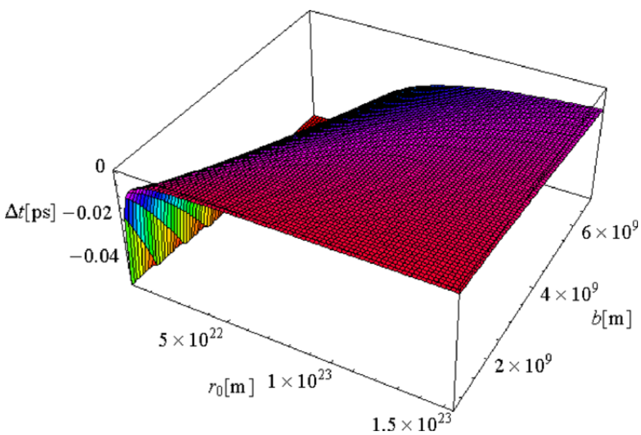

Fig. 2 Signal time delay in the vicinity of the Sun, related to the accelerating phase of the universe, is plotted as a function of the DGP parameter r_0 and the distance of closest approach b for the range $[-4\pi/10, 4\pi/10]$

4 Conclusions

The signal time delay in the vicinity of the Sun due the Dvali-Gabadadze-Porrati metric (DGP) has been calculated, for various subintervals of the range $(-\pi/2, \pi/2)$ of θ. Both algebraic signs have been considered in the metric element. The plus sign is related to the Friedman-Lemaitre-Robertson-Walker phase of the universe, while the negative one is related to the self accelerating one. The Friedman-Lemaitre-Robertson-Walker phase results in an increase to the signal delay, while the self accelerating phase of the universe results in a reduction of this delay. With reference to Haranas and Ragos (2011) we say that to get an idea of today's radar systems, somebody could talk about the sensitivity of radar, a property that is related to the power of the transmitting radar. Since we are interested in signal time delays and in order to substantiate our findings, we will refer to today's radar resolution instead, something that is related to the detectable times. Quoting Shapiro et al. (1968), Shapiro (1999), fractional system errors of echo time delays in solar system experiments can be up to 1 part in 10^{10} or smaller. Signal delays of this magnitude might be in the borderline of time detection of today's technology and, therefore, it will be difficult to be detected. Future technologies might be able to push for such delectability limit, and therefore delays attribute to (DGP) gravity might be measured.

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